Progress Report (Baker-Atlas)

A Fully Automatic Goal-Oriented hp-Adaptive Finite Element Strategy for Simulations of Resistivity Logging Instruments.

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OVERVIEW

1. Overview
2. Mathematical Formulation of Electrodynamic Problems
   - Maxwell's Equations and Boundary Conditions
   - Variational Formulation
   - Axial Symmetry
3. Error Estimation: Towards Certified Solutions
   - Control of Modeling and Discretization Errors
   - Automatic Built-in Error Estimation
   - Benchmarking Problems
4. Numerical Results
5. Conclusions and Future Work

MAXWELL'S EQUATIONS (FREQUENCY DOMAIN)

Time Harmonic Maxwell's Equations:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$
$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} + \mathbf{J}^{imp}$$

Reduced Wave Equation:

E-Formulation ; **H-Formulation**

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega\boldsymbol{J}^{imp} \quad ; \quad \boldsymbol{\nabla} \times \left(\frac{1}{\sigma + j\omega\epsilon}\boldsymbol{\nabla} \times \boldsymbol{H}\right) + j\omega\mu\boldsymbol{H} = \boldsymbol{\nabla} \times \frac{1}{\sigma + j\omega\epsilon}\boldsymbol{J}^{imp}$$

Boundary Conditions (BC):

- **Perfect Electric Conductor Surface:**

$$\mathbf{n} \times \boldsymbol{E} = 0 \quad ; \quad \mathbf{n} \cdot \boldsymbol{H} = 0$$

- **Idealized Antennas (Impressed Surface Electric Current):**

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega\mathbf{J}_S^{imp} \quad ; \quad \mathbf{n} \times \boldsymbol{H} = \mathbf{J}_S^{imp}$$

MAXWELL'S EQUATIONS (FREQUENCY DOMAIN)

Variational formulation

The reduced wave equation in Ω,

E-Formulation:
$$\nabla \times \left(\frac{1}{\mu} \nabla \times E \right) - (\omega^2 \epsilon - j\omega\sigma) E = -j\omega J^{imp}$$

H-Formulation:
$$\nabla \times \left(\frac{1}{\sigma + j\omega\epsilon} \nabla \times H \right) + j\omega\mu H = \nabla \times \frac{1}{\sigma + j\omega\epsilon} J^{imp}$$

Variational formulation:

E-Formulation:

Find $\mathrm{E} \in H_D(\mathrm{curl};\Omega)$ such that:
$$\int_\Omega \frac{1}{\mu} (\nabla \times \mathrm{E})(\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\omega^2 \epsilon - j\omega\sigma) \mathrm{E} \cdot \bar{\mathrm{F}}\, dV =$$
$$-j\omega \int_\Omega \mathrm{J}^{imp} \cdot \bar{\mathrm{F}}\, dV + j\omega \int_{\Gamma_N} \mathrm{J}_S^{imp} \cdot \bar{\mathrm{F}}\, dS \qquad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega)$$

H-Formulation:

Find $\mathrm{H} \in \tilde{\mathrm{H}}_S + H_D(\mathrm{curl};\Omega)$ with $\tilde{\mathrm{J}}_S^{imp} = \mathrm{n} \times \mathrm{H}|_S$ and such that:
$$\int_\Omega \frac{1}{\sigma + j\omega\epsilon} (\nabla \times \mathrm{H})(\nabla \times \bar{\mathrm{F}})\, dV + j\omega \int_\Omega \mu \mathrm{H} \cdot \bar{\mathrm{F}}\, dV =$$
$$\int_\Omega \nabla \times \left(\frac{1}{\sigma + j\omega\epsilon} \mathrm{J}^{imp} \right) \cdot \bar{\mathrm{F}}\, dV \qquad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega)$$

MAXWELL'S EQUATIONS (FREQUENCY DOMAIN)

Variational formulation in cylindrical coordinates

Using cylindrical coordinates (ρ, ϕ, z):

E_ϕ-Formulation:

Find $E_\phi \in \tilde{H}^1_D(\Omega)$ such that:

$$\int_\Omega \frac{1}{\mu}\left(\frac{\partial E_\phi}{\partial z}\frac{\partial \bar{F}_\phi}{\partial z} + \frac{1}{\rho^2}\frac{\partial(\rho E_\phi)}{\partial \rho}\frac{\partial(\rho \bar{F}_\phi)}{\partial \rho}\right) dV - \int_\Omega k^2 E_\phi \cdot \bar{F}_\phi \, dV =$$
$$-j\omega \int_\Omega J^{imp}_\phi \cdot \bar{F}_\phi \, dV + j\omega \int_{\Gamma_N} J^{imp}_{\phi,S} \cdot \bar{F}_\phi \, dS \quad \forall F_\phi \in \tilde{H}^1_D(\Omega) \, .$$

$\mathrm{E}_{\rho,z}$-Formulation:

Find $\mathrm{E} = (E_\rho, 0, E_z) \in \tilde{H}_D(\mathrm{curl}; \Omega)$ such that:

$$\int_\Omega \frac{1}{\mu}\left(\frac{\partial E_\rho}{\partial z}\frac{\partial \bar{F}_\rho}{\partial z} + \frac{\partial E_z}{\partial \rho}\frac{\partial \bar{F}_z}{\partial \rho}\right) - k^2 \int_\Omega E_\rho \bar{F}_\rho + E_z \bar{F}_z \, dV = -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z \, dV +$$
$$j\omega \int_{\Gamma_N} J^{imp}_{\rho,S} \bar{F}_\rho + J^{imp}_{z,S} \bar{F}_z \, dS \quad \forall \mathrm{F} = (F_\rho, 0, F_z) \in \tilde{H}_D(\mathrm{curl}; \Omega) \, .$$

H_ϕ-Formulation:

Find $H_\phi \in f_1(J^{imp}_{\rho,S}, J^{imp}_{z,S}) + \tilde{H}^1_D(\Omega)$ such that:

$$\int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial H_\phi}{\partial z}\frac{\partial \bar{F}_\phi}{\partial z} + \frac{1}{\rho^2}\frac{\partial(\rho H_\phi)}{\partial \rho}\frac{\partial(\rho \bar{F}_\phi)}{\partial \rho}\right) dV - j\omega \int_\Omega \mu H_\phi \cdot \bar{F}_\phi \, dV =$$
$$\int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial J^{imp}_\rho}{\partial z} - \frac{\partial J^{imp}_z}{\partial \rho}\right)\bar{F}_\phi dV \quad \forall F_\phi \in \tilde{H}^1_D(\Omega) \, .$$

$H_{\rho,z}$-Formulation:

Find $\mathrm{H} = H_\rho, 0, H_z) \in f_2(J^{imp}_{\phi,S}) + \tilde{H}_D(\mathrm{curl}; \Omega)$ such that:

$$\int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial H_\rho}{\partial z}\frac{\partial \bar{F}_\rho}{\partial z} + \frac{\partial H_z}{\partial \rho}\frac{\partial \bar{F}_z}{\partial \rho}\right) - j\omega \int_\Omega \mu(H_\rho \bar{F}_\rho + H_z \bar{F}_z) \, dV =$$
$$\int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial J_\phi}{\partial z}\bar{F}_\rho + \frac{1}{\rho}\frac{\partial(\rho J^{imp}_\phi)}{\partial \rho}\bar{F}_z\right) dV \quad \forall \mathrm{F} = (F_\rho, 0, F_z) \in \tilde{H}_D(\mathrm{curl}; \Omega) \, .$$

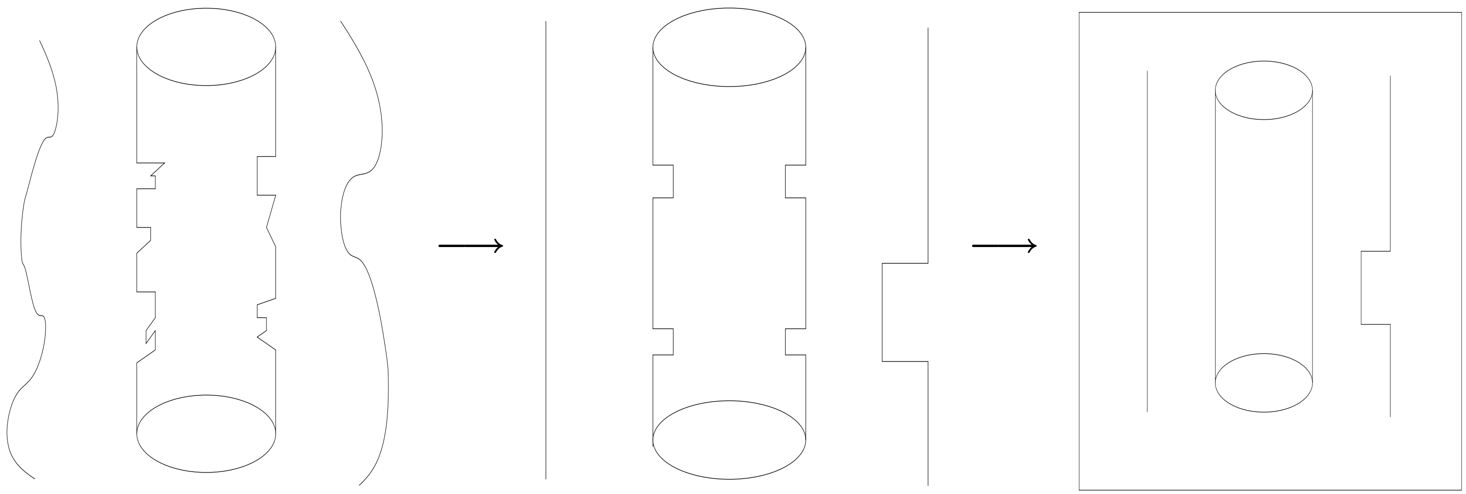
TOWARDS CERTIFIED SOLUTIONS

Sources of Error

Physical (Real) Problem
Exact Geometry
Exact Physics
Exact Instruments
No Boundaries

Mathematical Problem
Approx. Geometry
Maxwell's Equations
Perfect Instruments
Some Boundaries

Computational Problem
Approx. Geometry
Maxwell's Equations
Perfect Instruments
Boundary Conditions



No Errors in the Solution

Errors Associated to:
Geometry
Approximations
Coupling Physics
Material Coefficients

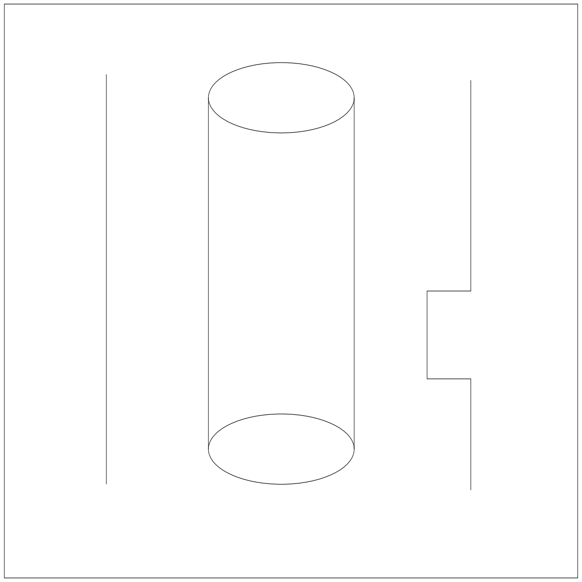
Errors Associated to:
Mathematical Modeling
Geometry and BC's
The Code (Bugs)
Discretization

TOWARDS CERTIFIED SOLUTIONS

Sources of Error

Computational Problem

Approx. Geometry
Maxwell's Equations
Perfect Instruments
Boundary Conditions



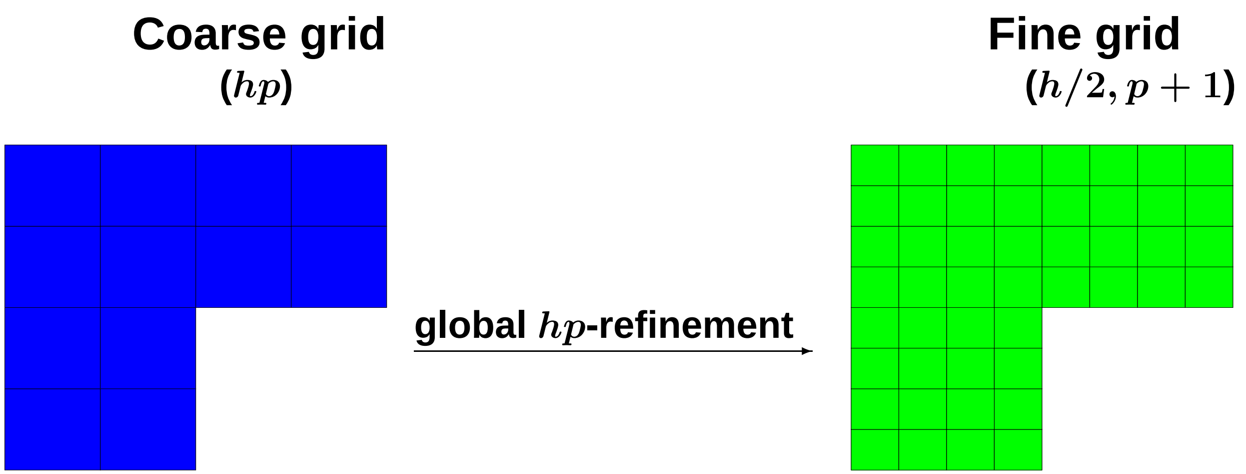
Solution is Not Exact Due to Errors in:

- **Mathematical Modeling**
- **Geometry and BC's**
- **The Code (Bugs)**
- **Discretization**

TOWARDS CERTIFIED SOLUTIONS

Discretization Error

The self-adaptive goal-oriented hp-adaptive strategy provides a very accurate built-in discretization error estimator.



