Fluid Mechanics & Aeroacoustics of Fans and Compressors

Day 1: Axial Flow Compressors & Fans

Short Course Offered at BCAM—July 2-4, 2013
Farzad Taghaddosi, Ph.D.
Course Objective

• Provide basic understanding of fluid flow in compressors and fans (axial & centrifugal)

• Understand sources of noise and methods for acoustic analysis
Definition

• Compressors & fans:
  – Use input mechanical energy to increase fluid total pressure
  – Fans ($\Delta p \sim 0.01$ atm), compressors ($\Delta p \geq 1$ atm)
  – Configs: axial, radial/centrifugal, or mixed Flow
Choosing the Right Fan/Compressor

• Performance variables: \( w_{stage}, \eta, \Delta p = f(\dot{m}, \rho, N, D, \mu, \alpha) \)

• Non-dimensional form:

  – Where

  \[
  \psi = \frac{w_s}{N^2 D^2}
  \]

  loading coefficient

  \[
  \phi = \frac{\dot{m}/\rho}{N D^3}
  \]

  flow coefficient

• Specific speed

  \[
  N_s = \frac{\phi^{1/2}}{\psi^{3/4}} = \frac{N(\dot{m}/\rho)^{1/2}}{w_s^{3/4}}
  \]

• Specific diameter

  \[
  D_s = \frac{\psi^{1/4}}{\phi^{1/2}} = \frac{D w_s^{1/4}}{(\dot{m}/\rho)^{1/2}}
  \]
\[ N_s \sim \frac{N \dot{m}}{w_s} \quad D_s \sim \frac{D w_s}{\dot{m}} \]

\[ 8.26 D_s^{-1.936} \]

\[ 2.5 D_s^{-0.916} \]
Sample $N_s D_s$ (Balje) Diagram

$N_s = \frac{N \sqrt{V_3}}{H_{ad}^{3/4}}$

$D_s = \frac{D H_{ad}^{3/4}}{\sqrt{V_3}}$

$S/D = \frac{S}{D}$

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Axial-flow Compressors
Axial-flow Compressors

• Applications: Industrial gas turbines, aircraft engines

http://www.youtube.com/watch?v=CSi4GXUojo
Axial-flow Compressors

- Best for applications with high $N_S$ and low $D_S$
  - High mass flow w/ relatively small $\Delta p$ per stage
  - Therefore large number of stages are needed
  - Each stage: rotor followed by stator

![Diagram of Axial-flow Compressor](image-url)
Terminology

• **Cylindrical coordinate system**

• **Velocity components**
  » $c_x$: axial
  » $c_r$: radial
  » $c_\theta$: tangential/circumferential
  » $c_m = \sqrt{c_x^2 + c_r^2}$: meridional

• **Relative frame of reference**
  » $\vec{U}$: blade tangential velocity $= r\vec{\omega}$
  » $\vec{w}$: relative velocity
  » $\vec{c}$: absolute velocity $= \vec{w} + \vec{U}$

• **Flow angles**
  » $\alpha$: absolute
  » $\beta$: relative
Flow through Compressor Stage

- Pressure increase through both rotor and stator
- Moderate pressure rise (flow deceleration) due to adverse pressure gradient
- As a result, blades have small curvature/camber; are very thin
- Need multiple stages to create large pressure increase
Stage Work (Loading)

• Euler’s equation:

$$w_s = U_2 c_{\theta_2} - U_1 c_{\theta_1}$$

$$w_s = U(c_{\theta_2} - c_{\theta_1})$$ at mean radius

» All work done in rotor, none in stator

• Loading coefficient

$$\psi = \frac{w_s}{U^2} = \frac{(c_{\theta_2} - c_{\theta_1})}{U} = \frac{\Delta c_{\theta}}{U}$$

