Introduction to the Mathematical Theory of the Navier-Stokes equations

I. The Cauchy problem for the Navier-Stokes equations. Existence of solutions: mild solutions, weak Leray-Hopf solutions, strong solutions, Lemarie-Reisset solutions, other type of solutions.

II. Regularity theory: ε-regularity, Caffarelli-Kohn-Nirenberg theorem, scale invariant quantities, type I blowups, Ladyzhenskaya-Prodi-Serrin condition, $L_{3,\infty}$-case, other scale invariant norms, ancient solutions and Liouville type theorems.

III. Initial boundary value problems for the Navier-Stokes equations in half space. Local boundary regularity and bounded mild ancient solutions.

References

[1] Ladyzhenskaya, O. A. The mathematical theory of viscous incompressible flow. Second English edition, revised and enlarged. Translated from the Russian by Richard A. Silverman and John Chu. Mathematics and its Applications, Vol. 2 Gordon and Breach, Science Publishers, New York-London-Paris 1969 xviii+224 pp.

[2] Lemarie-Rieusset, P. G., Recent developemnets in the Navier-Stokes problem, Chapman&Hall/CRC reseacrh notes in mathematics series, 431.

[3] Seregin, G., Lecture notes on regularity theory for the Navier-Stokes equations. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2015. x+258 pp. ISBN: 978-981-4623-40-7