Matemáticas y Turbulencia

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UPM, 17 de marzo, 2022

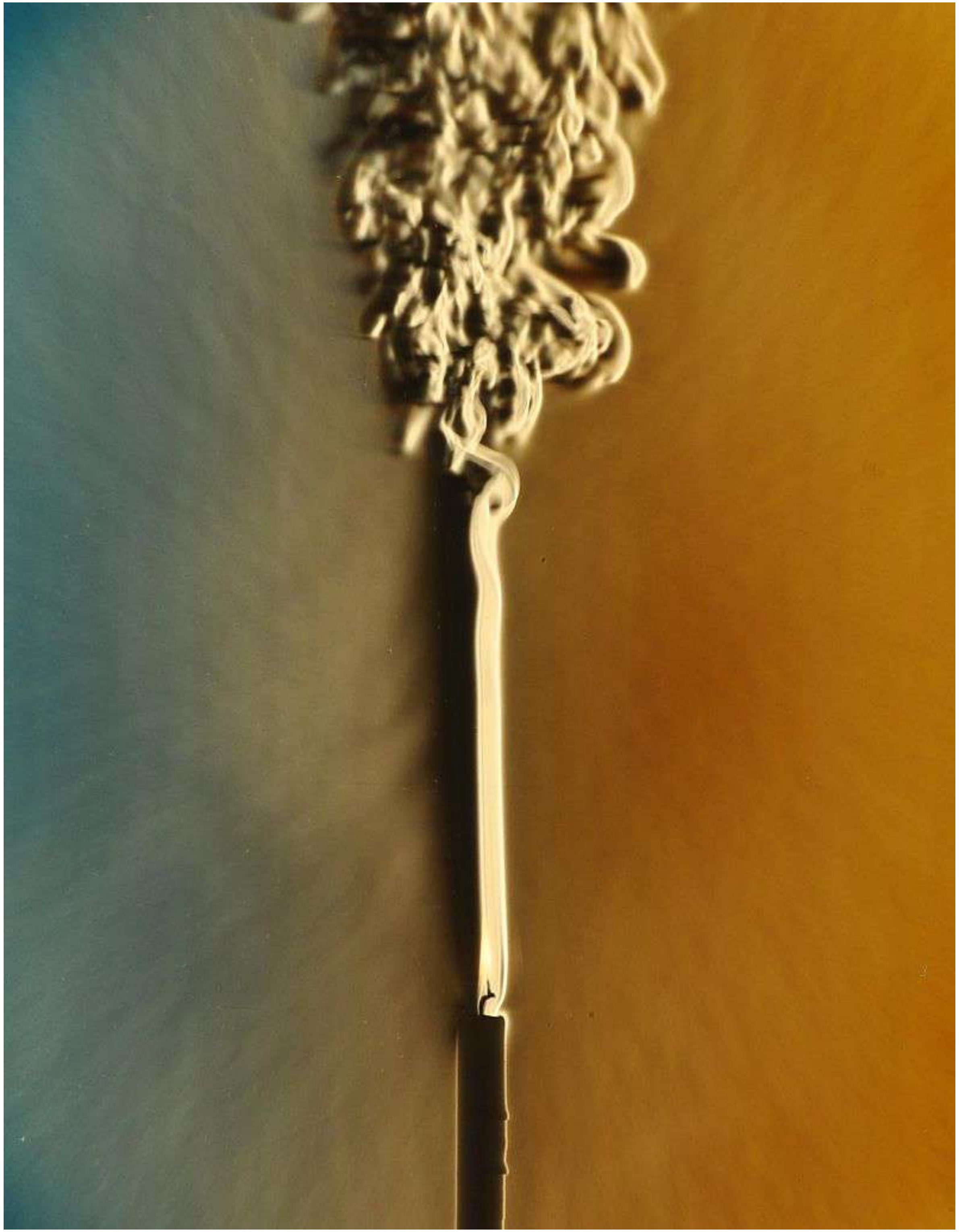
In fluid dynamics, **turbulence** or **turbulent flow** is fluid motion characterized by chaotic changes in pressure and flow velocity. It is in contrast to a laminar flow, which occurs when a fluid flows in parallel layers, with no disruption between those layers.

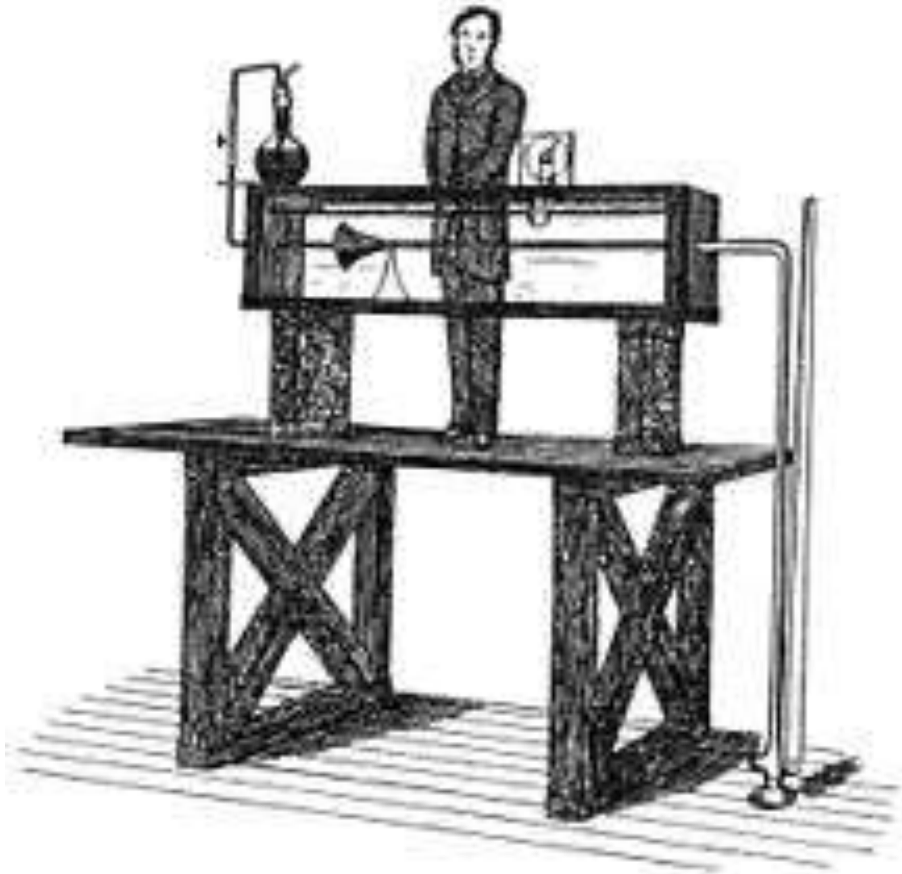
Turbulence is commonly observed in everyday phenomena such as surf, fast flowing rivers, billowing storm clouds, or smoke from a chimney, and most fluid flows occurring in nature or created in engineering applications are turbulent. Turbulence is caused by excessive kinetic energy in parts of a fluid flow, which overcomes the damping effect of the fluid's viscosity. For this reason turbulence is commonly realized in low viscosity fluids. In general terms, in turbulent flow, unsteady vortices appear of many sizes which interact with each other, consequently drag due to friction effects increases. This increases the energy needed to pump fluid through a pipe.