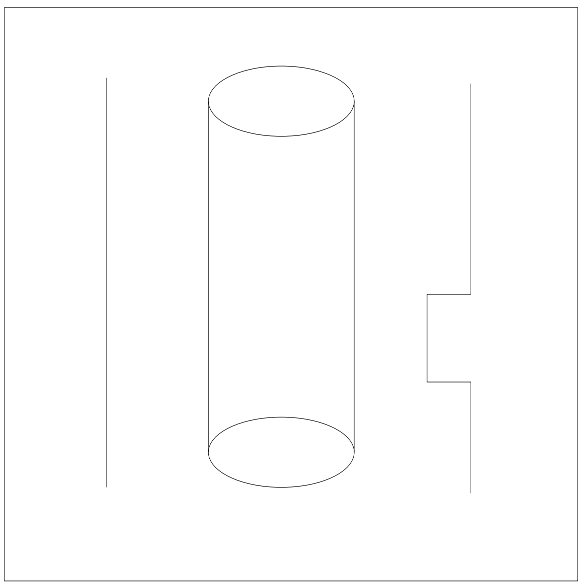
Discretization Error Estimate for Coarse Grid =
Fine Grid Solution - Coarse Grid Solution

TOWARDS CERTIFIED SOLUTIONS

Sources of Error

Computational Problem

Approx. Geometry
Maxwell's Equations
Perfect Instruments
Boundary Conditions



Solution is Not Exact Due to Errors in:

- **Mathematical Modeling**
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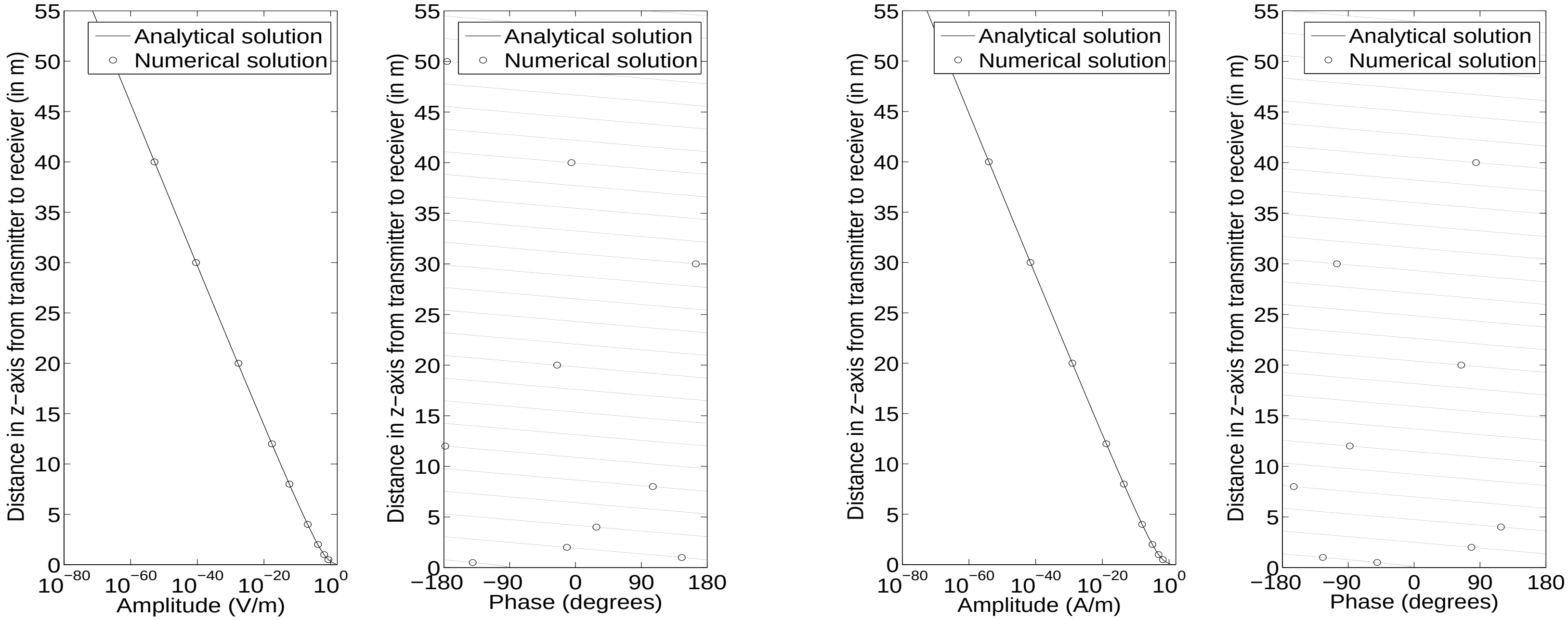
- **Discretization**

TOWARDS CERTIFIED SOLUTIONS

Avoiding Errors in the Code Using Benchmarking Examples

Solutions in a Homogeneous Lossy ($1\ \Omega\ m$) Media (2 Mhz)

Solenoid Antenna **Toroid Antenna**



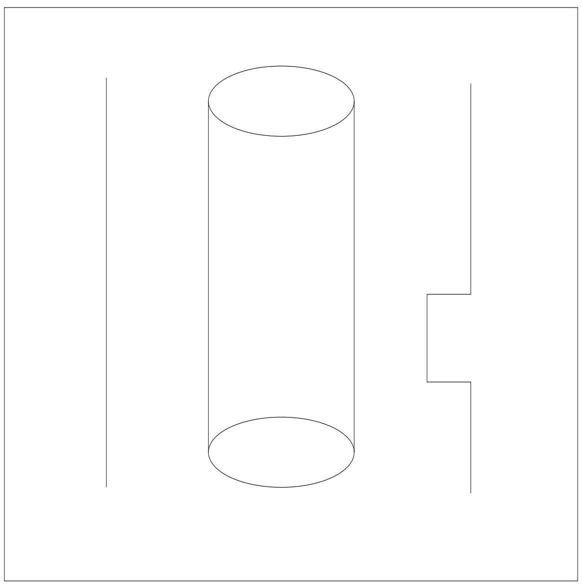
Electric Field **Magnetic Field**

TOWARDS CERTIFIED SOLUTIONS

Sources of Error

Computational Problem

Approx. Geometry
Maxwell's Equations
Perfect Instruments
Boundary Conditions



Solution is Not Exact Due to Errors in:

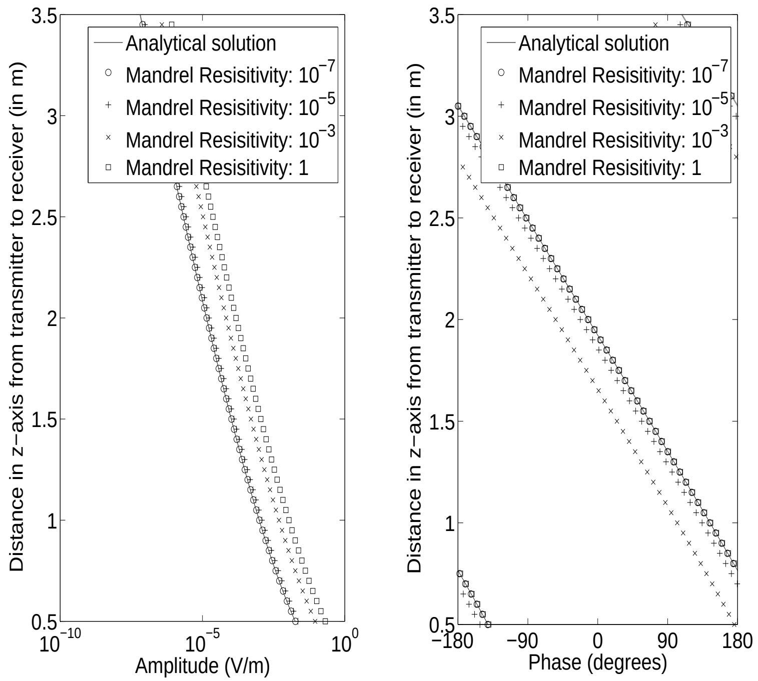
- **Mathematical Modeling**
- **Geometry and BC's**
- **The Code (Bugs)**
- **Discretization**

TOWARDS CERTIFIED SOLUTIONS

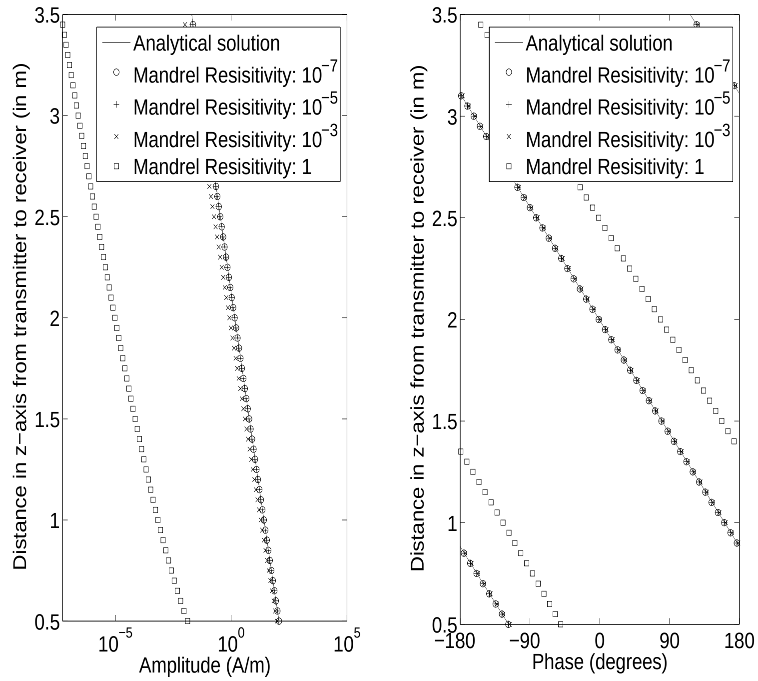
Avoiding Errors in the Code Using Benchmarking Examples

Solutions in a Homogeneous Lossy ($1\ \Omega\ m$) Media (2 Mhz) in Presence of a Conductive Mandrel

Solenoid Antenna **Toroid Antenna**



Electric Field



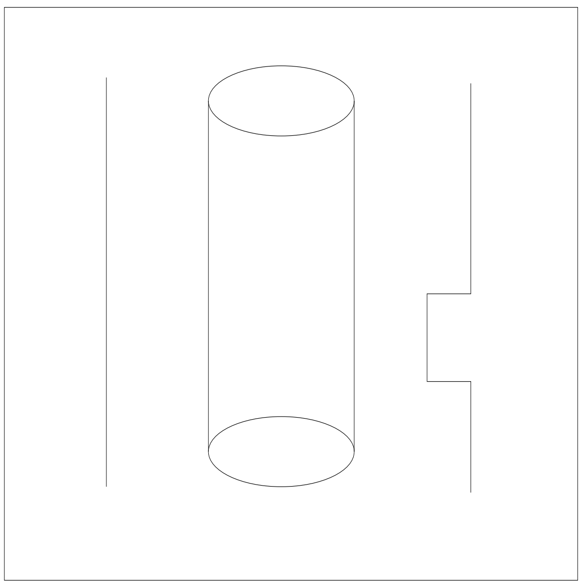
Magnetic Field

TOWARDS CERTIFIED SOLUTIONS

Summary

Computational Problem

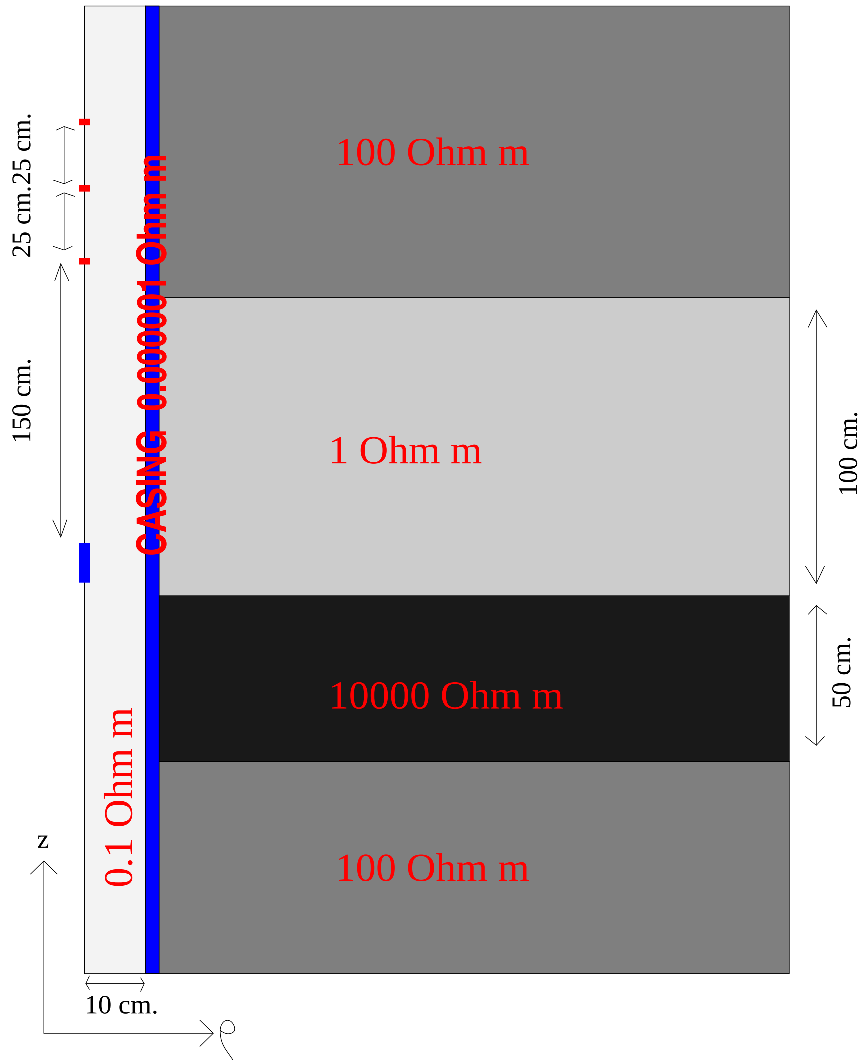
Approx. Geometry
Maxwell's Equations
Perfect Instruments
Boundary Conditions



Solution is Not Exact Due to Errors in:

- **Mathematical Modeling**
- **Geometry and BC's**
- **The Code (Bugs)**
- **Discretization**

THROUGH CASING RESISTIVITY INSTRUMENTS



Axisymmetric 3D problem.

