

SELF-ADAPTIVE *hp* FINITE-ELEMENT SIMULATION OF MULTI-COMPONENT INDUCTION MEASUREMENTS ACQUIRED IN DIPPING, INVADED, AND ANISOTROPIC FORMATIONS

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V. M. Calo, M. Paszynski, and P. J. Matuszlik

Presentation at Korean Institute of Geoscience and
Minerals (KIGAM), September 8, 2008.



Overview

1. Main Lines of Research and Applications

- Previous work
- Main features of our technology

2. Application 1: Tri-Axial Induction Instruments (M. J. Nam)

3. Application 2: Dual-Laterolog Instruments (M. J. Nam)

4. Multi-Physics Inversion (D. Pardo)

5. Sonic Instruments (L. Demkowicz)

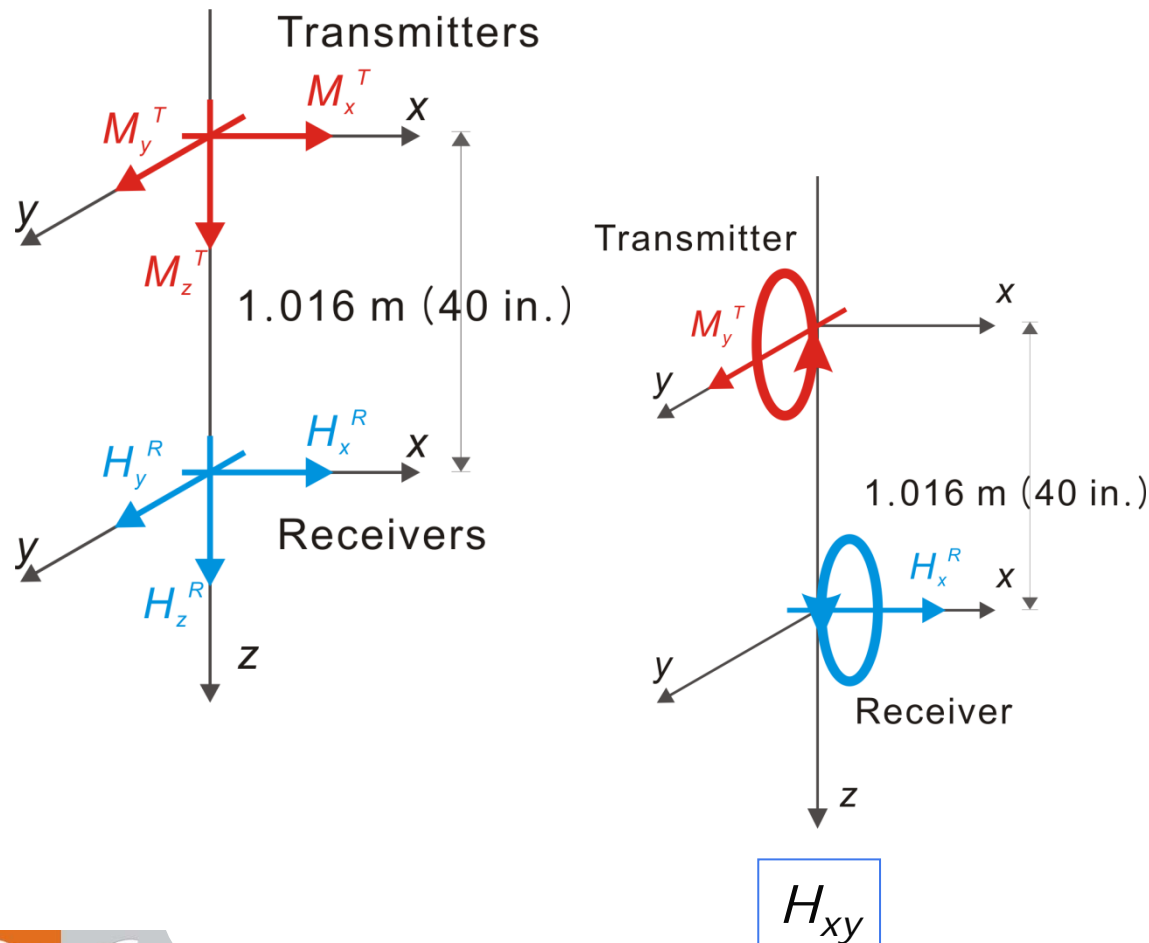


Outline

- **Introduction to Tri-Axial Induction**
- **Method**
- **Numerical Results:**
 - **Verification of 3D Method for Tri-Axial Induction Tool**
 - **Dipping, Invaded, Anisotropic Formations**
- **Conclusions**

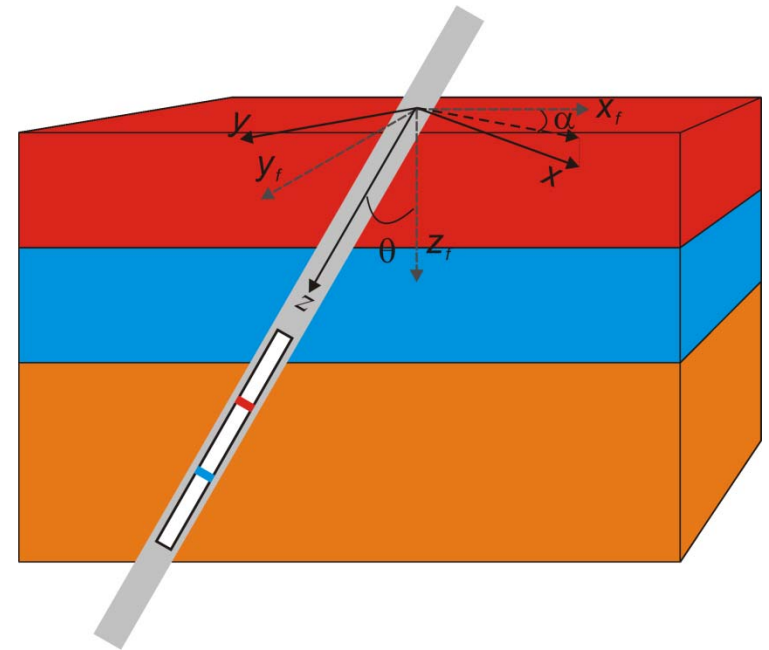


Tri-Axial Induction Tool



$L = 1.016 \text{ m (40 in.)}$

Operating frequency: 20 kHz



θ : dip angle

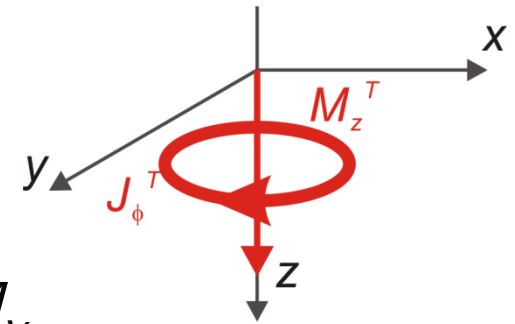
α : tool orientation angle



3D Source Implementation

1. Solenoidal Coil (J_ϕ) for M_z

→ becoming a 2D source in (ρ, ϕ, z)

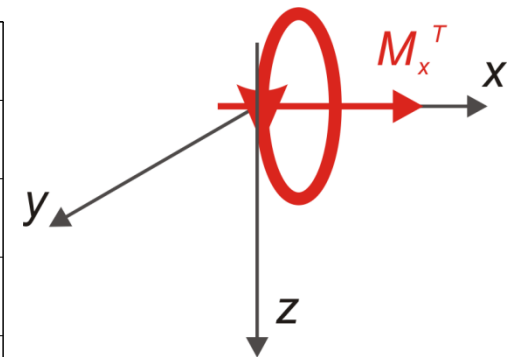
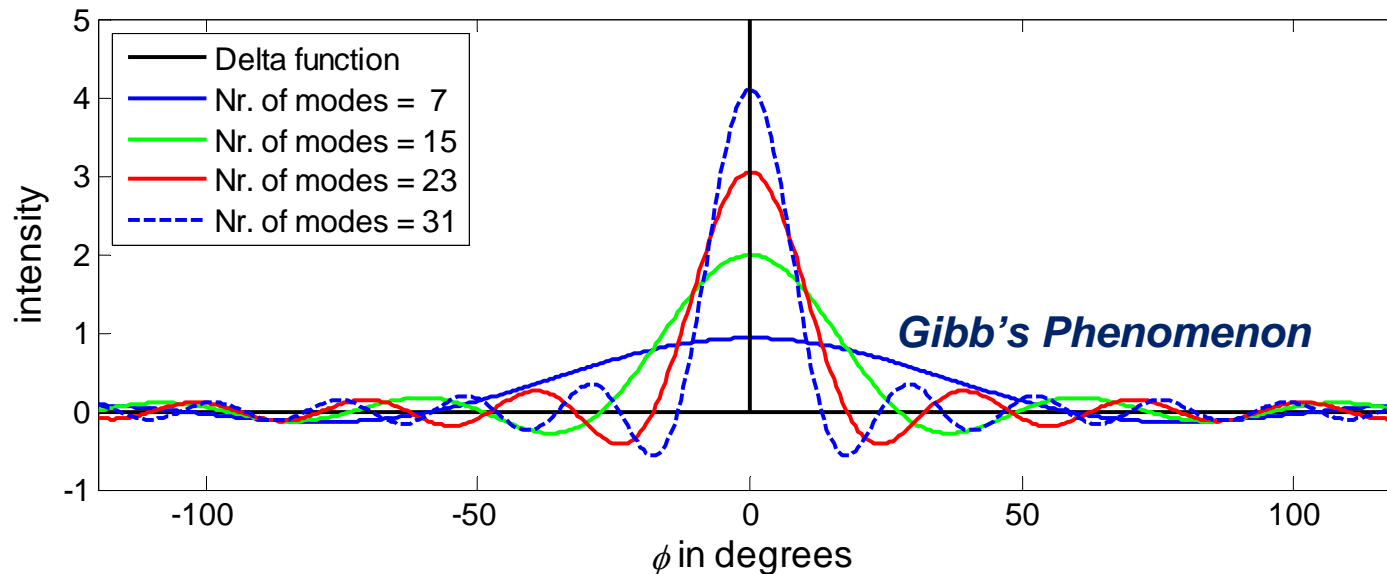


2. Delta Function for 3D source M_x or M_y

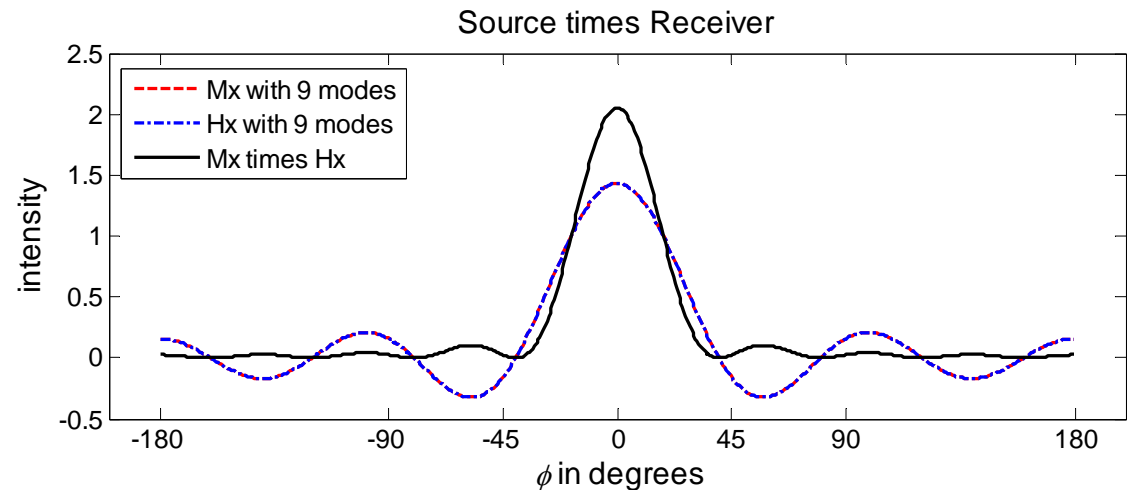
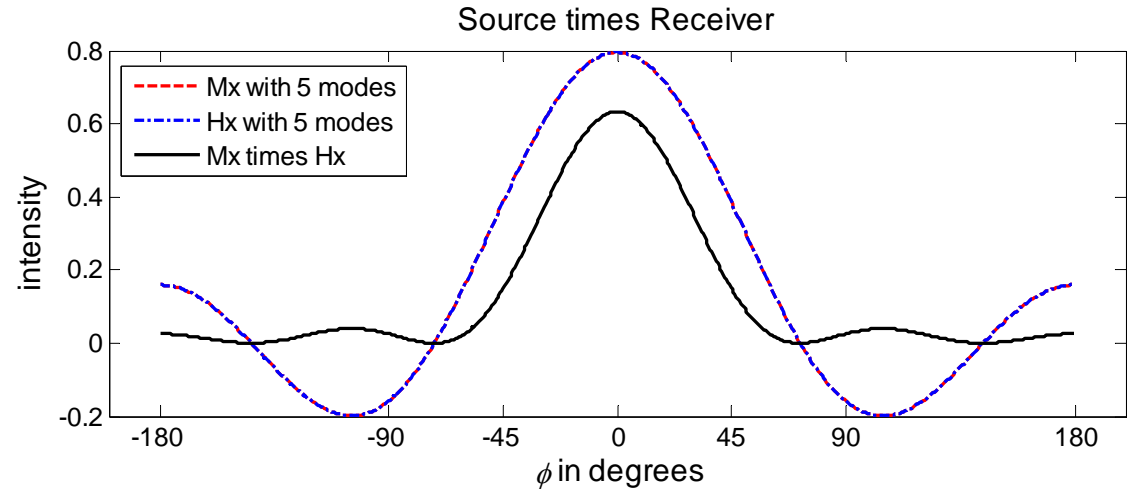
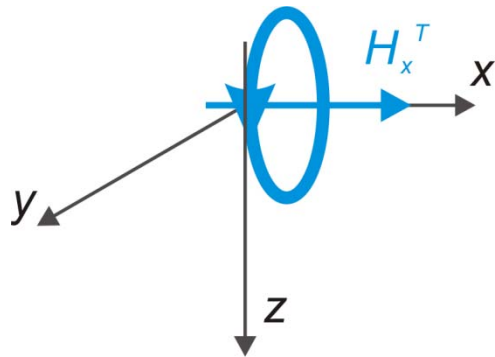
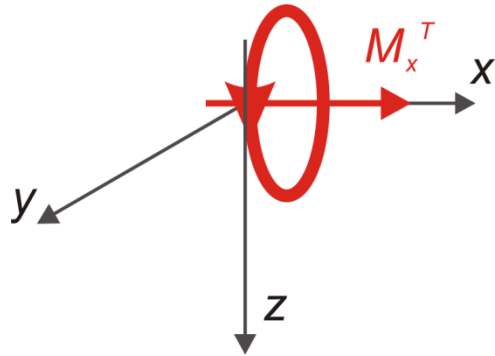
$$f(\phi) = \delta(\phi - \phi_0)$$

ϕ_0 : the position of the center of the peak
(0° for M_x ; 90° for M_y)

M_x : Delta function



3D Source and Receiver (Delta Functions)



**Coupling between source and receiver:
less Gibb's phenomenon**



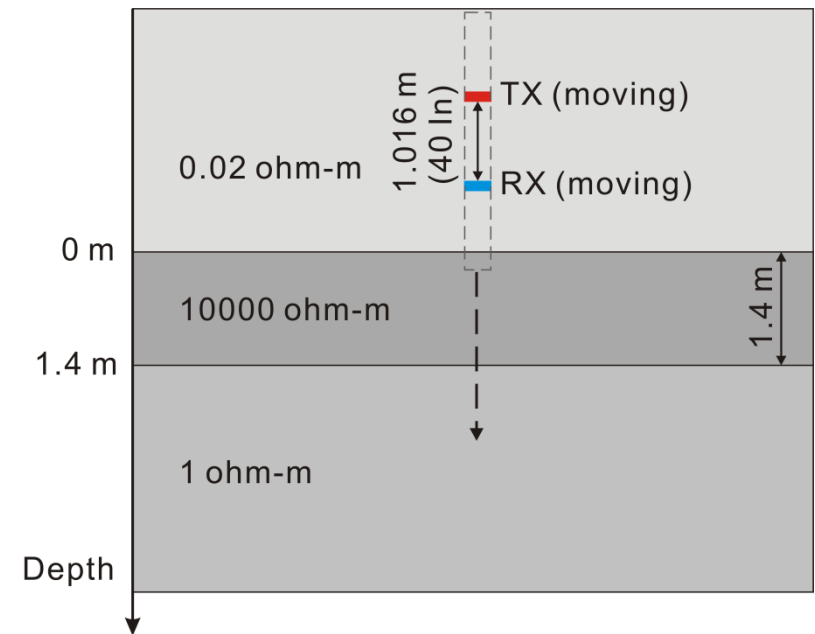
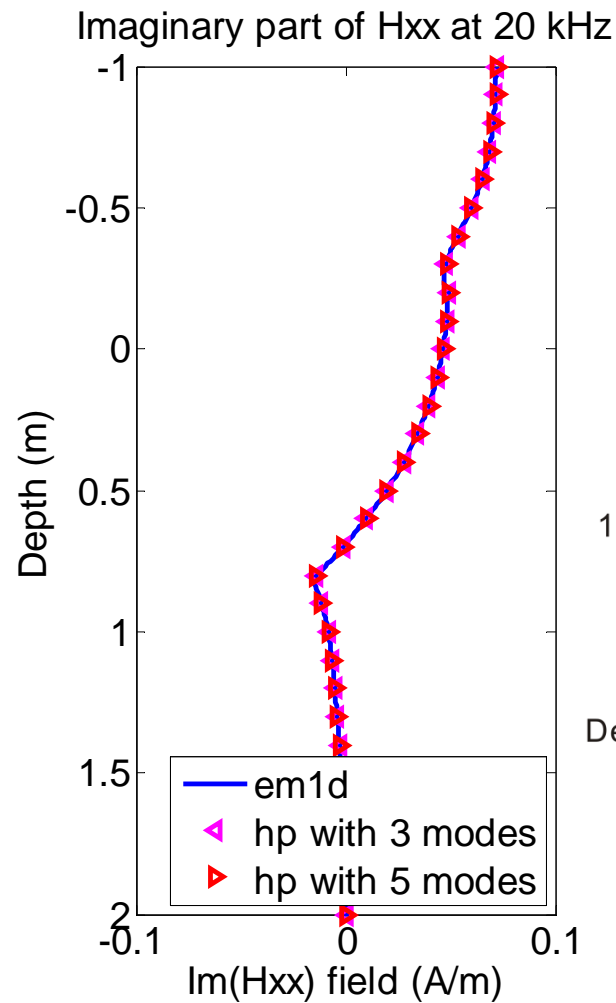
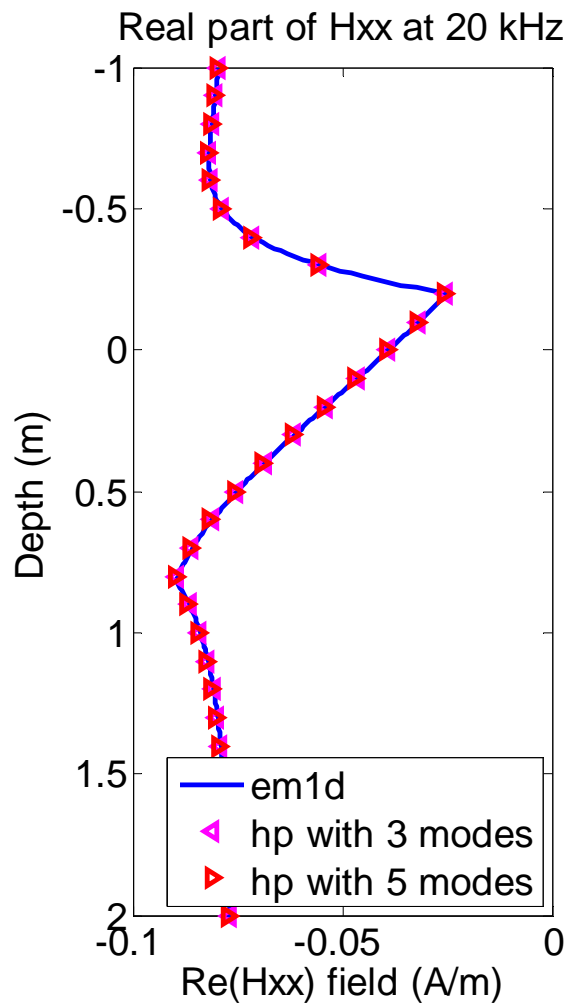
Method

Combination of:

- 1. A Self-Adaptive Goal-Oriented *hp*-FEM
for AC problems**
- 2. A Fourier Series Expansion
in a Non-Orthogonal System of Coordinates**
- 3. Parallel Implementation**



Verification of 2.5D Simulation ($H_{xx} = H_{yy}$)

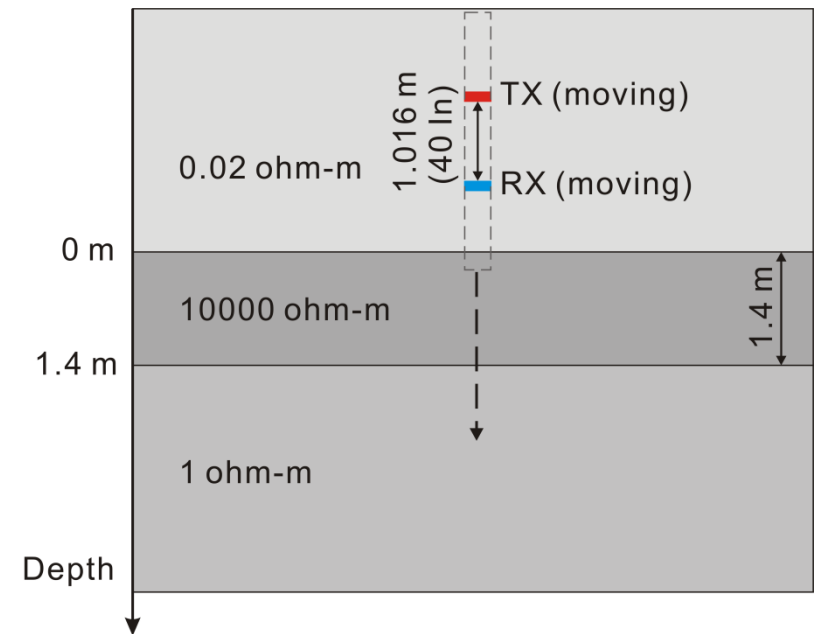
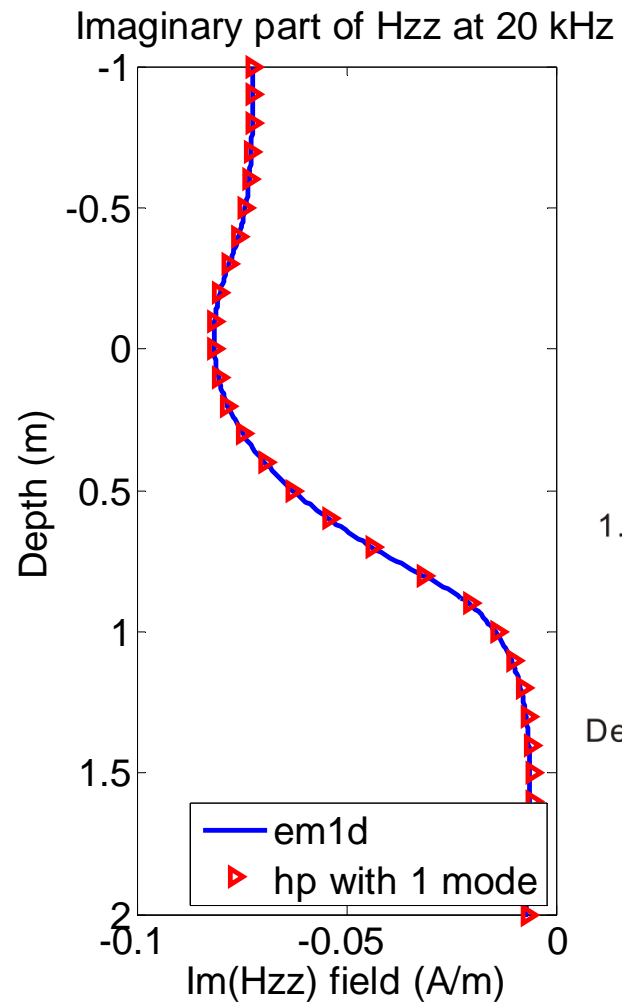
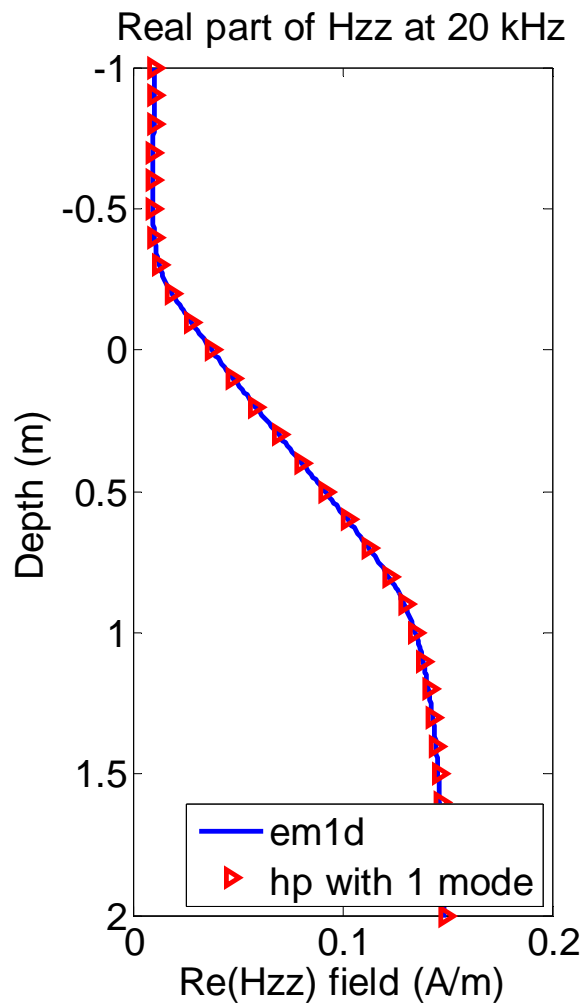


**Converged solutions
with 3 Fourier modes**

em1D: K. H. Lee 1984, pers. comm.



