2005 Finite Element Rodeo

A Fully Automatic Goal-Oriented hp-Adaptive Finite Element Strategy for Simulations of Resistivity Logging Instruments.

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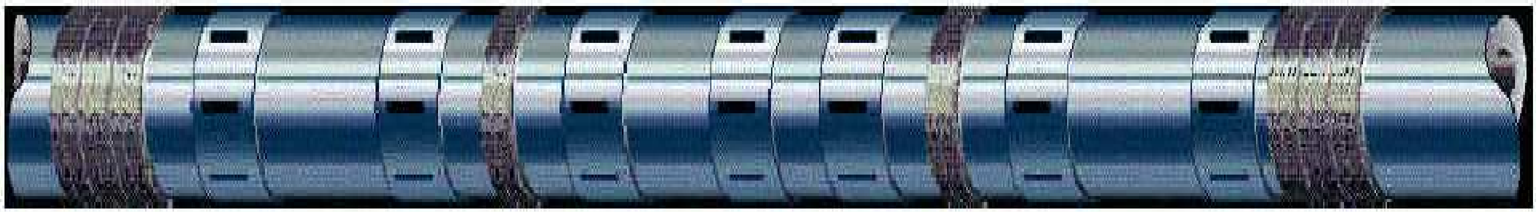
Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