» $\psi$ directly related to flow turning $\Delta c_{\theta}$

» Higher $\psi \rightarrow$ reduced no. of stages

» Reducing inlet swirl $\rightarrow$ higher $\psi$

» $\psi_{design} \sim 0.4$
Flow Coefficient

• Definition: $\phi = \frac{c_x}{U}$
  » Determines change in flow angles
  » Typically $c_x$ const. thru compressor
  » Higher $\phi \rightarrow$ reduced flow turning
  » $\phi_{design} \sim 0.4$-$0.8$
  » $\psi$ and $\phi$ are directly related:

$$\psi = \frac{(c_{\theta_2} - c_{\theta_1})}{U}$$

\[\vdots\]

$$\psi = \phi (\tan \alpha_2 - \tan \alpha_1) = \phi (\tan \beta_1 - \tan \beta_2)$$
Stage Reaction

\[ R = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}} = \frac{h_2 - h_1}{h_3 - h_1} \approx \frac{(\Delta p)_{\text{rotor}}}{(\Delta p)_{\text{stage}}}, \quad 0 \leq R \leq 1 \]

- Impacts asymmetry of velocity triangles hence blade shapes

- Typical range: \( R_{\text{design}} \approx 0.5 - 0.8 \)

- Relationship with \( \psi \) and \( \phi \):
  \[ \psi = 2(1 - R - \phi \tan \alpha_1) \]
  » Higher reaction tends to reduce stage loading
Stage Thermodynamics

• Work:

\[ w_s = U_2 c_{\theta 2} - U_1 c_{\theta 1} = h_{02} - h_{01} \]

\[ \rightarrow h_{01} - U_1 c_{\theta 1} = h_{02} - U_2 c_{\theta 2} \]

\[ I \equiv h_0 - U c_{\theta} \text{ rothalpy} \]

\[ I \equiv h + \frac{w^2}{2} - \frac{U^2}{2} \rightarrow I \equiv h_{0,rel} - \frac{U^2}{2} \]

• Enthalpy change across rotor:

\[ I_1 = I_2 \text{ or at mean radius } (U_1 = U_2): \]

\[ h_{01,rel} = h_{02,rel} \]

\[ h_{0,rel} = h + \frac{w^2}{2} \]

• Enthalpy change across stator \((U = 0)\):

\[ h_{02} = h_{03} \]

\[ h_0 = h + \frac{c^2}{2} \]
Stage Losses & Efficiency

- Efficiency of the compressor is impacted by the losses in each stage (rotor+stator)
- Losses are typically quantified using correlations obtained from experimental tests
• Profile/annulus losses
  – BL drag & wake mixing

• Secondary flow losses
  – Corner stalls, 3D effects

• Tip leakage losses
  » Tip vortex mixing

• Shock-induced losses
Stage Loss Metrics

• Enthalpy loss coefficients:

\[ \zeta_R = \frac{h_2 - h_{2s}}{w_2^2/2} \]

\[ \zeta_N = \frac{h_3 - h_{3s}}{c_3^2/2} \]

• Stagnation pressure loss coefficients:

\[ Y_R = \frac{p_{01,rel} - p_{02,rel}}{p_{01,rel} - p_1} \]

\[ Y_N = \frac{p_{02} - p_{03}}{p_{02} - p_2} \]

• Loss coefficients are related:
  – At low Mach numbers: \( Y \approx \zeta \)
  – But at higher Mach numbers: \( Y > \zeta \)
Stage Efficiency

• Efficiency

\[ \eta_{tt} \approx 1 - \frac{\gamma - 1}{\gamma} \frac{Y_R (1 - p_1/p_{01,rel}) + Y_N (1 - p_2/p_{02})}{1 - T_1/T_{03}} \]

Or

\[ \eta_{tt} \approx 1 - \frac{\zeta_R w_2^2 (T_3/T_2) + \zeta_N c_3^2}{2(h_{03} - h_{01})} \]