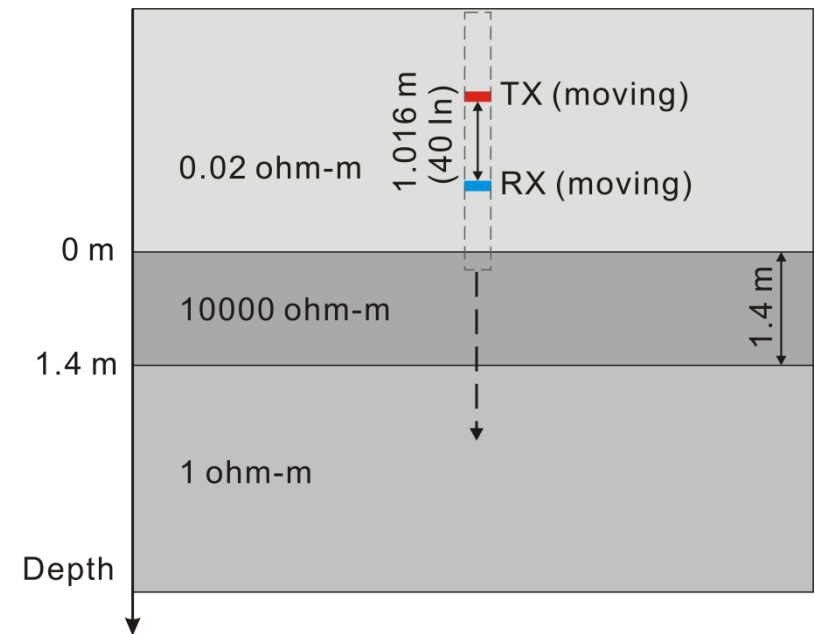
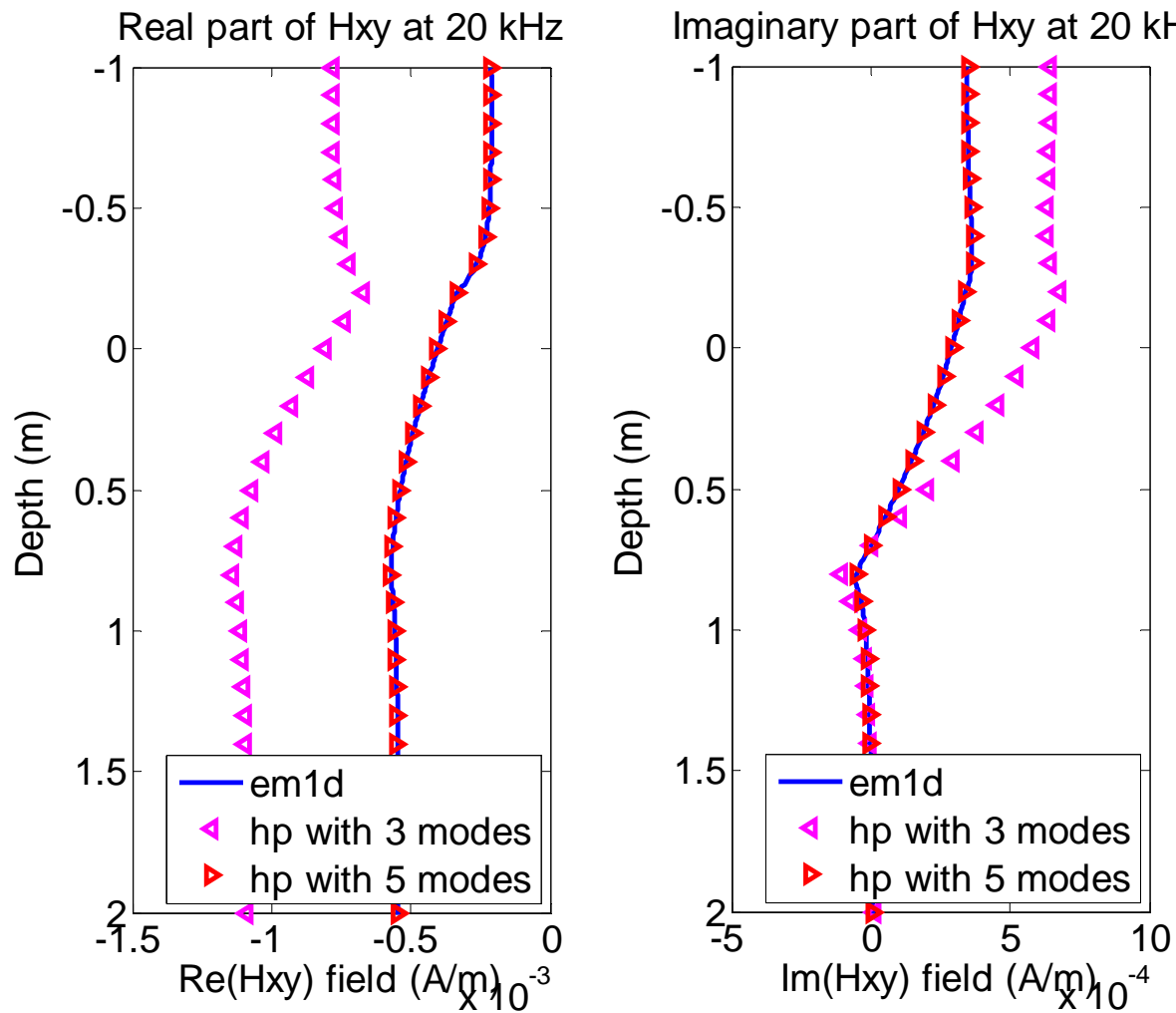
Verification of 2.5D Simulation (H_{zz})



The same solutions
with 1 Fourier mode



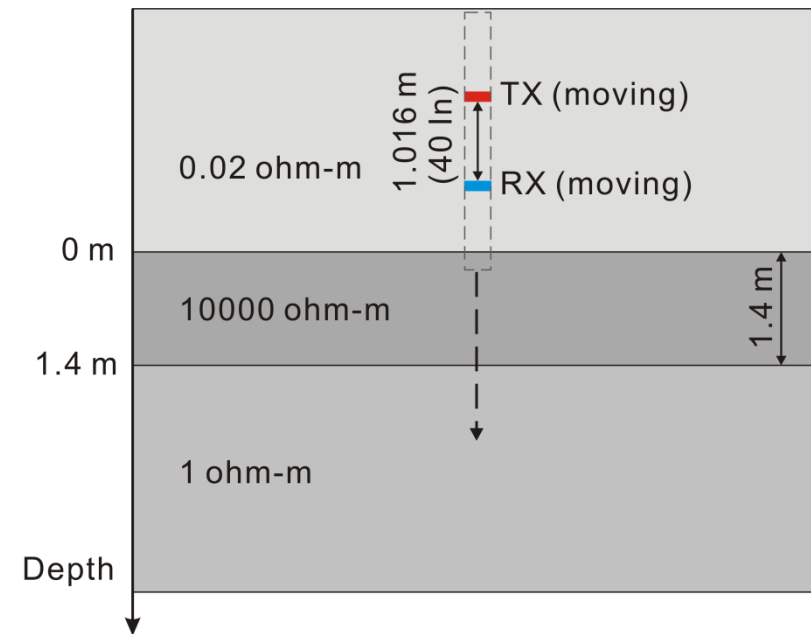
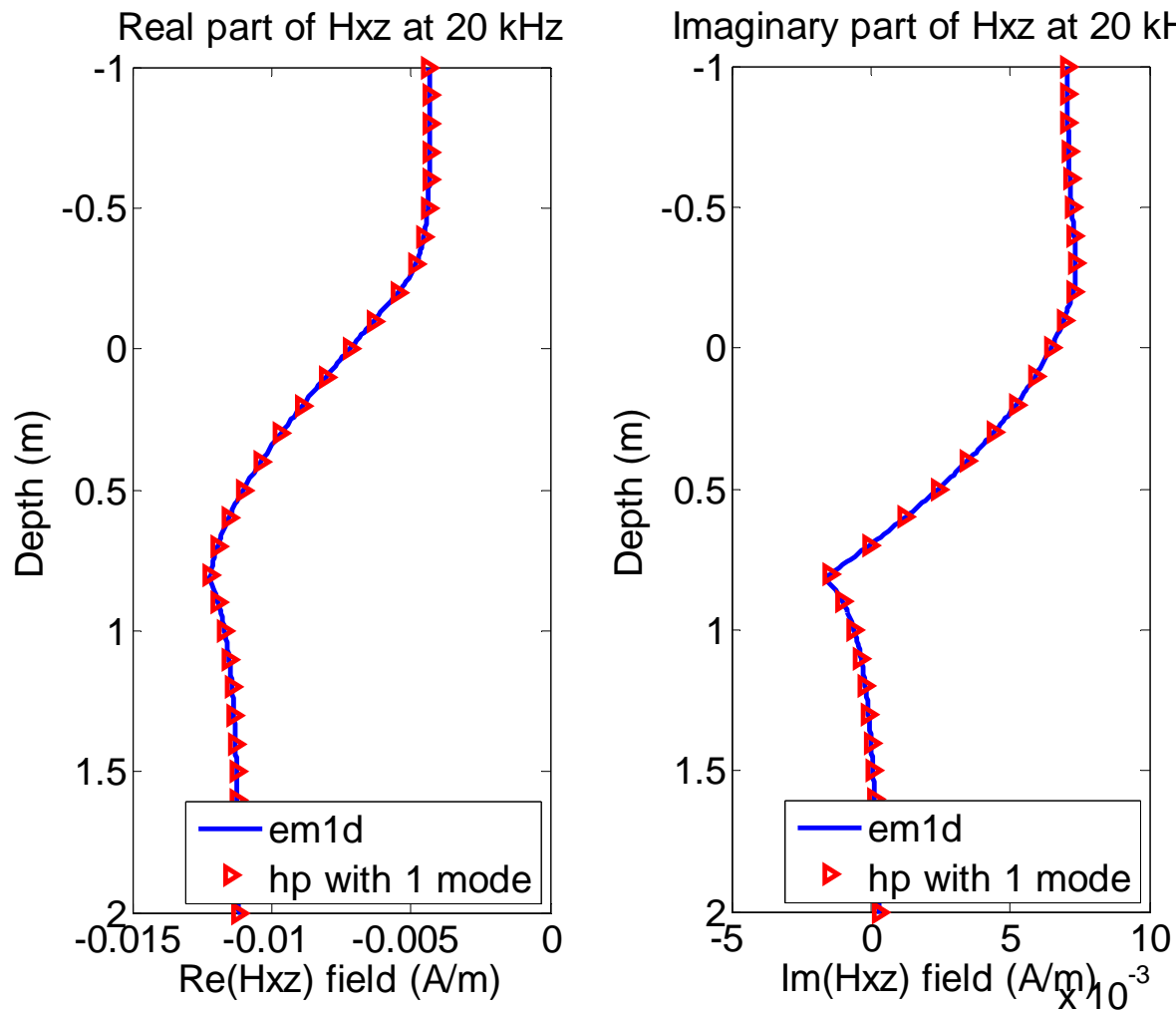
Verification of 2.5D Simulation ($H_{xy} = H_{yx}$)



**Converged solutions
with 5 Fourier modes**



Verification of 2.5D Simulation ($H_{xz} = H_{zx}$)

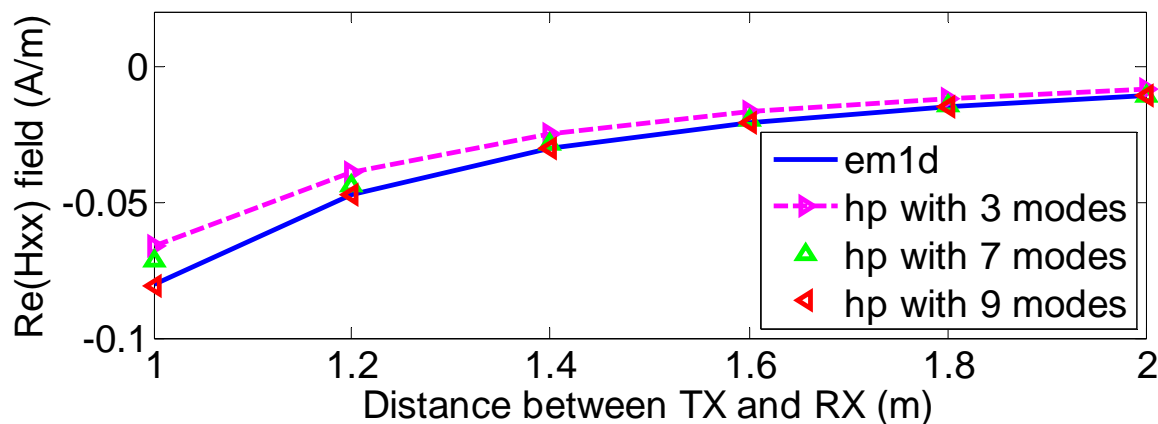


The same solutions with 1 Fourier mode

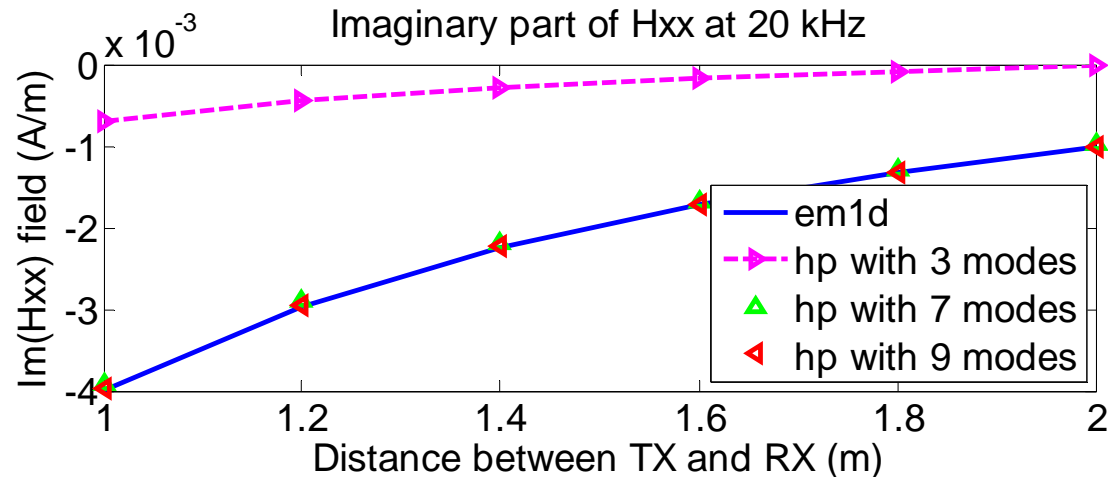


Verification of 3D Simulation ($H_{xx} = H_{yy}$)

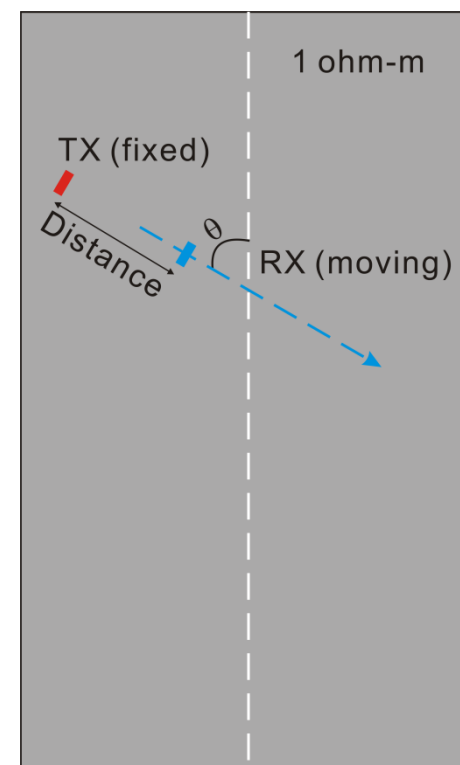
Real part of Hxx at 20 kHz



Imaginary part of Hxx at 20 kHz



Dip angle: 60 degrees

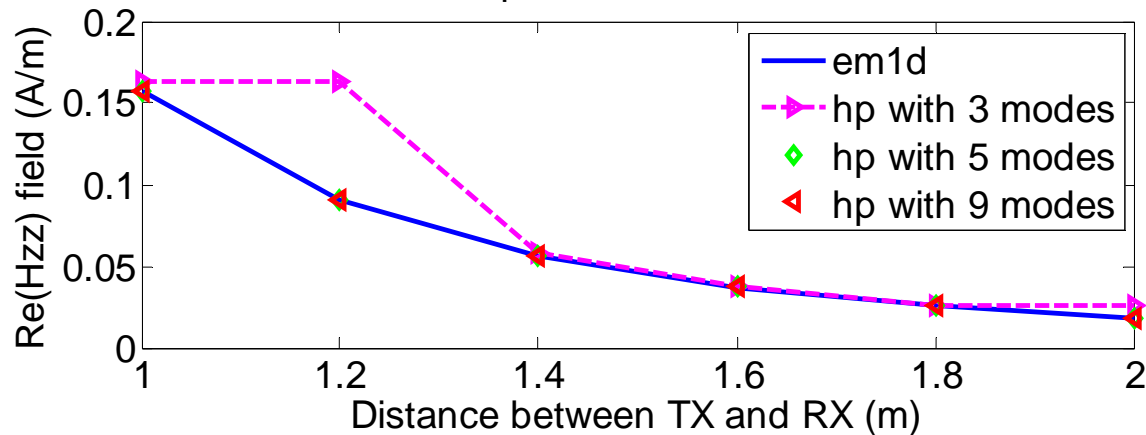


Converged solutions
with 9 Fourier mode

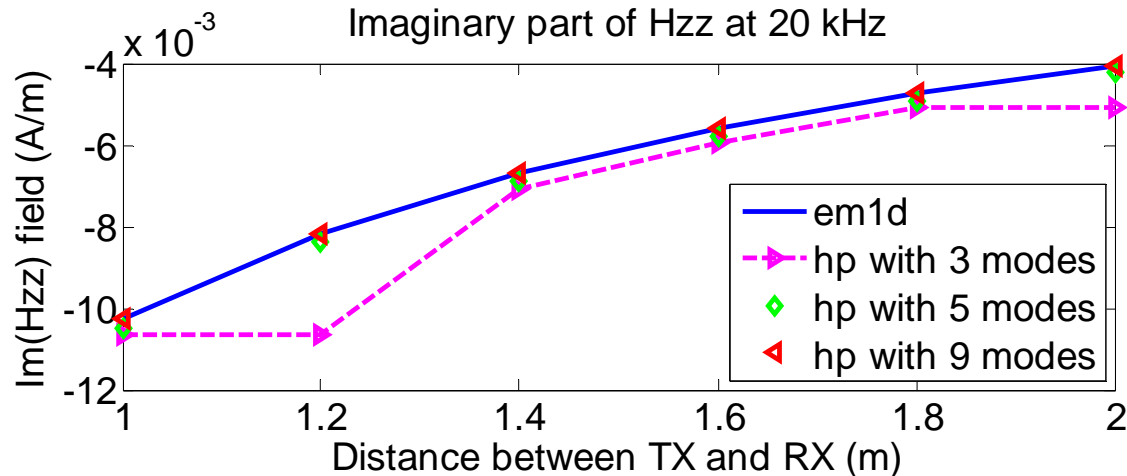


Verification of 3D Simulation (H_{zz})

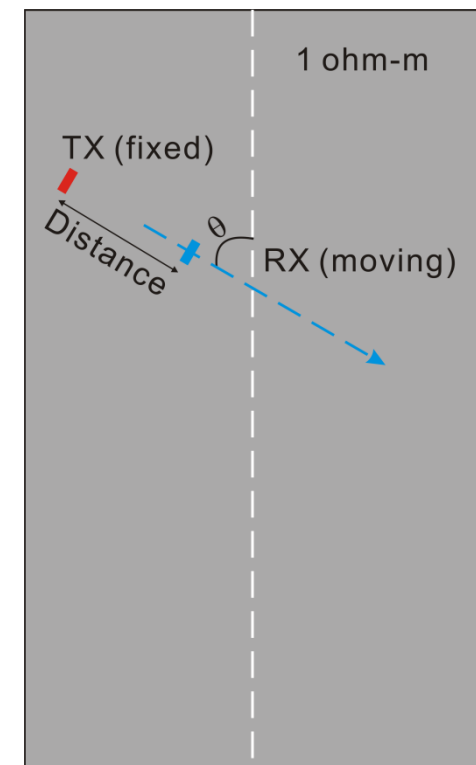
Real part of H_{zz} at 20 kHz



Imaginary part of H_{zz} at 20 kHz



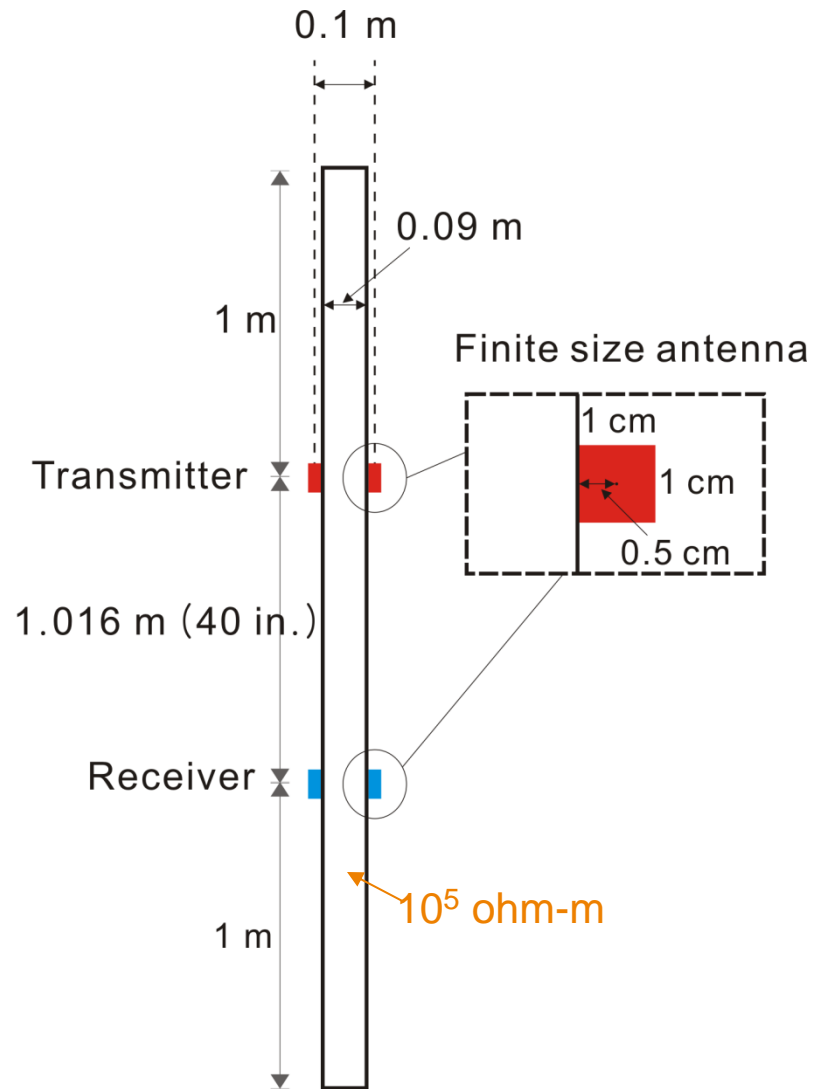
Dip angle: 60 degrees



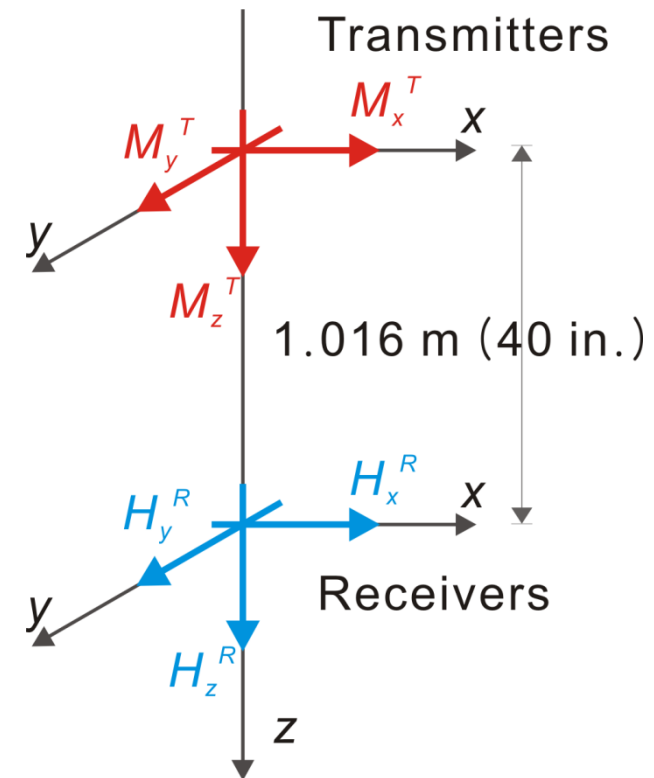
Converged solutions
with 5 Fourier mode



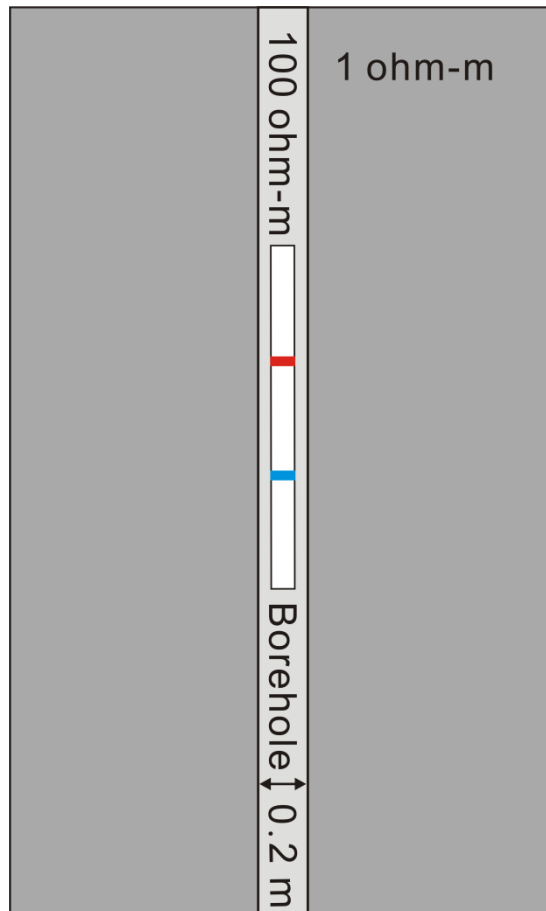
Description of the Tri-Axial Tool



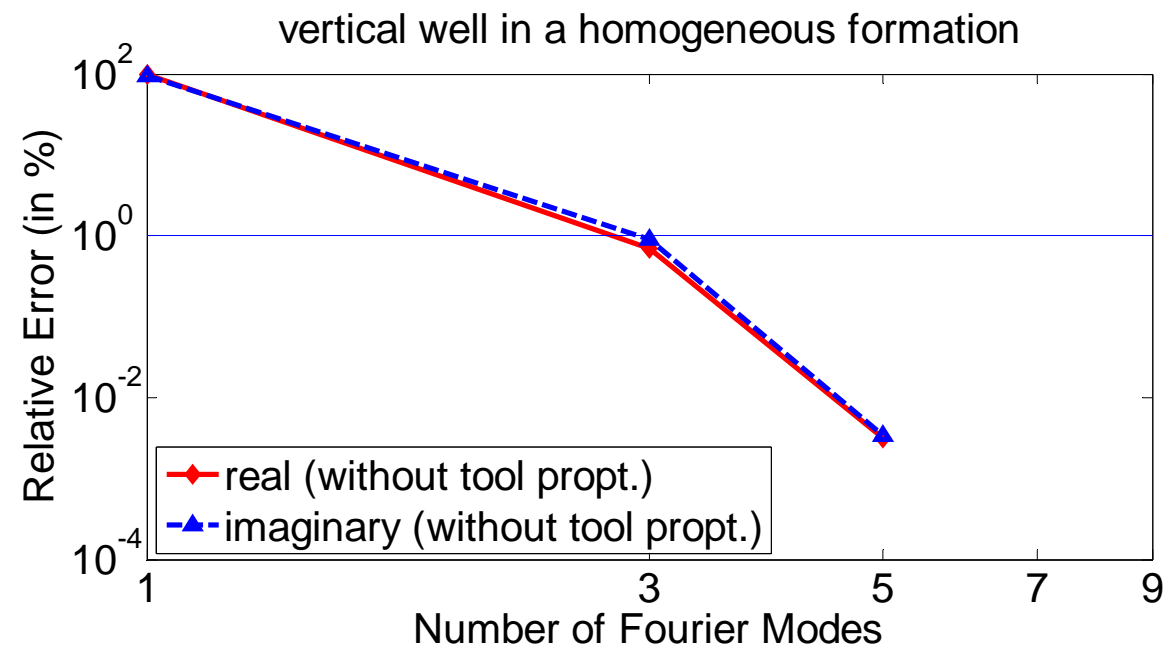
Operating frequency: 20 kHz



Verification of 2.5D Simulation (H_{xx})