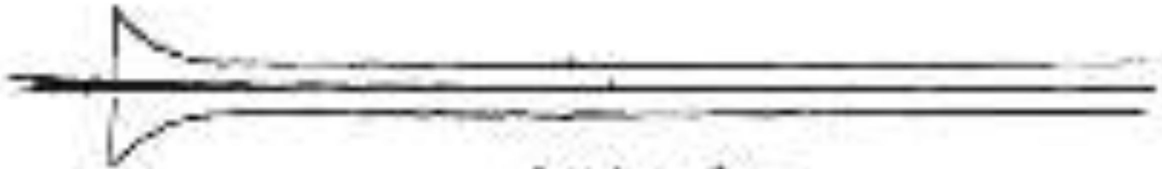
La turbulencia se observa comúnmente en fenómenos cotidianos como el **oleaje**, los **ríos de corriente rápida**, las **nubes de tormenta** ondulantes o el humo de una chimenea, y la mayoría de los flujos de fluidos que se producen en la naturaleza o se crean en aplicaciones de ingeniería son turbulentos La turbulencia está causada por un **exceso** de **energía cinética** en partes de un flujo de fluido, que **supera el efecto de amortiguación** de la viscosidad del fluido. Por esta razón, la turbulencia se produce habitualmente en fluidos de baja viscosidad. En términos generales, en el flujo turbulento aparecen **vórtices inestables de muchos tamaños que interactúan entre sí**, por lo que aumenta la **resistencia** debido a los efectos de la **fricción**. Esto aumenta la **energía necesaria para bombear el fluido a través de una tubería**.

En dinámica de fluidos, la turbulencia o flujo turbulento es un movimiento de fluidos caracterizado por **cambios caóticos** en la presión y la velocidad del flujo. Contrasta con el **flujo laminar**, que se produce cuando un fluido fluye en capas paralelas, sin interrupción entre ellas.

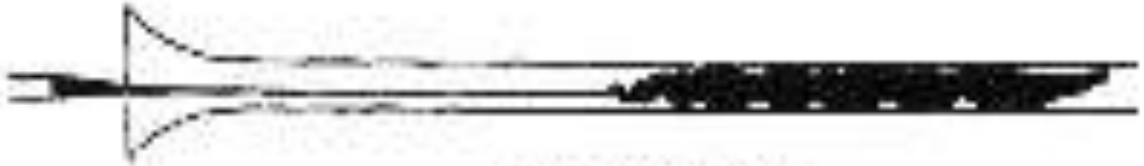
La aparición de turbulencias puede predecirse mediante el número adimensional de **Reynolds**, la relación entre la energía cinética y la amortiguación viscosa en un flujo de fluido. Sin embargo, la turbulencia se ha resistido durante mucho tiempo al análisis físico detallado, y las interacciones dentro de la turbulencia crean un fenómeno muy complejo. **Richard Feynman** ha descrito la turbulencia como el problema más importante sin resolver de la física clásica.