» For low-speed/incompressible machines:

\[ \eta_{tt} \approx 1 - \frac{T_{03} \Delta S_{stage}}{h_{03} - h_{01}} = 1 - \frac{\Delta p_{0,R} + \Delta p_{0,N}}{\rho (h_{03} - h_{01})} \]

Or

\[ \eta_{tt} \approx 1 - \frac{(w_1^2 Y_R + c_2^2 Y_N)}{2(h_{03} - h_{01})} \]
Measuring Losses: Cascade Flow Analysis

• Sample Cascade Tunnel

• Main objective:
  – Characterize losses
  – Measure exit flow angle

• Based on simplified 2D, steady flow
• Both design and off-design conditions are tested
**Cascade Nomenclature**

- $\alpha'_1 =$ blade inlet angle
- $\alpha'_2 =$ blade outlet angle
- $\theta =$ blade camber angle
  - $= \alpha'_1 - \alpha'_2$
- $\zeta =$ setting or stagger angle
- $s =$ pitch (or space)
- $\varepsilon =$ deflection
  - $= \alpha_1 - \alpha_2$
- $\alpha_1 =$ air inlet angle
- $\alpha_2 =$ air outlet angle
- $c_1 =$ air inlet velocity
- $c_2 =$ air outlet velocity
- $i =$ incidence angle
  - $= \alpha_1 - \alpha'_1$
- $\delta =$ deviation angle
  - $= \alpha_2 - \alpha'_2$
- $c =$ chord
- $b =$ axial chord
- $c/s =$ solidity
Some Design Criteria

- **Diffusion factor**
  
  \[
  DF = \left( \frac{w_1 - w_2}{w_1} \right) + \left( \frac{\Delta c \theta}{2w_1} \right) \left( \frac{s}{c} \right) \approx 0.45
  \]
  
  deceleration  turning

  » Helps determine space-chord ratio \((s/c)\)
  
  » For given DF, higher turning requires reduced blade spacing to avoid separation
  
  » Typical values: \(s/c \approx 0.8 - 1.2\)

- **Inlet swirl angle**: \(\alpha_1 \approx 20^\circ - 30^\circ\)
  
  » Helps reduce relative inlet Mach number
  
  » Reduces flow turning hence stage loading

- **Blade aspect ratio**: \(H/c \approx 1 - 2\)

- **Blade spacing**: \(s/b \approx 0.5\)
Multi-stage Compressors

• Effective annulus area is reduced because of BL growth
  – Axial velocity is adversely impacted

  \[ w_s = \lambda U (c_{\theta 2} - c_{\theta 1}) \]

  – The effect is taken into account by introducing work-done factor (\( \lambda \)):
  – American design practice: apply blockage factor to account for reduced annulus area
Radial Flow Variations

- 2D flow assumption only valid when $r_{hub}/r_{tip}$ is large ($\geq 0.8$) – typically last stages blades

- For $r_{hub}/r_{tip} \approx 0.4-0.8$, blade speed ($U$) & flow angles will significantly vary from hub to tip
  - Blades require significant twist
Radial Flow Variations

• Change in annulus shape means $c_r$ cannot be ignored, although still smaller than $c_x$ and $c_\theta$

• Pressure increase from hub to tip to counter centrifugal forces acting on the fluid will cause slight variation in the radial direction

• Radial flow variation is taken into account by solving “radial equilibrium equation”
Flow Instability - Surge

- It is caused by drop in delivery pressure due to reduction in \( \dot{m} \)
- If \( p_{\text{exit}} \) does not drop fast enough, air will reverse direction and flow upstream due to resulting pressure gradient. This will cause sudden drop of compressor exit pressure, reversing air flow direction...
- The cycle can then continue at high frequency
- Surge characterized by vibration in “axial” direction, causes excessive blade vibration, and can lead to flame-out (flame extinction)

http://www.youtube.com/watch?v=osAT6mwkr94
http://www.youtube.com/watch?v=9KhZwsYTNDE
Flow Instability – Rotating Stall

