Solving Helmholtz Equation Using a New Family of Discontinuous Petrov-Galerkin Methods

To approximate numerically Helmholtz equation in the high frequency regime is computationally expensive, regardless the choice of the solver (direct or iterative). More precisely, in most numerical methods, the computational cost increases as the wave number increases, which explains why high frequency approximations are difficult to obtain. One of the main reasons of the computational burden is the so-called dispersion effect. The dispersion (pollution) error can be defined as the difference between the finite element solution and the best solution over the considered finite element subspace. It is well known that the dispersion error quickly increases as we consider higher frequencies. A possible solution to minimize the dispersion error for high frequency approximations is to consider high order polynomials and a limited number of elements per wavelength. Unfortunately, this method is rather expensive. As you increase the order of the polynomials, you increase the cost of the method. In addition, iterative solvers of linear equations such as multigrid fails if the coarsest grid does not contain a rather large number of elements per wavelength (cf D. Pardo's PhD thesis). During the last decades, several methods have been proposed to minimize the dispersion effect, despite the progress that have been made to reduce computational costs when solving high frequency Helmholtz equation, the existing methods are still rather expensive.

We are working on a new discontinuous Petrov-Galerkin method for the efficient simulation of wave propagation problems based on an extension of the method applied to the transport equation and described in [1, 2]. The method combines a new inter-element unknown (flux) with the use of optimal test functions. Local "optimal" test basis functions are constructed in such a way that the constants of the continuity and the inf-sup conditions are equal to 1. The new method for wave propagation problems presents several advantages over traditional finite element or discontinuous Galerkin methods : (a) it is always stable, (b) it has no dispersion error in a newly defined norm, and (c) it always provide a symmetric and positive definite system structure that can be exploited by iterative solvers.

Références

[1] L. Demkowicz, J. Gopalakrishnan, *A class of discontinuous Petrov-Galerkine methods. Part I : The transport equation*, Institute for computational Engineering and Sciences report, Austin, Texas, USA, 2009.

[2] L. Demkowicz, J. Gopalakrishnan, *A class of discontinuous Petrov-Galerkine methods. Part II : Optimal test functions*, Institute for computational Engineering and Sciences report, Austin, Texas, USA, 2009.