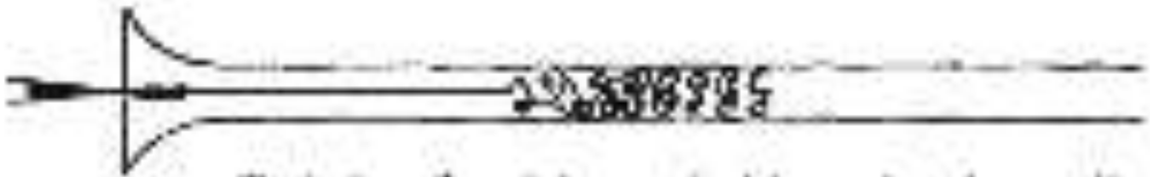




Laminar flow

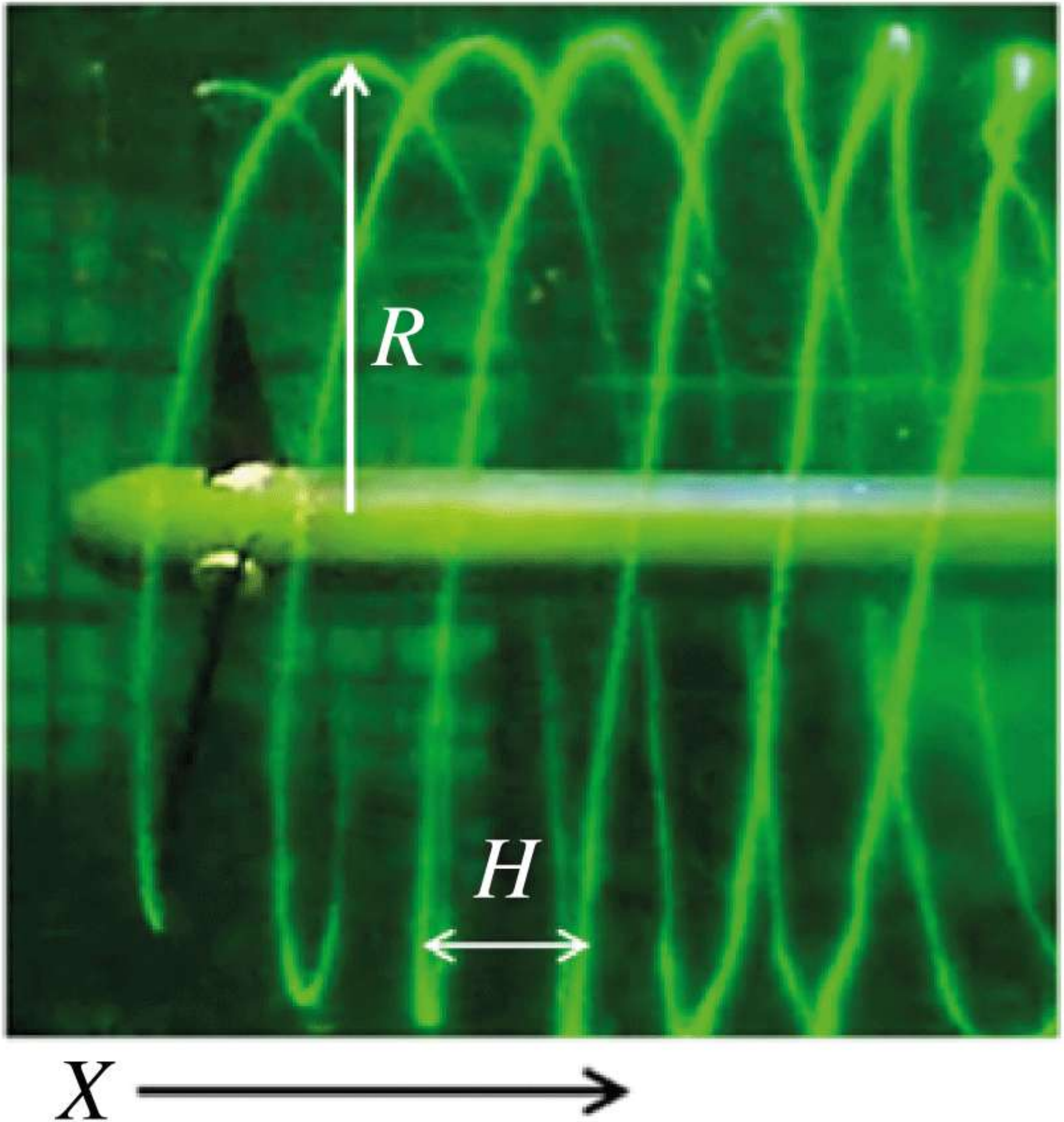


Turbulent flow

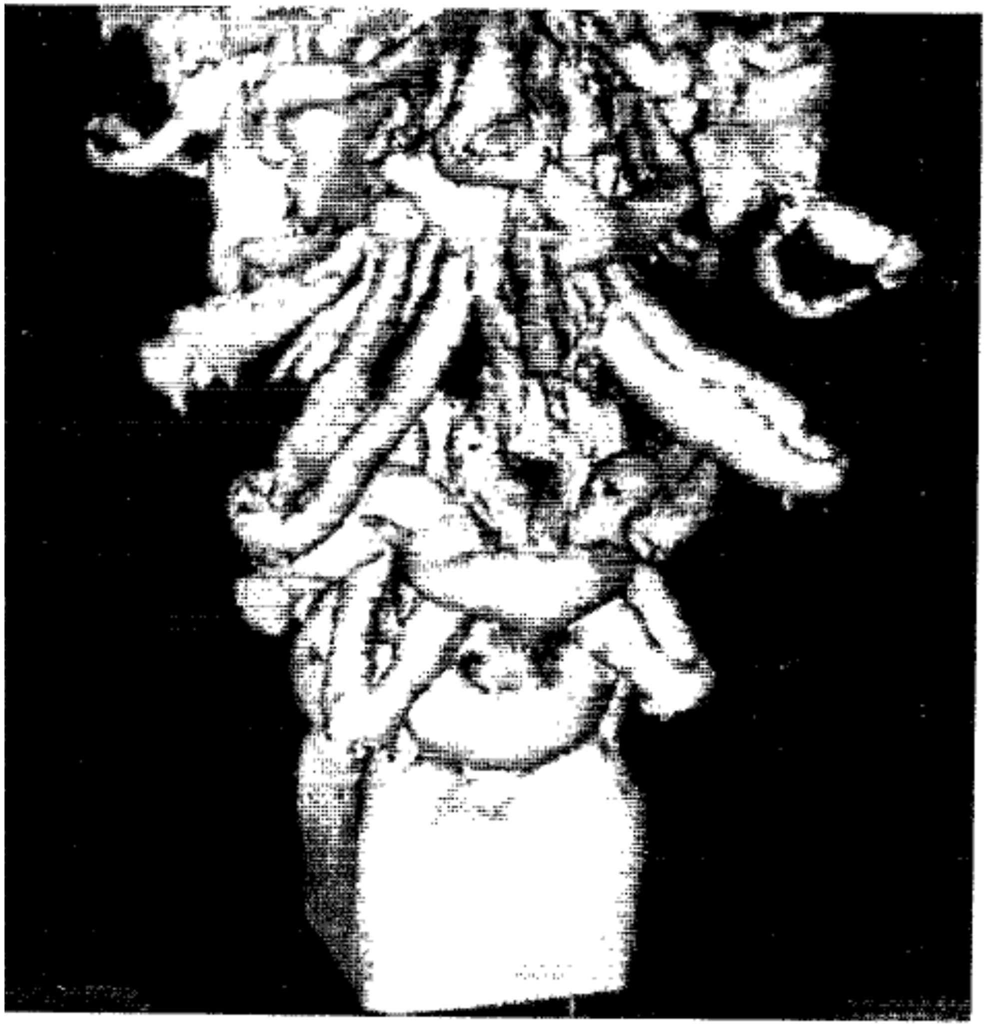


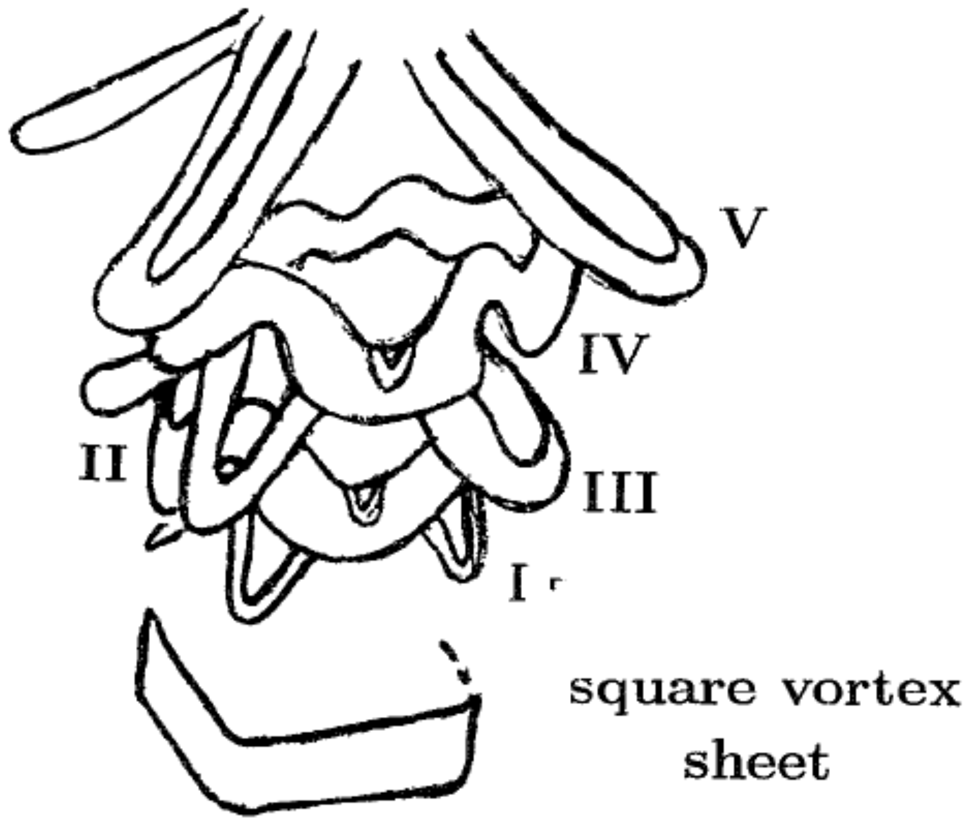
Turbulent flow (observed with an electric spark)

(*a*)



I, III, V : hairpin (braid) vortices