• An instability usually observed at low operating speed ($N$)
• Is caused by blade stall (due to increased loading, tip vortex or corner stall), leading to flow blockage and change in angle-of-attack of neighboring blades (increase on one side and decrease on another side)
• This causes neighboring blade stall and recover creating stall patches that will travel around compressor annulus
• Rotating stall can exist in normal operating conditions; both part-span and full-span stall has been observed
• It causes vibration in circumferential direction
Low-speed Ducted Fans
Introduction

- Ducted fans are essentially single-stage compressors but with low pressure ratio

- Two configurations may be used:
  a) IGV – Rotor
  b) Rotor – OGV
**Introduction**

- Ducted fans have typically higher space-chord ratio (low solidity) compared to compressors

- Isolated airfoil theory is often used since the influence of neighboring blades is small
Fluid Mechanics & Aeroacoustics of Fans and Compressors

Day 2: Centrifugal Compressors & Fans

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Introduction

- Efficiency of axial flow compressors sharply drops at low flow rates
  - Increased losses due to larger surface/volume ratio of annulus
  - Manufacturing of small parts, high maintenance cost, etc.

Compressor chart will be similar
Centrifugal compressors best choice for high pressure rise for small flow rate
Introduction

• Centrifugal compressors
  » Smaller number of components
  » More compact design
  » Pressure ratio’s as high as 8:1

• Centrifugal fans/blowers: $\Delta p$ small, about a few inches of water ($\rho_e/\rho_i \leq 1.05$)
  » Usually treated as incompressible
First Jet Engine (Frank Whittle) - 1930

• Used a centrifugal compressor

• Soon became apparent that they are not suitable for higher mass flows
  – Larger frontal area, lower efficiency, etc.)
Some Applications

- Automobile turbochargers
- Auxiliary Power Units (APU’s)
- Gas pipeline, refrigeration, process plants
Components & Operation

- Impeller: pressure rise due to centrifugal action & diffusion
- Diffuser (vaned / vaneless): pressure rise by diffusion (velocity almost reduced to inlet value)
- Design practice: 50-50 pressure rise across impeller & diffuser
- Scroll or Volute: collects and delivers the air
Flow Path

- Air enters through impeller eye in axial direction
- Unless inlet guide-vanes (IGV’s) are used, vanes must be curved to allow smooth inflow
- Air leaves impeller tip with absolute velocity $c_2$
- Some impellers have shroud to reduce leakage & losses
Stage Work

• Impeller: \( I_1 = I_2 \) (constant rothalpy)

\[
I \equiv h + \frac{w^2}{2} - \frac{U^2}{2}
\]

\[
\rightarrow h_2 - h_1 = \frac{1}{2}(U_2^2 - U_1^2) + \frac{1}{2}(w_1^2 - w_2^2)
\]

- centrifugal action \(~ 75\%~
- flow deceleration \(~ 25\%~

» \( \Delta h \) directly related to \( \Delta p \)

• Diffuser: \( h_{02} = h_{03} \) (constant stagnation enthalpy)

\[
h_0 \equiv h + \frac{c^2}{2} \quad \rightarrow h_3 - h_2 = \frac{1}{2}(c_2^2 - c_3^2)
\]

\( \Delta p \) due to flow deceleration
Stage Thermodynamics

- Impeller
  - Rothalpy:
    \[ I \equiv h + \frac{w^2}{2} - \frac{U^2}{2} \]

- Diffuser
  - Stagnation enthalpy:
    \[ h_0 \equiv h + \frac{c^2}{2} \]
Stage Thermodynamics

• Work:

\[ w = U_2 c_{\theta 2} - U_1 c_{\theta 1} \]

\[ c_{\theta 1} = 0 \quad \text{for axial inflow} \]

\[ \rightarrow w = U_2 c_{\theta 2} = h_{02} - h_{01} \]