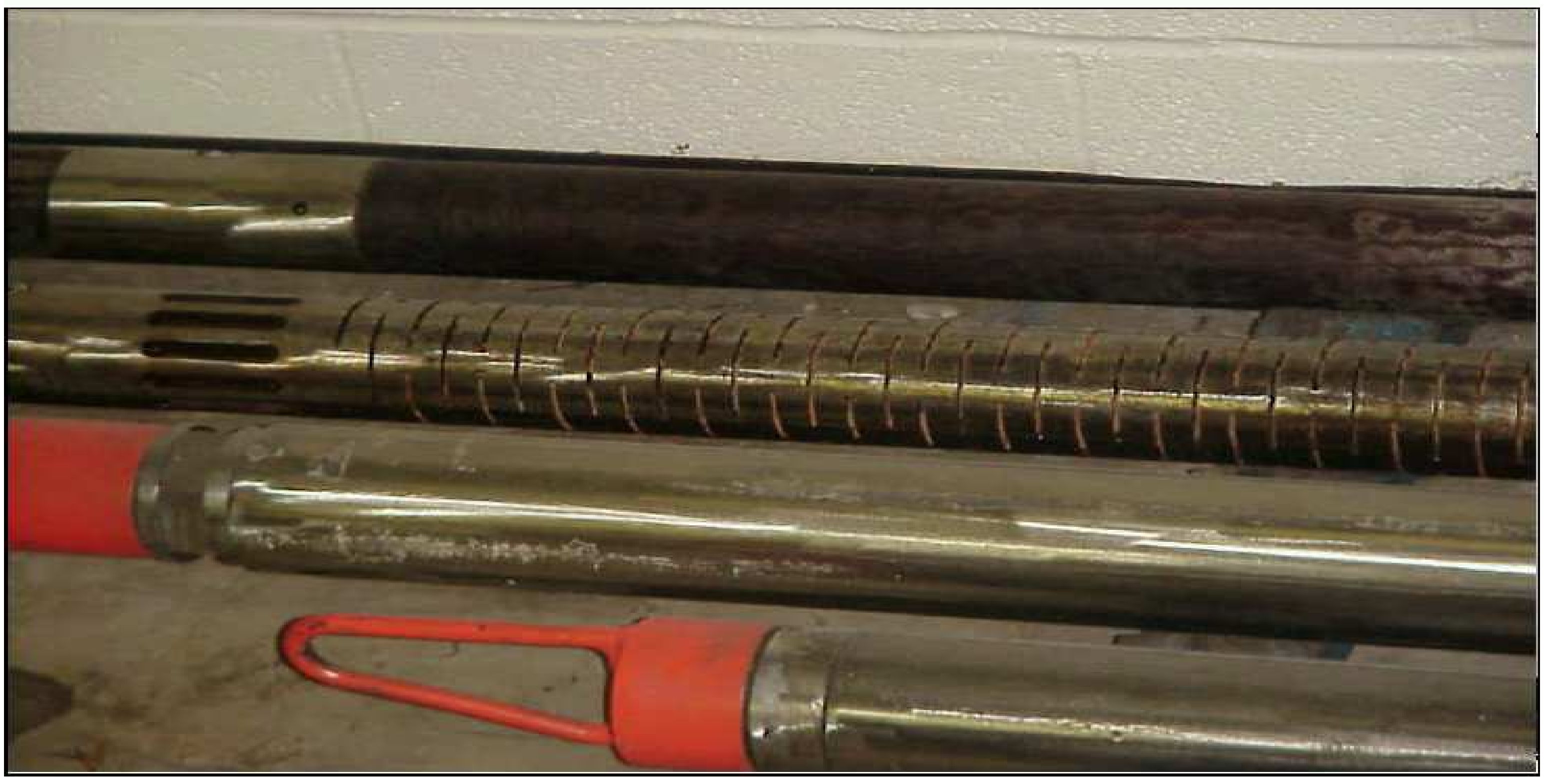
OVERVIEW

1. Overview
2. Resistivity Logging Instruments: An Introduction
3. Maxwell's Equation
4. Self-Adaptive Goal-Oriented hp-FEM
5. Numerical Results
6. Conclusions and Future Work