Five different materials.

Size of computational domain: SEVERAL MILES.

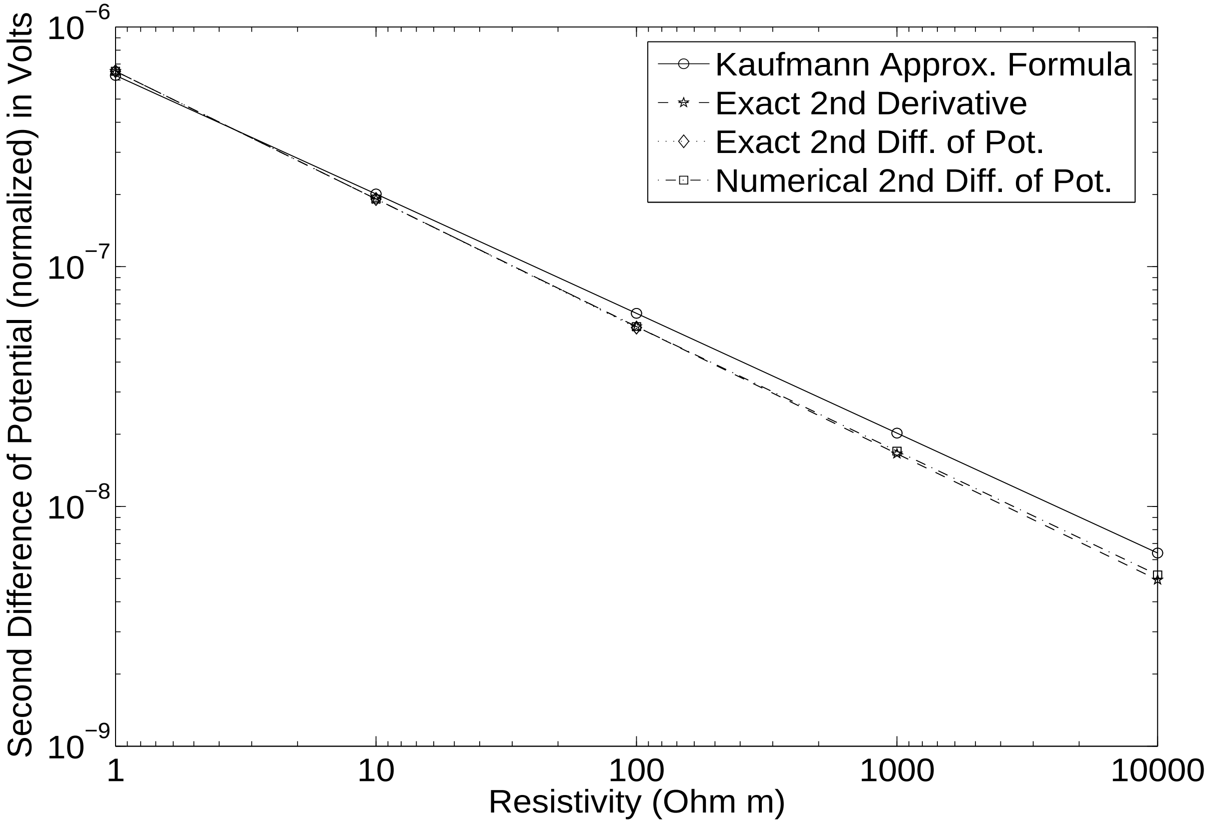
Material properties varying by up to TEN orders of magnitude (10000000000!!!).

Objective: Determine Second Difference of Potential Receiving Electrodes.

THROUGH CASING RESISTIVITY INSTRUMENTS

Logging Through Casing (Benchmark Problem)

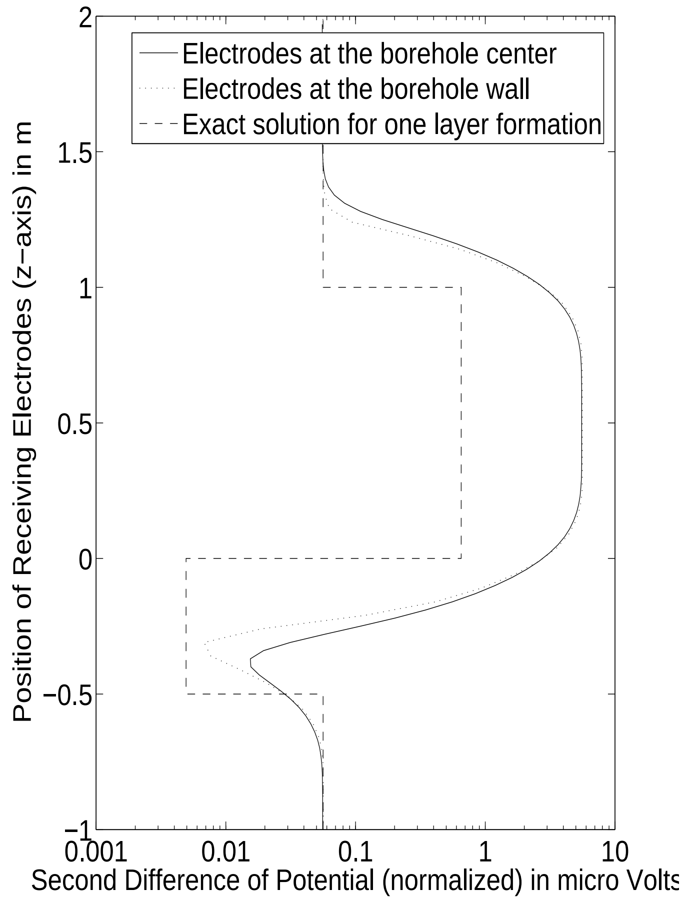
Rock Formation: Homogeneous Media

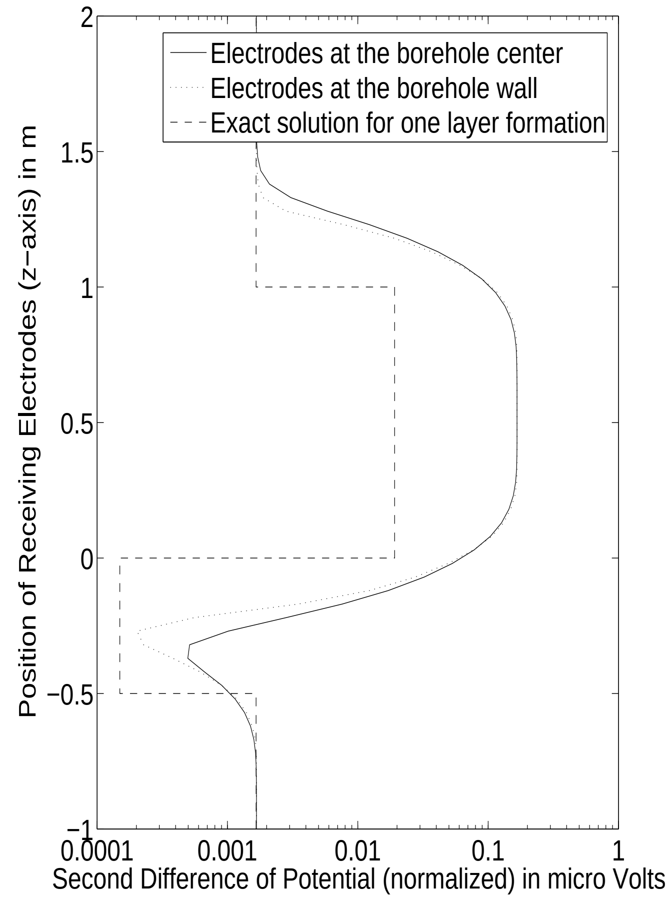


The second vertical difference of the Electric Potential is proportional to the formation conductivity.