• Slip factor

» Ideally: \( c_{\theta 2} = U_2 \), but in reality: \( c_{\theta 2} < U_2 \) due to less than perfect guidance received because of finite no. of vanes

» Define \( \sigma_s = \frac{c_{\theta 2}}{U_2} \) as slip factor:

\[ \sigma_s \approx 1 - \frac{0.63\pi}{N_{\text{vane}}} \approx 1 - \frac{2}{N_{\text{vane}}} \]

Stanitz formula
Stage Thermodynamic

• Power input factor ($\lambda$)
  » Correction factor to account for losses in the impeller only
  » $\lambda \approx 1.035 - 1.04$

• Overall stagnation pressure ratio:

$$\frac{p_{03}}{p_{01}} = \left[ 1 + \frac{\eta_c \lambda (\sigma_s U_2^2 - U_1 c_{\theta 1})}{c_p T_{01}} \right]^{\gamma/(\gamma-1)}$$

$\eta_c$ : isentropic efficiency  \hspace{1cm} T_{01} : inlet stagnation temperature
$c_p$ : specific heat
Impeller Design Considerations

Backward Swept Vanes

» Radial impeller designs lead to high exit velocity \( c_2 \), which may lead to flow separation in diffuser

» Backward swept vanes will reduce (increase \( w_2 \)) hence reduce diffusion in both impeller & diffuser

» Because of more controlled diffusion in impeller & diffuser both overall efficiency and operating margin improve

» To maintain pressure ratio, however, rpm has to be increased. Therefore, centrifugal stresses will be higher

» Swept vanes will also experience bending stresses

» Typical bend angles: \( \beta = 30^\circ-40^\circ \)
Impeller Design Considerations

• Inlet pre-whirl
  » Without pre-swirling inflow (using inlet guide vanes), and hence relative Mach no. will be very high
  » The flow can become supersonic, creating shock waves, which in interaction with the BL may cause flow separation
  » Adding pre-whirl will help reduce inlet Mach number
  » But, as a result, $c_{\theta 1}$ will no longer be zero, which mean more work is needed to create the same pressure ratio:

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta c \lambda (\sigma_s U_2^2 - U_1 c_{\theta 1})}{c_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

  » Since $w_1$ will be highest at the tip of the eye (highest $U_1$), one can minimize the impact by adding pre-whirl near tip of the eye only
Diffuser Design Considerations

• Can further increase pressure in diffuser by reducing $c_2$ (or $c_{\theta 2}$)
  – Since past impeller exit, angular momentum stays constant:

\[ r c_{\theta} = \text{const.} \]

increasing radius will achieve this

• Vaneless diffuser: reduce $c_{\theta}$ by increasing radius

• Vaned diffuser: use vanes to reduce $c_{\theta}$ faster
Diffuser Design Considerations

- **Volute or Scroll**
  - Collects and delivers the flow
  - Spiral-shaped channel of increasing cross-sectional area

![Diagram of a volute or scroll diffuser with impeller, vaned diffuser, symmetric volute, and overhung volute.]
• Centrifugal compressors can also suffer from instabilities such as rotating stall & surge
Day 3: Introduction to Aeroacoustics

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Introduction

• What is aero-acoustics?
  » Study of sound generated by aerodynamic sources

• Examples:

  Courtesy ISVR

  Courtesy NASA

  Courtesy NASA
Introduction

• There is obviously a need to reduce man-made noise
Physical Nature of Sound

- Sound: pressure disturbances/fluctuations of very small amplitude \((p' := p - \bar{p})\)
  - Sound waves require a medium to travel

- For any \(p'\), there is an associated fluctuations of velocity particles \((\nu')\)

- Speed of sound: speed of sound propagation in a medium; in undisturbed medium \(c_0 = (\partial p' / \partial \rho')_s\)
  - Note that \(\nu'\) and \(c_0\) are not the same
Noise Signal Analysis

• Noise signals are measured in the “time domain” but are analyzed in the “frequency domain” using Fourier transform

• Any complex signal can be decomposed this way
Noise Signal Analysis

- Human ear can hear a sound, if the frequency content of the signal is in the range: 20 Hz – 20 kHz, provided the signal amplitude is higher than threshold of hearing. The amplitude is measured using sound pressure level (SPL).