RESISTIVITY LOGGING INSTRUMENTS

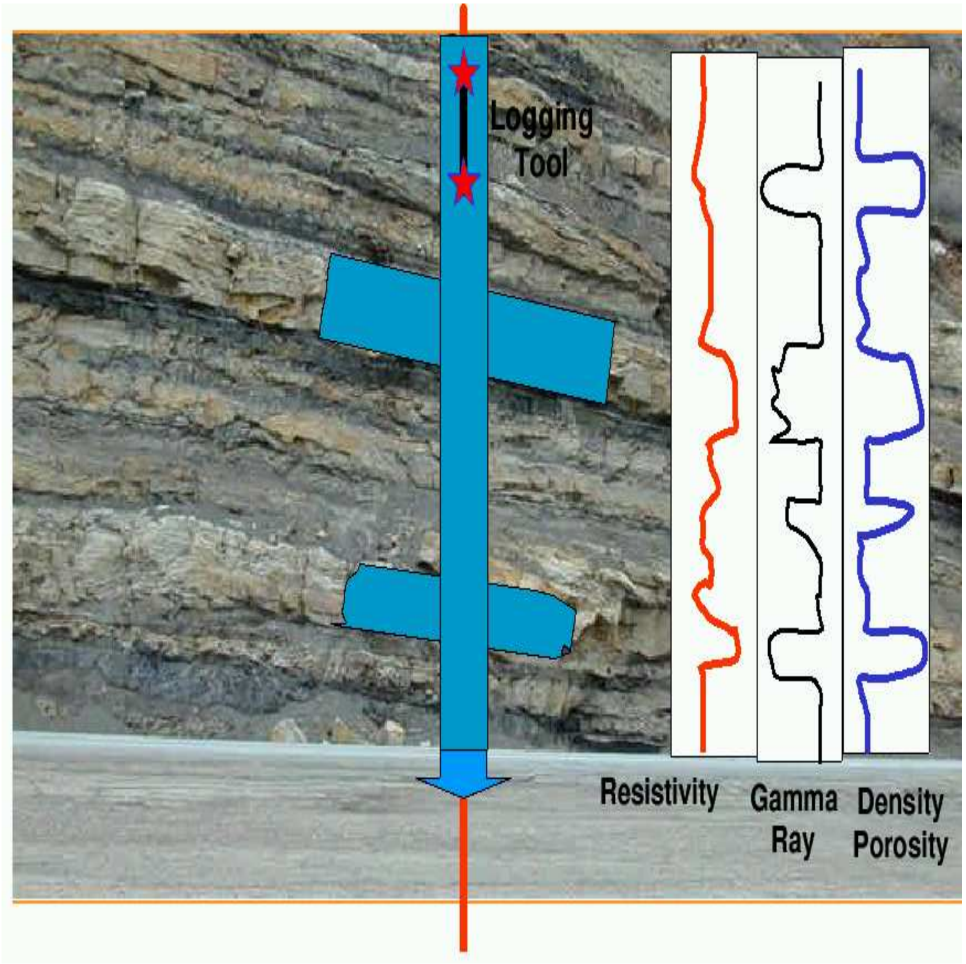
Logging Instruments: Definition





RESISTIVITY LOGGING INSTRUMENTS

Utility of Logging Instruments



OBJECTIVES: To determine

- **Payzones (oil and gas).**
- **Amount of oil/gas.**
- **Ability to extract oil/gas.**

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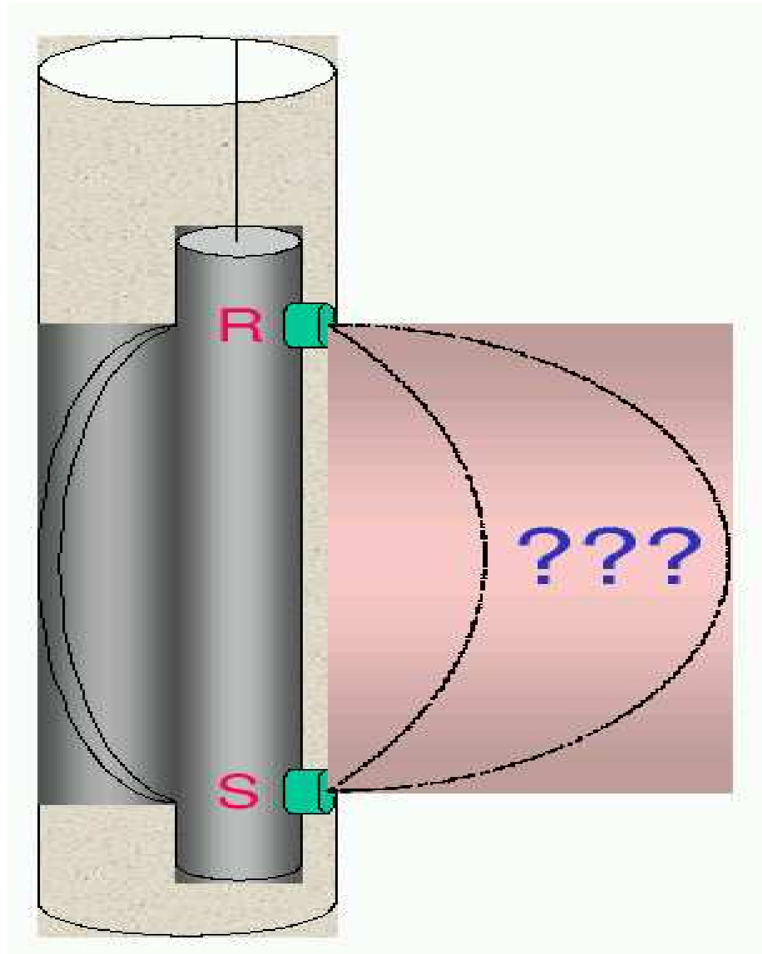
RESISTIVITY LOGGING INSTRUMENTS

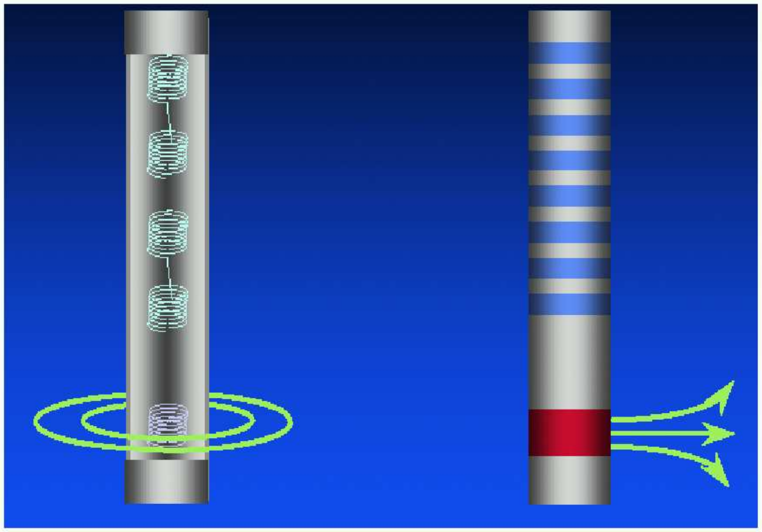
Main Objective: To Solve an Inverse Problem