THROUGH CASING RESISTIVITY INSTRUMENTS

Final Log Obtained by Our Finite Element Software

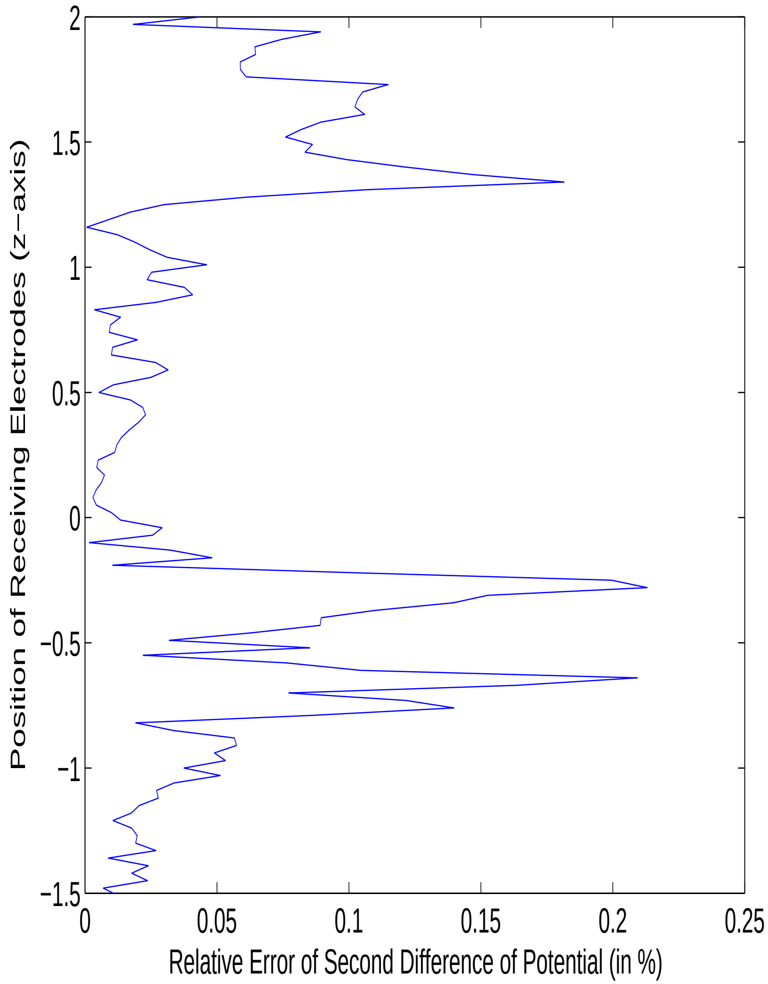


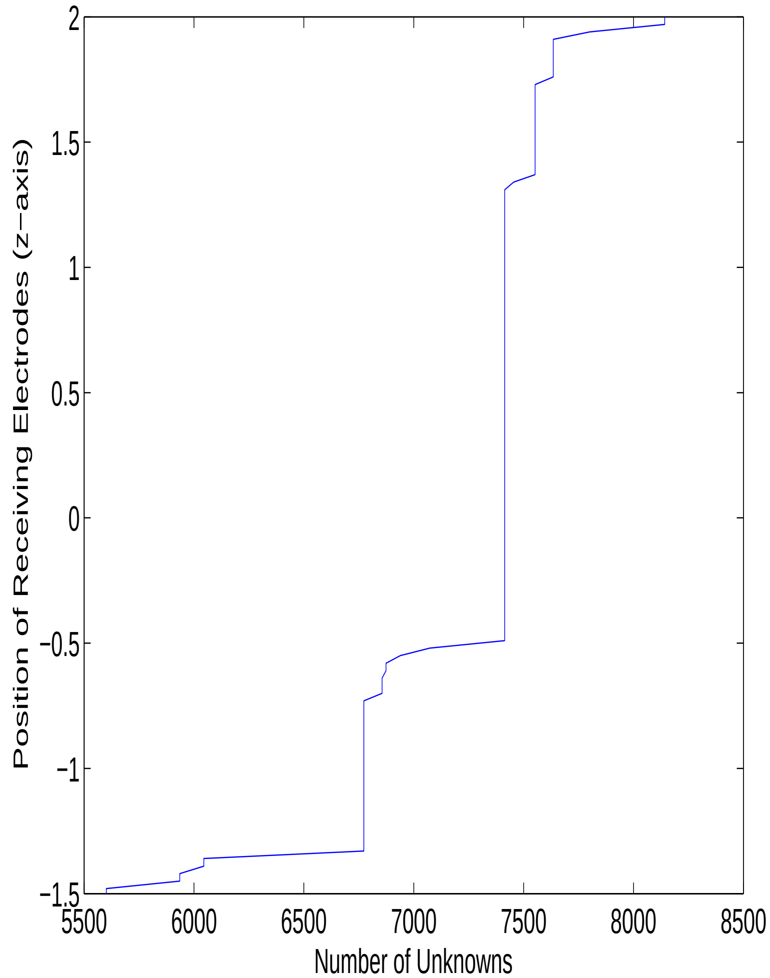


Resistivity of casing $= 10^{-6}$ **Resistivity of casing** $= 10^{-7}$

THROUGH CASING RESISTIVITY INSTRUMENTS

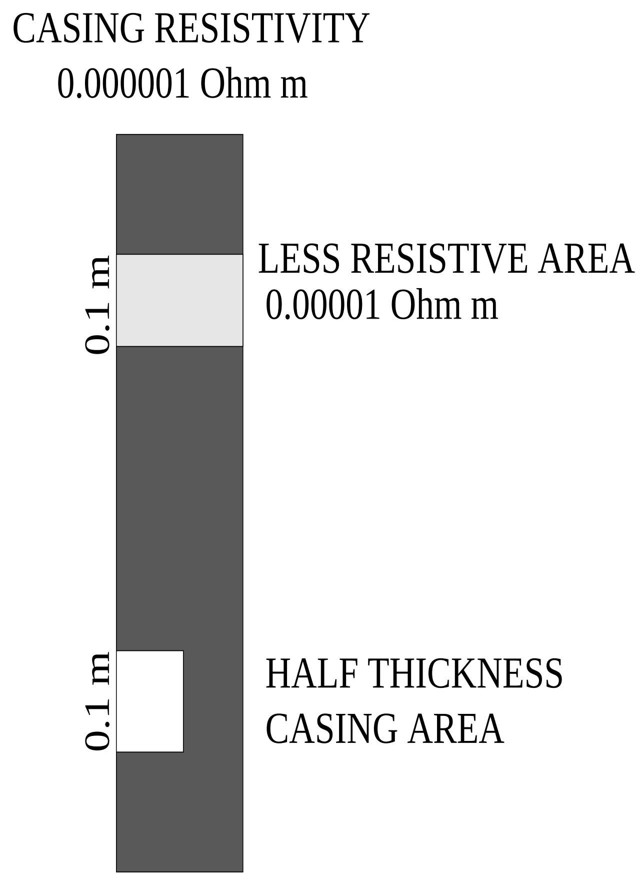
Approximation Error



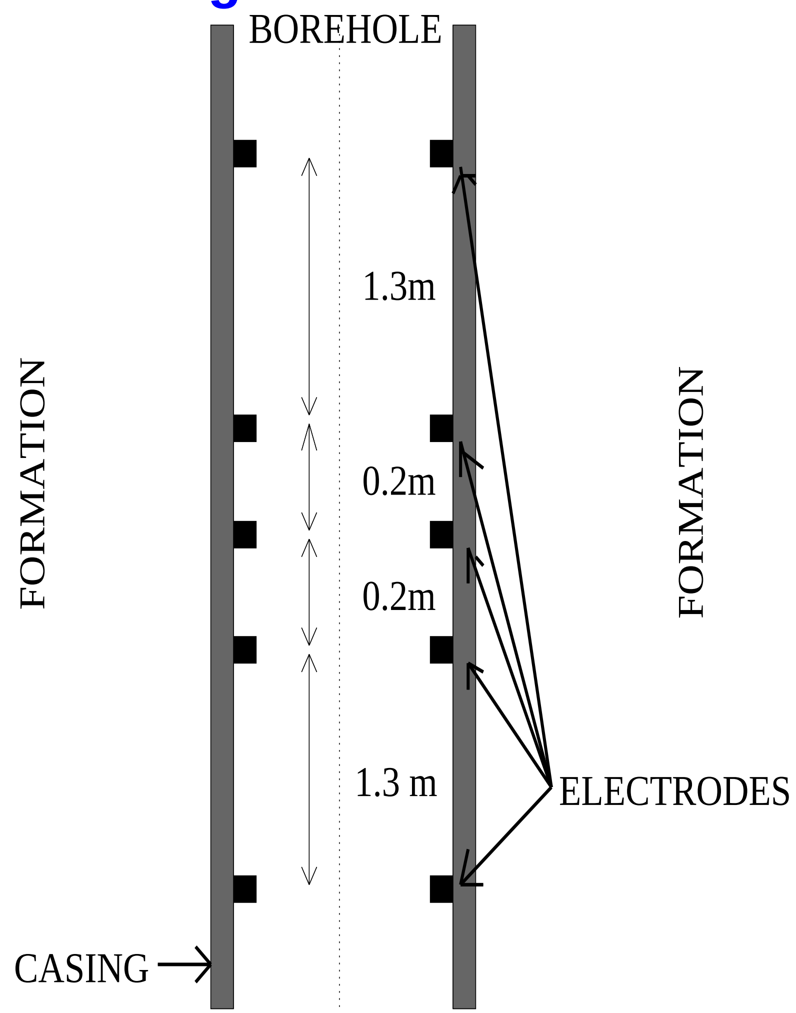


THROUGH CASING RESISTIVITY INSTRUMENTS

Damaged Casing

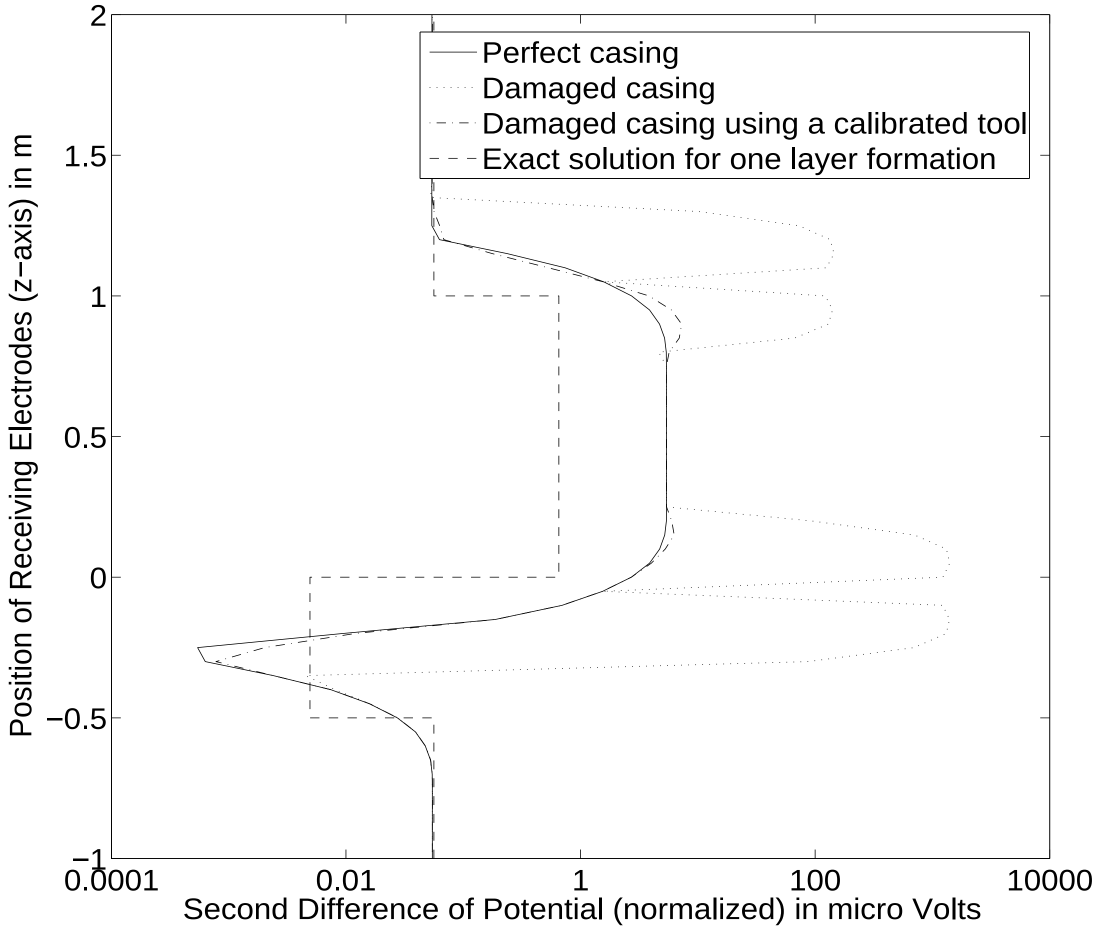


Imperfect Casing



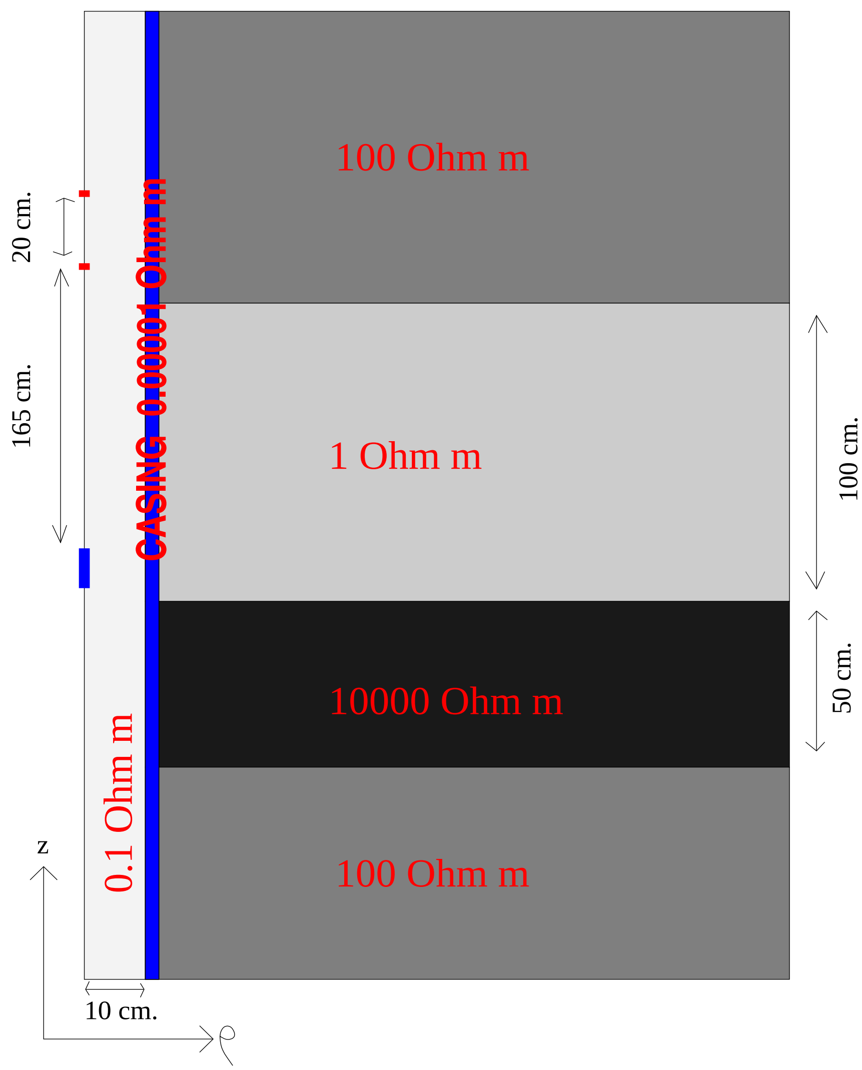
THROUGH CASING RESISTIVITY INSTRUMENTS

Damaged Casing



In the presence of damaged casing, the use of calibrated instruments is essential.

THROUGH CASING RESISTIVITY INSTRUMENTS



Axisymmetric 3D problem.

Toroid Antennas.

Size of computational domain: SEVERAL MILES.

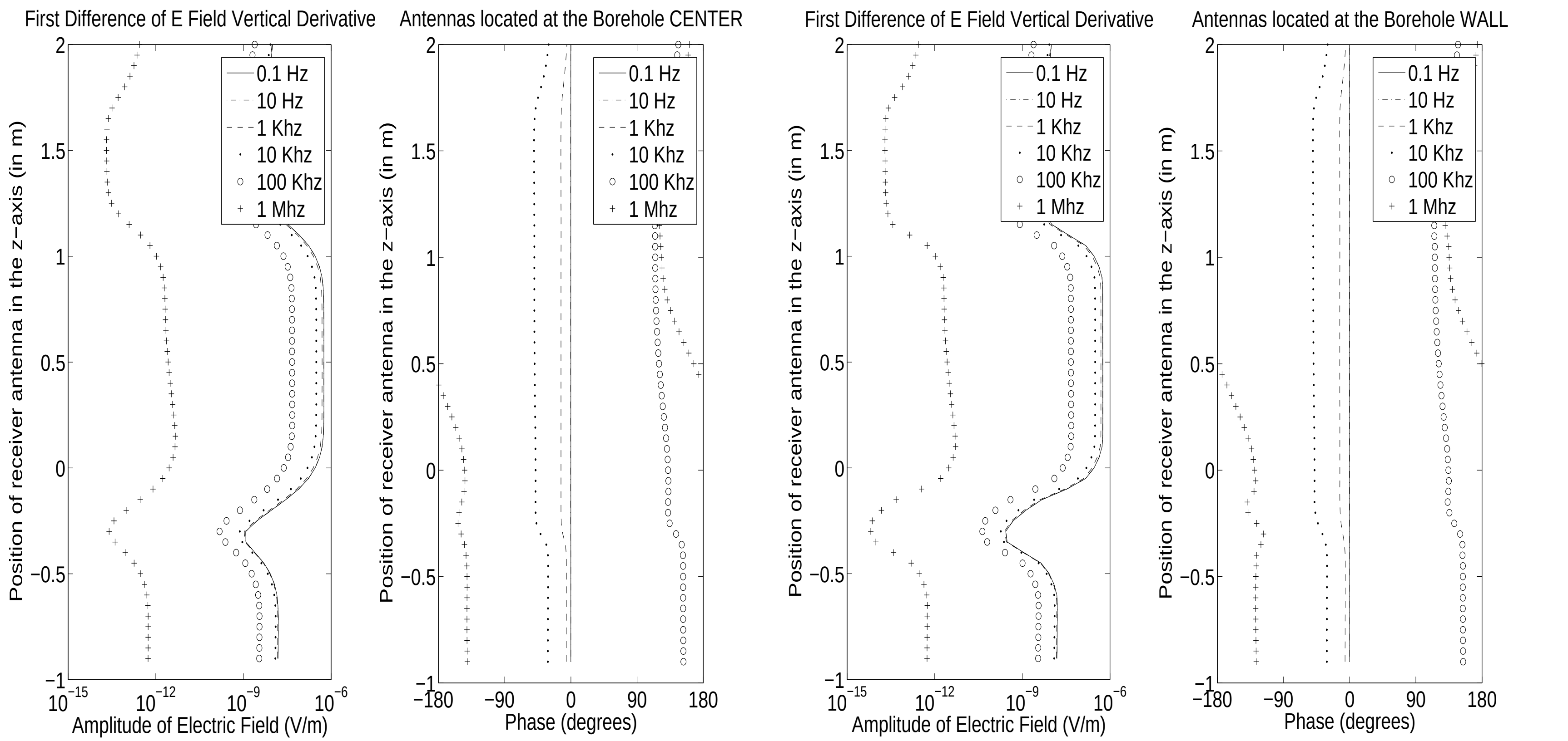
Different frequencies.