- Amplitude is usually weighted within above freq. range to replicate human ear sensitivity
  » A-weighting (dBA) is most commonly used
Metrics

• Strength of acoustic signal is measured using rms (root-mean-square) value, defined as: \[ p'_{rms} = \sqrt{(p')^2} \]

  » Threshold of hearing: \[ p'_{rms} \approx 10^{-5} \text{ Pa} \]
  » Threshold of pain: \[ p'_{rms} \approx 10^2 \text{ Pa} \]
  » Because of large range of values, logarithmic scale is used
  » Acoustic signal strength is measured using sound pressure level (SPL)

• Sound pressure level (\textit{SPL} or \( L_p \)):

\[
SPL = 10 \log \left( \frac{p'_{rms}^2}{p_{ref}^2} \right) = 20 \log \left( \frac{p'_{rms}}{p_{ref}} \right) \text{ (dB)}
\]

where \( p_{ref} = 2 \times 10^{-5} \text{ Pa} \) for air and \( 10^{-6} \text{ Pa} \) in other media.

» Doubling \( p'_{rms} \) will increase the noise by only \( \sim 6 \text{dB} = 20 \log 2 \)
Loud noise of short duration is less annoying to human ear than a persistent noise of lower amplitude.
Metrics

• Sound intensity Level ($IL$)
  – Sound intensity (energy flux): $\tilde{I}(x) = p'v'$; or time-averaged: $I = \bar{p}'\bar{v}'$
  – The direction of the intensity is the average direction in which the acoustic energy is flowing

\[
IL = 10 \log \left( \frac{I}{I_{ref}} \right) \text{ (dB)} \quad \text{where} \quad I_{ref} = 10^{-12} \text{ W/m}^2
\]

• Sound power level ($L_W$)
  – Is the power of sound sources enclosed within an area, $A$
  – Sound power is thus obtained by integrating intensity over the area
  – It is independent of integration area as long as $A$ encloses all sources

\[
L_W = 10 \log \left( \frac{P}{P_{ref}} \right) \text{ (dB)} \quad \text{where} \quad P_{ref} = 10^{-12} \text{ W}
\]
Directivity

- In general, noise is a directional phenomena, i.e., it radiates more intensely in certain direction(s).

- Examples:
Wave Equation

• General form of governing equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]

continuity

\[
\rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \nabla \cdot \vec{\tau}_{ij} + \vec{f}
\]
momentum

• Assumptions:
  » Neglect body \(\vec{f}\) and viscous forces \(\vec{\tau}_{ij}\)
  » Small perturbations: \(\rho = \rho_0 + \rho', \ p = p_0 + p', \) etc.
  » Stagnant fluid \((\vec{v}_0 = 0)\) with uniform properties \((\rho_0 = \text{const})\) at observer

• Combine continuity & momentum equations:

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x^2} = 0
\]

or

\[
\frac{\partial^2 p'}{\partial t^2} - c_0^2 \frac{\partial^2 p'}{\partial x^2} = 0
\]

» This is the homogenous wave equation
More on Wave Equation

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x^2} = 0 \quad \text{or} \quad \frac{\partial^2 p'}{\partial t^2} - c_0^2 \frac{\partial^2 p'}{\partial x^2} = 0
\]

• It is both linear and homogeneous
• Solutions can be sought using Green’s function
• Only governs sound propagation without any references to sound sources
Lighthill’s Equation