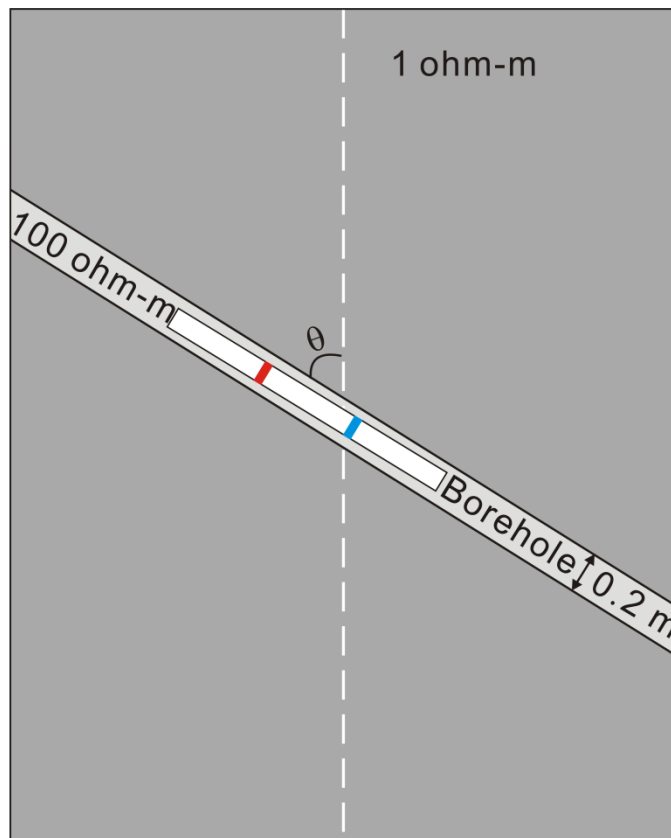


Relative errors of tri-axial induction solutions with respect to the solution with 9 Fourier modes

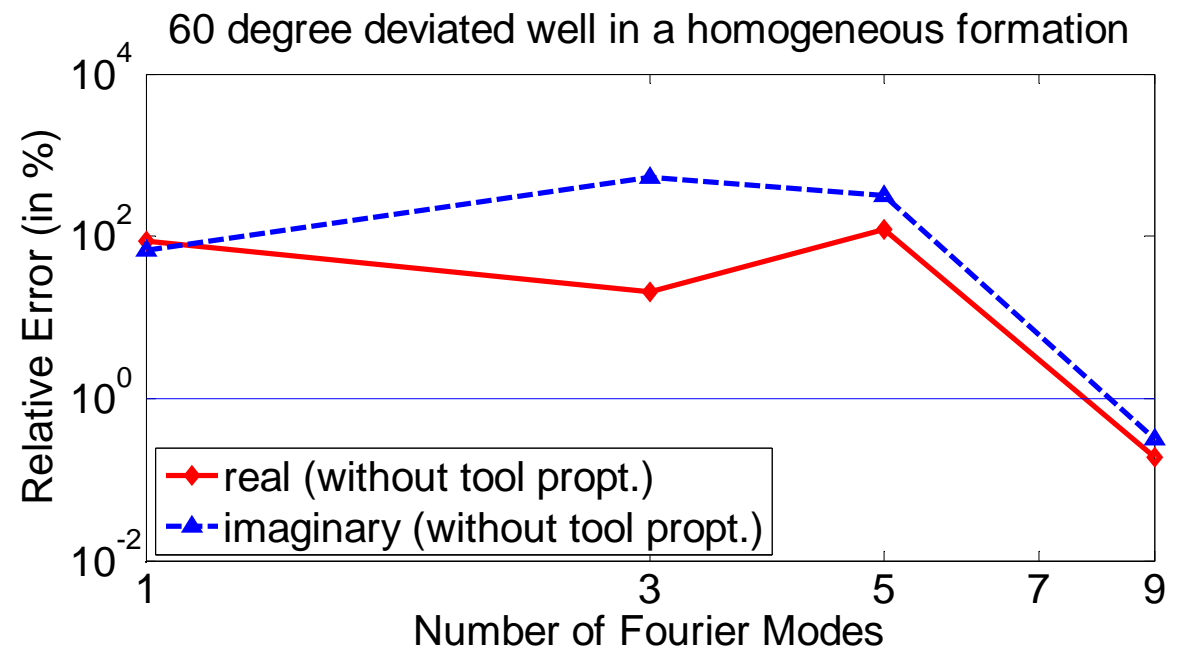


Verification of 3D Simulation (H_{xx})

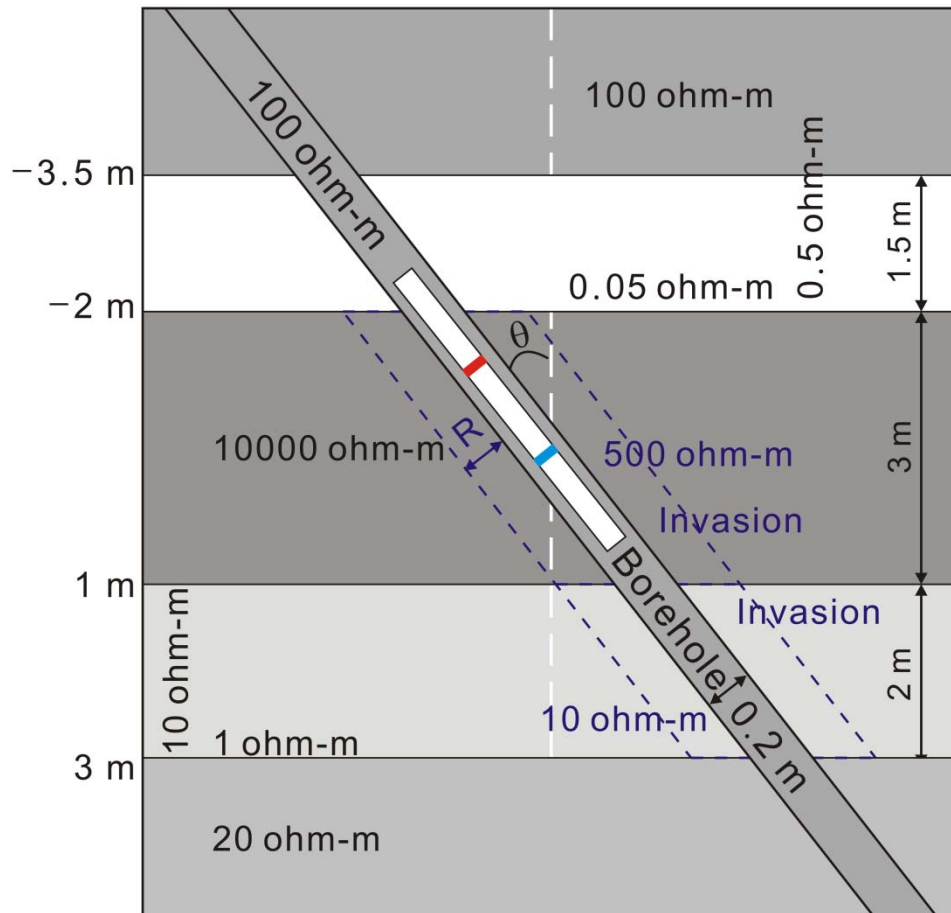
$\theta = 60$ degrees



Relative errors of tri-axial Induction solutions with respect to the solution for the vertical well



Model for Numerical Experiments



Five layers: 100, 0.05, 10000, 1 and 20 ohm-m from top to bottom

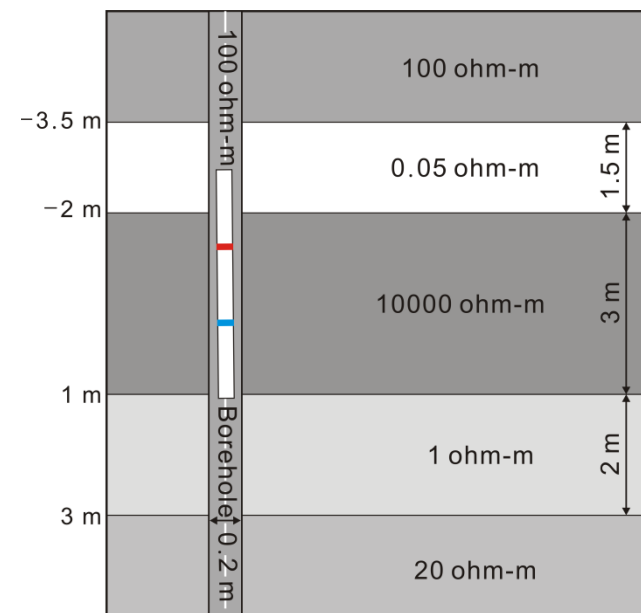
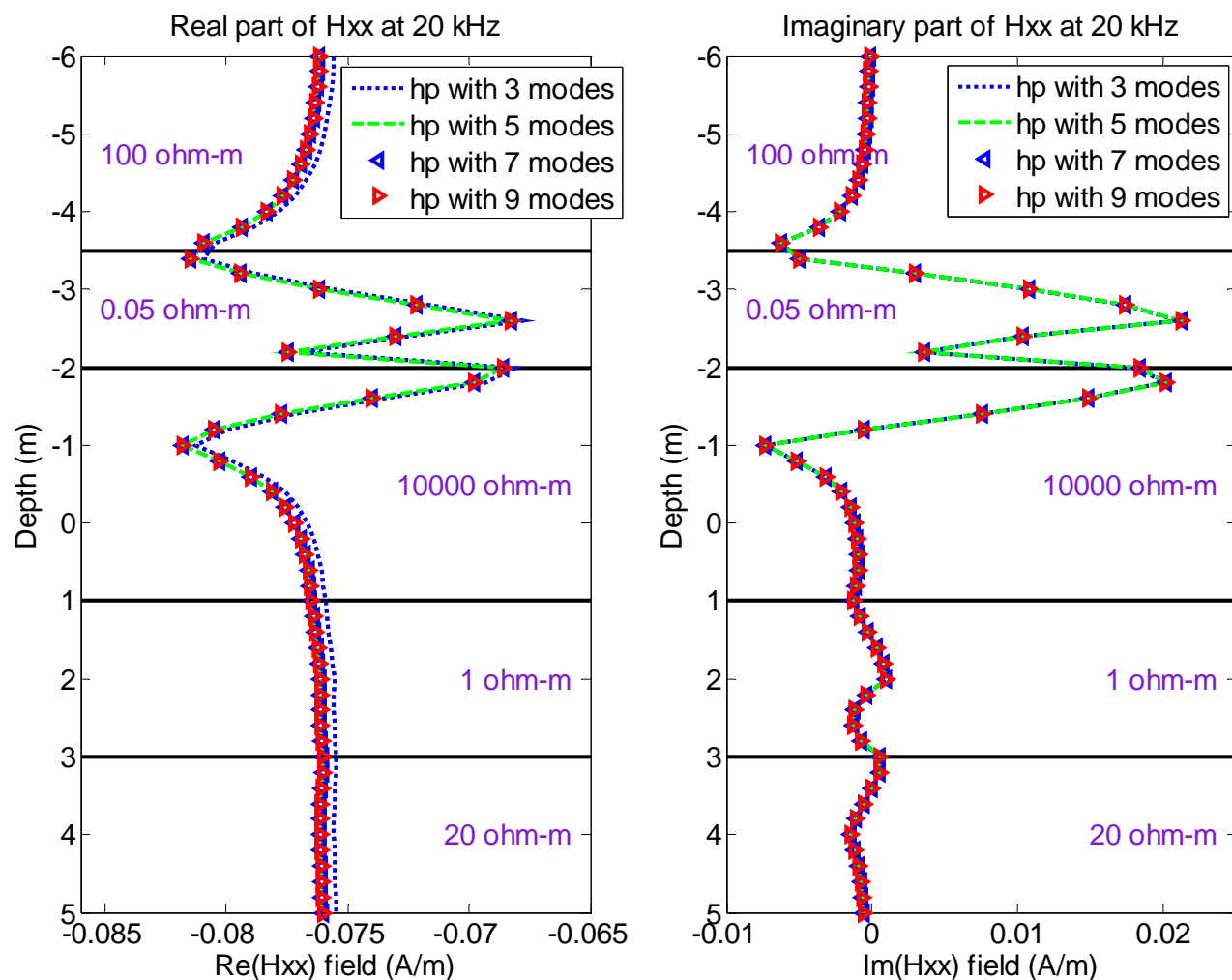
**Borehole: 0.1 m in radius
100 ohm-m in resistivity**

Invasion in the third and fourth layers

Anisotropy in the second and fourth layers

$\theta = 0, 30$ and 60 degrees

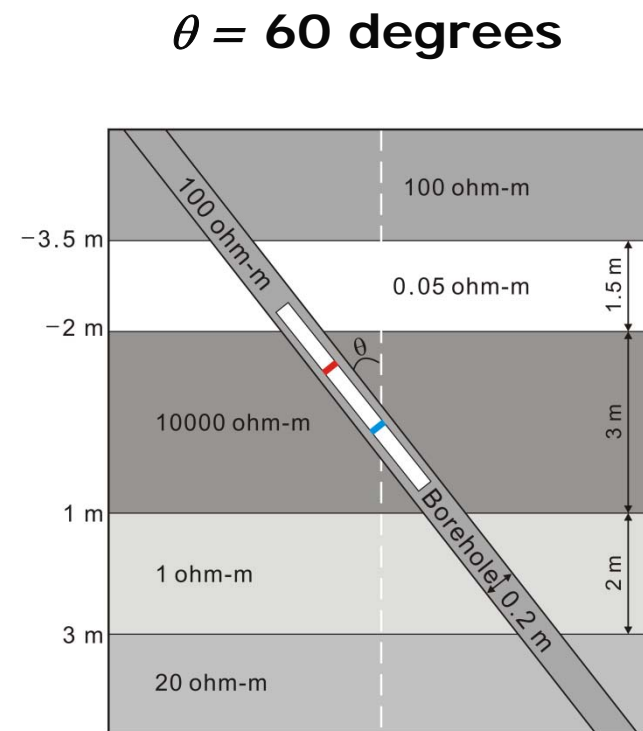
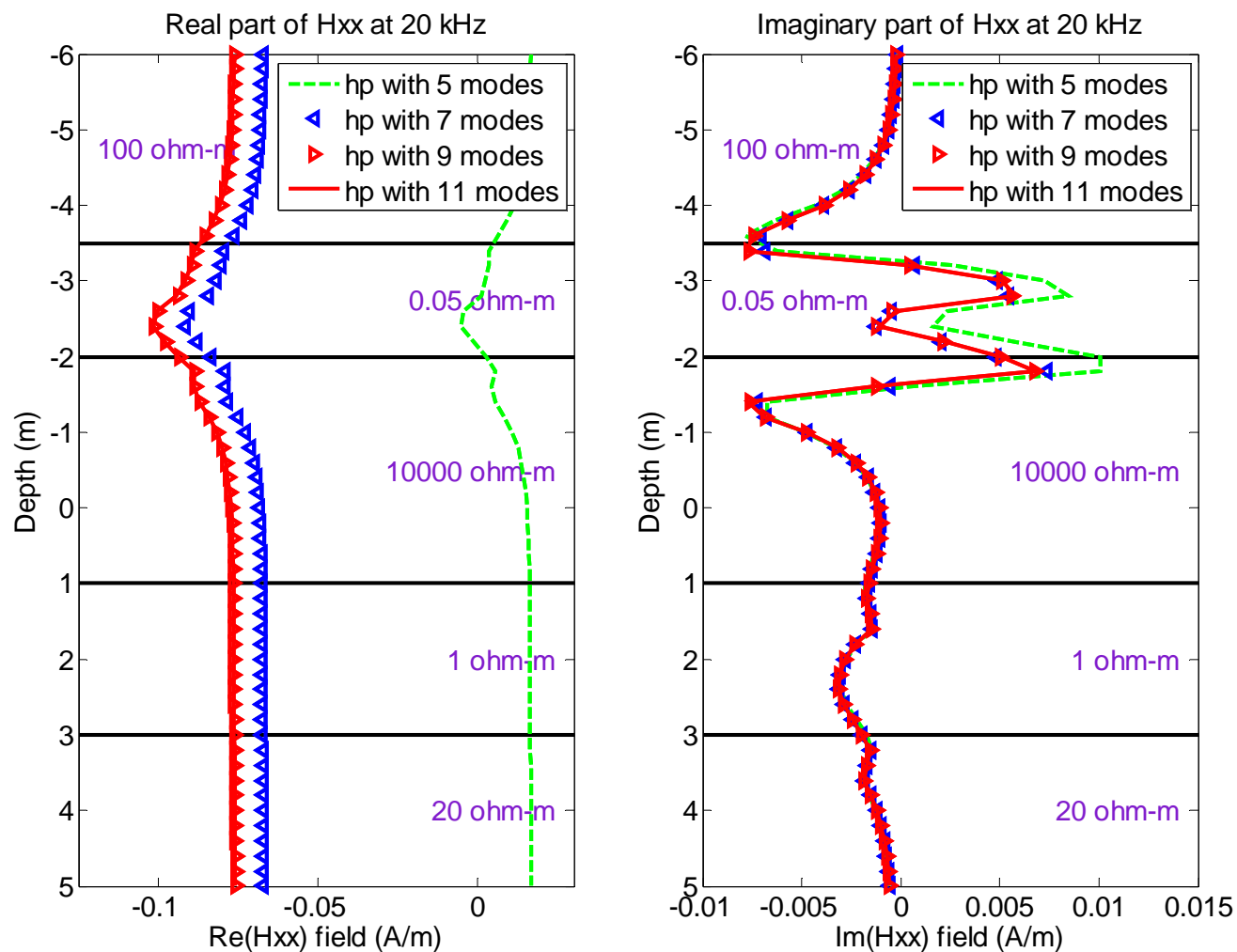
Convergence History of H_{xx} in Vertical Well



**Converged solutions
with 5 Fourier modes**

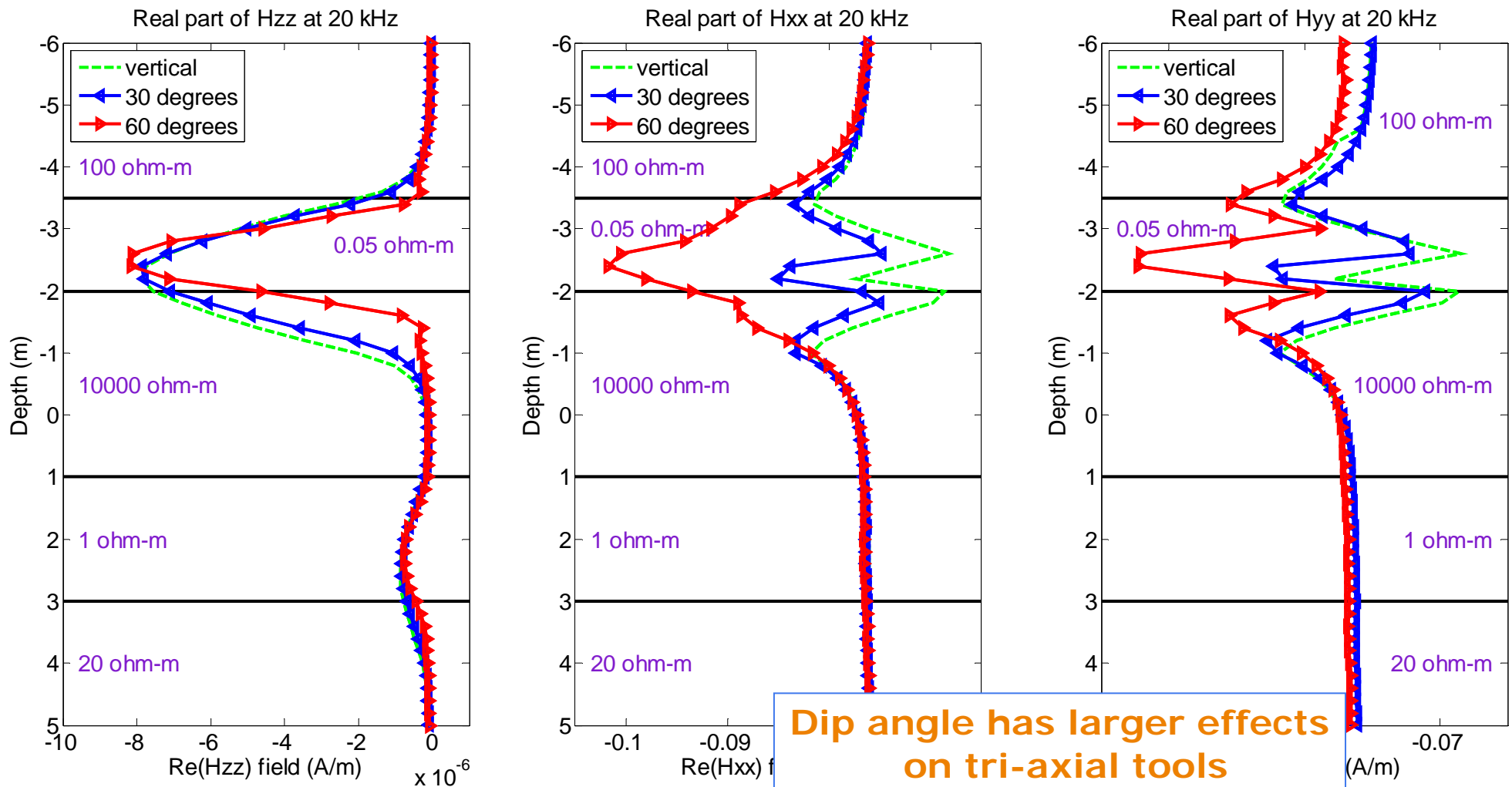


Convergence History of H_{xx} in Deviated Well

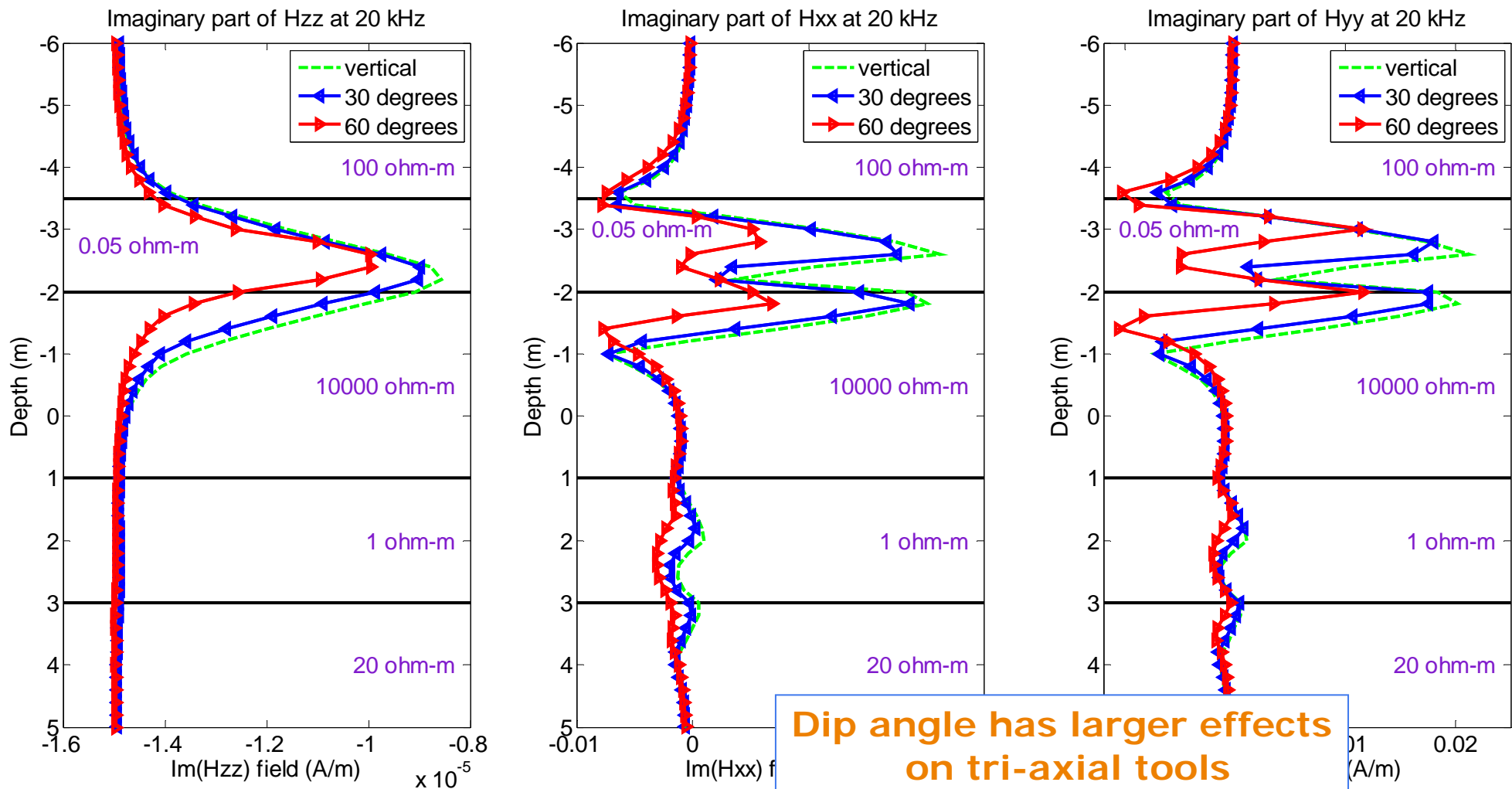


**Converged solutions
with 9 Fourier modes**

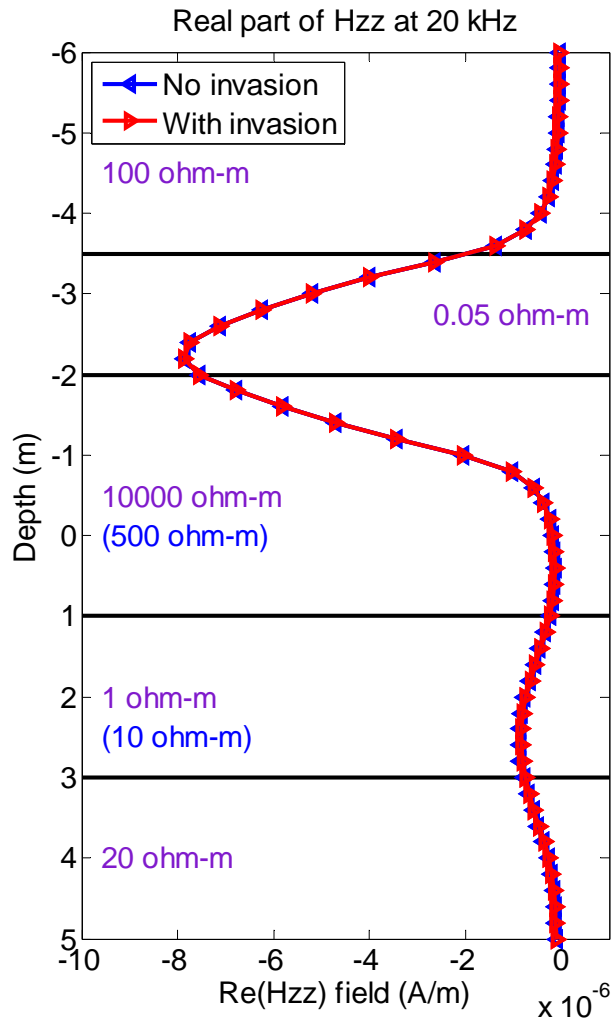
Deviated Wells (0, 30 & 60 degrees)



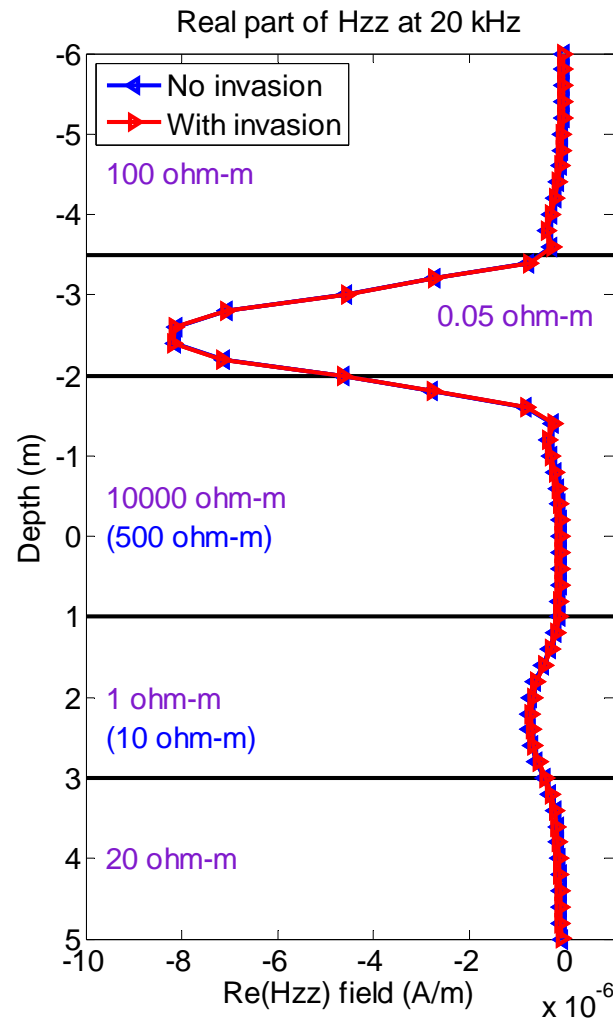
Deviated Wells (0, 30 & 60 degrees)



H_{zz} in Deviated Wells with Invasion (Re.)