Material properties varying by up to NINE orders of magnitude (1000000000!!!).

Objective: Determine First Difference of Electric and Magnetic Fields.

THROUGH CASING RESISTIVITY INSTRUMENTS

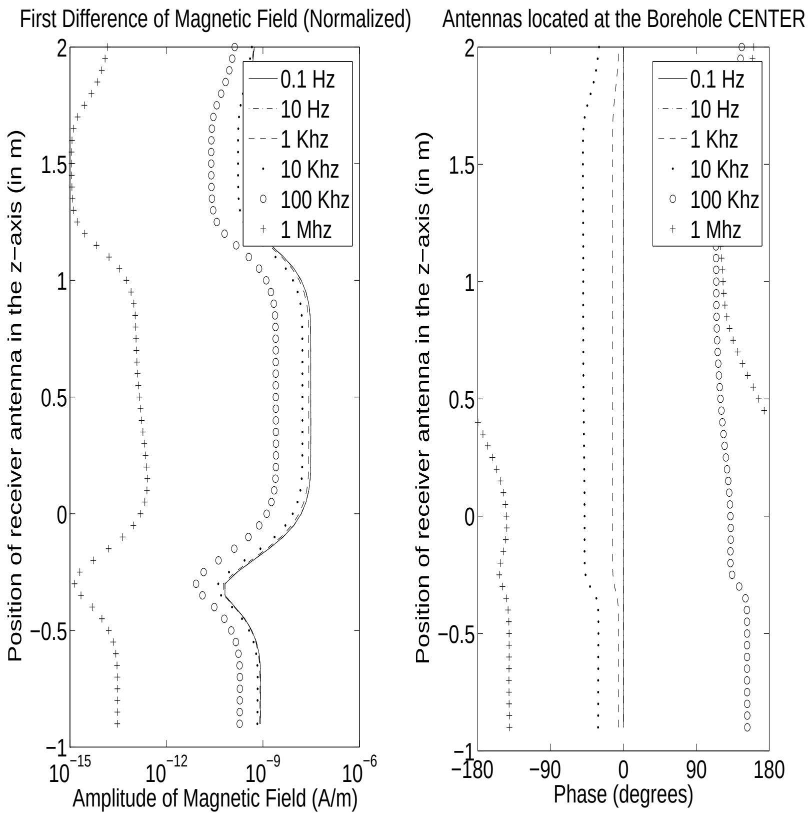
First Difference of Electric Field at Different Frequencies

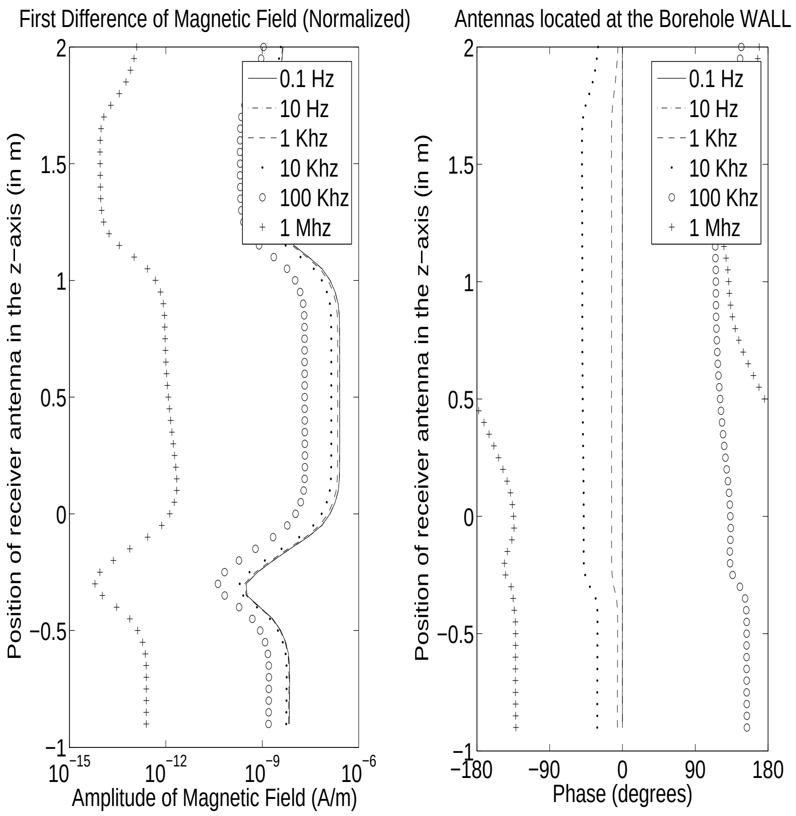


Toroid antennas are more sensitive to the rock formation resistivity when located on the borehole's wall

THROUGH CASING RESISTIVITY INSTRUMENTS

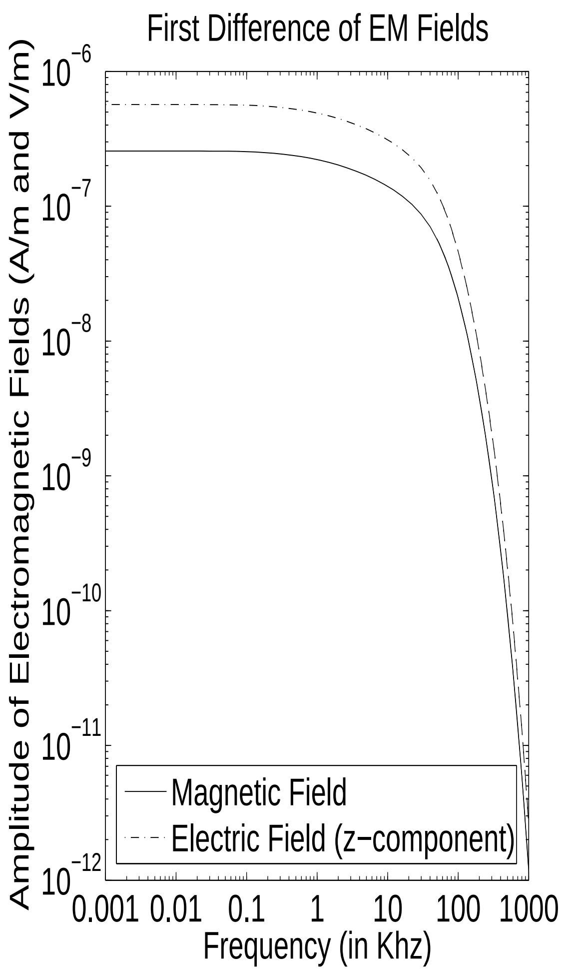
First Difference of Magnetic Field at Different Frequencies

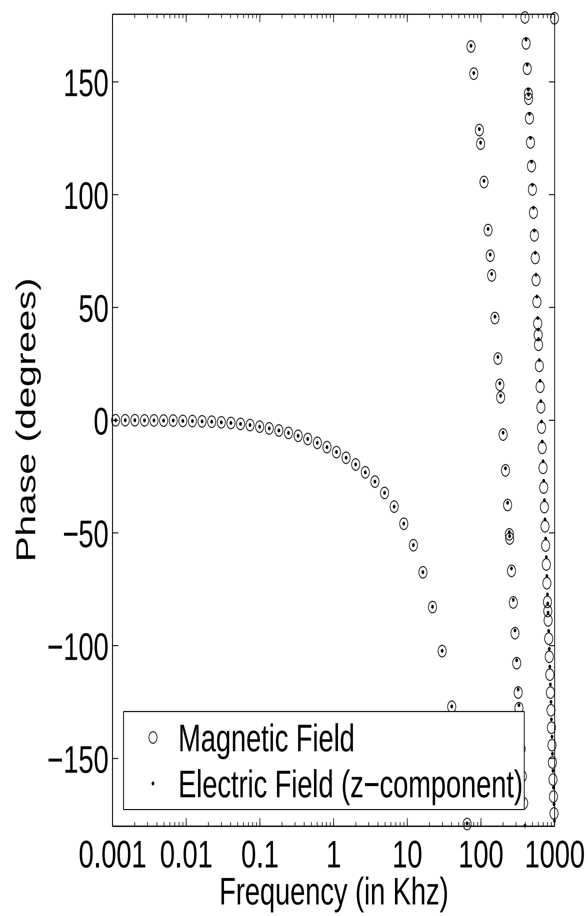




THROUGH CASING RESISTIVITY INSTRUMENTS

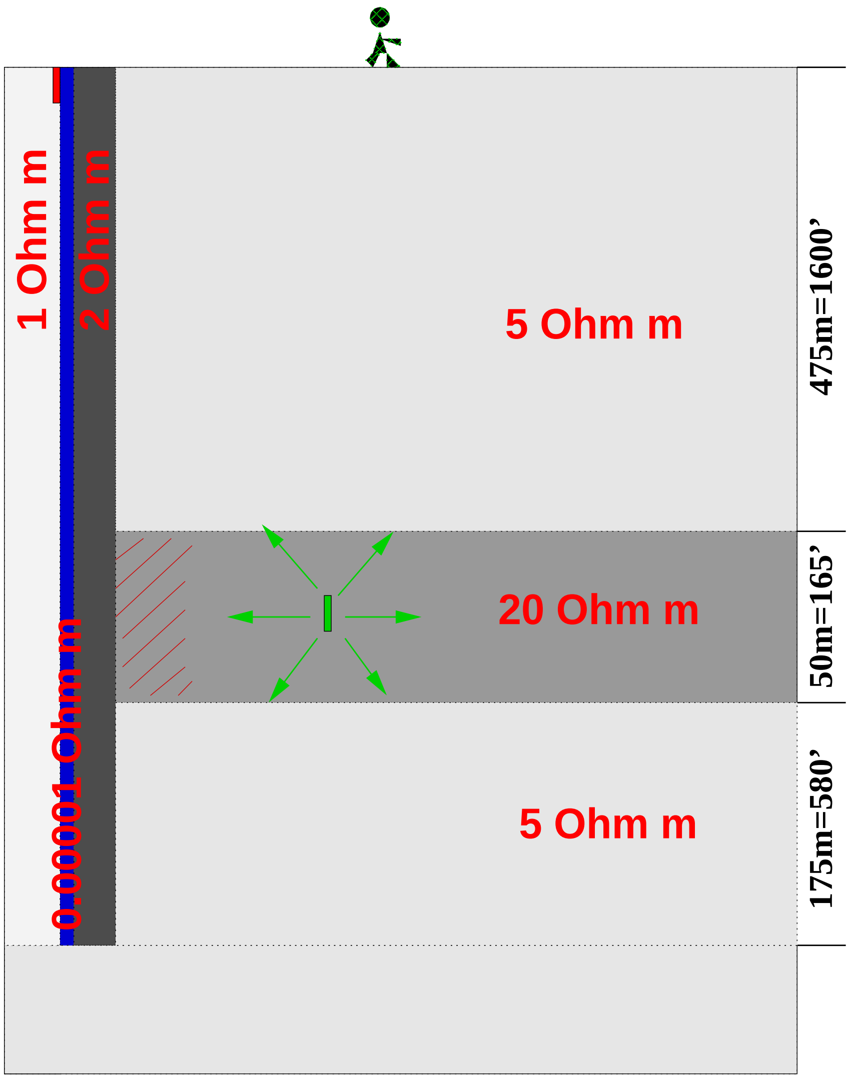
Electromagnetic Fields at Different Frequencies





Electromagnetic Fields are almost constant for frequencies below 1 kHz. A sudden drop in the amplitude occurs at frequencies above 20 kHz.

THROUGH CASING RESISTIVITY INSTRUMENTS



5.5" Borehole radio ; 0.5" Casing ; 2" Cement

Axisymmetric 3D problem.

Five different materials.

Different positions of receiving antenna.