• General form of governing equations:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) &= -\nabla p + \nabla \cdot \mathbf{\tau}_{ij} + \mathbf{f}
\end{align*}
\]

continuity momentum

• Assumptions:

» **DO NOT** neglect body \((\mathbf{f})\) and viscous forces \((\mathbf{\tau}_{ij})\)
» Small perturbations: \(\rho = \rho_0 + \rho', p = p_0 + p', \text{ etc.}\)
» Stagnant fluid \((\mathbf{v}_0 = 0)\) with uniform properties \((\rho_0 = \text{const})\) at observer

• Combine continuity & momentum equations:

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 \mathbf{T}_{ij}}{\partial x_i \partial x_j} - \frac{\partial f_i}{\partial x_i}
\]

\[
\mathbf{T}_{ij} = \rho u_i u_j - \tau_{ij} + (p' - c_0^2 \rho') \delta_{ij}
\]

Lighthill stress tensor

» This is called **Lighthill’s equation** (1952)
Lighthill’s Acoustic Analogy

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i x_j} - \frac{\partial f_i}{\partial x_i}
\]

\[T_{ij} = \rho u_i u_j - \tau_{ij} + (p' - c_0^2 \rho') \delta_{ij}\]

Lighthill stress tensor

• Lighthill’s equation is exact (based on Navier-Stokes eqs)

• The word “analogy” refers to the fact that we can determine the sound field of a complex noise generating phenomena by treating it as source terms of the wave eq.

• It is a non-homogeneous equation where the right-hand side represents aeroacoustic sources

• Solution can be sought using Green’s function, if the source terms can be suitably modeled
Sources of Aerodynamic Sound

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i x_j} - \frac{\partial f_i}{\partial x_i}
\]

\[T_{ij} = \rho u_i u_j - \tau_{ij} + (p' - c_0^2 \rho')\delta_{ij}\]

Lighthill stress tensor

The right-hand side represents the sources:

– Monopole:
  » Any changes in the entropy (where \(s' = p' - c_0^2 \rho'\) will be non-zero) or deviation from uniform speed of sound \((c_0)\)

– Dipole:
  » Acoustic field due to external forces exerted on the flow \((\partial f_i / \partial x_i)\)

– Quadrupole:
  » Induced by non-linear convective forces represented by the Reynolds stress tensor \((\rho u_i u_j)\), such as turbulence
  » Due to viscous forces \((\tau_{ij})\)
Modeling Acoustic Sources

- **Monopole**
  - Thickness noise

- **Dipole**
  - Loading noise

- **Quadrupole**
  - BL/viscous effects

http://www.acs.psu.edu/drussell/demos/rad2/mdq.html
FW-H Equation

- Ffowcs Williams–Hawkings equation is a generalization of the Lighthill analogy to add sound field associated with sources in arbitrary motion.
- Like Lighthill’s equation, FW-H equation is derived using full Navier-Stokes equations w/o simplifying assumptions.
- FW-H vs. Lighthill:
The first term on the RHS (volume integral) is the Lighthill tensor.

Surface integrals are associated with moving source assumption. So, in absence of moving sources, FW-H reduces to Lighthill eqn.

If the source term represented by $T_{ij}$ is moved inside the control surface $\partial V_H$, volume integral will vanish b/c of Heaviside function. This has very important practical implications.
FW-H Equation

• Practical applications:
Kirchhoff Integral

- Based on solution of the homogeneous wave equation using the free-space Green’s function

- Is equivalent to the FW-H integral, if integration surface is placed in the linear region of the flow

- FW-H is superior because is valid in both linear and nonlinear flow regions

- It can yield wrong answers if homogenous wave equation not satisfied on the control surface

- Linear assumption usually valid in a region far enough from the sources
Noise Prediction Using CAA

- Computational Aero-Acoustics (CAA) refers to the numerical simulation of sound propagation/radiation, with noise sources either modeled or resolved as part of the simulation.
- Although CAA relies on existing CFD methodology, it requires special treatment in certain aspects of simulation.
Two main issues arise in acoustic simulation:

1. Acoustic perturbations \( (p', \rho', \vec{u}') \) are usually several orders of magnitude \((\sim 10^{-6})\) smaller than background flow variables \((p, \rho, \vec{u})\).