II, IV : deformed vortex rings





Annu. Rev. Fluid Mech. 1999. 31:239–72

FLOW CONTROL WITH NONCIRCULAR JETS[1]

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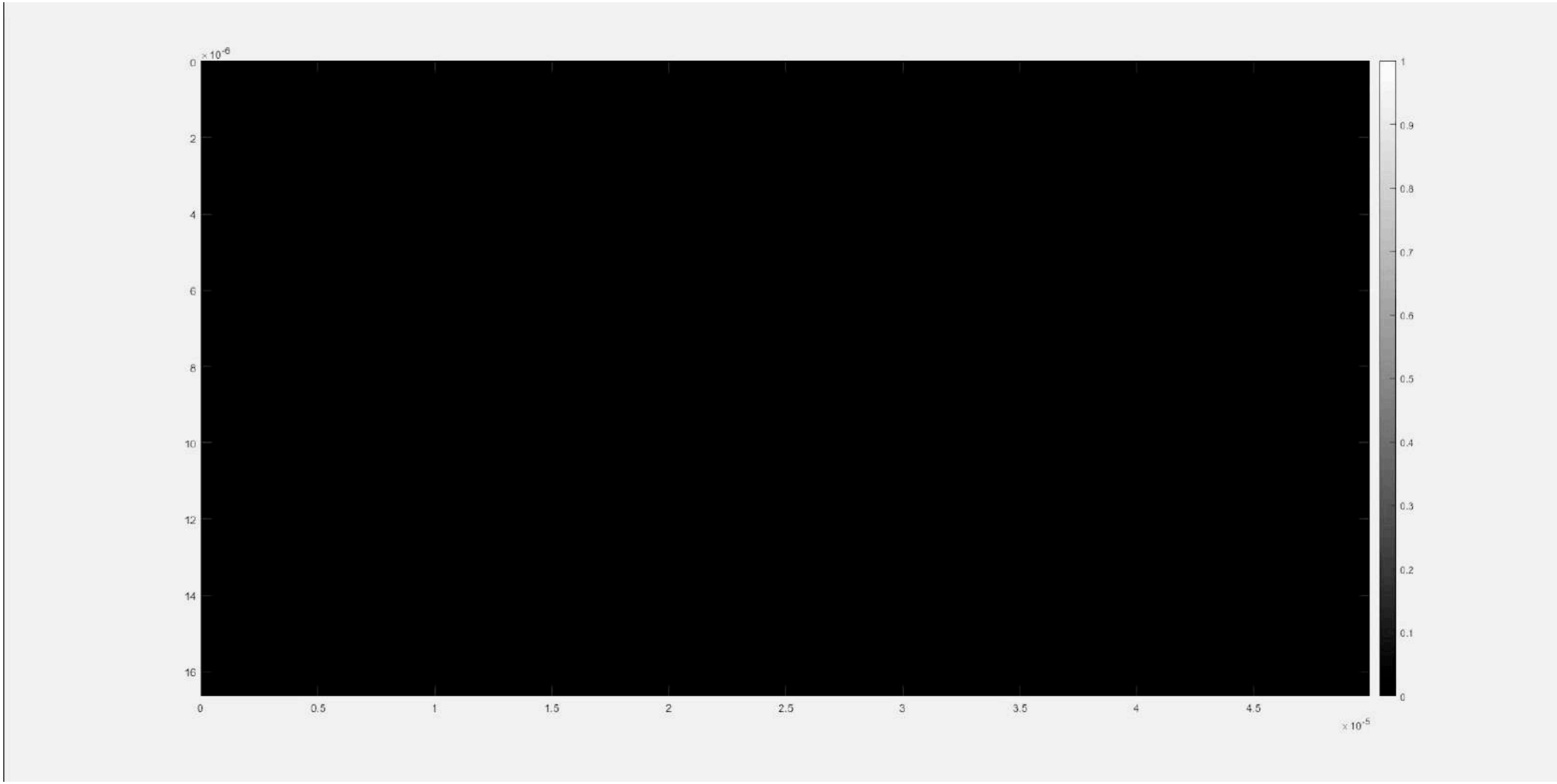
KEY WORDS: vortices, mixing, combustion, entrainment