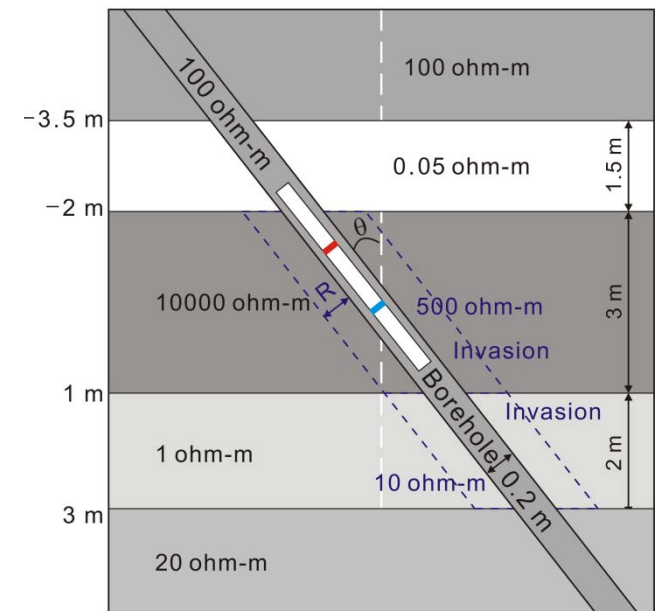


vertical

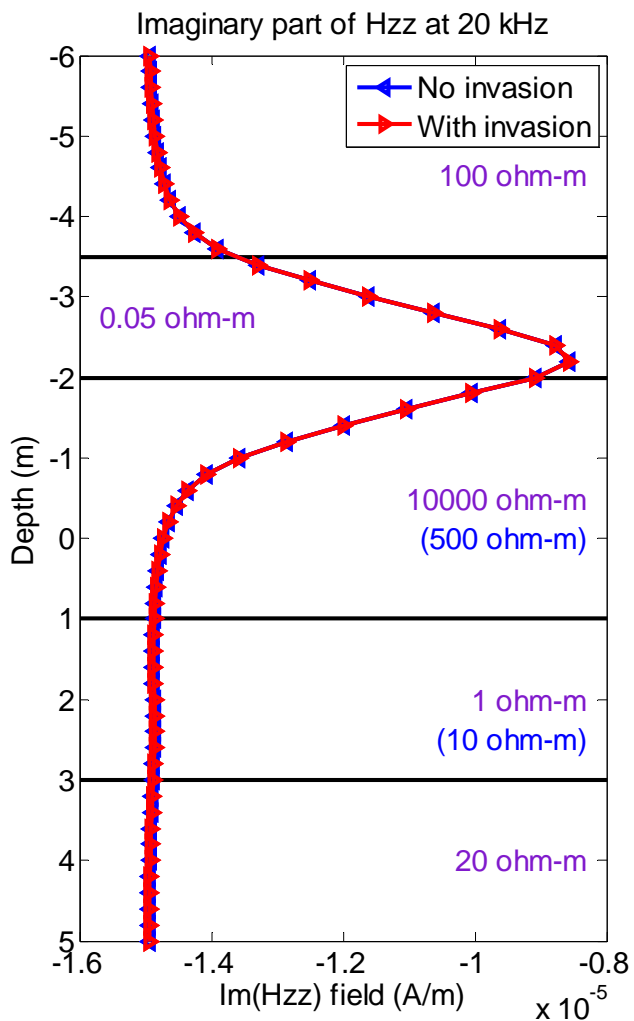


60 degrees

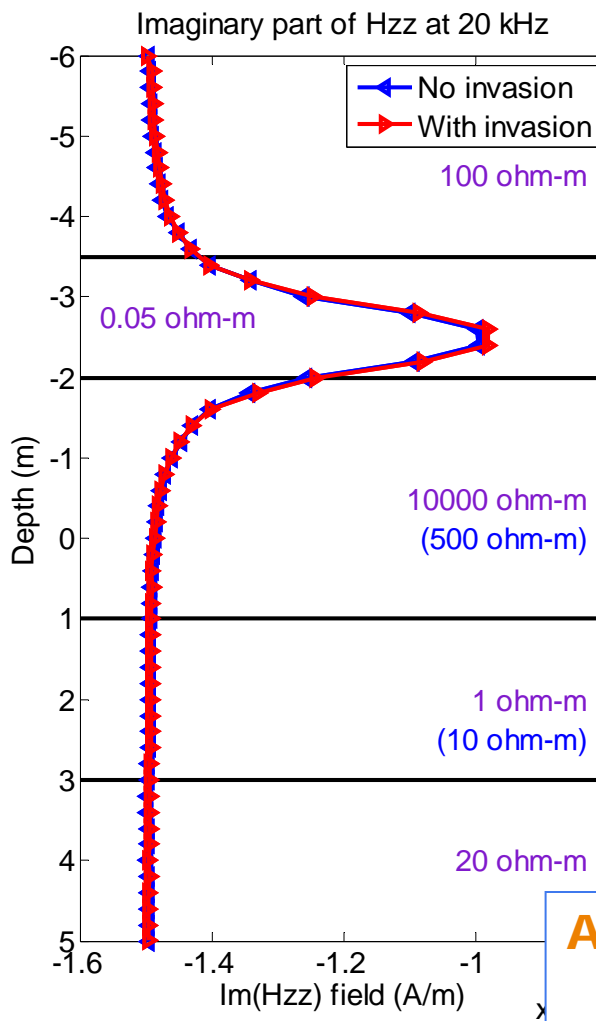
Shallow invasion with $R = 0.1$ m



H_{zz} in Deviated Wells with Invasion (Im.)

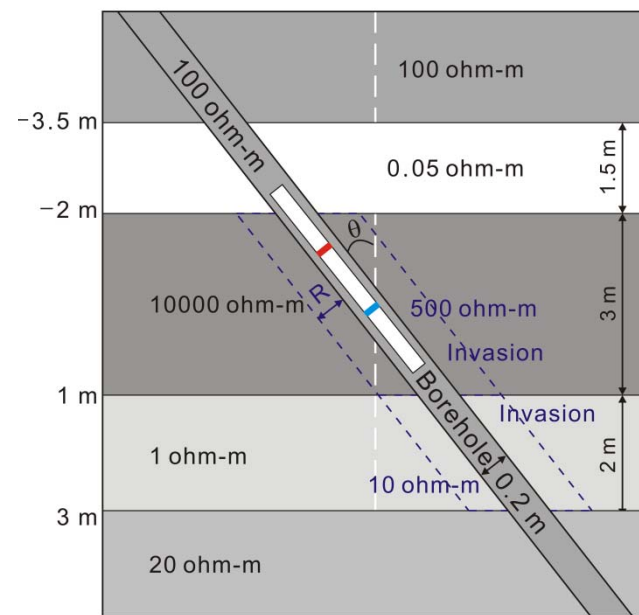


vertical



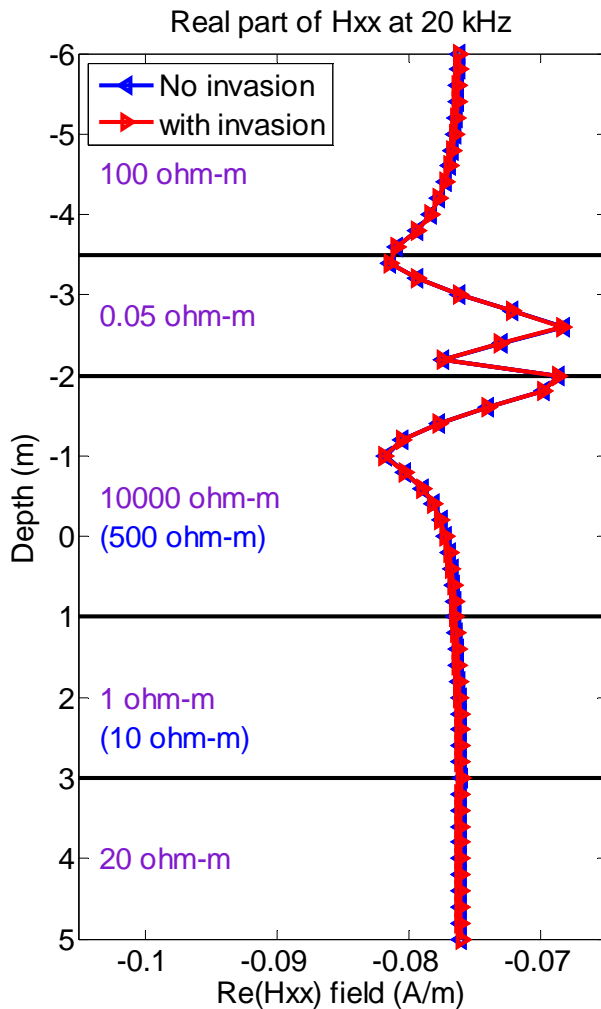
60 degrees

Shallow invasion with $R = 0.1$ m

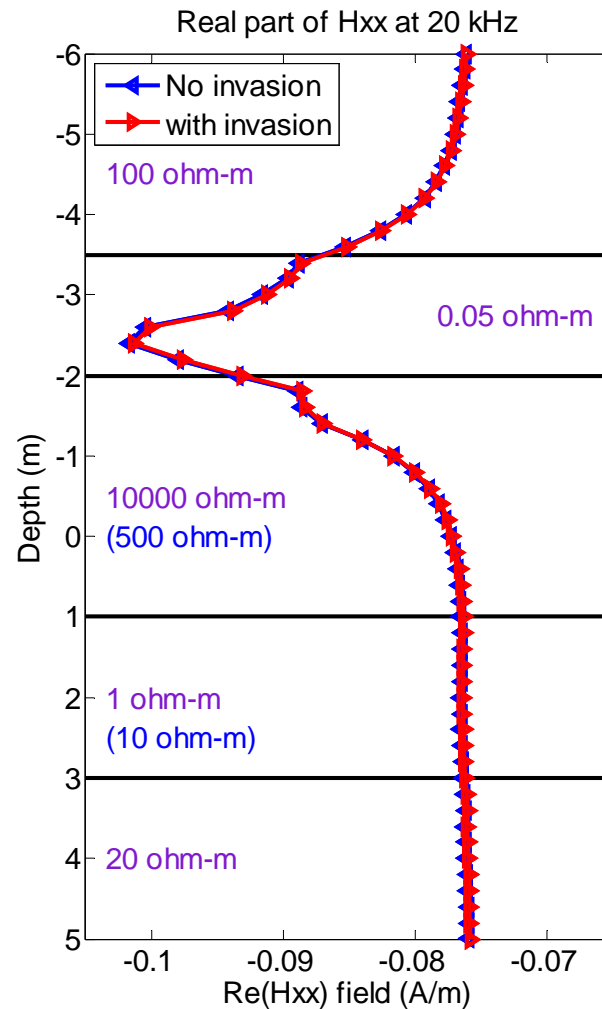


Almost no effects of invasion regardless of the dip angle

H_{XX} in Deviated Wells with Invasion (Re.)

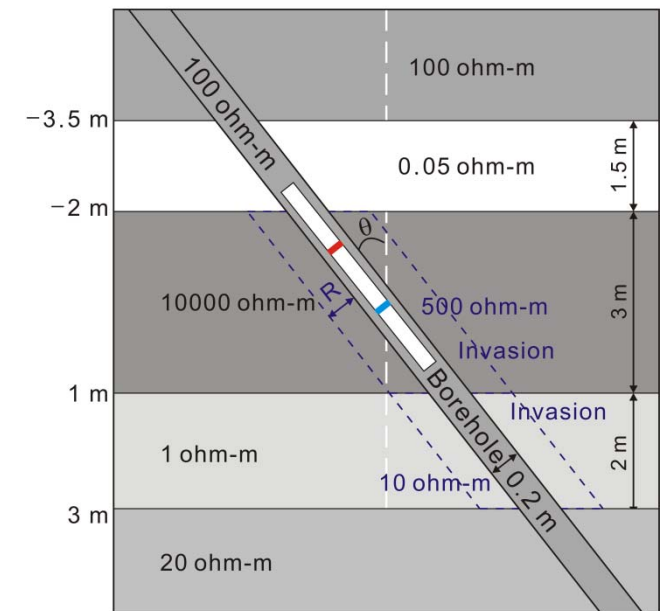


vertical

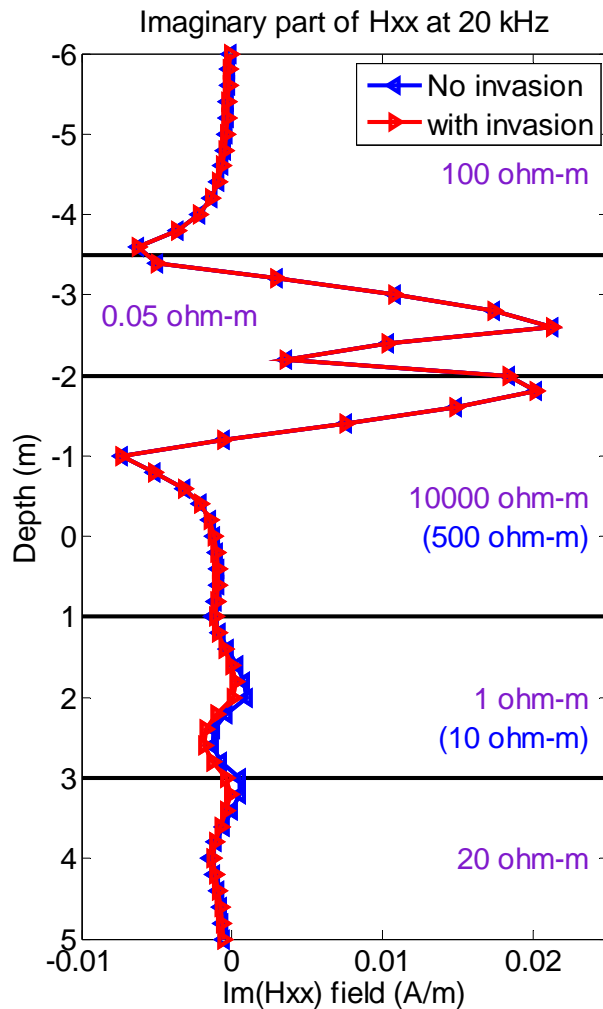


60 degrees

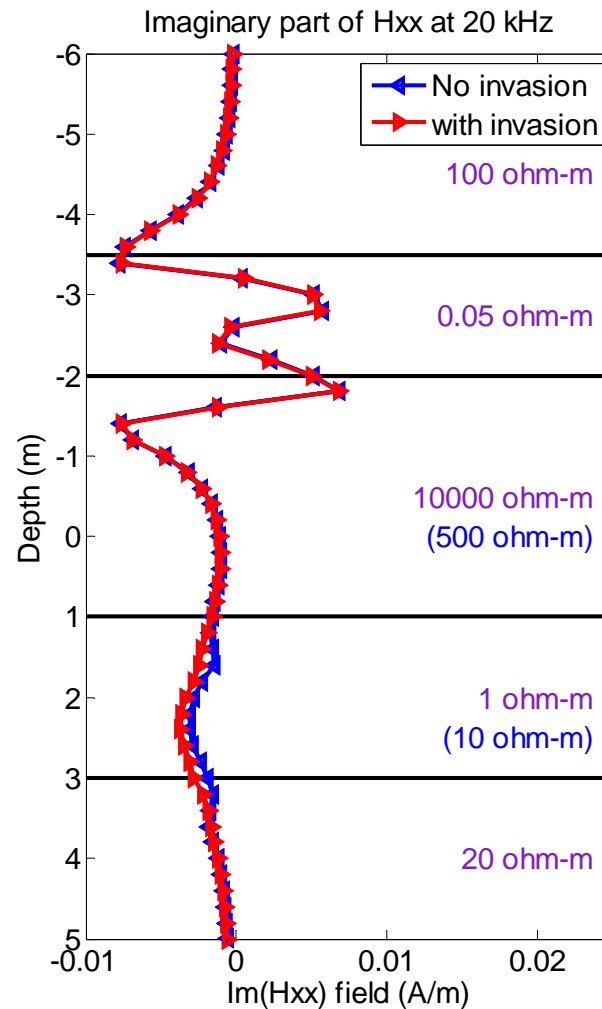
Shallow invasion
with $R = 0.1$ m



H_{XX} in Deviated Wells with Invasion (Im.)

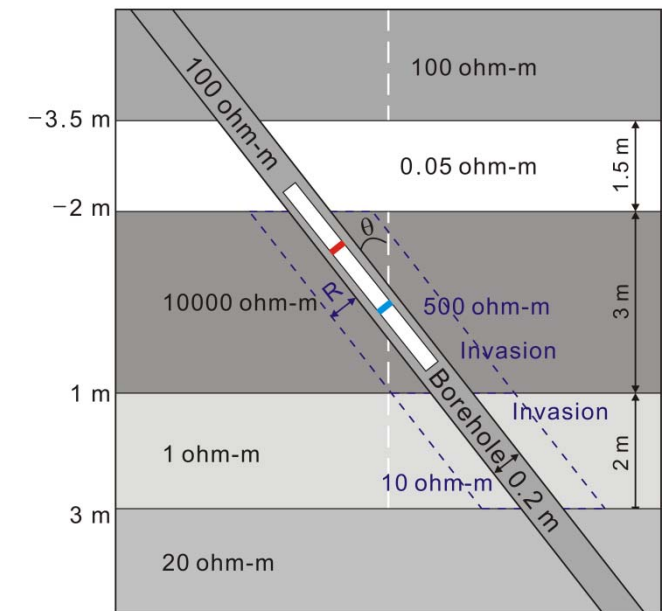


vertical



60 degrees

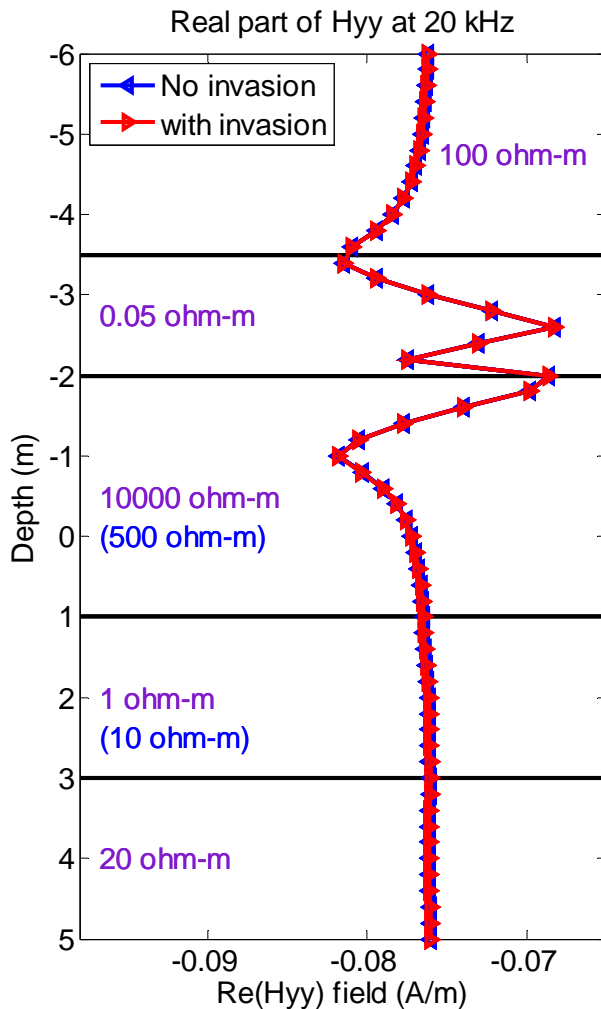
Shallow invasion
with $R = 0.1$ m



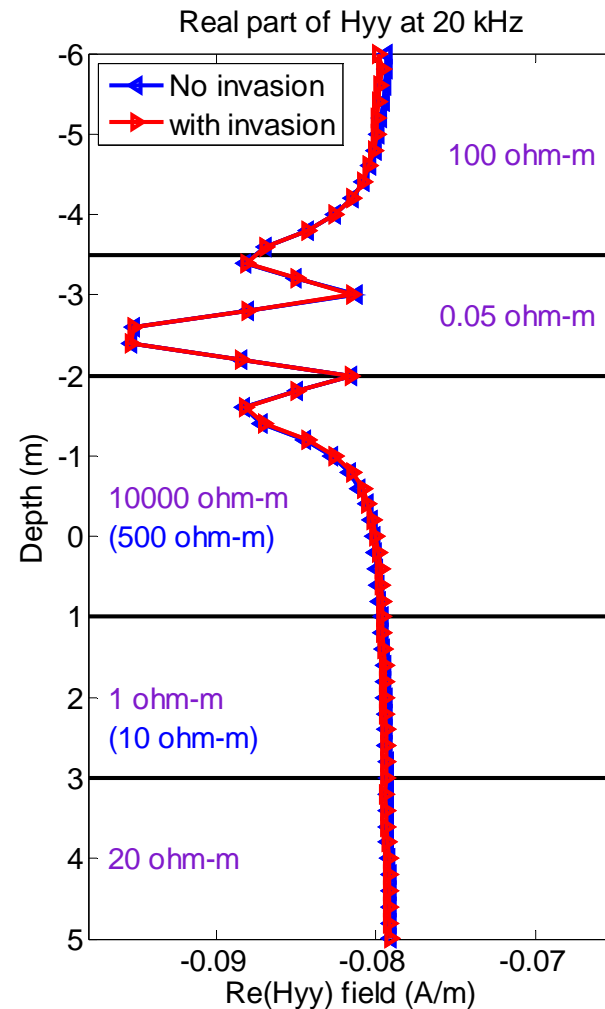
Small effects of invasion



H_{yy} in Deviated Wells with Invasion (Re.)

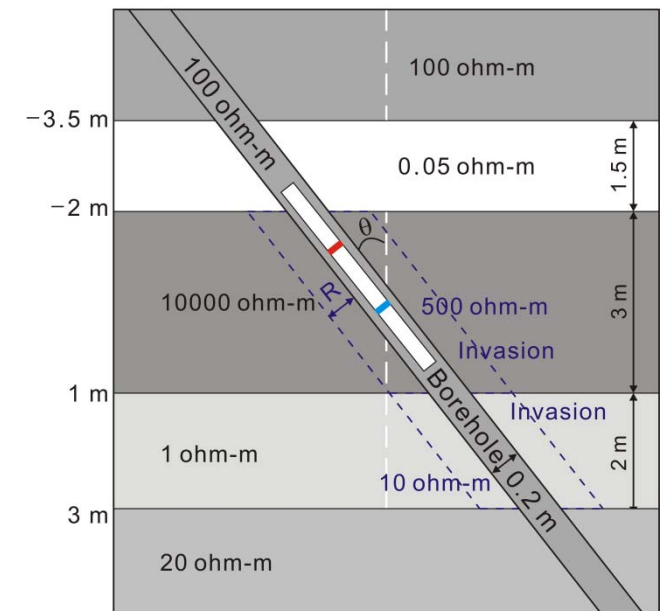


vertical

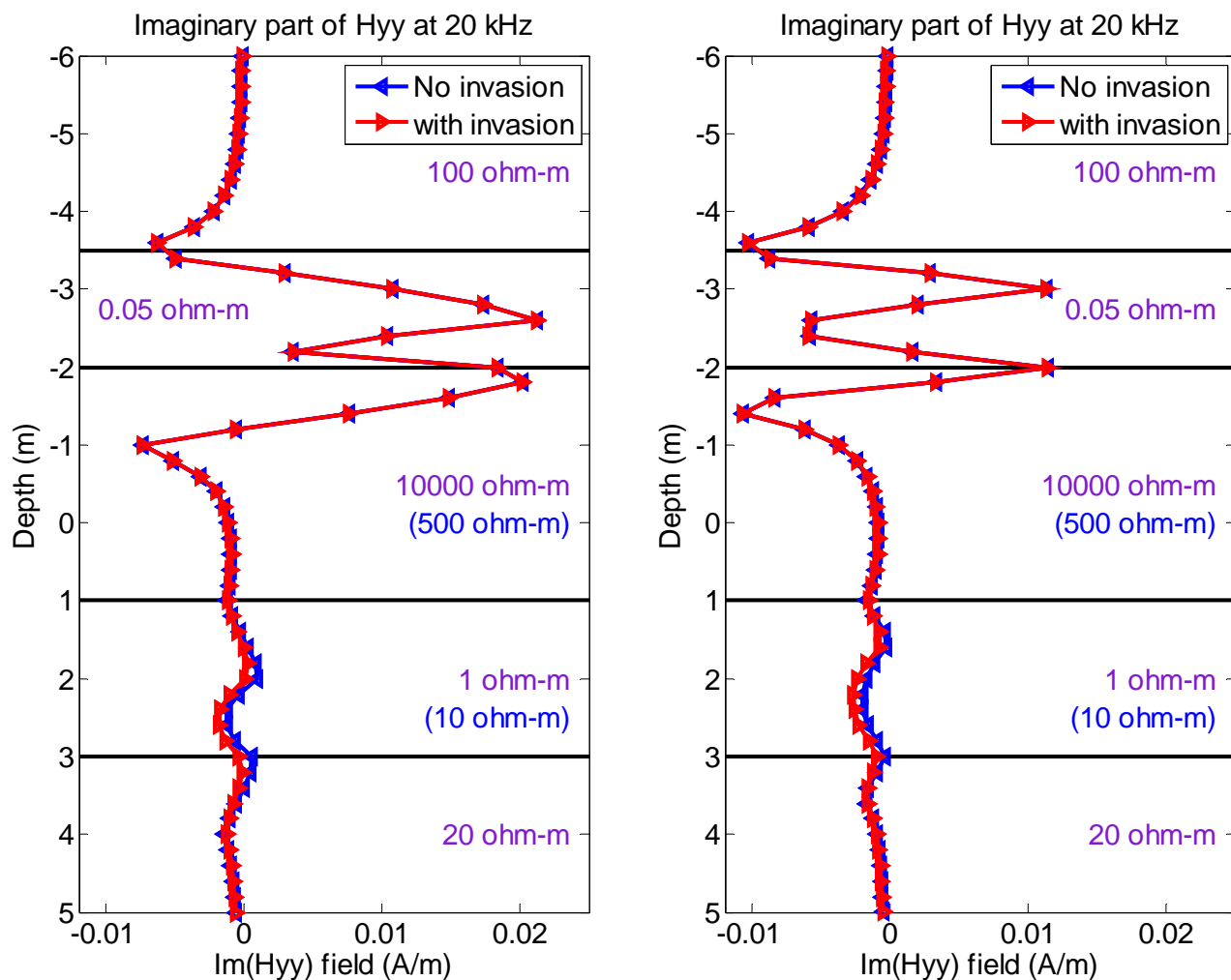


60 degrees

Shallow invasion
with $R = 0.1$ m



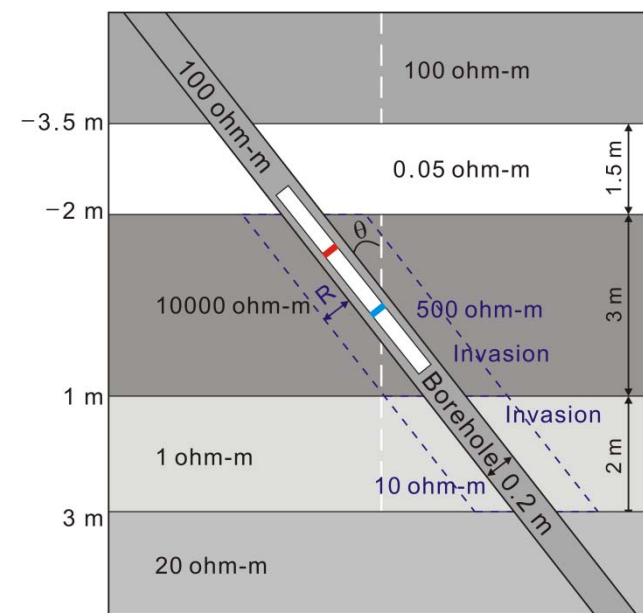
H_{yy} in Deviated Wells with Invasion (Im.)