A software for solving the DIRECT problem is essential in order to solve the INVERSE problem

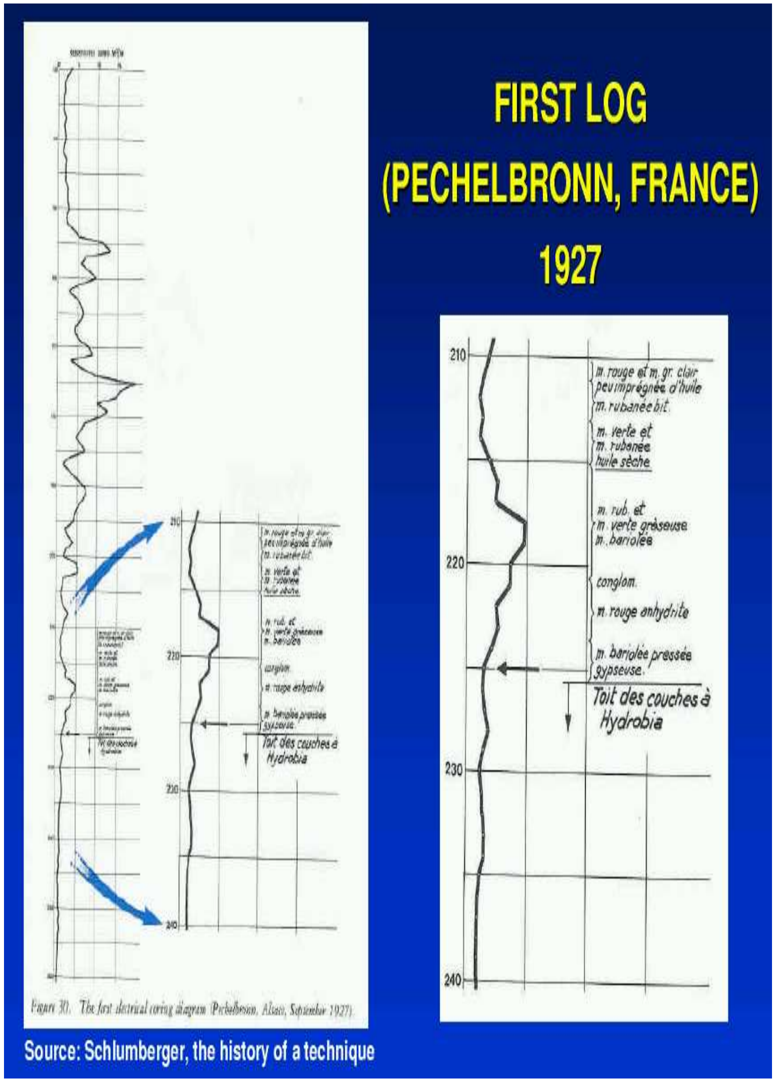
RESISTIVITY LOGGING INSTRUMENTS

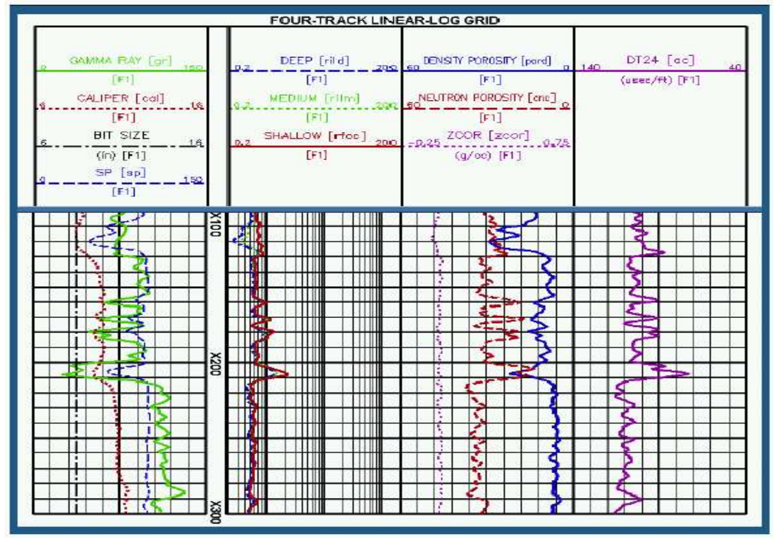
Resistivity Logging Instruments

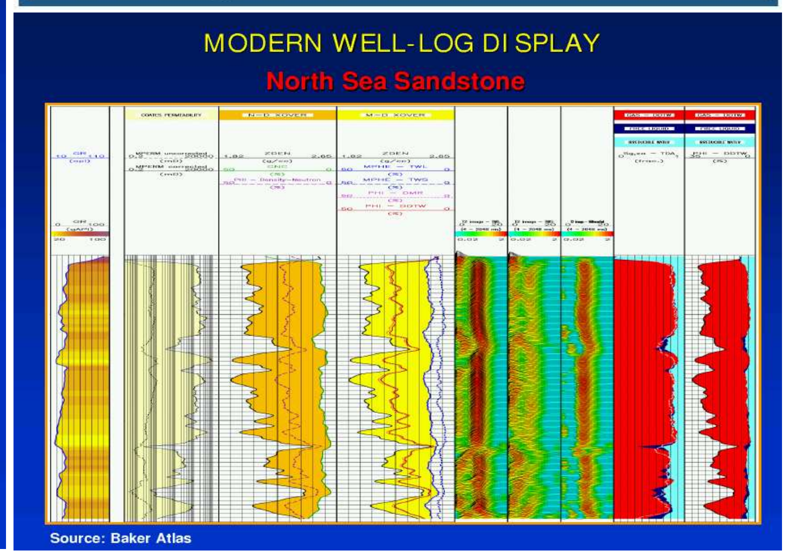


RESISTIVITY LOGGING INSTRUMENTS

Final Result Obtained from the Logging Instruments







MAXWELL'S EQUATIONS (FREQUENCY DOMAIN)

Time Harmonic Maxwell's Equations:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} + \mathbf{J}^{imp}$$

Reduced Wave Equation:

E-Formulation **H-Formulation**

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega\boldsymbol{J}^{imp} \quad ; \quad \boldsymbol{\nabla} \times \left(\frac{1}{\sigma + j\omega\epsilon}\boldsymbol{\nabla} \times \boldsymbol{H}\right) + j\omega\mu\boldsymbol{H} = \boldsymbol{\nabla} \times \frac{1}{\sigma + j\omega\epsilon}\boldsymbol{J}^{imp}$$

Boundary Conditions (BC):

- **Perfect Electric Conductor Surface:**

$$\mathbf{n} \times \boldsymbol{E} = 0 \quad ; \quad \mathbf{n} \cdot \boldsymbol{H} = 0$$

- **Idealized Antennas (Impressed Surface Electric Current):**

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega\mathbf{J}_S^{imp} \quad ; \quad \mathbf{n} \times \boldsymbol{H} = \mathbf{J}_S^{imp}$$

MAXWELL'S EQUATIONS (FREQUENCY DOMAIN)

Variational formulation

The reduced wave equation in Ω,

E-Formulation: $$\nabla \times \left(\frac{1}{\mu}\nabla \times E\right) - (\omega^2\epsilon - j\omega\sigma)E = -j\omega J^{imp}$$

H-Formulation: $$\nabla \times \left(\frac{1}{\sigma + j\omega\epsilon}\nabla \times H\right) + j\omega\mu H = \nabla \times \frac{1}{\sigma + j\omega\epsilon} J^{imp}$$