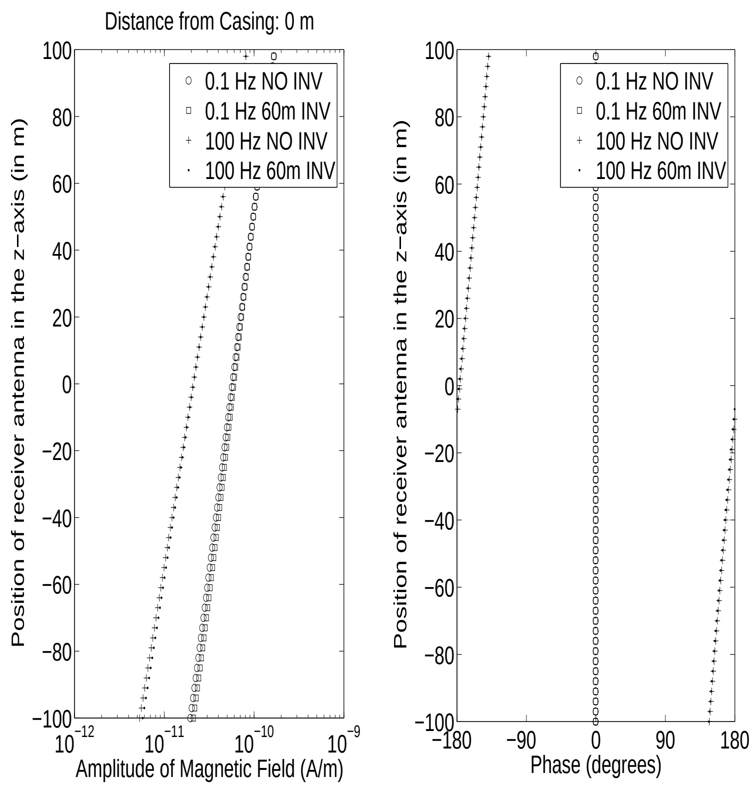
Toroidal Transmitter and receiver separated by up to half a mile.

Objective: Determine First Difference of Potential Receiving Electrodes.

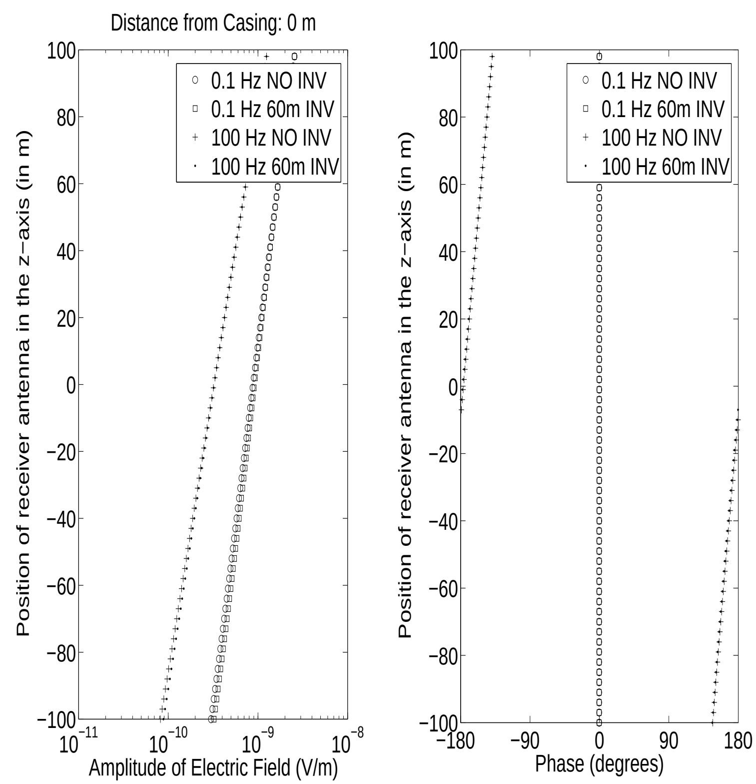
THROUGH CASING RESISTIVITY INSTRUMENTS

Final Log Obtained by Our Finite Element Software

Magnetic Field



Electric Field

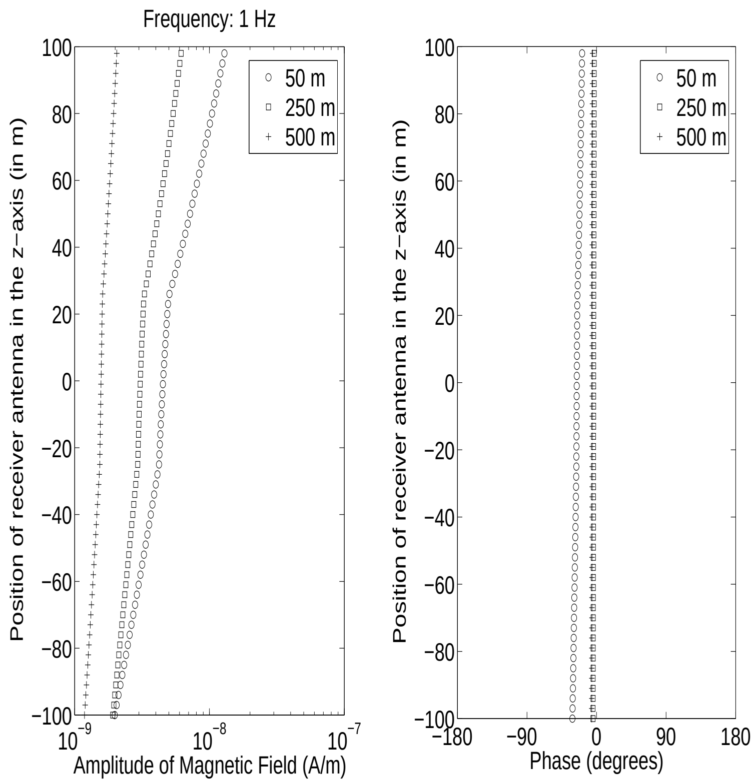


If the transmitter antenna is located on the surface of a cased well, the received EM signal within the borehole is too weak.

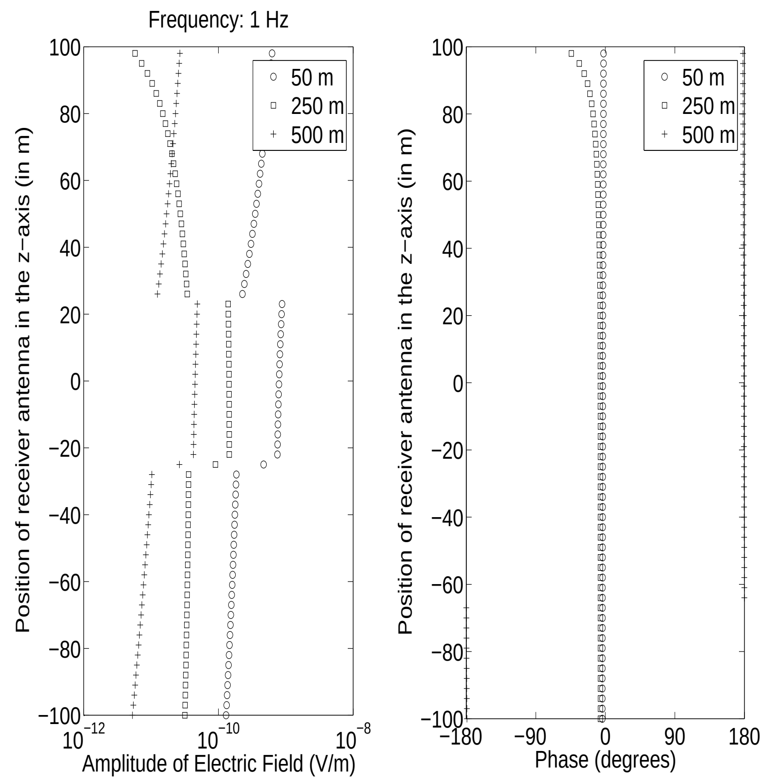
THROUGH CASING RESISTIVITY INSTRUMENTS

Final Log Obtained by Our Finite Element Software

Magnetic Field



Electric Field

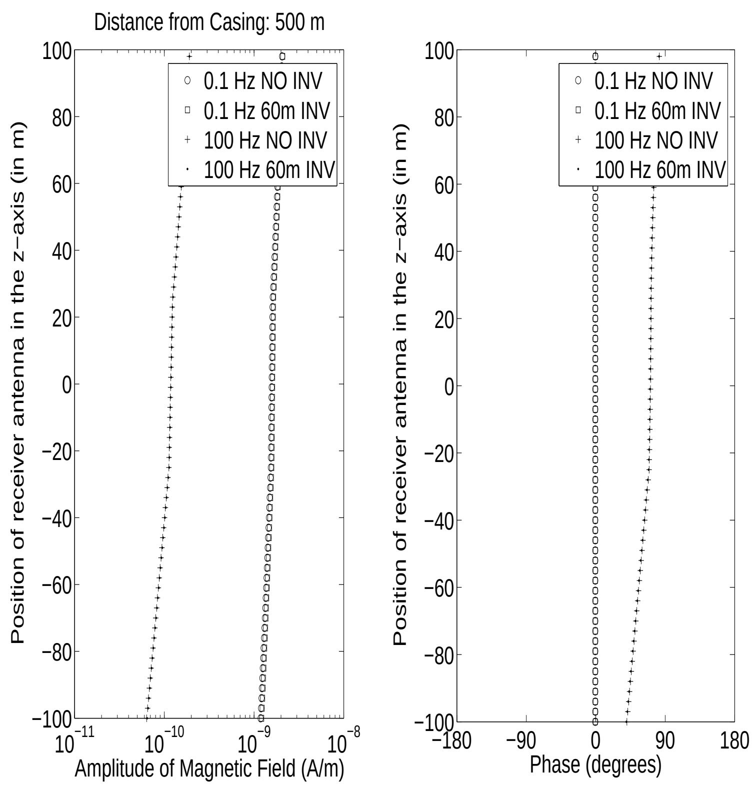


If the transmitter antenna is located on the surface of a cased well, the received cross-well EM signal is too weak.

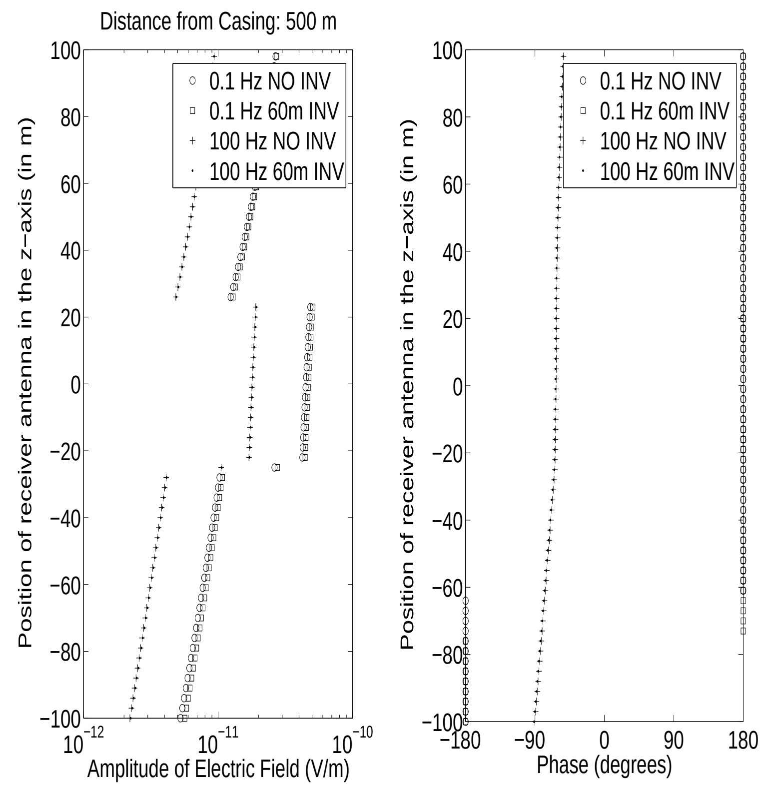
THROUGH CASING RESISTIVITY INSTRUMENTS

Final Log Obtained by Our Finite Element Software

Magnetic Field



Electric Field

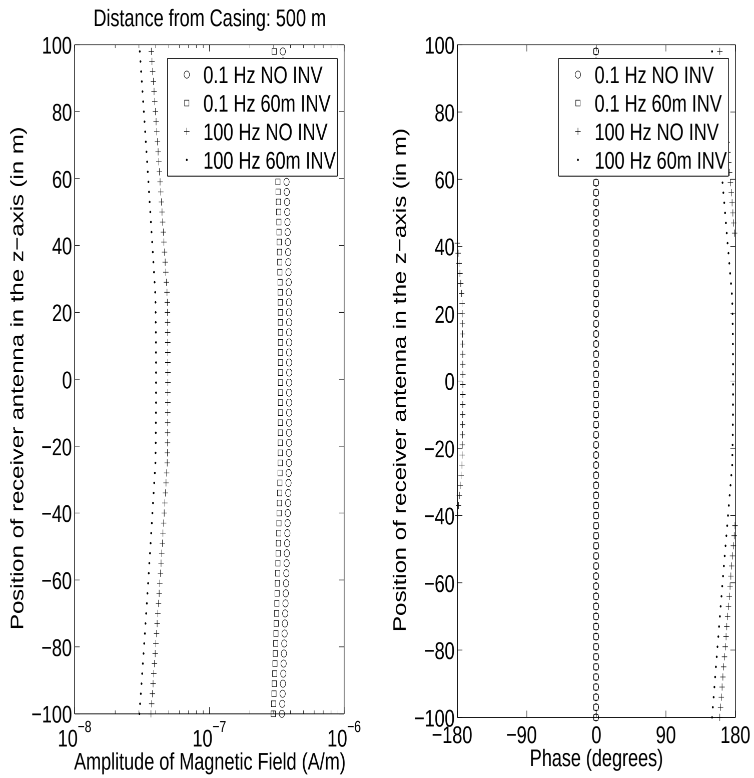


If the transmitter antenna is located on the surface of a cased well, a 60 meters layer of water cannot be detected by using cross-well EM signals.