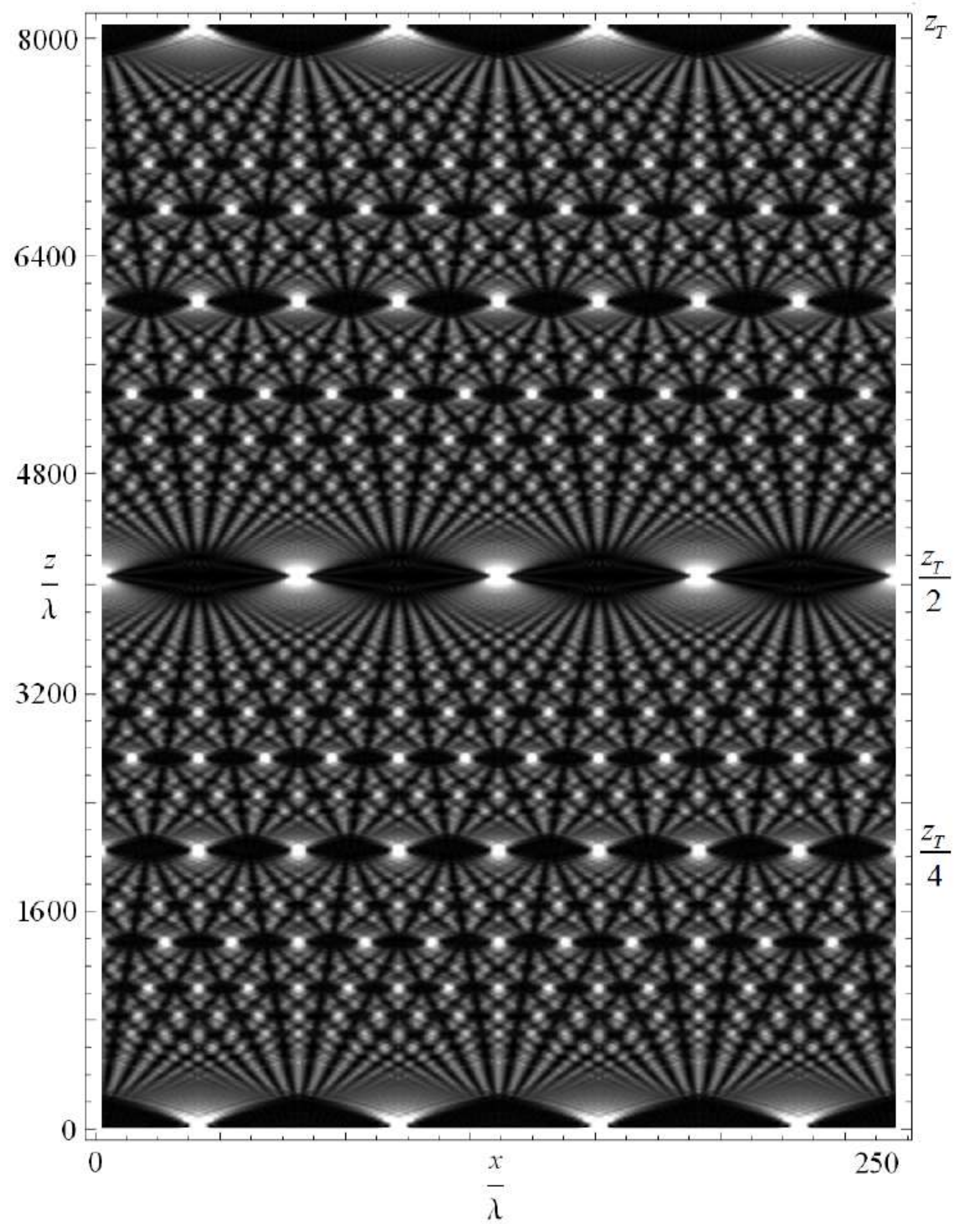
ABSTRACT

Noncircular jets have been the topic of extensive research in the last fifteen years. These jets were identified as an efficient technique of passive flow control that allows significant improvements of performance in various practical systems at a relatively low cost because noncircular jets rely solely on changes in the geometry of the nozzle. The applications of noncircular jets discussed in this review include improved large- and small-scale mixing in low- and high-speed flows, and enhanced combustor performance, by improving combustion efficiency, reducing combustion instabilities and undesired emissions. Additional applications include noise suppression, heat transfer, and thrust vector control (TVC).