Variational formulation:

E-Formulation:
$$\left\{\begin{array}{l}
\text{Find } \mathrm{E} \in H_D(\mathrm{curl};\Omega) \text{ such that:} \\
\displaystyle\int_\Omega \frac{1}{\mu}(\nabla \times \mathrm{E})(\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\omega^2\epsilon - j\omega\sigma)\mathrm{E}\cdot\bar{\mathrm{F}}\, dV = \\
\qquad \displaystyle -j\omega\int_\Omega \mathrm{J}^{imp}\cdot\bar{\mathrm{F}}\, dV + j\omega\int_{\Gamma_N} \mathrm{J}_S^{imp}\cdot\bar{\mathrm{F}}\, dS \qquad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega)
\end{array}\right.$$

H-Formulation:
$$\left\{\begin{array}{l}
\text{Find } \mathrm{H} \in \tilde{\mathrm{H}}_S + H_D(\mathrm{curl};\Omega) \text{ with } \tilde{\mathrm{J}}_S^{imp} = \mathrm{n}\times \mathrm{H}|_S \text{ and such that:} \\
\displaystyle\int_\Omega \frac{1}{\sigma + j\omega\epsilon}(\nabla \times \mathrm{H})(\nabla \times \bar{\mathrm{F}})\, dV + j\omega\int_\Omega \mu\mathrm{H}\cdot\bar{\mathrm{F}}\, dV = \\
\qquad \displaystyle\int_\Omega \nabla \times \left(\frac{1}{\sigma + j\omega\epsilon}\mathrm{J}^{imp}\right)\cdot\bar{\mathrm{F}}\, dV \qquad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega)
\end{array}\right.$$

MAXWELL'S EQUATIONS (FREQUENCY DOMAIN)

Variational formulation in cylindrical coordinates

Using cylindrical coordinates (ρ, ϕ, z):

E_ϕ-Formulation:

$$\begin{cases} \text{Find } E_\phi \in \tilde{H}^1_D(\Omega) \text{ such that:} \\ \int_\Omega \frac{1}{\mu}\left(\frac{\partial E_\phi}{\partial z}\frac{\partial \bar{F}_\phi}{\partial z} + \frac{1}{\rho^2}\frac{\partial(\rho E_\phi)}{\partial \rho}\frac{\partial(\rho \bar{F}_\phi)}{\partial \rho}\right) dV - \int_\Omega k^2 E_\phi \cdot \bar{F}_\phi \, dV = \\ \quad -j\omega \int_\Omega J^{imp}_\phi \cdot \bar{F}_\phi \, dV + j\omega \int_{\Gamma_N} J^{imp}_{\phi,S} \cdot \bar{F}_\phi \, dS \quad \forall F_\phi \in \tilde{H}^1_D(\Omega) \, . \end{cases}$$

$\mathrm{E}_{\rho,z}$-Formulation:

$$\begin{cases} \text{Find } \mathrm{E} = (E_\rho, 0, E_z) \in \tilde{H}_D(\mathrm{curl};\Omega) \text{ such that:} \\ \int_\Omega \frac{1}{\mu}\left(\frac{\partial E_\rho}{\partial z}\frac{\partial \bar{F}_\rho}{\partial z} + \frac{\partial E_z}{\partial \rho}\frac{\partial \bar{F}_z}{\partial \rho}\right) - k^2 \int_\Omega E_\rho \bar{F}_\rho + E_z \bar{F}_z \, dV = -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z \, dV + \\ j\omega \int_{\Gamma_N} J^{imp}_{\rho,S} \bar{F}_\rho + J^{imp}_{z,S} \bar{F}_z \, dS \quad \forall \mathrm{F} = (F_\rho, 0, F_z) \in \tilde{H}_D(\mathrm{curl};\Omega) \, . \end{cases}$$