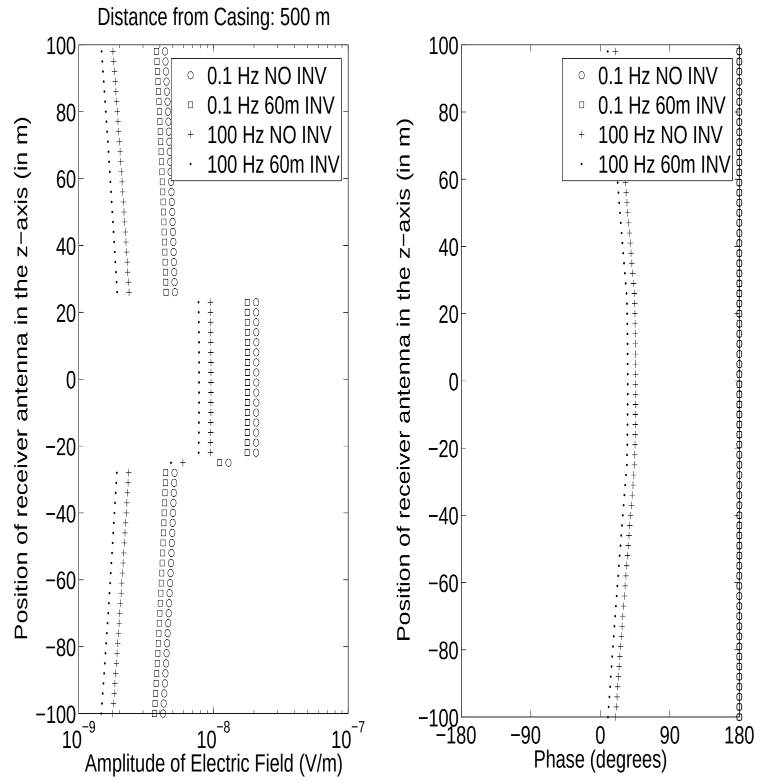
THROUGH CASING RESISTIVITY INSTRUMENTS

Final Log Obtained by Our Finite Element Software

Magnetic Field

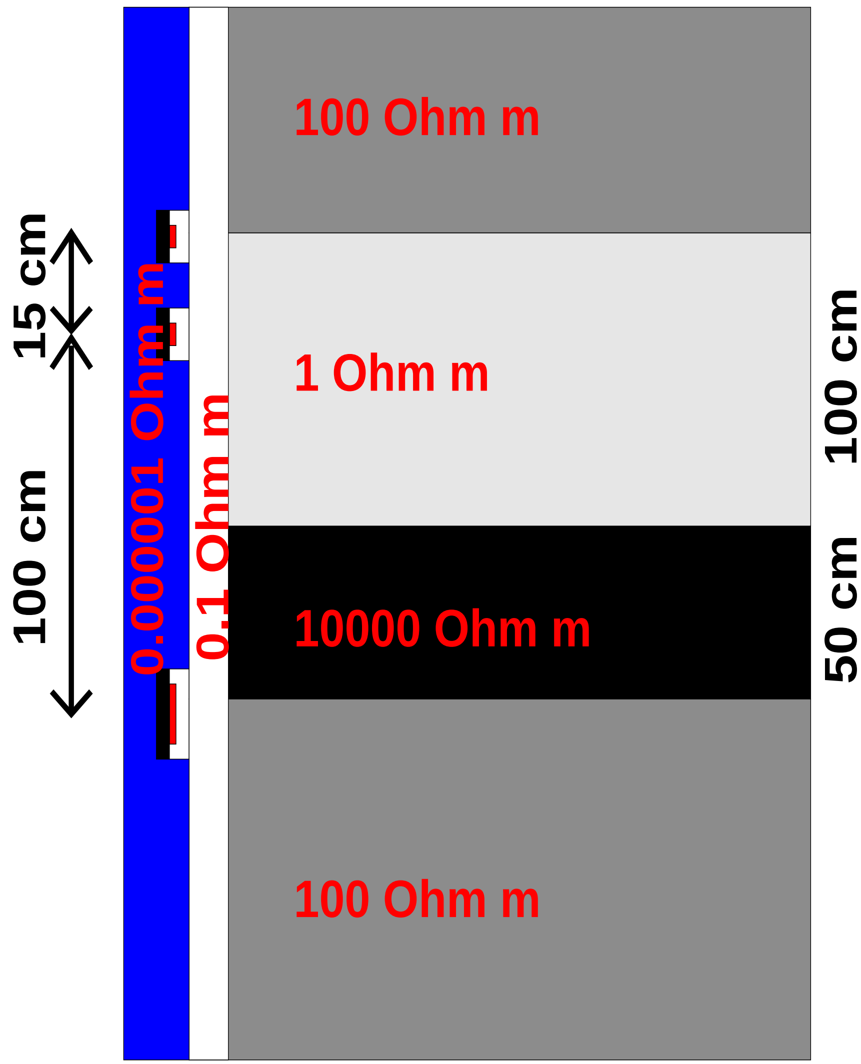


Electric Field

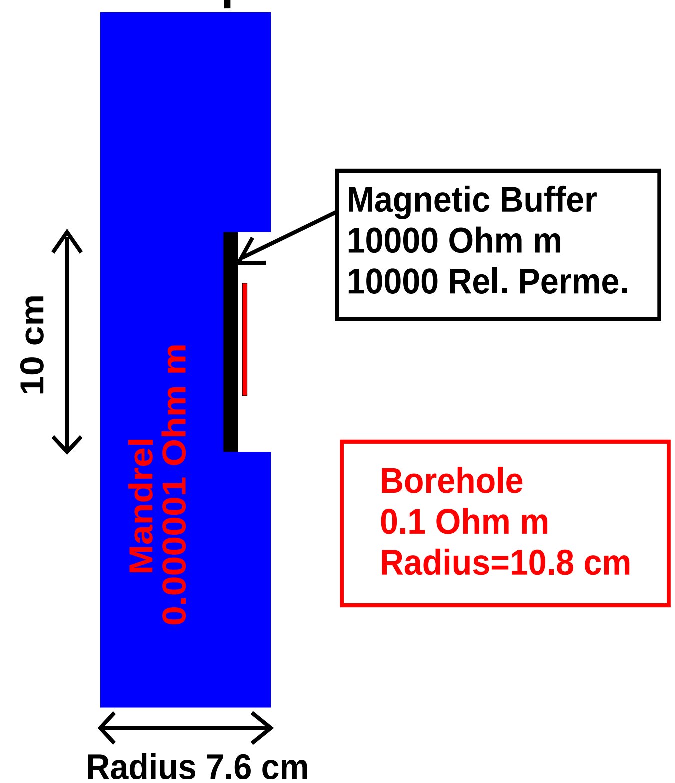


If the transmitter antenna is located downhole a cased well, it is feasible to perform meaningful cross-well EM measurements.