vertical

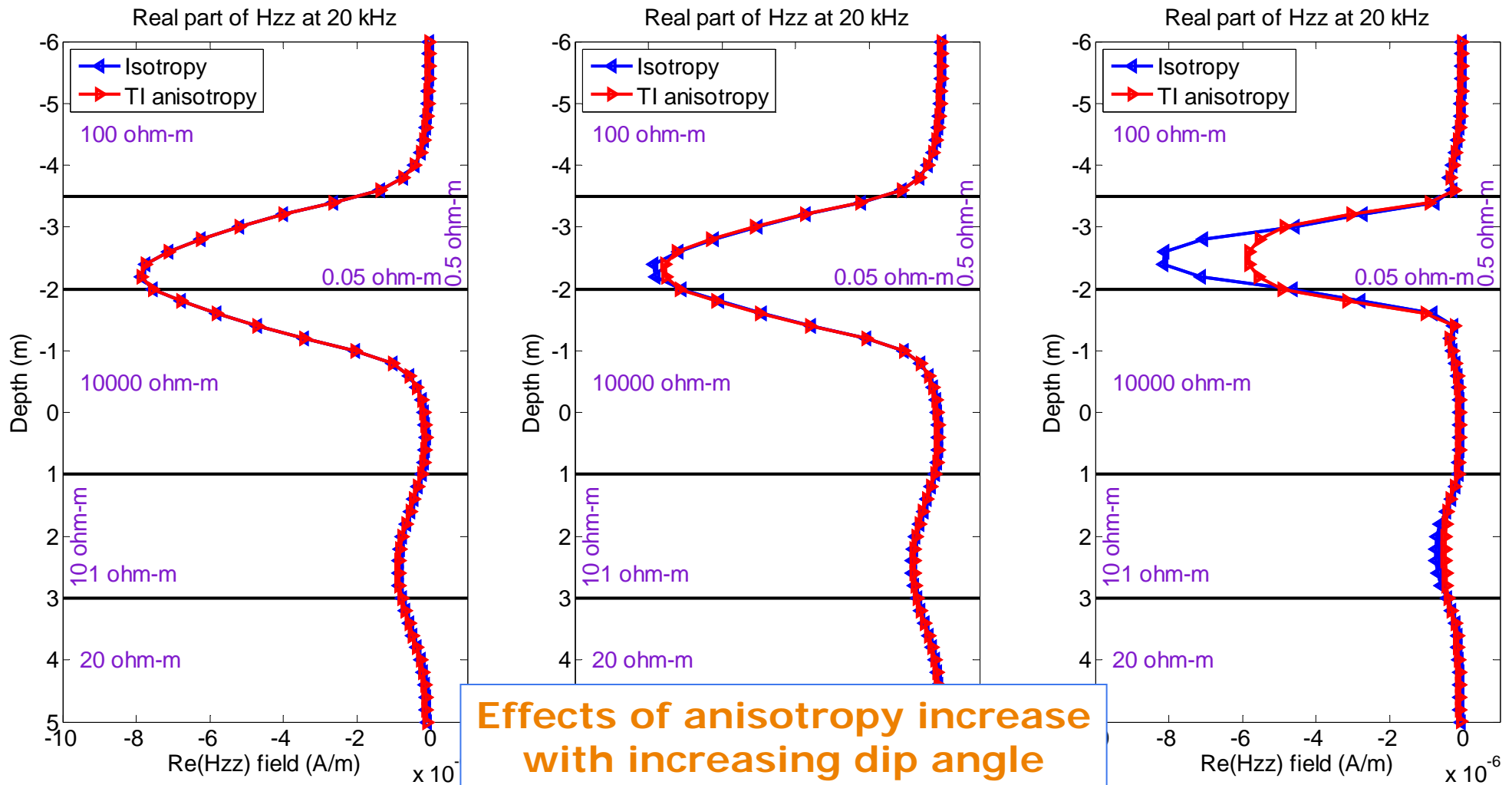
60 degrees

Shallow invasion
with $R = 0.1$ m



Small effects of invasion

H_{ZZ} in Deviated Wells with Anisotropy (Re.)

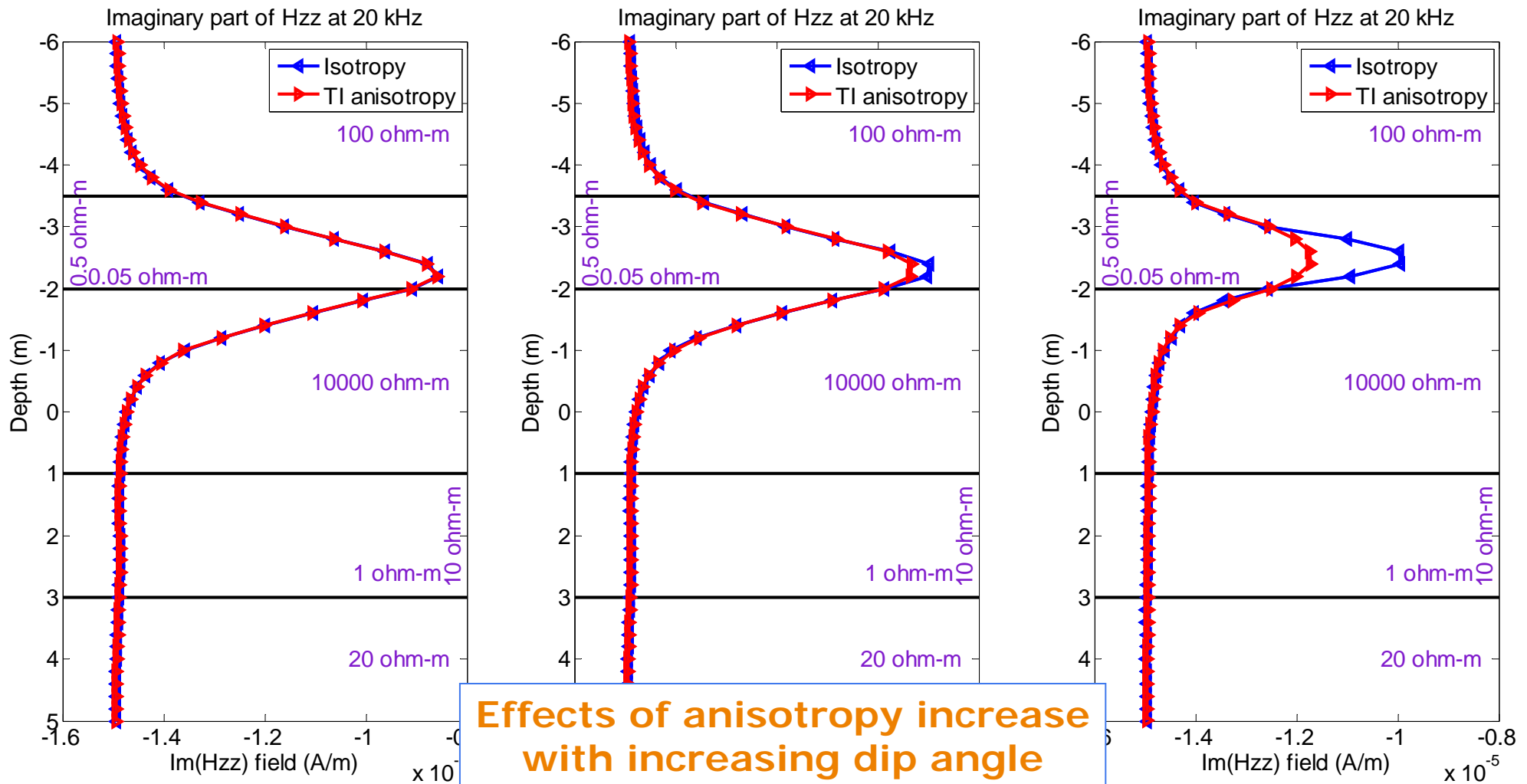


vertical

30 degrees

60 degrees

H_{ZZ} in Deviated Wells with Anisotropy (Im.)

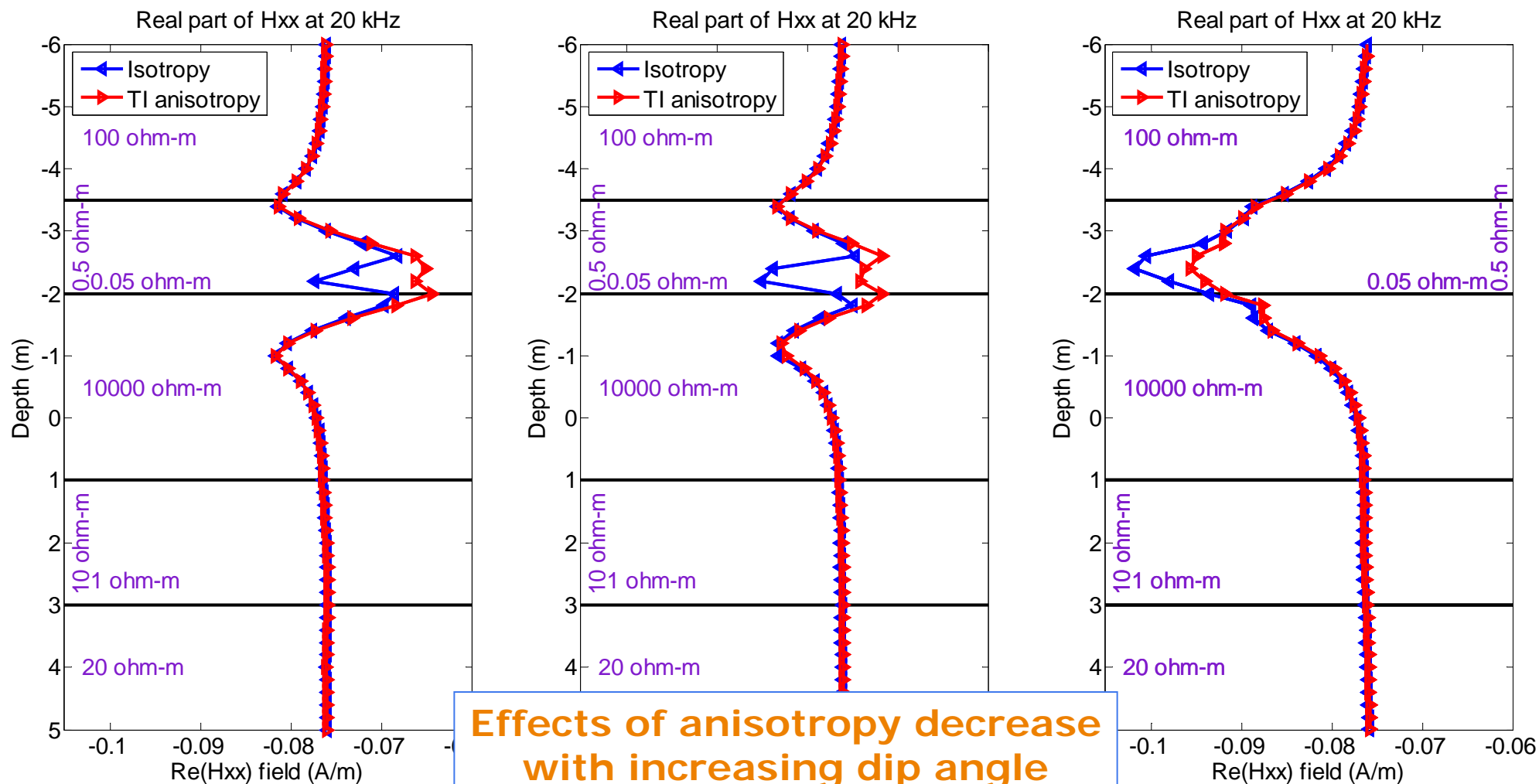


vertical

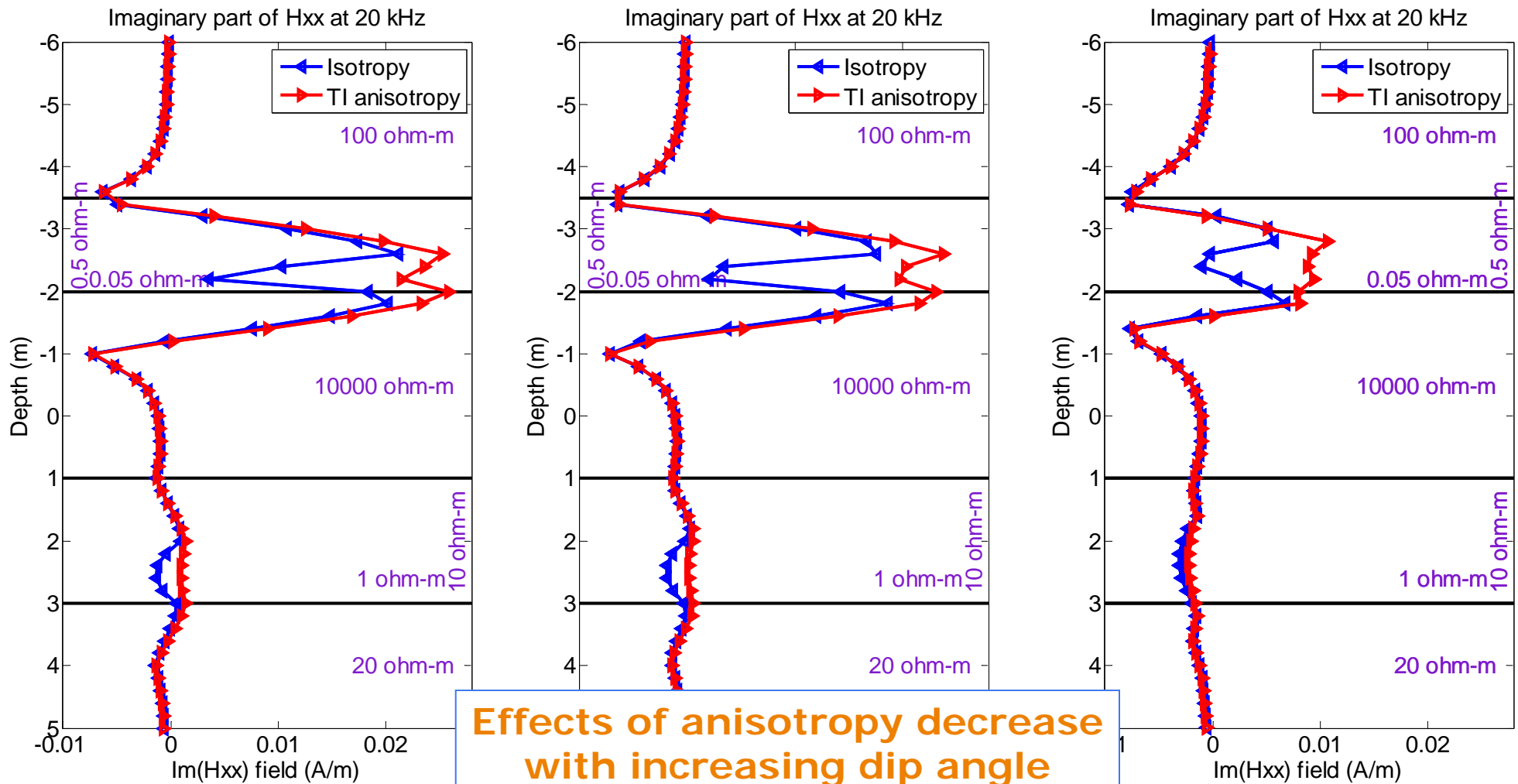
30 degrees

60 degrees

H_{xx} in Deviated Wells with Anisotropy (Re.)



H_{xx} in Deviated Wells with Anisotropy (Im.)

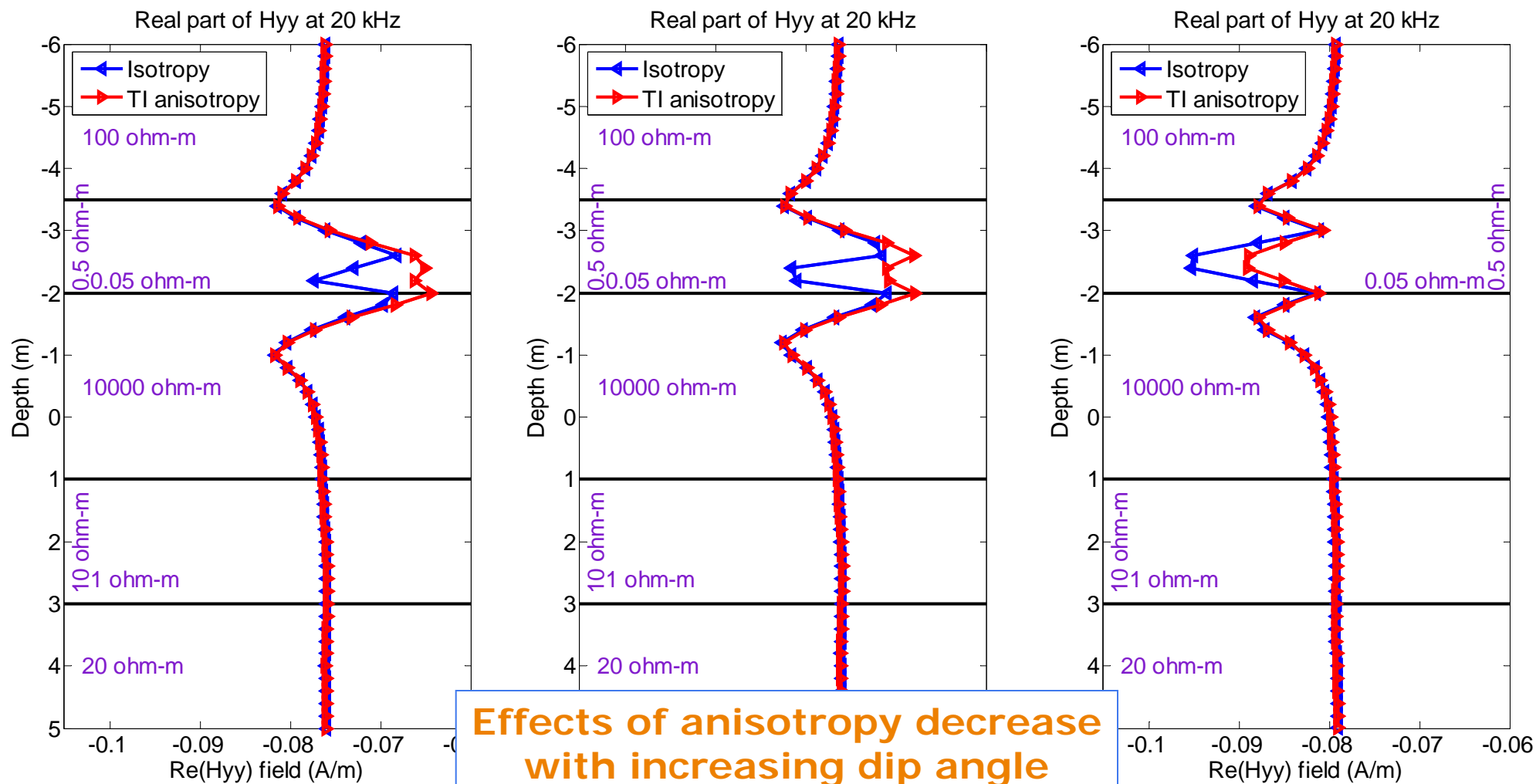


vertical

30 degrees

60 degrees

H_{yy} in Deviated Wells with Anisotropy (Re.)

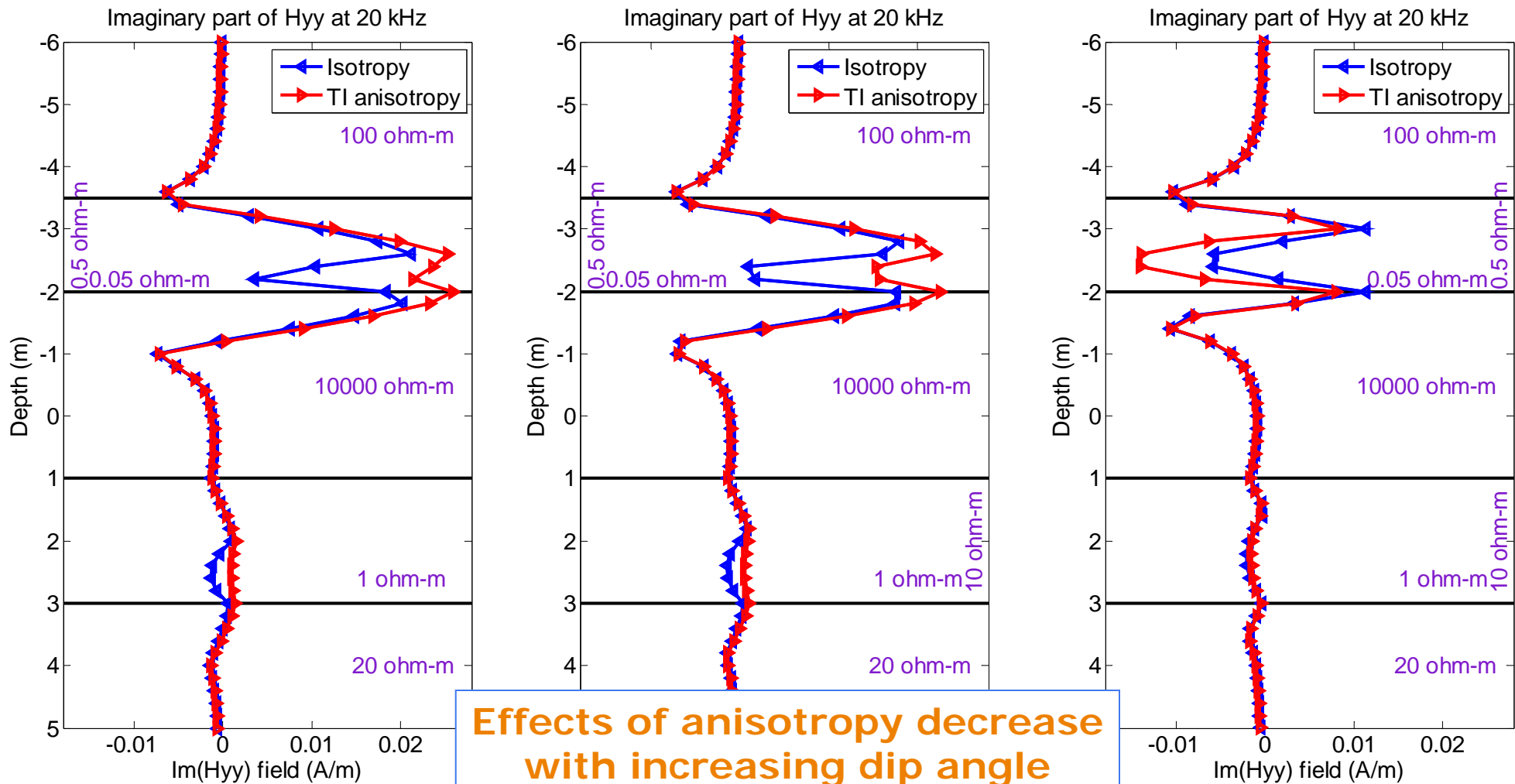


vertical

30 degrees

60 degrees

H_{yy} in Deviated Wells with Anisotropy (Im.)

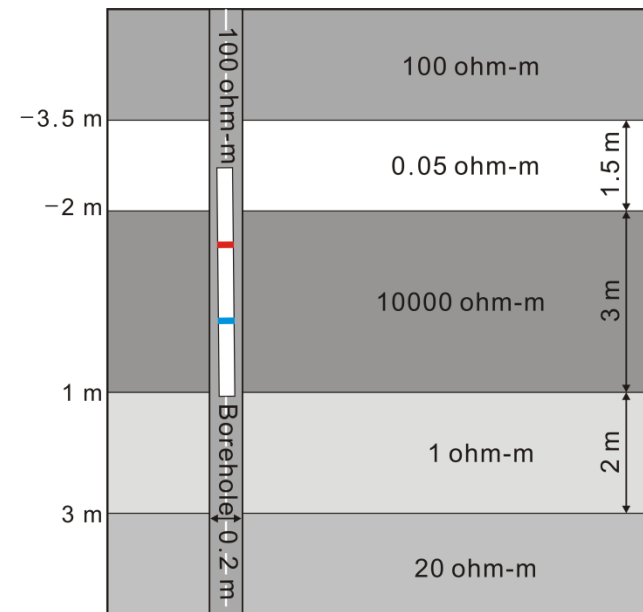
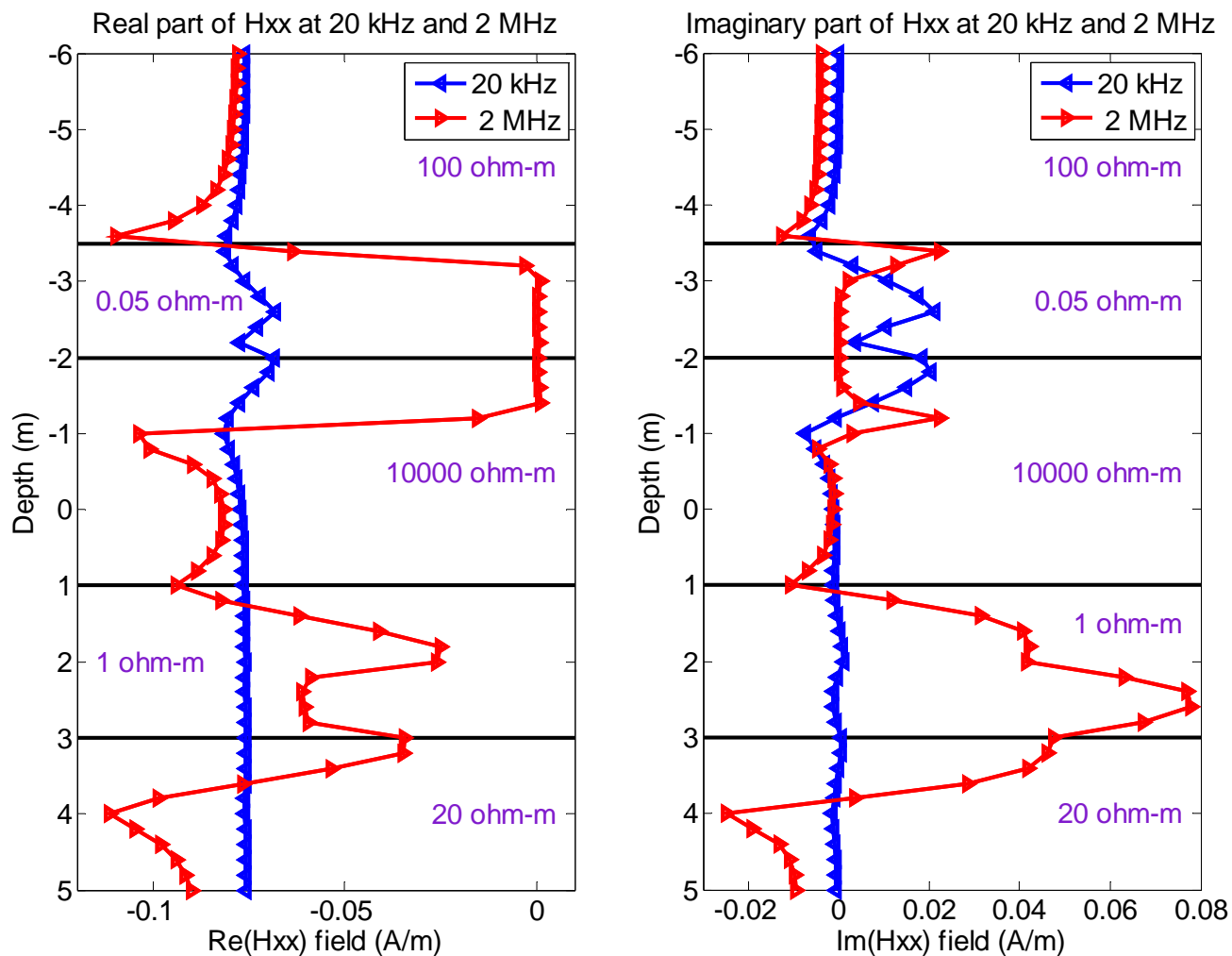


vertical

30 degrees

60 degrees

H_{xx} at 20 KHz and 2 MHz in Vertical Well



Larger variations at 2 MHz than at 20 kHz



Conclusions

- We successfully simulated 3D tri-axial induction measurements by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a 2D high-order, self-adaptive *hp* finite-element method.
- Dip angle effects on tri-axial tools are larger than on more traditional induction logging instruments.
- Anisotropy effects on H_{xx} and H_{yy} decrease with increasing dip angle, while those on H_{zz} increase.
- H_{xx} at 20 kHz exhibits smaller variations than at 2 MHz.