H_ϕ-Formulation:

$$\begin{cases} \text{Find } H_\phi \in f_1(J^{imp}_{\rho,S}, J^{imp}_{z,S}) + \tilde{H}^1_D(\Omega) \text{ such that:} \\ \int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial H_\phi}{\partial z}\frac{\partial \bar{F}_\phi}{\partial z} + \frac{1}{\rho^2}\frac{\partial(\rho H_\phi)}{\partial \rho}\frac{\partial(\rho \bar{F}_\phi)}{\partial \rho}\right) dV - j\omega \int_\Omega \mu H_\phi \cdot \bar{F}_\phi \, dV = \\ \quad \int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial J^{imp}_\rho}{\partial z} - \frac{\partial J^{imp}_z}{\partial \rho}\right)\bar{F}_\phi dV \quad \forall F_\phi \in \tilde{H}^1_D(\Omega) \, . \end{cases}$$

$H_{\rho,z}$-Formulation:

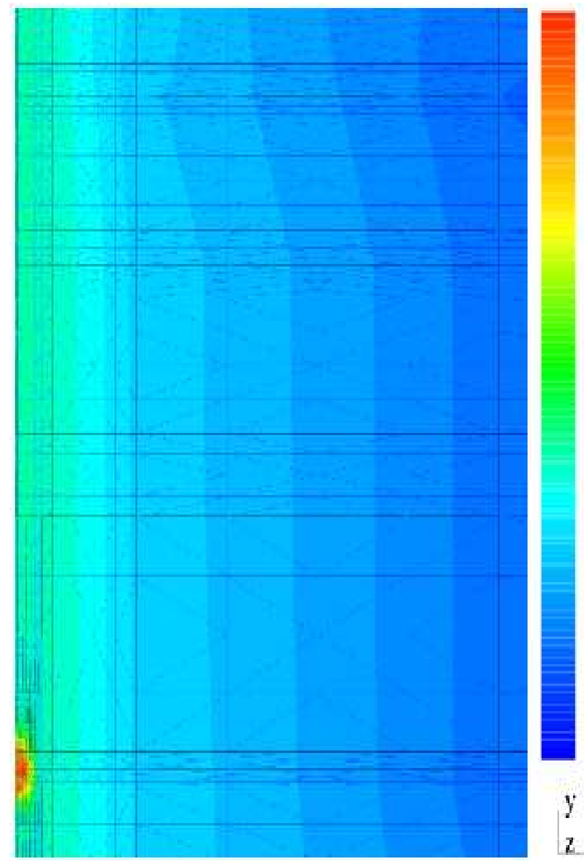
$$\begin{cases} \text{Find } \mathrm{H} = H_\rho, 0, H_z) \in f_2(J^{imp}_{\phi,S}) + \tilde{H}_D(\mathrm{curl};\Omega) \text{ such that:} \\ \int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial H_\rho}{\partial z}\frac{\partial \bar{F}_\rho}{\partial z} + \frac{\partial H_z}{\partial \rho}\frac{\partial \bar{F}_z}{\partial \rho}\right) - j\omega \int_\Omega \mu(H_\rho \bar{F}_\rho + H_z \bar{F}_z) \, dV = \\ \quad \int_\Omega \frac{1}{\sigma + j\omega\epsilon}\left(\frac{\partial J_\phi}{\partial z}\bar{F}_\rho + \frac{1}{\rho}\frac{\partial(\rho J^{imp}_\phi)}{\partial \rho}\bar{F}_z\right) dV \quad \forall \mathrm{F} = (F_\rho, 0, F_z) \in \tilde{H}_D(\mathrm{curl};\Omega) \, . \end{cases}$$

SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

What does it mean *Goal-Oriented* Adaptivity?