   Therefore, discretization method used should be able to resolve such disparity, while maintaining amplitude & phase characteristics of the waves.

   This requires the use of high-order methods, which are computationally expensive (compact FD schemes, DRP schemes, spectral methods).

2. Boundary conditions should be non-reflective to avoid contamination of interior solution.
Noise Prediction – Hybrid Methods

- Typically, it is desired to calculate the noise at the far-field.
- Using CAA, it is generally impractical or impossible to extend the computational domain to the far-field.
- A hybrid approach is therefore the best (often only) choice:
  - Use CAA in the near-field.
  - Use FW-H equation for far-field noise propagation. Note that accuracy of far-filed predictions will heavily depend on accuracy of predicted sources on the FW-H surface.
Turbofan Engine Noise

• Different sources:
  – Fan/OGV interaction (tone & broadband)
  – Core noise (broadband)
  – Jet noise (broadband)
Fan Noise Mechanism

Caused by interaction of rotor with downstream stator/OGV. It consists of:

– Tone noise associated with periodic aerodynamic interactions
  » Tonal noise is radiated at multiples of blade-passage frequency (BPF)
– Broadband noise associated with turbulence
Fan Noise Mechanism

- It creates a pressure field, locked to the rotor, which is made of \( m \)-lobe patterns each rotating at the speed \( nB\Omega/m \):
  - \( n \): harmonics of the blade-passing frequency
  - \( B \) and \( V \) are the number of rotor and stator blades, respectively
  - \( \Omega \) is the rotor’s angular speed
  - \( m = nB \pm kV \), where \( k \) is a positive integer

- According to Tyler-Sofrin theory, the pressure field at the fan face for a circular duct is then given by:

\[
p'_{ms}(x, r, \theta, t) = \sum_s A_{ms} J_m(k_{ms}r) e^{i(m\theta+k_x x-\omega t)}
\]

- \( J_m \): Bessel function of the first kind and order \( m \)
- \( k_{ms} \): eigenvalues defined by \( J'_m(k_{ms}R) = 0 \)
- \( s \): radial mode number
- \( k_x \): axial wave number
- \( \omega = \Omega R/c_0 \): non-dimensional frequency with \( R \) being the duct radius
Fan Noise Mechanism

- $m$ is also called engine order

rotor-locked pressure fluctuations
Fan Noise Mechanism

• Acoustic field is made up of $m$-lobe patterns

• For rotors with spinner, radial variation is given by:

$$A_{ms} [J_m(k_{ms}r) + Y_m(k_{ms}r)]$$

$Y_m$: Bessel function of second kind

Courtesy: Sjoerd W. Rienstra
Fan Noise Propagation

• The acoustic waves given by

\[ p'_ms(x, r, \theta, t) = \sum_s A_{ms} J_m(k_{ms}r) e^{i(m\theta + k_x x - \omega t)} \]

will propagate down the duct only if \( k_x \) is real-valued, which happens if \( \omega / k_{ms} > 1 \) (assuming no mean flow). Otherwise, \( k_x \) will be complex and the corresponding mode will be damped and not propagate (cut-off mode).

• The pressure field given by the above equation could alternately be obtained by direct simulation of rotor/stator flow interaction.
Fan Noise Propagation

• Typically, a hybrid approach is used to determine far-field noise of the fan. CAA is used (LEE, potential flow) to simulate flow propagation inside the duct and in a small region surrounding duct exit, where FW-H is located.

• More complex CAA analysis should include the effect of duct boundary layer and sound refraction

• The effect of liners on duct wall can be simulated by defining impedance boundary conditions