Inverse problems for hyperbolic PDEs

Lecturer: Rakesh

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Abstract:
Inverse problems for hyperbolic PDEs arise naturally in geophysics, oil prospecting, in the design of optical devices, and in many other areas where the interior of an object is to be imaged using the response of the object to acoustic waves (satisfying hyperbolic PDEs). There are many unresolved theoretical, algorithmic and computational issues and the course will focus mainly on the theoretical issues for such problems.

Many of these problems may be regarded as the study of the inversion of the nonlinear map $F$ from the coefficients of PDEs to the trace of some special solutions of the PDEs on the boundary of the object. Towards attempting an inversion of $F$ one also studies the uniqueness question (the injectivity of $F$) and the stability question (the continuity of $F^{-1}$).

We will explore the well understood work on the one dimensional problem (the object is one dimensional) regarding the use of the downward continuation method and then study the results and the many unsolved problems in the multidimensional case for formally determined data (data and the coefficient depend on the same number of parameters). There have been attempts to push through the downward continuation argument for the inversion in the multidimensional case, with limited success. Carleman estimates and tools from Harmonic Analysis have been used in the study of multidimensional formally determined problems but many problems remain unresolved. We will describe these tools, some of the results, and the many unsolved problems.

The course will be geared towards researchers with a basic knowledge of the study of Partial Differential Equations using Sobolev Spaces and Functional Analysis.

Program:

1. Some inverse problems for hyperbolic PDEs
2. Inversion from the Dirichlet to Neumann map data
3. The Gelfand–Levitan–Krein theory for the solution of the one dimensional inverse problem
4. Solution of inverse problems for layered mediums but which do not reduce to one-dimensional problems
5. Romanov’s results for problems with coefficients analytic in some direction
6. Carleman estimates for second order PDEs
7. The Bukhgeim and Klibanov result and its improvements for a formally determined multidimensional problem
8. Unsolved problems

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