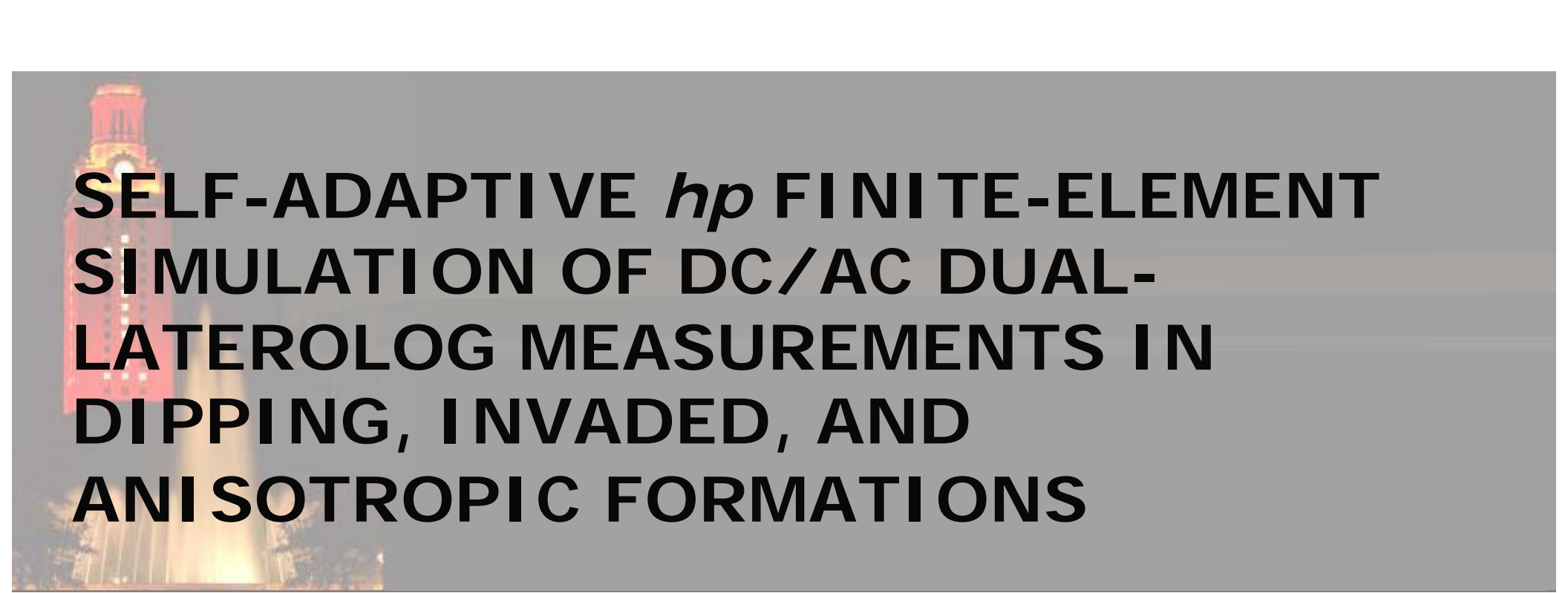
Acknowledgements

Sponsors of UT Austin's consortium on Formation Evaluation:



INSTITUTO MEXICANO DEL PETRÓLEO





SELF-ADAPTIVE *hp* FINITE-ELEMENT SIMULATION OF DC/AC DUAL- LATEROLOG MEASUREMENTS IN DIPPING, INVASED, AND ANISOTROPIC FORMATIONS

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V. M. Calo, M. Paszynski, and P. J. Matuszyk

Presentation at Korean Institute of Geoscience and
Minerals (KIGAM), September 8, 2008.



Overview

- 1. Main Lines of Research and Applications**
 - Previous work
 - Main features of our technology
- 2. Application 1: Tri-Axial Induction Instruments (M. J. Nam)**
- 3. Application 2: Dual-Laterolog Instruments (M. J. Nam)**
4. Multi-Physics Inversion (D. Pardo)
5. Sonic Instruments (L. Demkowicz)

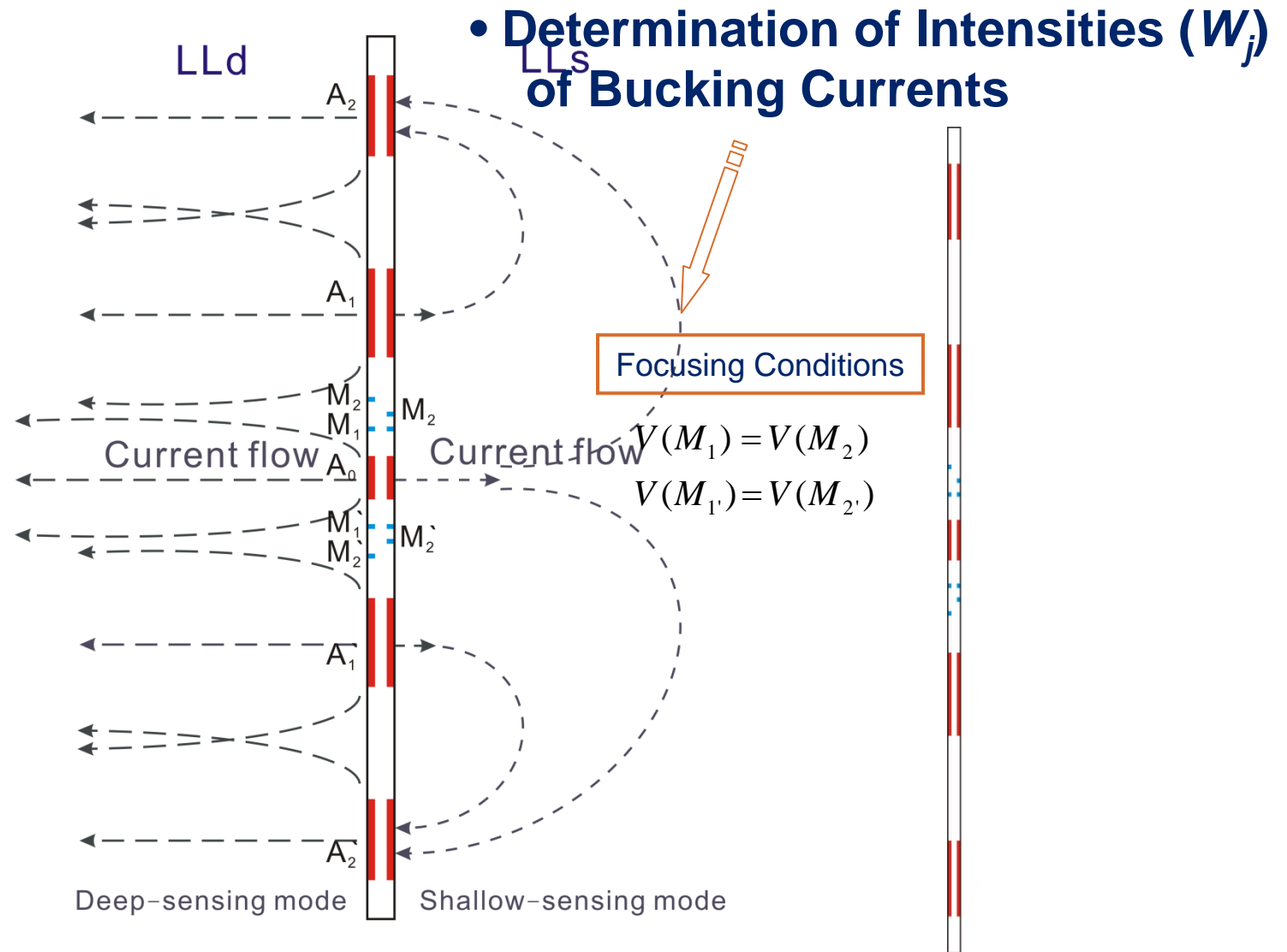


Outline

- **Introduction to Dual Laterolog**
- **Previous Work**
- **Method**
- **Numerical Results:**
 - **Groningen Effects on AC DLL**
 - **Dipping, Invaded, Anisotropic Formations (DC)**
 - **Eccentricity (DC)**
- **Conclusions**

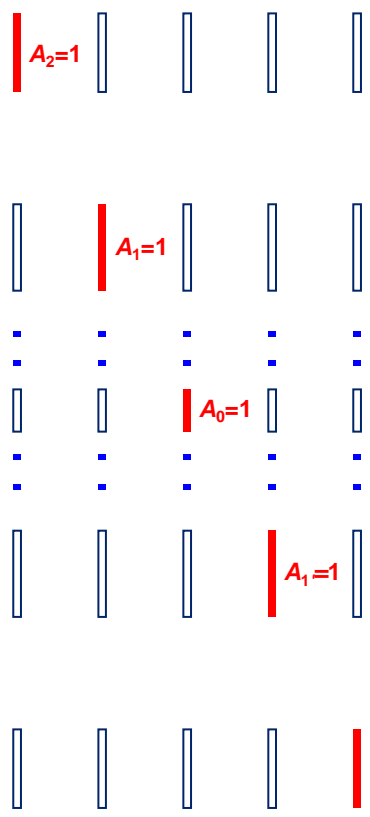


Dual Laterolog (DLL)



Summary from Last Year

Post-Processing Method



(1) Focusing conditions

$$V(M_1) = V(M_2)$$

$$V(M_{1'}) = V(M_{2'})$$

(2) Relationships between W_j

$$W_2 = (W_1 + c), \quad W_{2'} = (W_{1'} + c) \text{ for LLd}$$

$$W_2 = -(W_1 + c), \quad W_{2'} = -(W_{1'} + c) \text{ for LLs}$$

with $c = 0.5$

W_j for LLd: < from (1) and (2) with the LLd relationship of (3) >

$$\begin{bmatrix} V_{1,2} + V_{1,1} - V_{2,2} - V_{2,1} & V_{1,1'} + V_{1,2'} - V_{2,1'} - V_{2,2'} \\ V_{2,2} + V_{2,1} - V_{1,2} - V_{1,1} & V_{2,1'} + V_{2,2'} - V_{1,1'} - V_{1,2'} \end{bmatrix} \begin{bmatrix} W_1 \\ W_{1'} \end{bmatrix} = \begin{bmatrix} V_{2,0} - V_{1,0} + c(V_{2,2} + V_{2,2'} - V_{1,2} - V_{1,2'}) \\ V_{1,0} - V_{2,0} + c(V_{1,2} + V_{1,2'} - V_{2,2} - V_{2,2'}) \end{bmatrix}$$

W_j for LLs: < from (1) and (2) with the LLs relationship of (3) >

$$\begin{bmatrix} V_{2,2} + V_{1,1} - V_{2,1} - V_{1,2} & V_{2,2'} + V_{1,1'} - V_{1,2'} - V_{2,1'} \\ V_{1,2} + V_{2,1} - V_{1,1} - V_{2,2} & V_{1,2'} + V_{2,1'} - V_{2,2'} - V_{1,1'} \end{bmatrix} \begin{bmatrix} W_1 \\ W_{1'} \end{bmatrix} = \begin{bmatrix} V_{2,0} - V_{1,0} + c(V_{2,2} + V_{2,2'} - V_{1,2} - V_{1,2'}) \\ V_{1,0} - V_{2,0} + c(V_{1,2} + V_{1,2'} - V_{2,2} - V_{2,2'}) \end{bmatrix}$$

Total potential on M_i

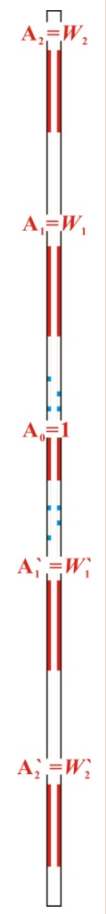
→ Superposition principle

$$V(M_2) = W_2 V_{2,2} + W_1 V_{2,1} + V_{2,0} + W_1' V_{2,1'} + W_2' V_{2,2'}$$

$$V(M_1) = W_2 V_{1,2} + W_1 V_{1,1} + V_{1,0} + W_1' V_{1,1'} + W_2' V_{1,2'}$$

$$V(M_{1'}) = W_2 V_{1',2} + W_1 V_{1',1} + V_{1',0} + W_1' V_{1',1'} + W_2' V_{1',2'}$$

$$V(M_{2'}) = W_2 V_{2',2} + W_1 V_{2',1} + V_{2',0} + W_1' V_{2',1'} + W_2' V_{2',2'}$$

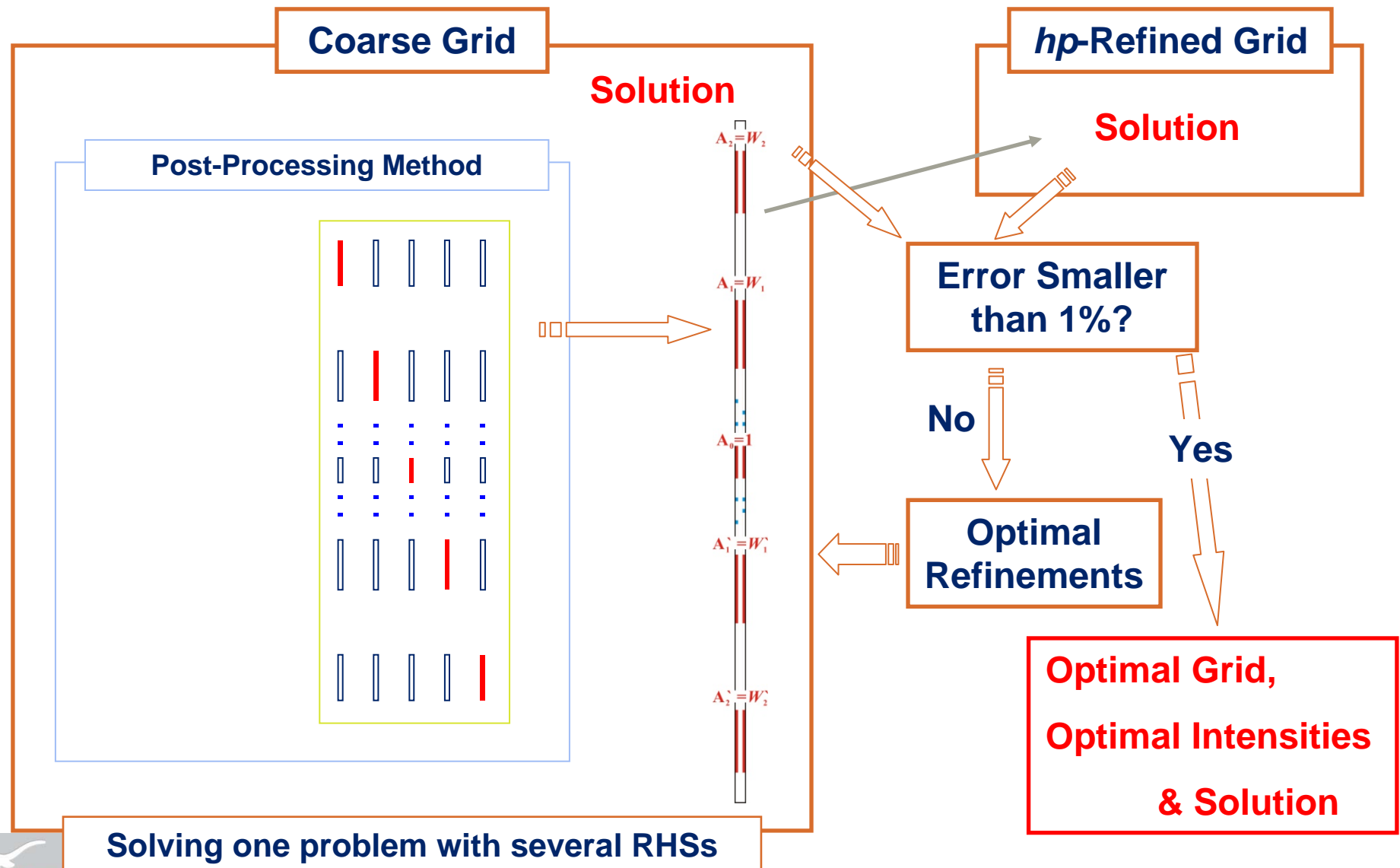


One problem with several RHSs



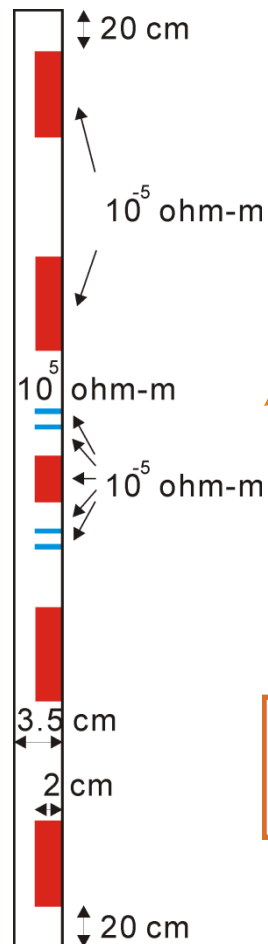
Summary from Last Year

Embedded
Post-Processing Method



Summary from Last Year

What we modeled in simulating the DLL tool



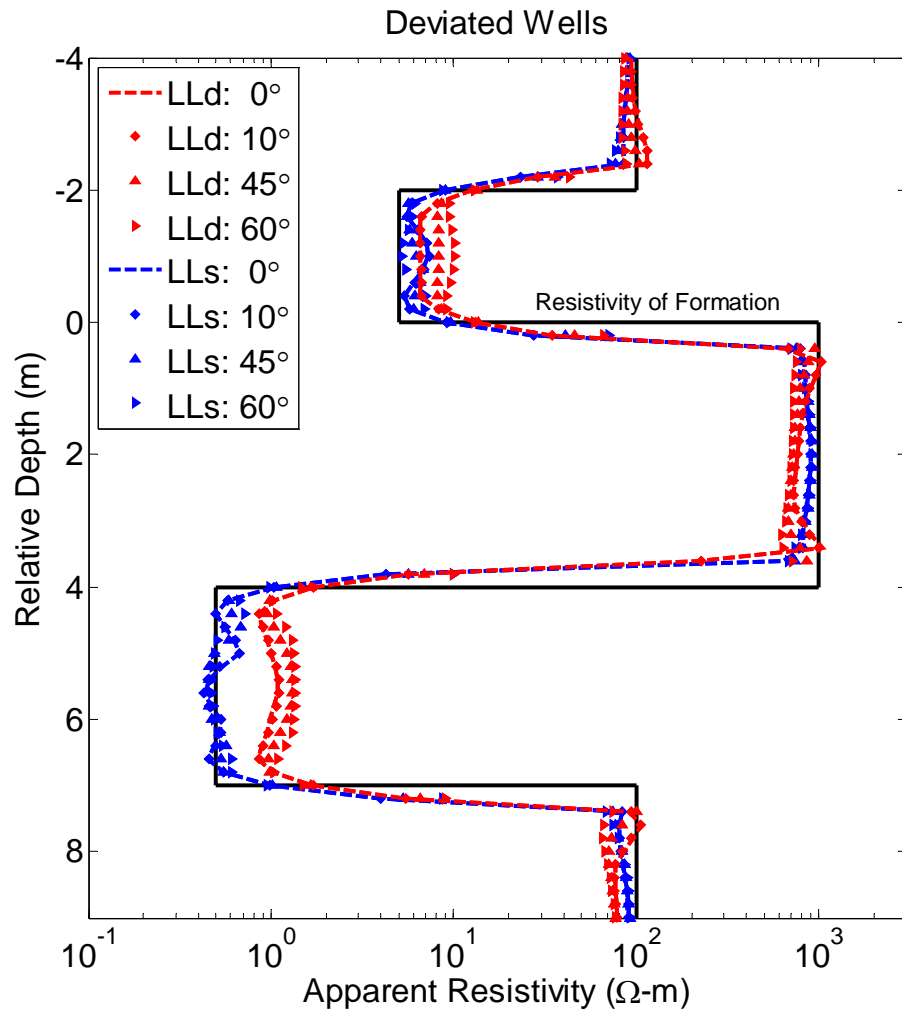
The resistivities and radial lengths of electrode and mandrel.

The vertical dimensions and locations of each electrode:
We followed the vertical tool configuration of a commercial tool

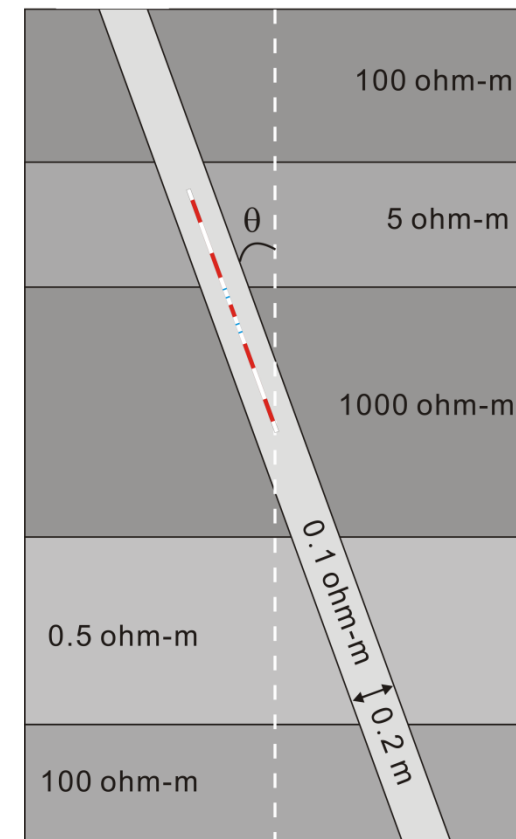


Summary from Last Year

Deviated Wells (0, 10, 45, and 60 degrees) at DC

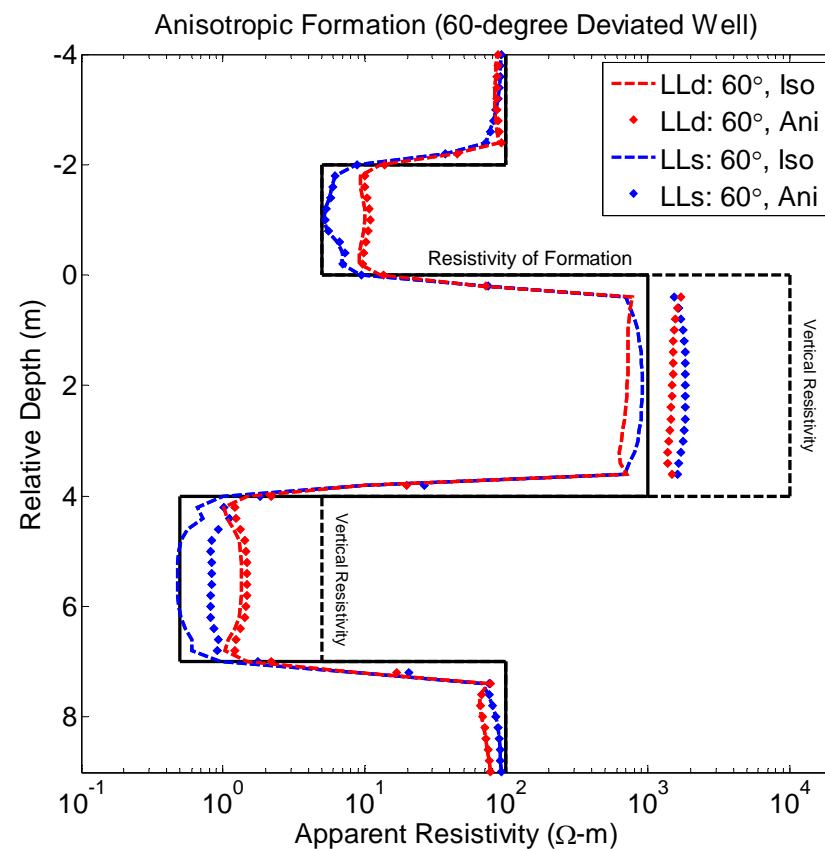
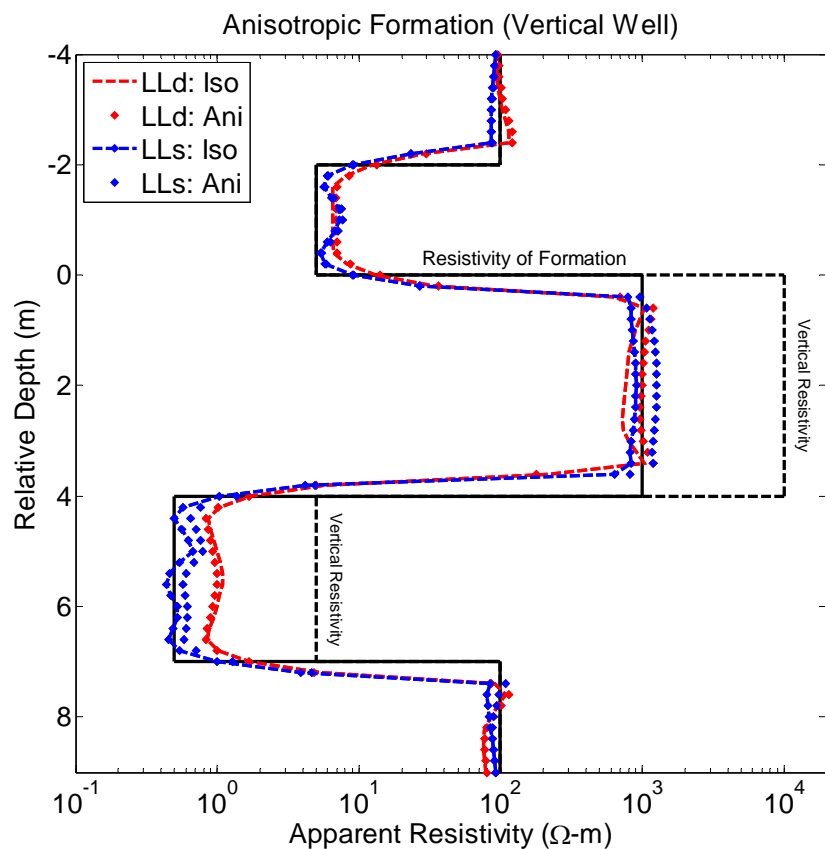


Effects of dip angle: Thin layer \uparrow



Summary from Last Year

Anisotropic Formation (60- and 0-degree Deviated Wells) at DC



Effects of anisotropy increase with increase of dip angle



Method for Simulating AC DLL Measurements

Combination of:

1. A Self-Adaptive Goal-Oriented *hp*-FEM
for AC problems
2. Embedded Post-Processing Method (EPPM)
3. Parallel Implementation



Simulating AC DLL Measurements 1

Main challenges when simulating AC DLL measurements 1:

Introducing in the AC formulation a source equivalent to $\nabla \cdot \mathbf{J}$

To avoid simulating the inner wiring system!!