LOGGING INSTRUMENTS WITH A MANDREL



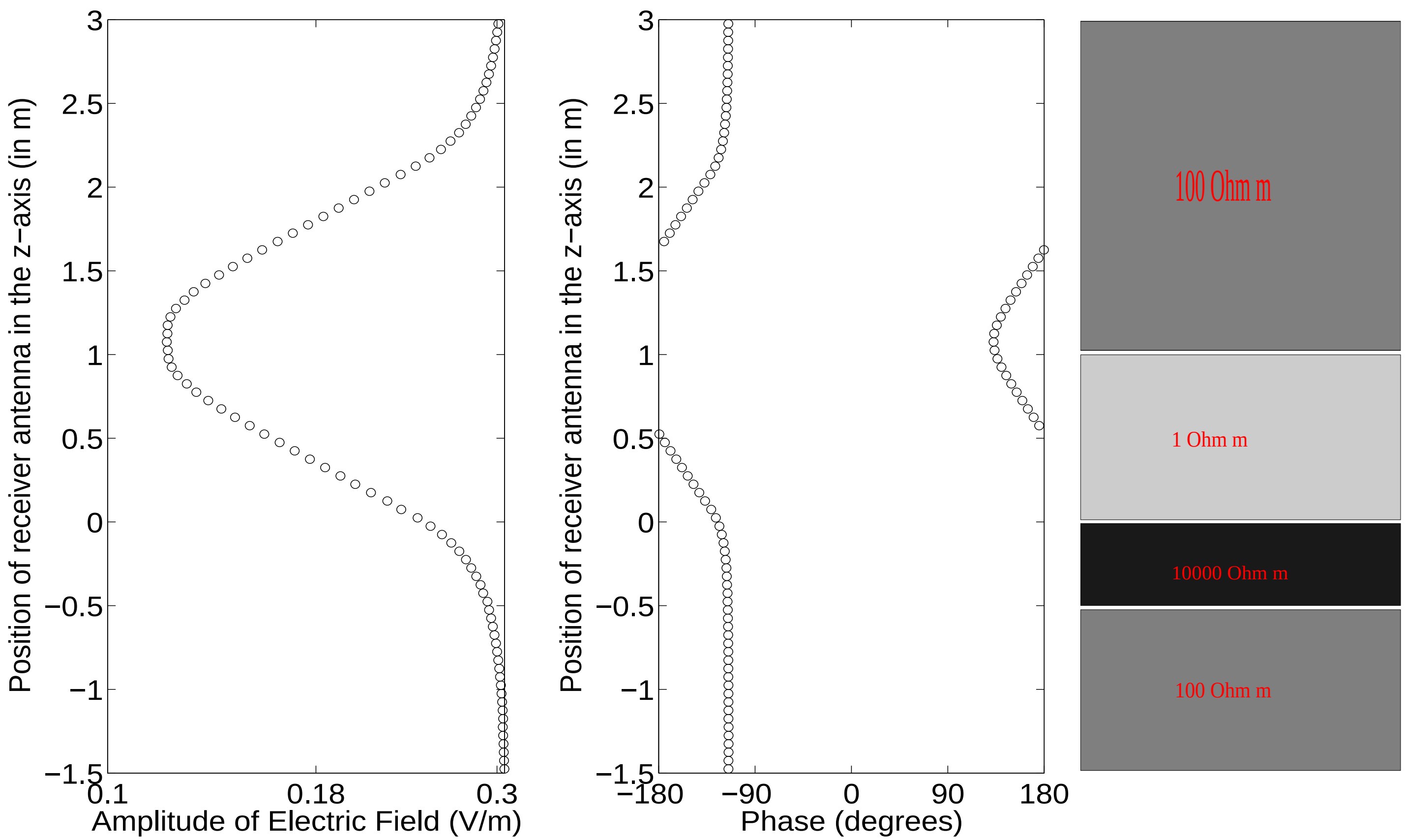
Description of Antennas



Goal: To Compute First Difference of Potential on Receiving Antennas

LOGGING INSTRUMENTS WITH A MANDREL

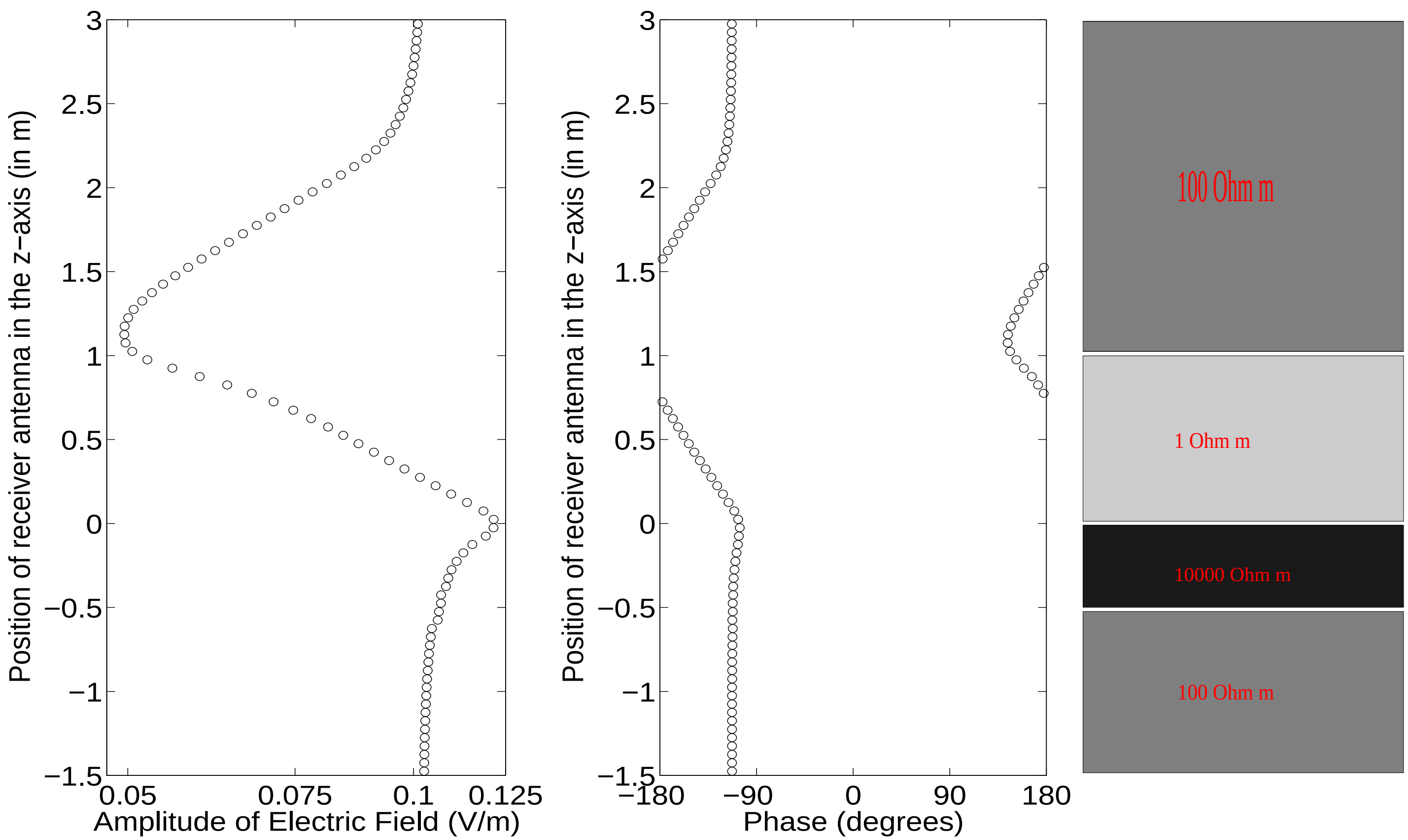
E_ϕ (normalized) for a solenoid antenna



Frequency: 2 Mhz

LOGGING INSTRUMENTS WITH A MANDREL

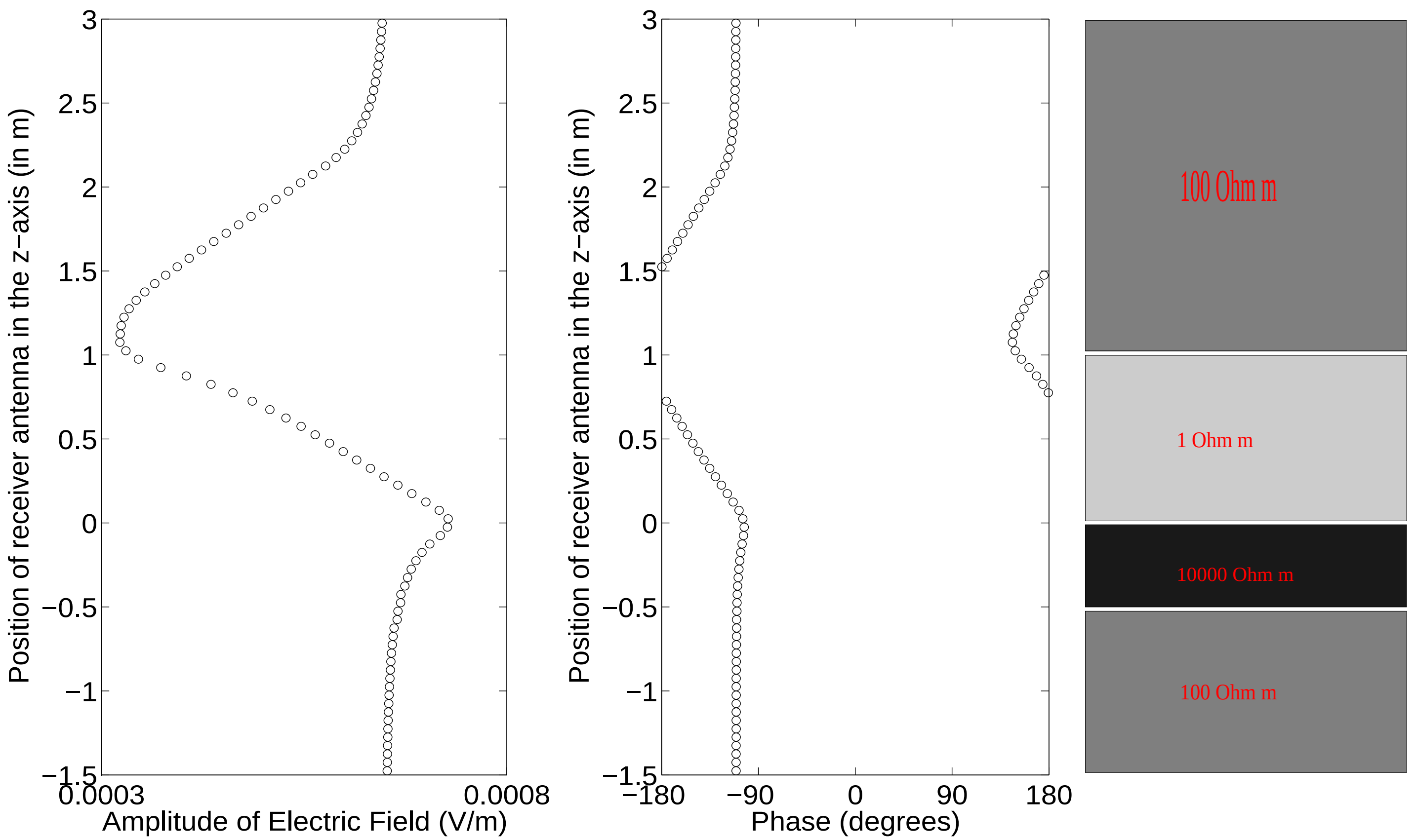
First Difference of E_ϕ (normalized) for a solenoid antenna



Frequency: 2 Mhz

LOGGING INSTRUMENTS WITH A MANDREL

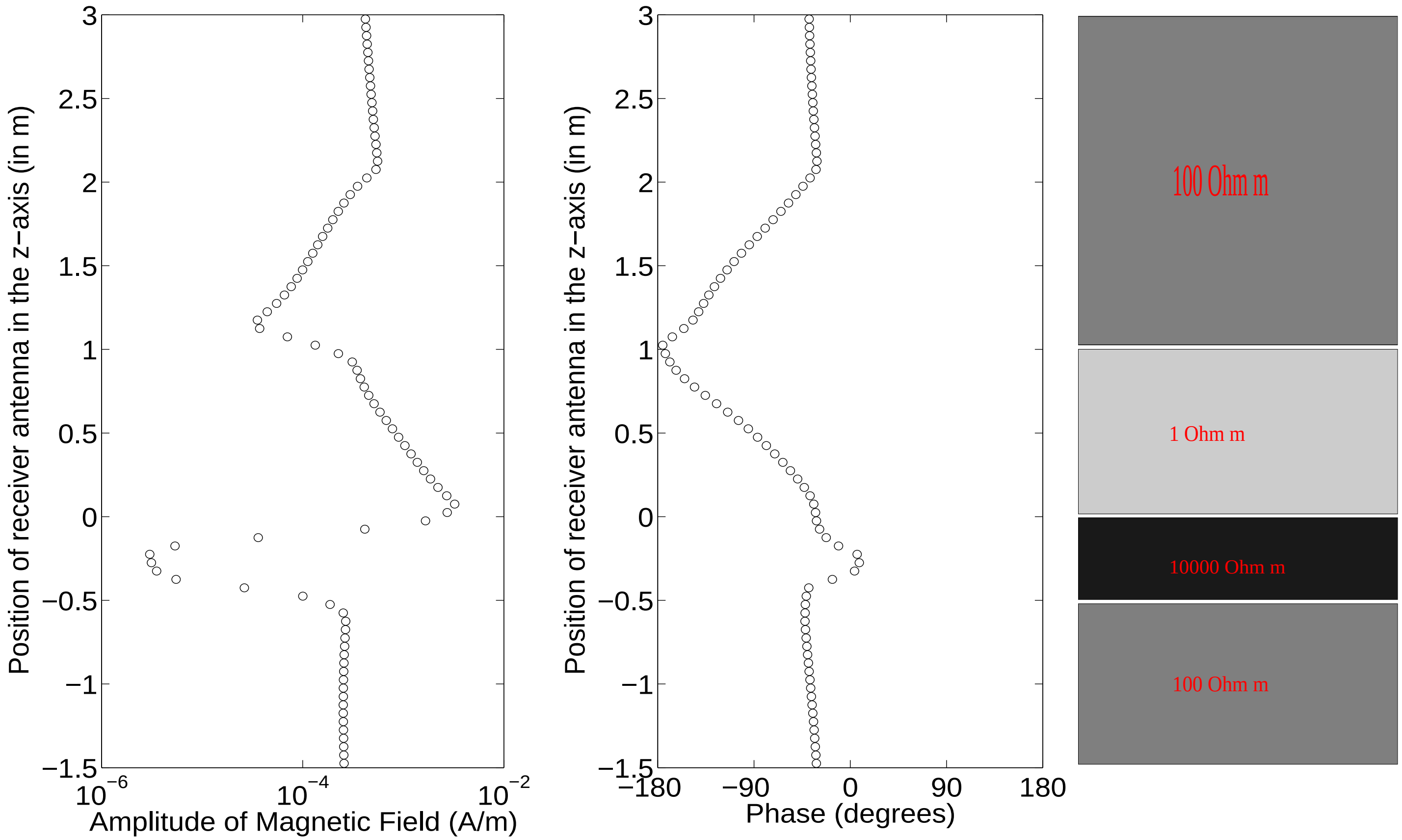
First Difference of E_ϕ (normalized) for a solenoid antenna



Frequency: 2 Mhz, NO MAGNETIC BUFFER

LOGGING INSTRUMENTS WITH A MANDREL

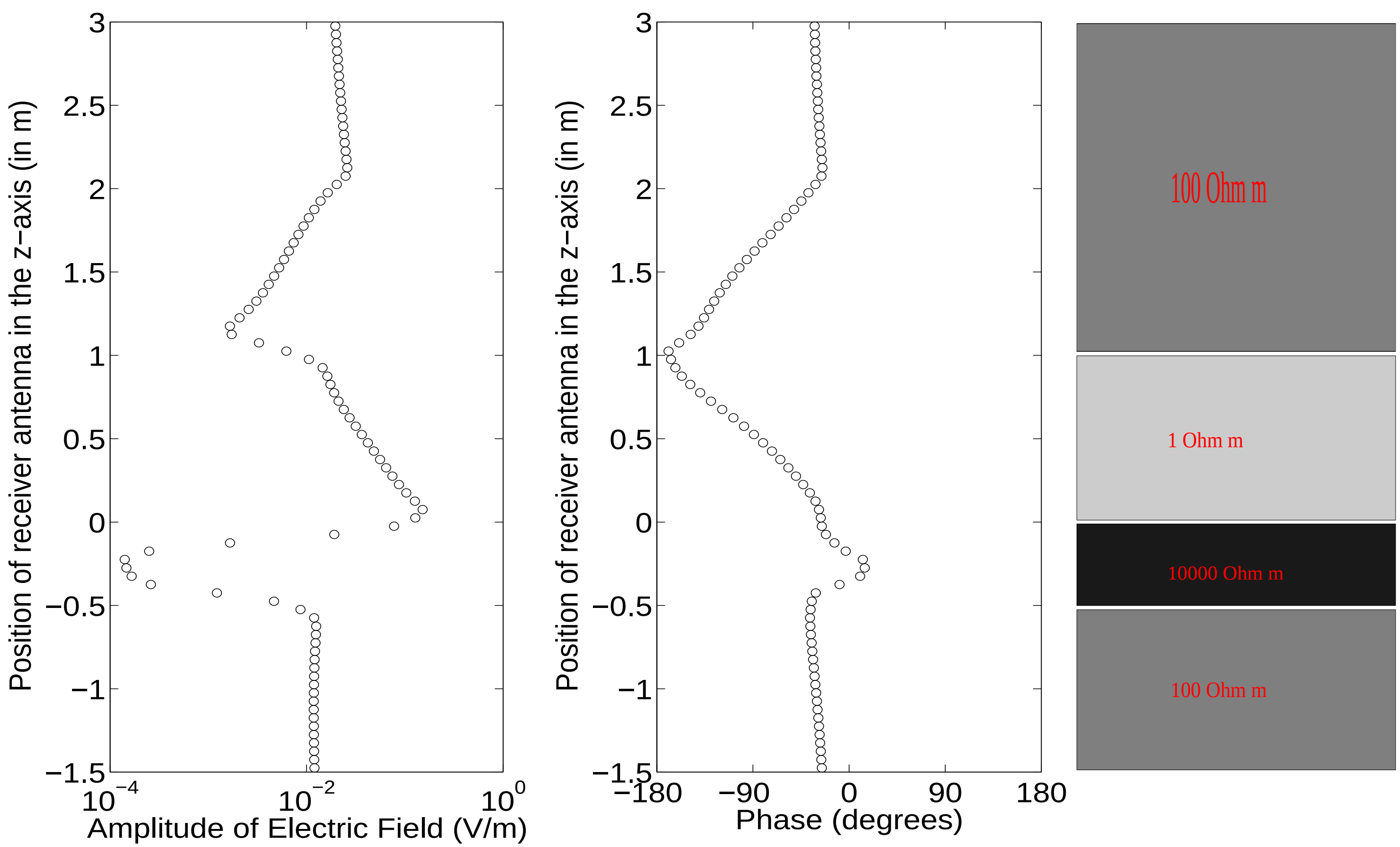
First Difference of H_ϕ (normalized) for a toroid antenna



Frequency: 2 Mhz

LOGGING INSTRUMENTS WITH A MANDREL

First Difference of E_z (normalized) for a toroid antenna



Frequency: 2 Mhz

CONCLUSIONS

- **It is possible to simulate a variety of EM logging instruments by using the self-adaptive goal-oriented hp-FEM.**

- **For TCRT, preliminary simulations suggest to**
  1. Place antennas on the borehole's wall,
  2. use calibrated instruments,
  3. use low frequencies (typically below 5 kHz),
  4. use downhole antennas for cross-well EM measurements, and
  5. avoid the use of cross-well EM measurements for assesment of water injection.

- **For LWD instruments, preliminary simulations suggest to**
  1. Use both solenoid and toroid antennas,
  2. use magnetic buffers to amplify EM signals, and
  3. compute first differences of EM fields.

FUTURE WORK

Within the next 3 months

- Implementation of the goal-oriented self-adaptive algorithm for 2D edge elements. It would allow us to solve 2.5 D problems.
- Parallel implementation of the 2D code (Maciek Paszynski).

Long term goals

- Solve 3D problems with casing at DC.
- Solve 3D problems with mandrel at AC.
- Invert coupled sonic and EM measurements in 2D.