The flow patterns associated with noncircular jets involve mechanisms of vortex evolution and interaction, flow instabilities, and fine-scale turbulence augmentation. Stability theory identified the effects of initial momentum thickness distribution, aspect ratio, and radius of curvature on the initial flow evolution. Experiments revealed complex vortex evolution and interaction related to self-induction and interaction between azimuthal and axial vortices, which lead to axis switching in the mean flow field. Numerical simulations described the details and clarified mechanisms of vorticity dynamics and effects of heat release and reaction on noncircular jet behavior.

The **Talbot effect** is a near-field diffraction effect first observed in 1836 by Henry Fox Talbot. When a **plane wave** is incident upon a periodic **diffraction grating**, the image of the grating **is repeated at regular distances** away from the grating plane. The regular distance is called the **Talbot length**, and the repeated images are called self images or Talbot images. Furthermore, **at half the Talbot length**, a self-image also occurs, but **phase-shifted by half a period**. **At one quarter** of the Talbot length, the self-image is halved in size, and appears with **half the period of the grating (thus twice as many images are seen)**. At **one eighth** of the Talbot length, the period and size of the images is **halved again**, and so forth creating a **fractal pattern** of sub images with ever decreasing size, often referred to as a **Talbot carpet**.


×10^-6
0
2
4
6
8
10
12
14
16
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
×10^-5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0


8000
6400
4800
3200
1600
0
z/λ
x/λ
0
250
z_T
z_T/2
z_T/4

Talbot effect and linear Schrödinger equation

$$\psi_t = i\psi_{xx}$$

$$\psi(x,0) = \frac{2\pi}{M} \sum_{k=-\infty}^{\infty} \delta\left(x - \frac{2\pi k}{M}\right)$$

$$\widehat{\psi}(k,t) = \frac{2\pi}{M} e^{-i(Mk)^2 t}$$

$$t_{pq} = (2\pi/M^2)(p/q)$$

$$\begin{aligned}
\theta_M(x,t_{pq}) := \psi(x,t_{pq}) &= \sum_{k=-\infty}^{\infty} e^{-i(Mk)^2 2\pi p/(M^2 q)+iMkx} \\
&= \sum_{k=-\infty}^{\infty} e^{-2\pi i(p/q)k^2+iMkx} \\
&= \sum_{l=0}^{q-1} \sum_{k=-\infty}^{\infty} e^{-2\pi i(p/q)(qk+l)^2+iM(qk+l)x} \\
&= \sum_{l=0}^{q-1} e^{-2\pi i(p/q)l^2+iMlx} \sum_{k=-\infty}^{\infty} e^{iMqkx}.
\end{aligned}$$

The generalized quadratic Gauss sums are defined by

$$\sum_{l=o}^{|c|-1} e^{2\pi i(al^2+bl)/c},$$

for given integers a, b, c, with $c \neq 0$.

$$G(-p, m, q) = \begin{cases} \sqrt{q}e^{i\theta m}, & \text{if } q \text{ is odd,} \\ \sqrt{2q}e^{i\theta m}, & \text{if } q \text{ is even and } q/2 \equiv m \mod 2, \\ 0, & \text{if } q \text{ is even and } q/2 \not\equiv m \mod 2, \end{cases}$$

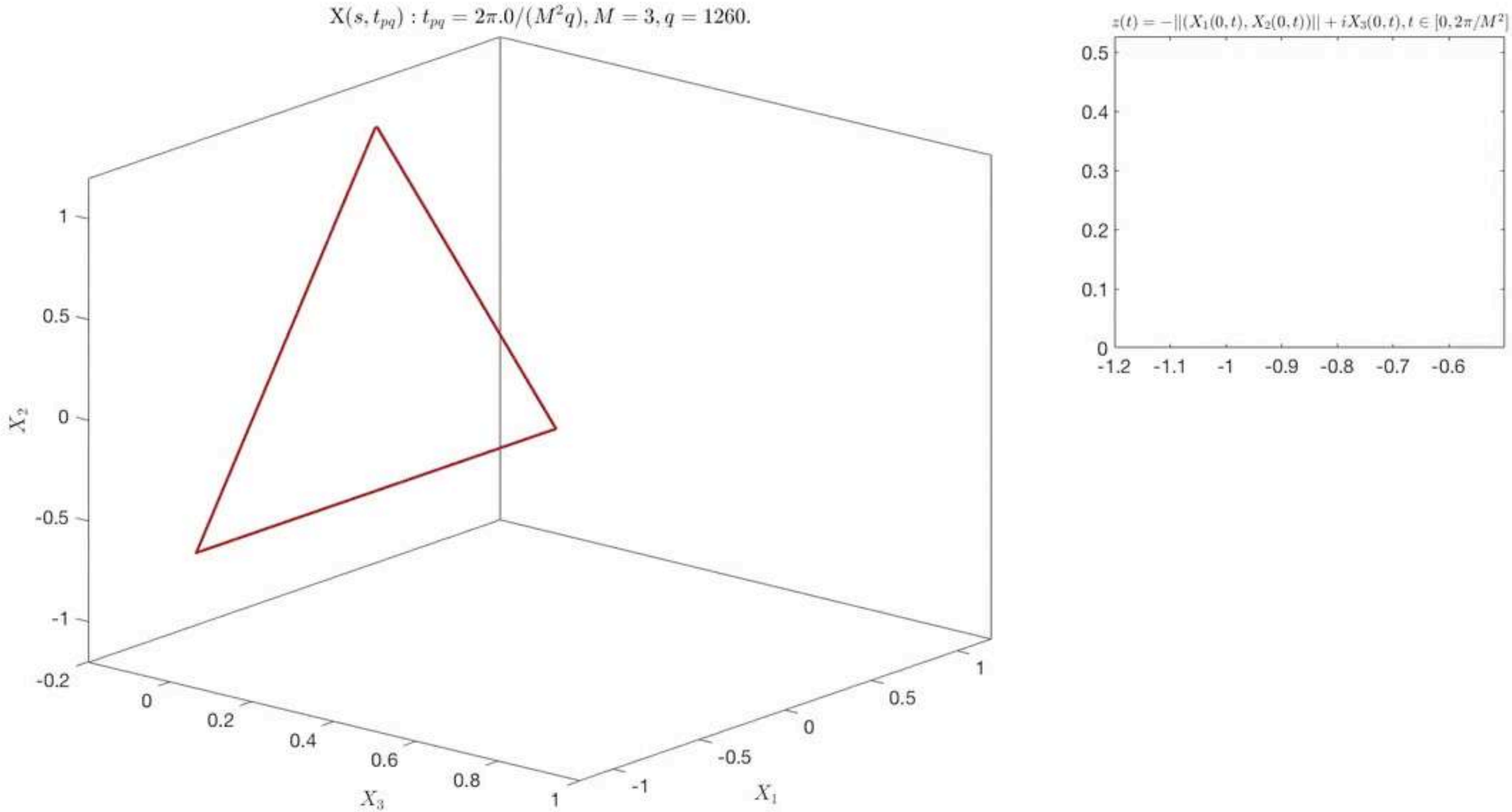
for a certain angle θ_m that depends on m (and, of course, on p and q, too).

