basque center for applied mathematics

Control theory: Introduction and numerics

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Date and time: 6–10 June 2011, from 9:00 to 11:00

Abstract:

In this series of lectures we shall discuss several topics related with the modelling, analysis, numerical simulation and control of Partial Differential Equations (PDE) arising in various contexts of Science and Technology.

Roughly speaking, the problem of controllability we shall discuss may be formulated as follows: To analyze whether by means of a suitable (and feasible!) controller the solution can be driven to a desired final configuration (or close to it). Of course this problem can be made precise in several ways, and the appropriate choice of the control problem to be analyzed is part of the modelling work that needs to be done to make the mathematical results of real practical use.

It is classical in control theory to attack these issues through its dual version. In this case the dual notion of controllability is the so called observability problem. It concerns the possibility of measuring or observing by suitable sensors the whole dynamics of the system through partial measurements made on the region which is accesible to the controllers. This problem, as we said, is relevant for control purposes, but also in other contexts like inverse problems and identification issues. Once again, there are different degrees of observability, and we shall consider the particularly important one of the genuinely infinite-dimensional problems.

In these lectures we shall try to summarize some of the most relevant work that has been done in the subject in recent years. We shall first describe and document some of the most relevant applications to Sciences, Engineering and Technology in which these problems arise. After a short introduction to the finite-dimensional theory, we shall then describe the basic theory for the wave and heat equation to later address some important coupled models, as the system of thermoelasticity, following the presentation in [3].

Following [1] we shall also analyze these problems for networks of flexible strings and beams and, in particular, we shall show how the geometry of the network and the mutual lengths of the various strings/beams entering on it may influence the properties of the system in what concerns control and observation.

Then we shall address the problem of the numerical approximation of control problems, following [2]. To begin with, we shall show that the control and the numerical approximation process do not conmute so that, in general, when controlling a finite-dimensional approximation of the continuous model, one does not actually compute an approximation of the control that one is looking for. We shall see what the possible remedies to these pathologies are: space discretizations, numerical damping, filtering of high frequencies, multi-grid algorithms, etc.

The latter fact is of great impact from a modelling point of view. Indeed, numerical approximation schemes may also be used (and they are often used that way) as discrete models. Our analysis shows that these two modelling approaches yield different results from a control theoretical point of view, and this should be taken into account.

Finally, following [4] we shall present the switching strategy, which provides a natural and efficient manner of adapting classical splitting strategies for the control of multiphysics systems.

To conclude, we shall present a list of open problems and directions of possible future research.

Programme:

1. Introduction to the control of finite-dimensional systems: The Kalman rank condition
2. Control and observation for the wave equation
3. Control and observation for the heat equation
4. Control of multiphysics systems
5. Control of waves on networks
6. Numerical approximation of control and observation problems
7. Switching strategies
8. Open problems and future directions of research

Bibliography:

[1] R. Dáger and E. Zuazua, *Wave propagation, observation and control in 1-d flexible multi-structures.* Springer Verlag. Mathématiques et Applications, vol. 50, 2006.

[2] E. Zuazua, Propagation, observation, and control of waves approximated by finite difference methods. *SIAM Review* **47** (2005) 197–243.

[3] E. Zuazua, Controllability and Observability of Partial Differential Equations: Some results and open problems. In “Handbook of Differential Equations: Evolutionary Equations, vol. 3” C. M. Dafermos and E. Feireisl eds. Elsevier Science, 2006, pp. 527–621.

[4] E. Zuazua, Switching control. *J. Eur. Math. Soc.* **13** (2011) 85–117.