Simulating AC DLL Measurements 1

Main challenges when simulating AC DLL measurements 1:

Governing equation

DC $\nabla \cdot (\sigma \nabla \cdot u) = \nabla \cdot \mathbf{J}^{imp}$

AC $\begin{cases} \nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} + \mathbf{J} \\ \nabla \times \mathbf{E} = -j\omega\mathbf{H} \end{cases}$

Variational formulation

$$\langle \nabla v, \sigma \nabla u \rangle_{L^2(\Omega)} = \langle v, \nabla \cdot \mathbf{J}^{imp} \rangle_{L^2(\Omega)} + \langle v, g \rangle_{L^2(\Gamma_N)} \quad \forall v \in H_D^1(\Omega)$$

$$\langle \nabla \times \bar{\mathbf{F}}, \mu^{-1} \nabla \times \mathbf{E} \rangle_{L^2(\Omega)} - \langle \bar{\mathbf{F}}, (\omega^2 \epsilon - j\omega\sigma) \mathbf{E} \rangle_{L^2(\Omega)} - \langle \bar{\mathbf{F}}, (\omega^2 \epsilon - j\omega\sigma) \nabla p \rangle =$$

$$- j\omega \langle \bar{\mathbf{F}}, \mathbf{J}^{imp} \rangle_{L^2(\Omega)} + j\omega \langle \bar{\mathbf{F}}_t, \mathbf{J}_S^{imp} \rangle_{L^2(\Gamma_H)} \quad \forall \mathbf{F} \in H_{\Gamma_E}(\text{curl}; \Omega)$$

Scalar potential eq. \rightarrow

$$- \langle \nabla \bar{q}, (\omega^2 \epsilon - j\omega\sigma) \mathbf{E} \rangle_{L^2(\Omega)} =$$

$$- j\omega \langle \nabla \bar{q}, \mathbf{J}^{imp} \rangle_{L^2(\Omega)} + j\omega \langle \nabla \bar{q}, \mathbf{J}_S^{imp} \rangle_{L^2(\Gamma_H)} \quad \forall q \in H_D^1$$

introducing $\nabla \cdot \mathbf{J}$

$$j\omega \langle \bar{q}, \nabla \cdot \mathbf{J}^{imp} \rangle_{L^2(\Omega)}$$

$$\therefore - \langle \nabla \bar{q}, (\omega^2 \epsilon - j\omega\sigma) \mathbf{E} \rangle_{L^2(\Omega)} = j\omega \langle \bar{q}, \nabla \cdot \mathbf{J}^{imp} \rangle_{L^2(\Omega)} \quad \forall q \in H_D^1$$



Simulating AC DLL Measurements 1

Main challenges when simulating AC DLL measurements 1:

Governing equation

DC $\nabla \cdot (\sigma \nabla \cdot u) = \nabla \cdot \mathbf{J}^{imp}$

Variational formulation

$$\langle \nabla v, \sigma \nabla u \rangle_{L^2(\Omega)} = \langle v, \nabla \cdot \mathbf{J}^{imp} \rangle_{L^2(\Omega)} + \langle v, g \rangle_{L^2(\Gamma_N)} \quad \forall v \in H_D^1(\Omega)$$

AC

$$\begin{cases} \nabla \times \mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E} + \mathbf{J} \\ \nabla \times \mathbf{E} = -j\omega\mathbf{H} \end{cases}$$

Final AC variational formulations we use:

$$\begin{aligned} \langle \nabla \times \bar{\mathbf{F}}, \mu^{-1} \nabla \times \mathbf{E} \rangle_{L^2(\Omega)} - \langle \bar{\mathbf{F}}, (\omega^2 \varepsilon - j\omega\sigma) \mathbf{E} \rangle_{L^2(\Omega)} - \langle \bar{\mathbf{F}}, (\omega^2 \varepsilon - j\omega\sigma) \nabla p \rangle \\ = 0 \quad \forall \mathbf{F} \in H_{\Gamma_E}(\text{curl}; \Omega) \\ - \langle \nabla \bar{q}, (\omega^2 \varepsilon - j\omega\sigma) \mathbf{E} \rangle_{L^2(\Omega)} = j\omega \langle \bar{q}, \nabla \cdot \mathbf{J}^{imp} \rangle_{L^2(\Omega)} \quad \forall q \in H_D^1 \end{aligned}$$



Simulating AC DLL Measurements 2

Main challenges when simulating AC DLL measurements 2:

Simulation of current return at earth surface

1. No current return results in no Groningen effects.

(Numerical results will be shown)

2. We have to simulate the earth surface.

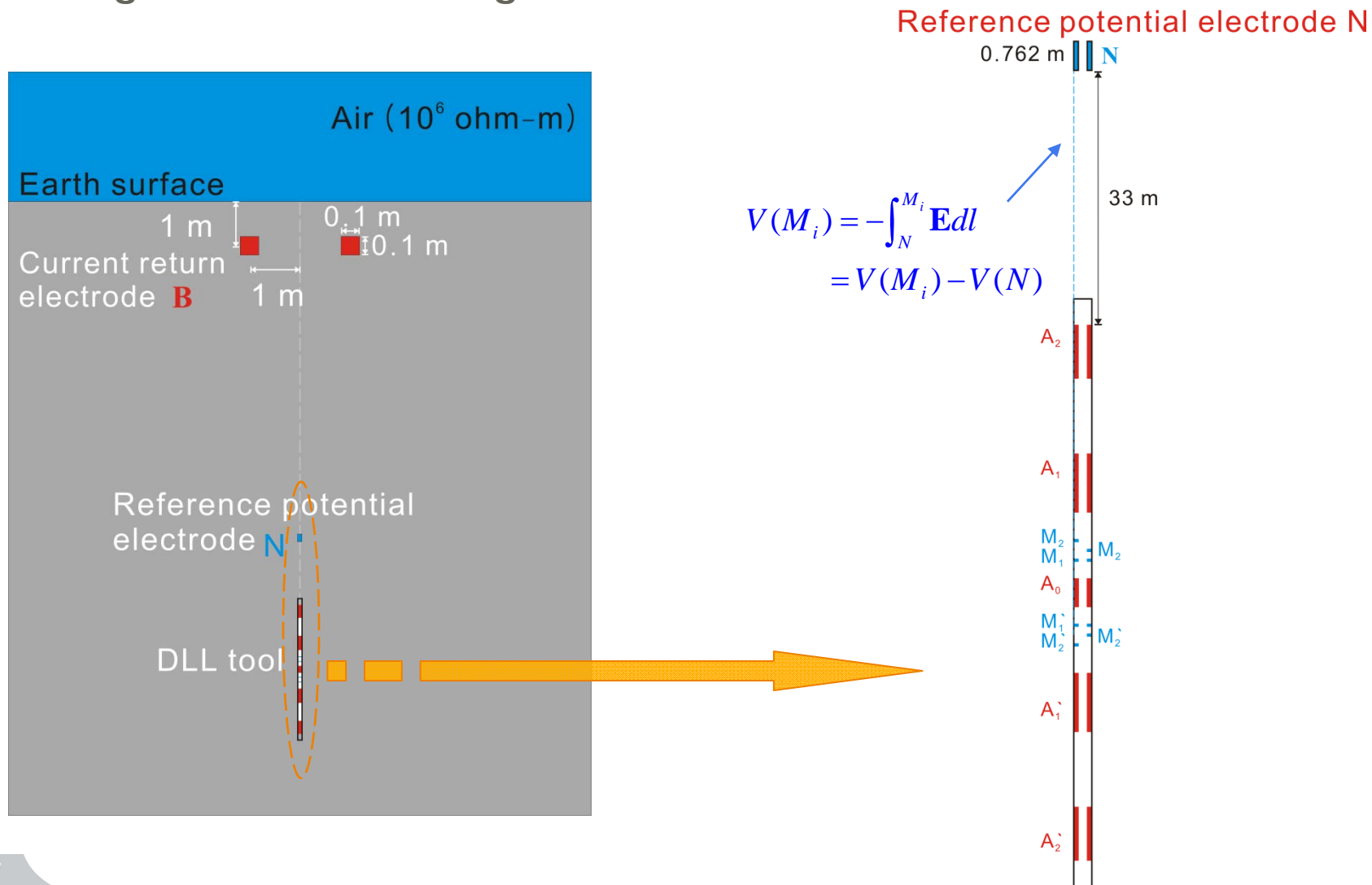
→ Our computing domain is larger than

2 km in the vertical direction.

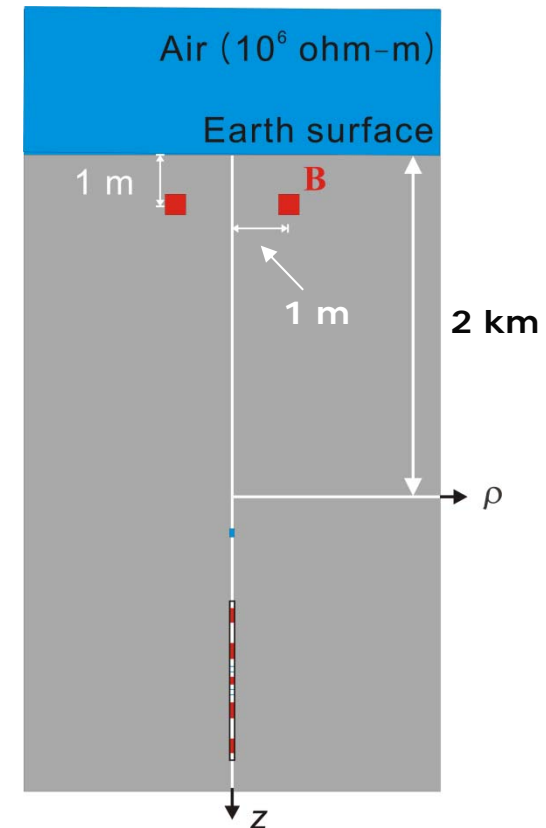
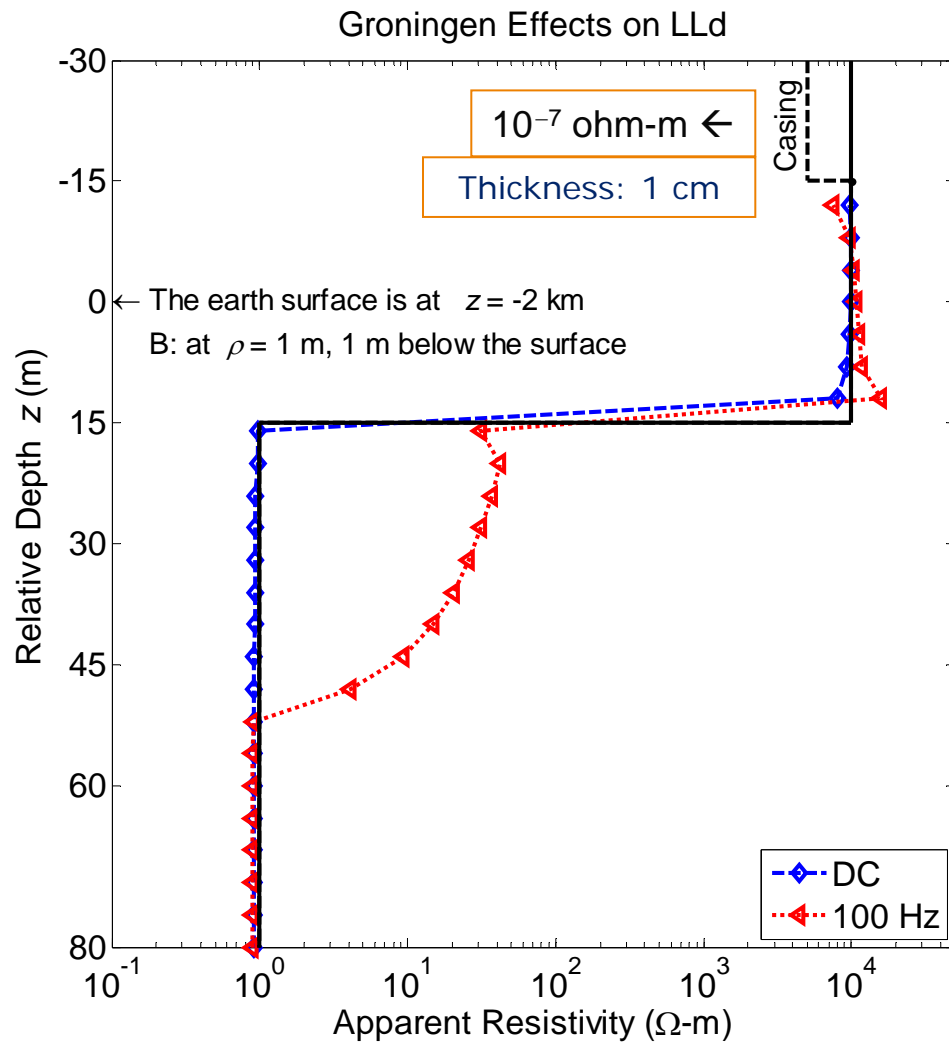


Simulating AC DLL Measurements 2

Main challenges when simulating AC DLL measurements 2:



Groningen Effects on LLd at DC and AC

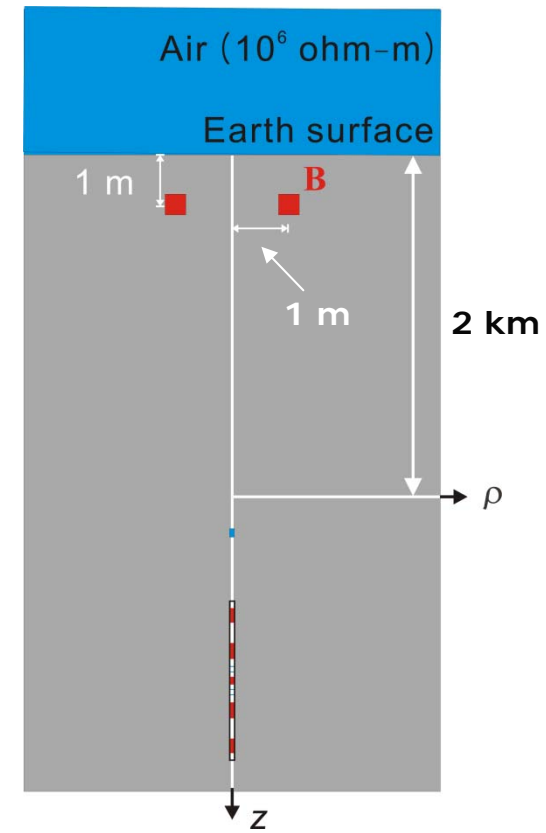
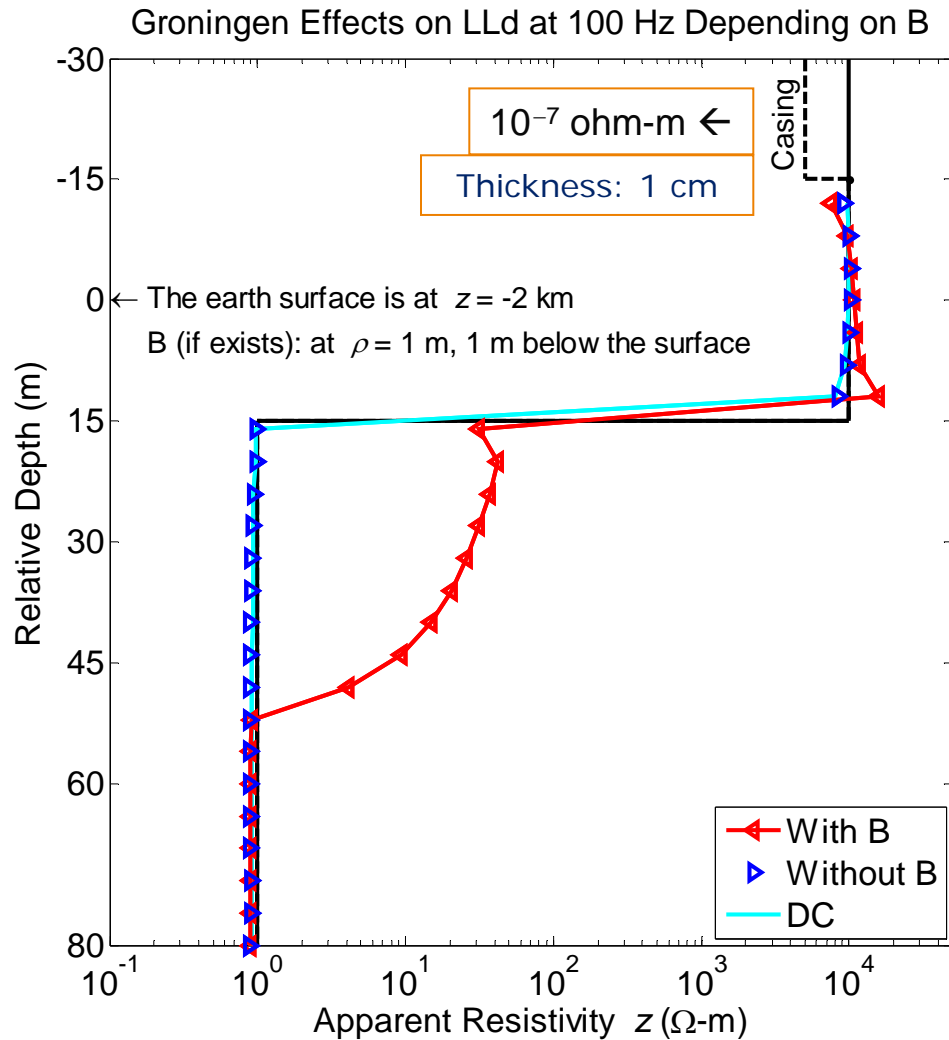


DC: No Groningen effects

AC: Groningen effects



Groningen Effects on LLd at 100 Hz (I)

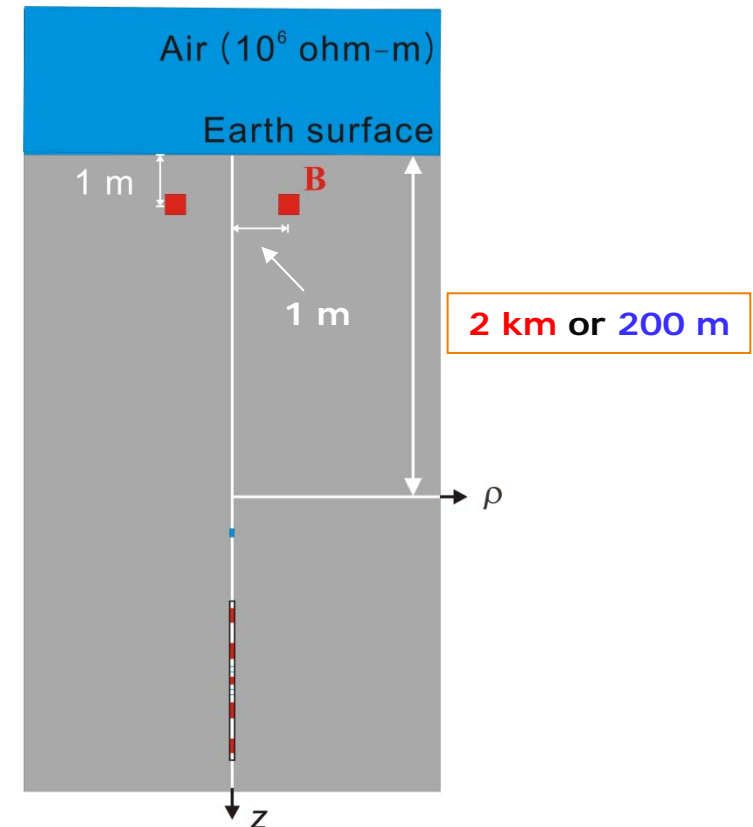
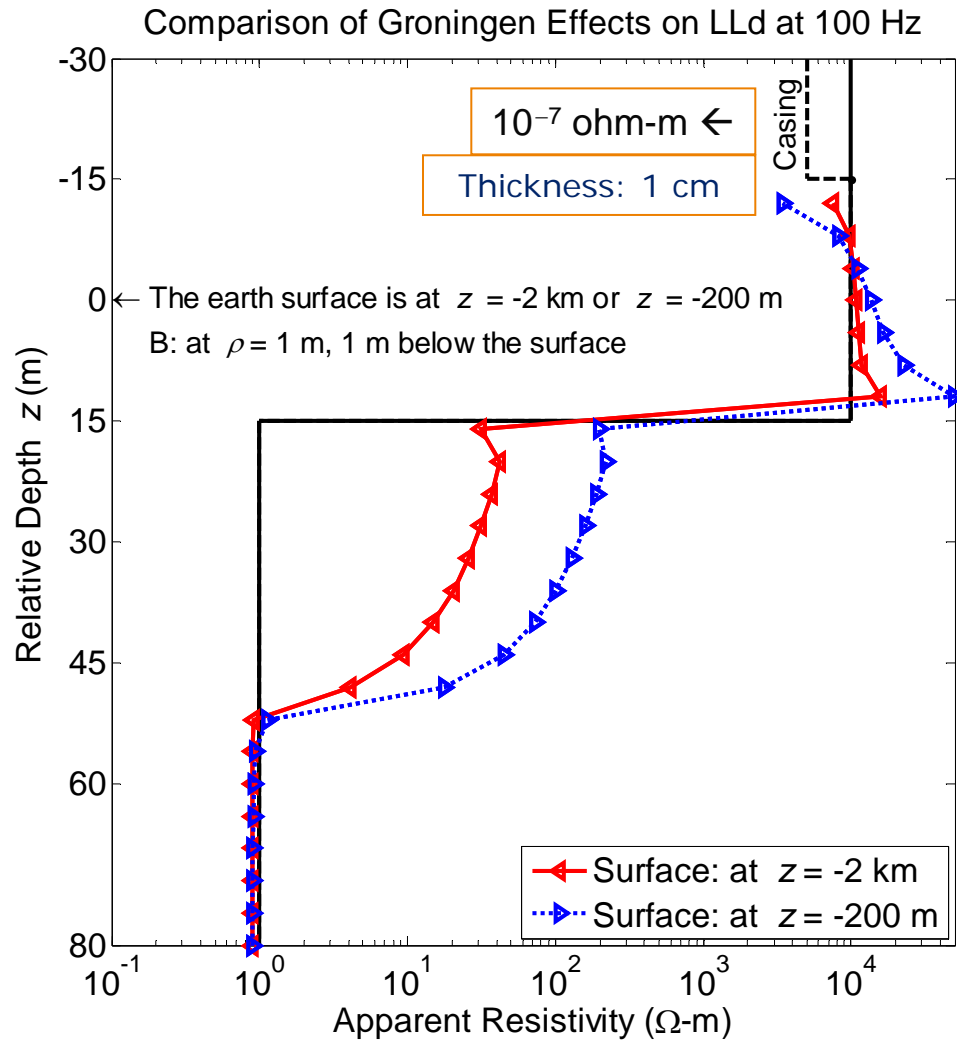


No B:

No Groningen effects



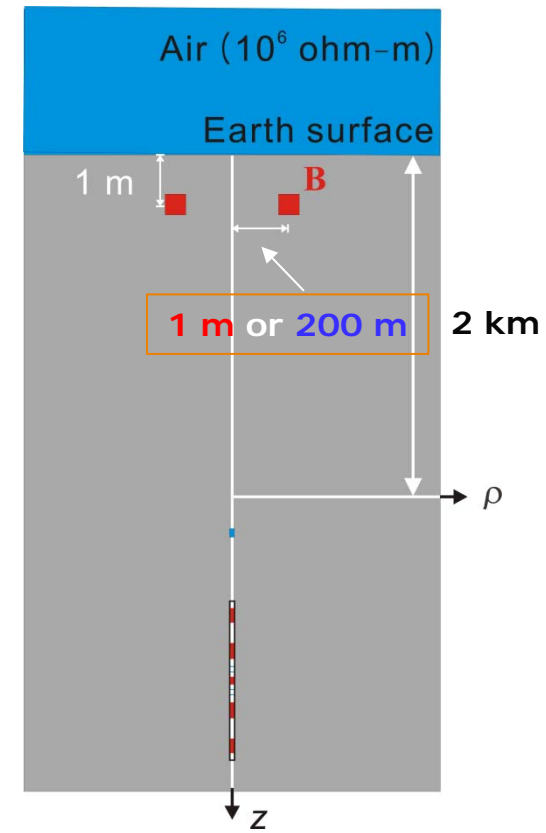
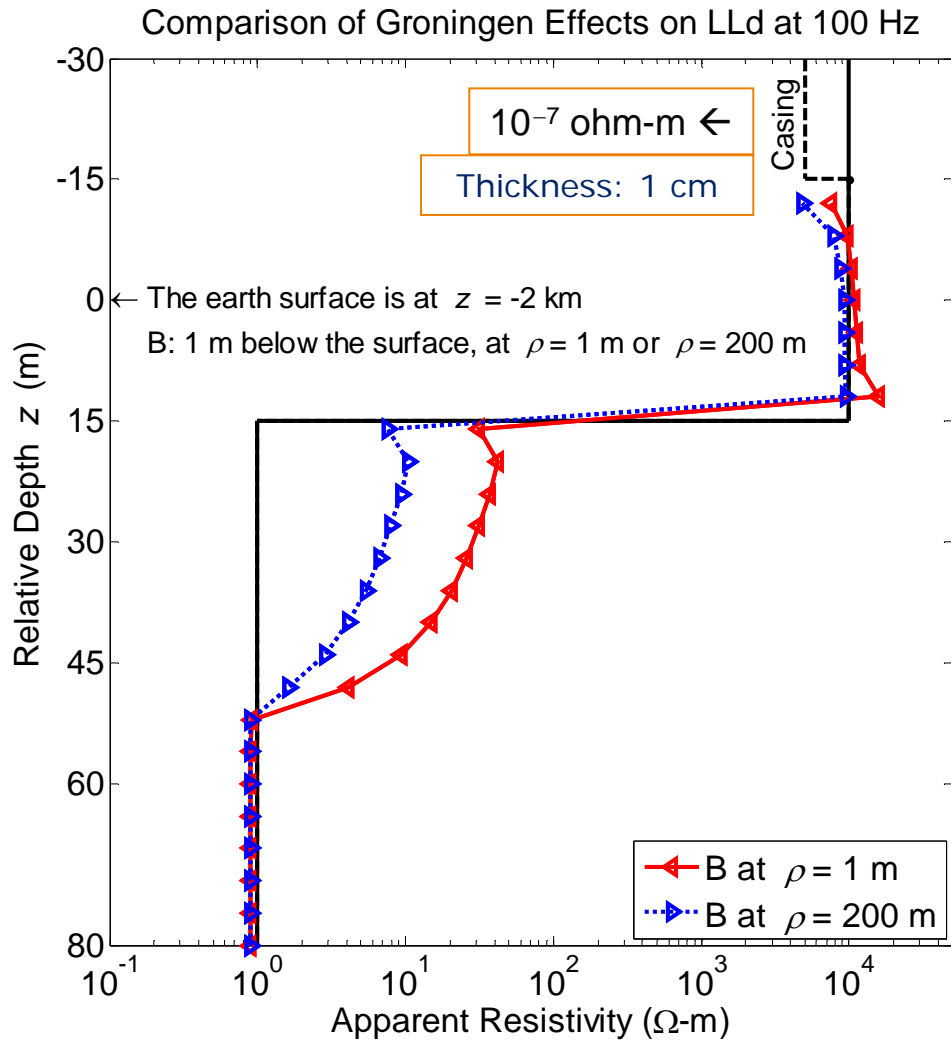
Groningen Effects on LLd at 100 Hz (II)