The Talbot effect

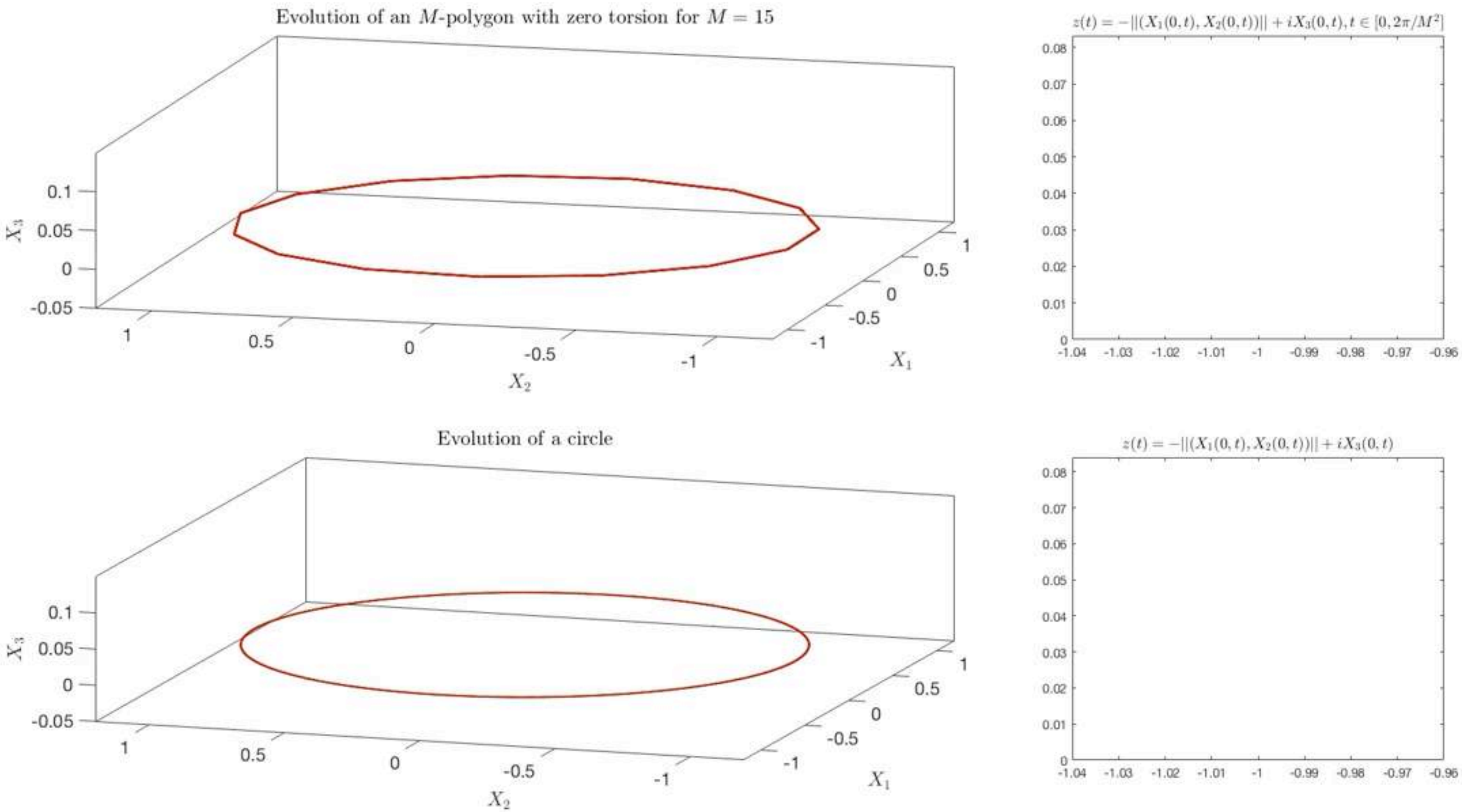
$$t_{pq} = (2\pi/M^2)(p/q)$$

$$\theta_M(x,0) = \frac{2\pi}{M} \sum_{k=-\infty}^{\infty} \delta\left(x - \frac{2\pi k}{M}\right)$$

$$\theta_M(x,t_{pq}) = \frac{2\pi}{Mq} \sum_{k=-\infty}^{\infty} \sum_{m=0}^{q-1} G(-p,m,q)\delta\left(x - \frac{2\pi k}{M} - \frac{2\pi m}{Mq}\right)$$

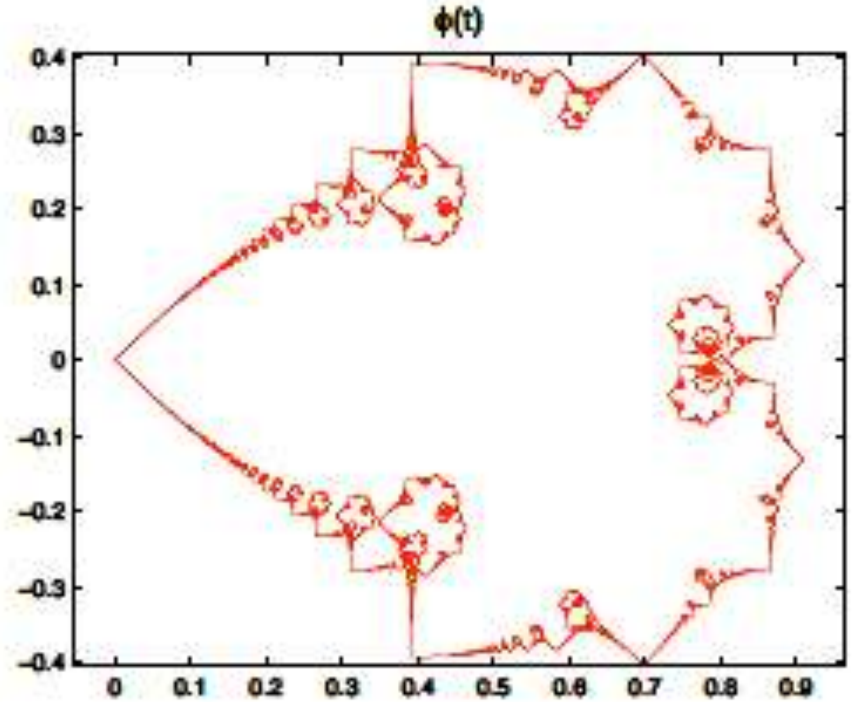


$X(s, t_{pq}) : t_{pq} = 2\pi.0/(M^2 q), M = 3, q = 1260.$
X_2
X_3
X_1
1
0.5
0
-0.5
-1
-0.2
0
0.2
0.4
0.6
0.8
1
-1
-0.5
0
0.5
1
$z(t) = -\|(X_1(0,t), X_2(0,t))\| + iX_3(0,t), t \in [0, 2\pi/M^2]$
0.5
0.4
0.3
0.2
0.1
0
-1.2
-1.1
-1
-0.9
-0.8
-0.7
-0.6



Evolution of an M-polygon with zero torsion for $M = 15$
$z(t) = -\|(X_1(0,t), X_2(0,t))\| + iX_3(0,t), t \in [0, 2\pi/M^2]$
Evolution of a circle
$z(t) = -\|(X_1(0,t), X_2(0,t))\| + iX_3(0,t)$
X_1
X_2
X_3

$$\phi(t) = \sum_{k \neq 0} \frac{e^{\pi i k^2 t}}{i\pi k^2}, \quad t \in [0, 2]$$



- Jaffard
- Multifractal (Frisch–Parisi conjecture)

Riemann's non–differentiable function

Integrating the Fourier series in time and evaluating at $x = 0$ we get

$$\phi(t) = i \int_0^t \theta_{2\pi}(0,\tau)d\tau = \sum_{k\in\mathbb{Z}} \frac{e^{-4\pi^2 ik^2 t} - 1}{-4\pi^2 k^2},$$

which is essentially Riemann's non–differentiable function.

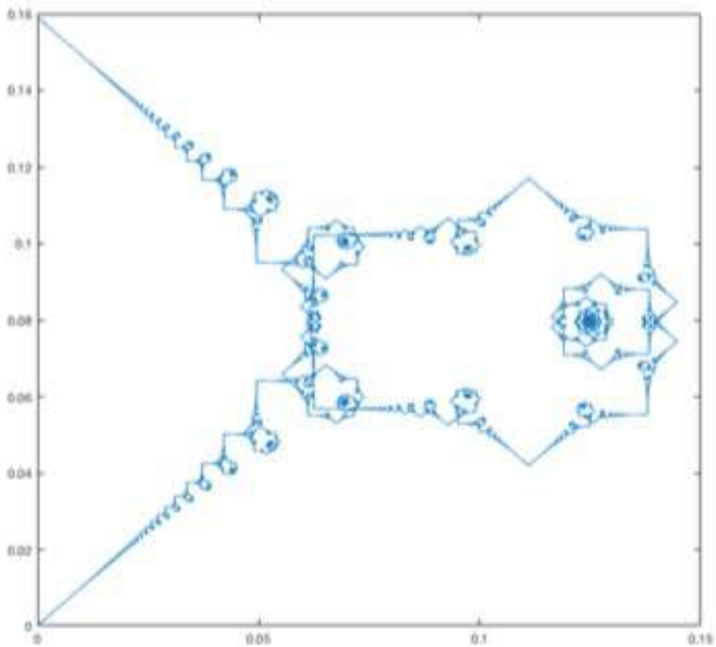


Figure: De la Hoz, Vega: *Vortex filament equation for a regular polygon*, Nonlinearity **27**(2014), 3031-3057

Muchas gracias por su atención