Further B in the z direction:
smaller Groningen effects



Groningen Effects on LLd at 100 Hz (III)

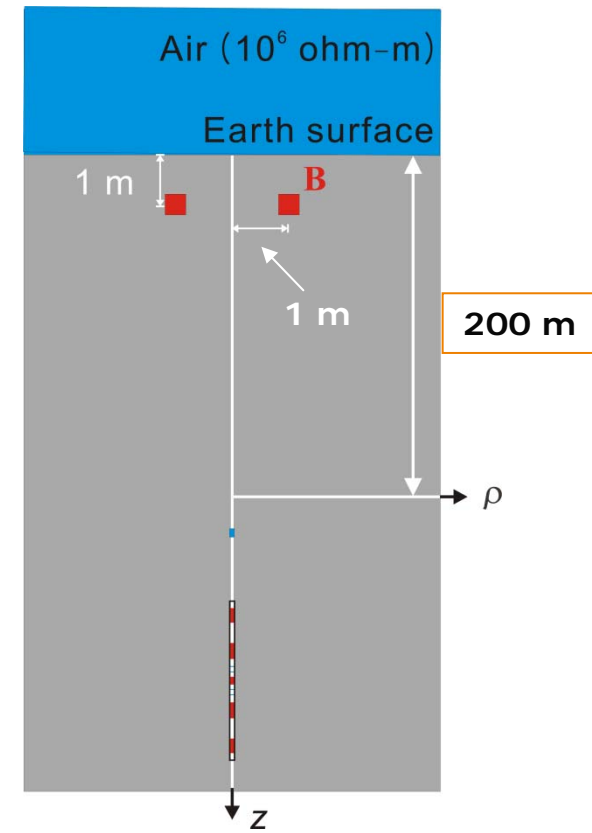
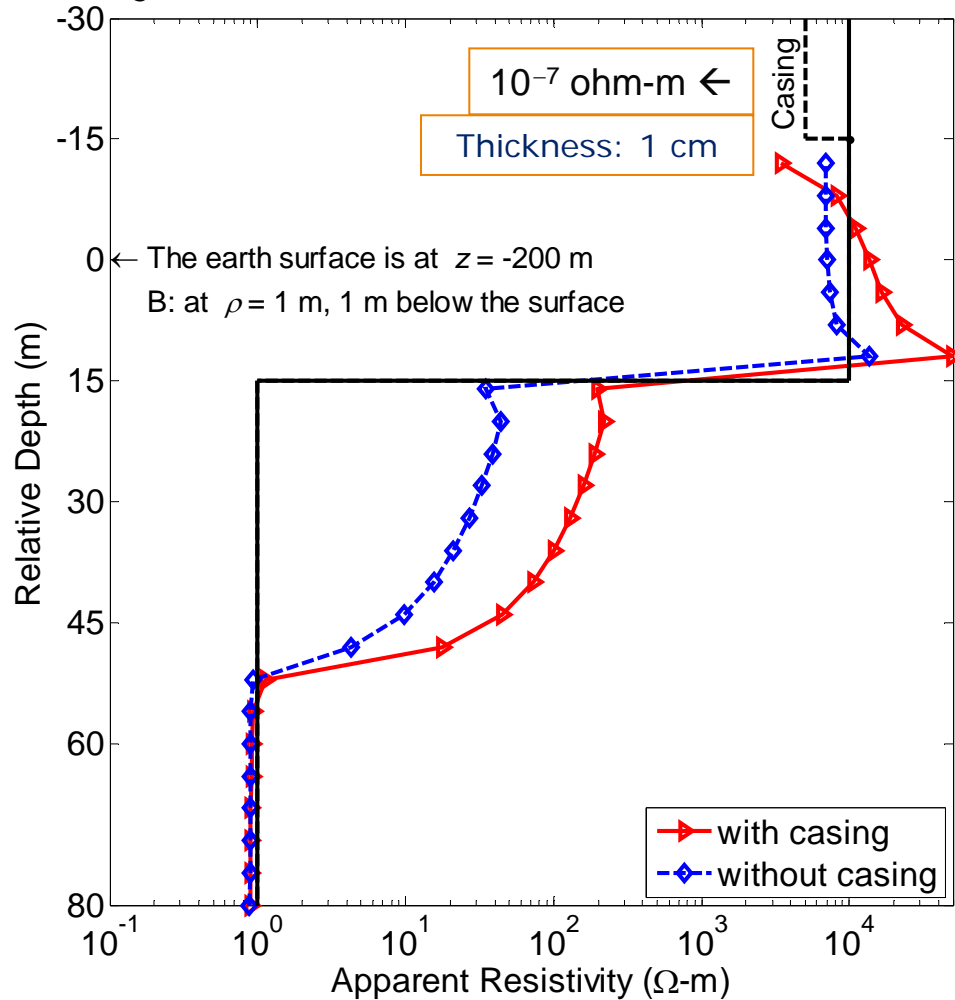


Further B in the ρ direction:
smaller Groningen effects



Groningen Effects on LLd at 100 Hz (IV)

Groningen Effects on LLd at 100 Hz with the Surface at $z = -200\text{m}$

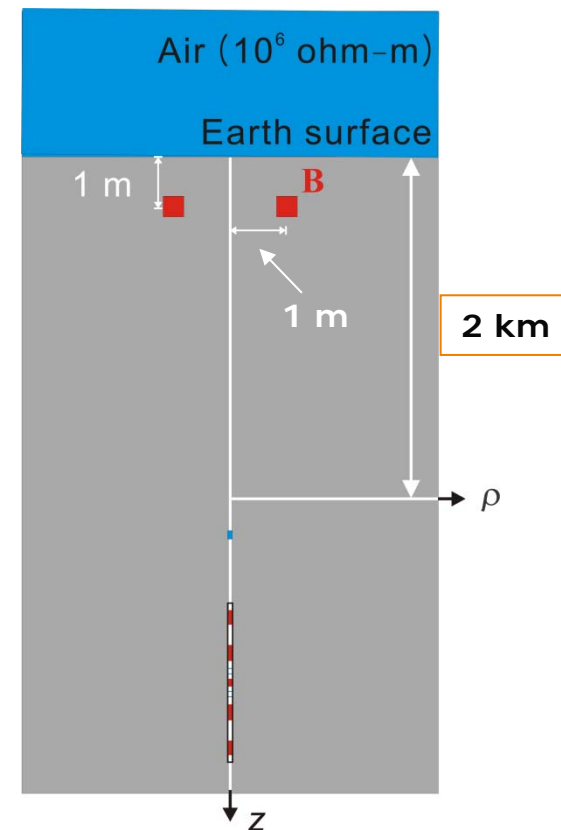
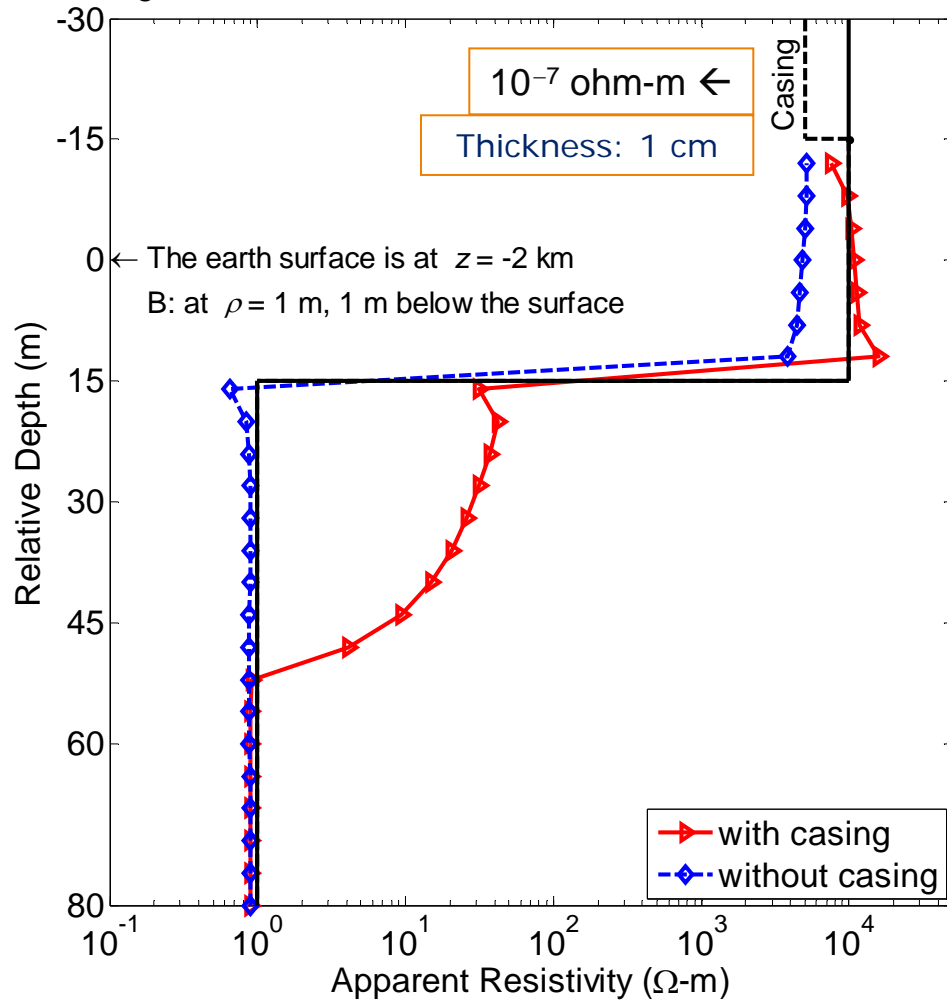


**No casing with B 200 m apart:
smaller Groningen effects**



Groningen Effects on LLd at 100 Hz (V)

Groningen Effects on LLd at 100 Hz with the Surface at $z = -2$ km

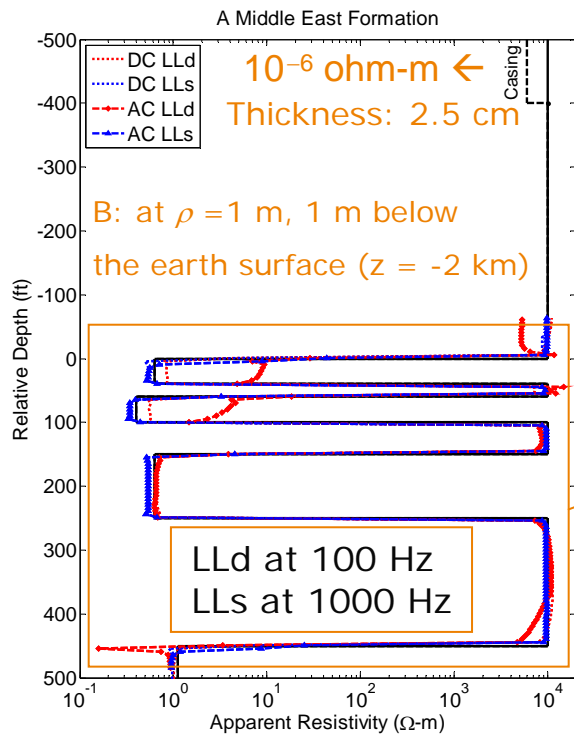


No casing with B 2 km apart:

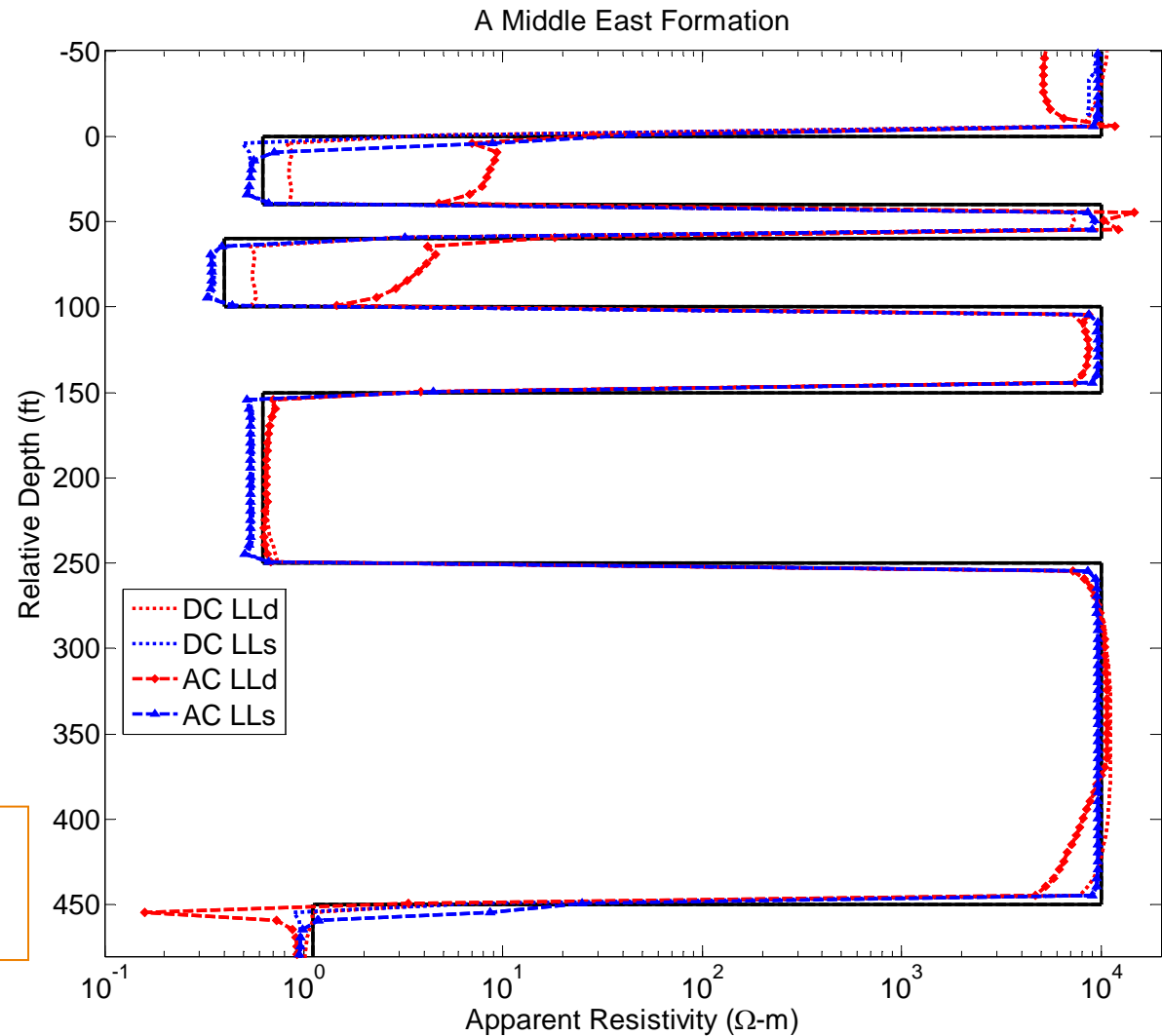
No Groningen effects



A Middle East Formation Model



Frequency effects
on AC DLL measurements

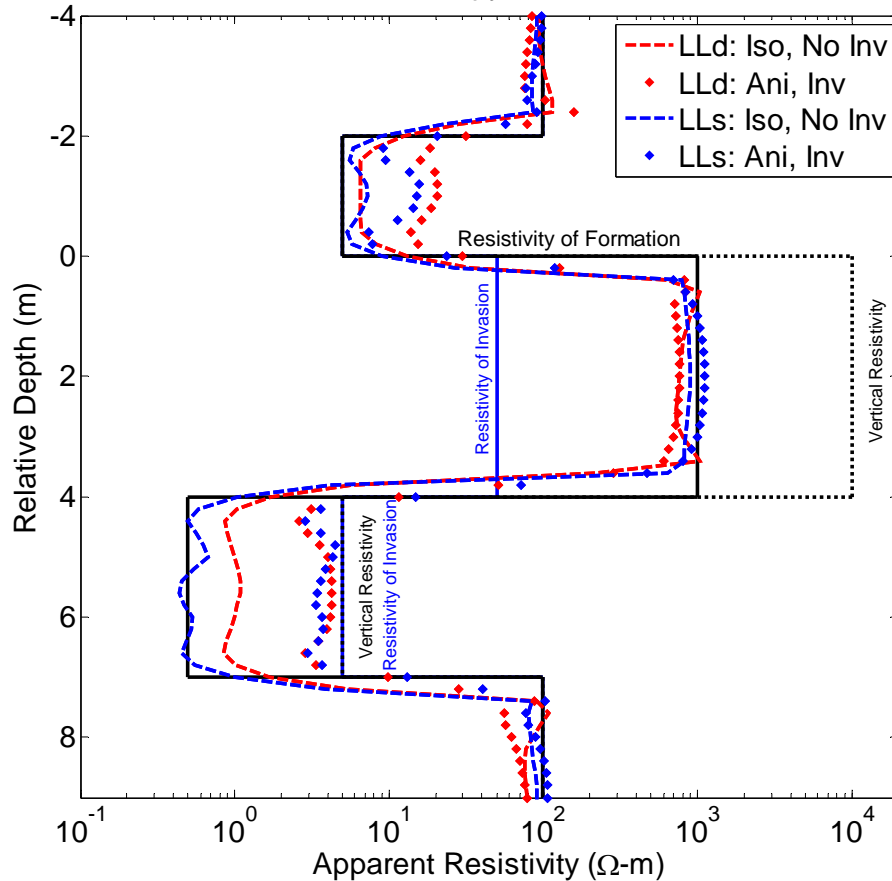


The resistivities of Layers: 10^4 , 0.625, 10^4 , 0.4, 10^4 , and 0.625 ohm-m (from top to bottom)



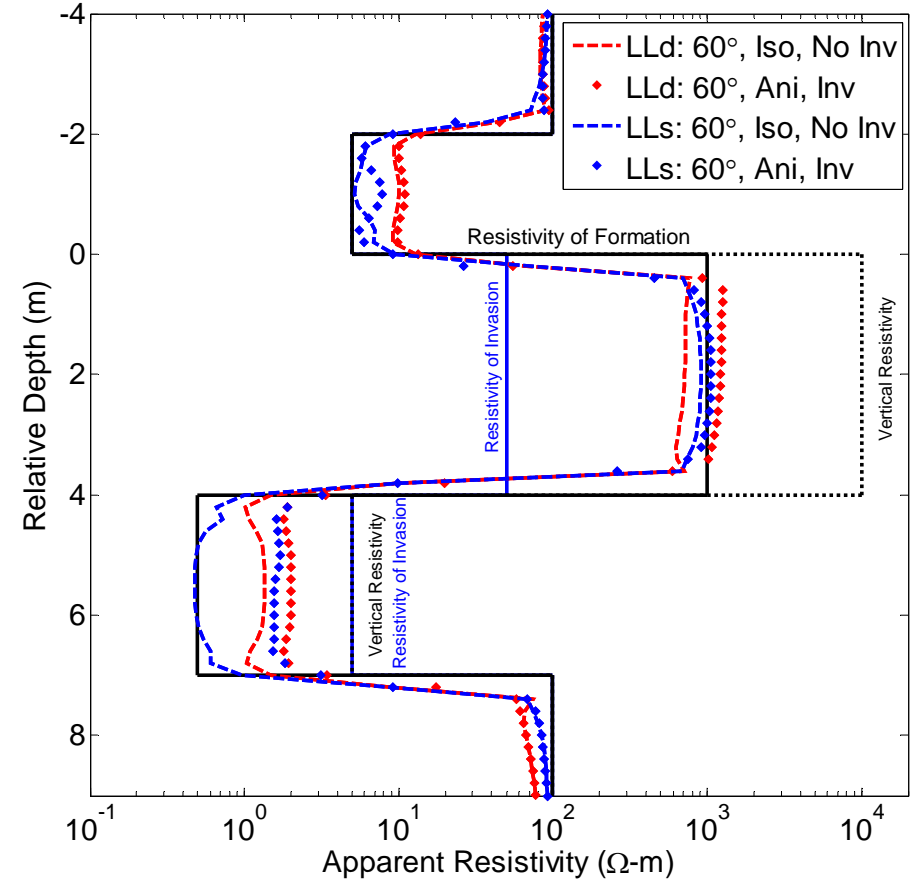
Invaded Anisotropic Formation (DC DLL)

Vertical Well in Anisotropy Formation with Invasion



Vertical Well

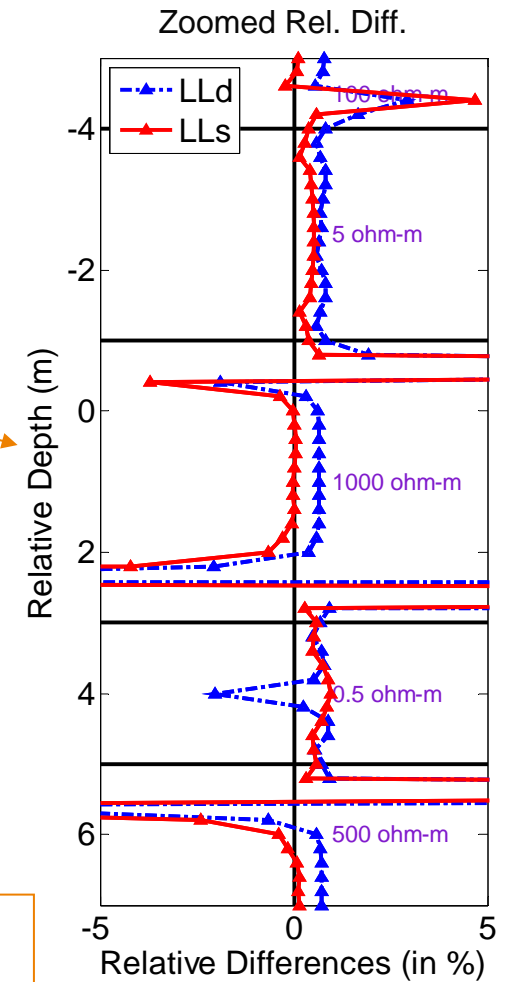
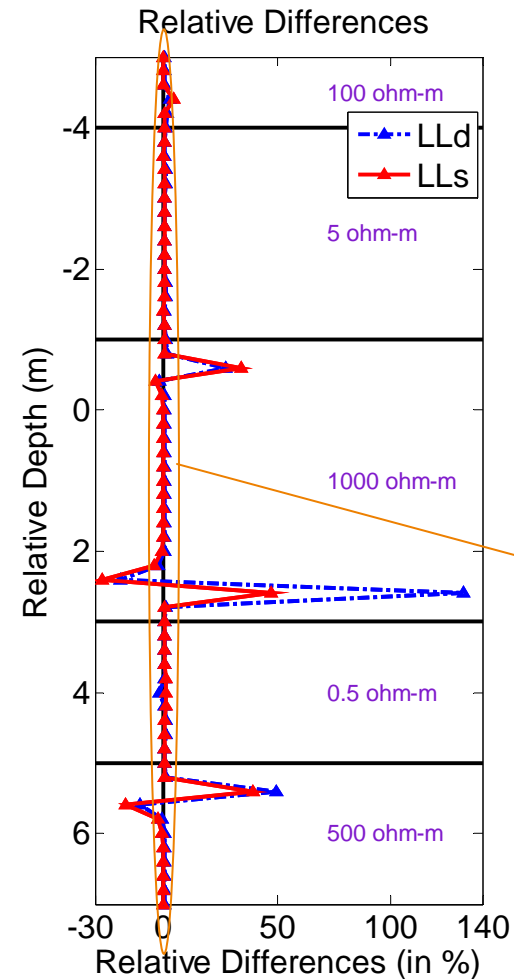
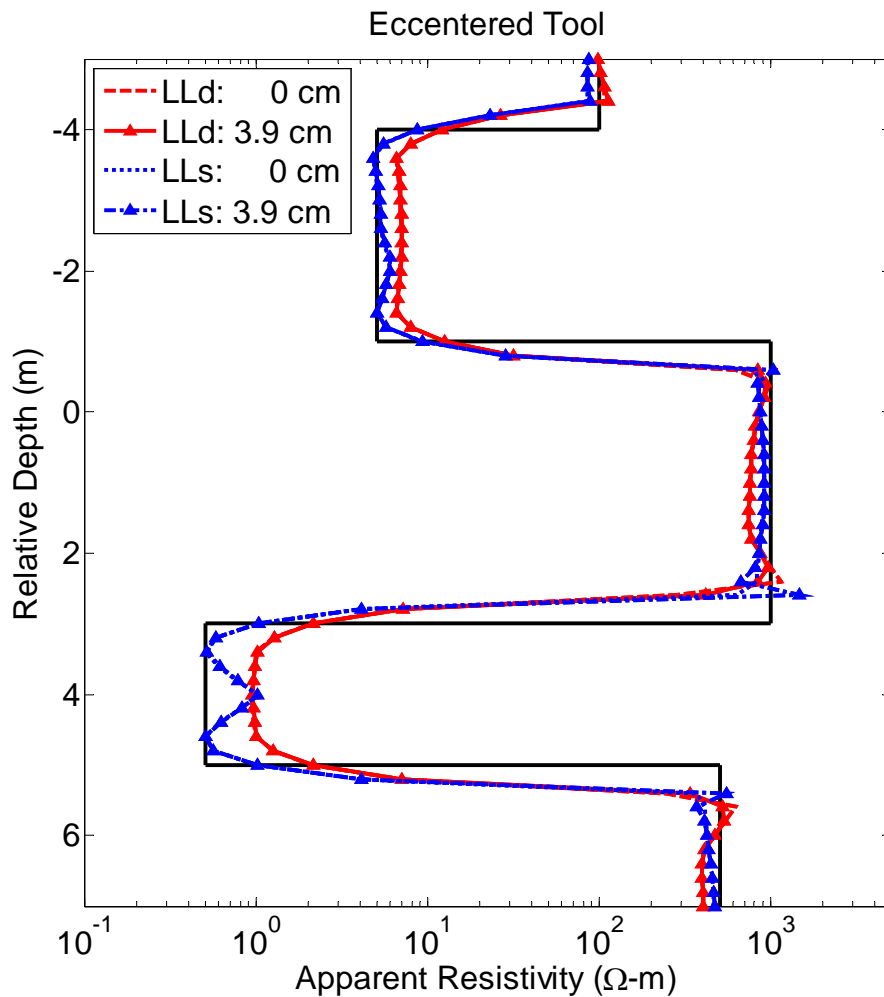
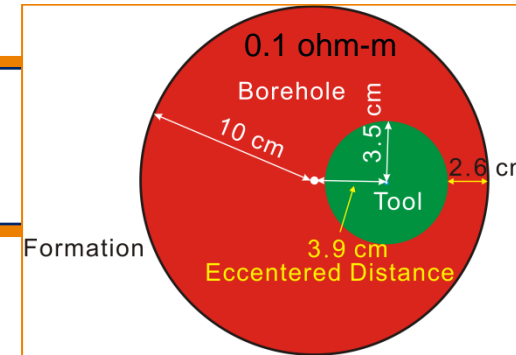
Deviated Well in Anisotropy Formation with Invasion



60 degree deviated Well



Eccentered Tool Effects (DC DLL)



Eccentered-tool effects are larger around layer boundaries in resistive layers



Conclusions

- We successfully simulated AC DLL measurements by explicitly incorporating the term $\nabla \cdot \mathbf{J}$ for non-zero frequency Maxwell's equations.
- The simulation employed a high-order self-adaptive *hp* finite-element method with an embedded post-processing technique.
- Numerical experiments indicate that the inclusion of a current return electrode is critical to simulate Groningen effects.
- Groningen effects decrease as the current return is placed farther away from either the logging points or the borehole.



Acknowledgements

Sponsors of UT Austin's consortium on Formation Evaluation:



INSTITUTO MEXICANO DEL PETRÓLEO