We consider the following problem:

$$\begin{cases} \text{Find } \Psi \in V \text{ such that :} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \; . \end{cases}$$



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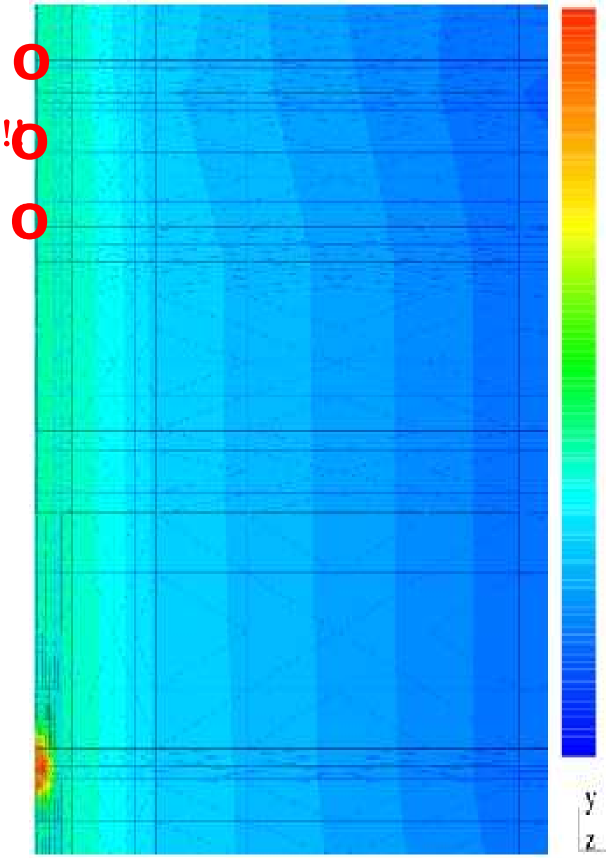
$MISLEADING!!!$



The problem we *really* want to solve is:

$$\begin{cases} \text{Find } L(\Psi), \text{ where } \Psi \in V \text{ such that :} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \ , \end{cases}$$

where $L(\Psi)$ is our goal.

SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

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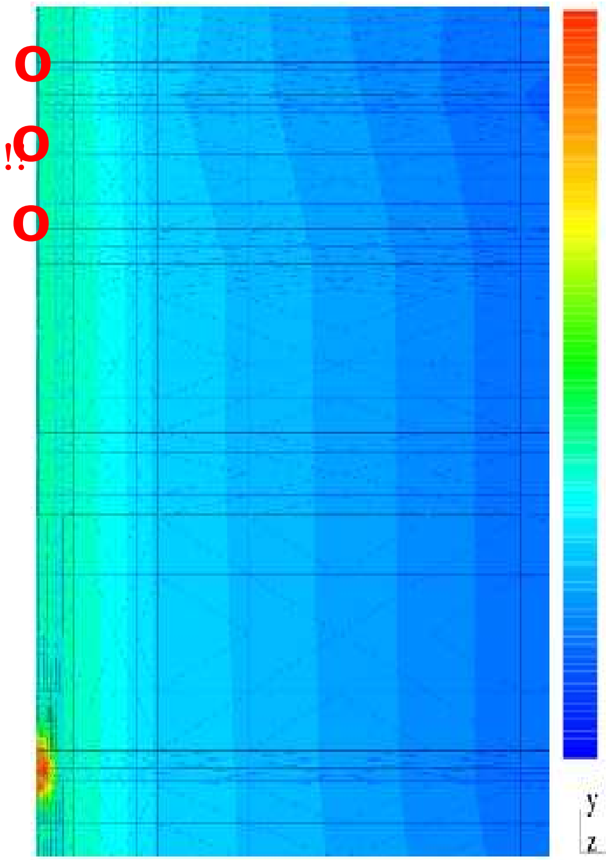

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where $L(\Psi)$ is our goal.

HP goal-oriented adaptivity consists of constructing an optimal grid:

$$\arg \min_{hp:|L(e_{hp})| \leq TOL} N_{hp}$$

SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

Mathematical Formulation (Goal-Oriented Adaptivity)

We consider the following problem (in variational form):

$$\begin{cases} \text{Find } L(\Psi), \text{ where } \Psi \in V \text{ such that :} \\ b(\Psi,\xi) = f(\xi) \quad \forall \xi \in V \; . \end{cases}$$

We define residual $r_{hp}(\xi) = b(e_{hp},\xi)$. We seek for solution G of:

$$\begin{cases} \text{Find } G \in V \text{ such that :} \\ r(G) = L(e_{hp}) \; . \end{cases}$$

This is necessarily solved if we find the solution of the *dual* problem:

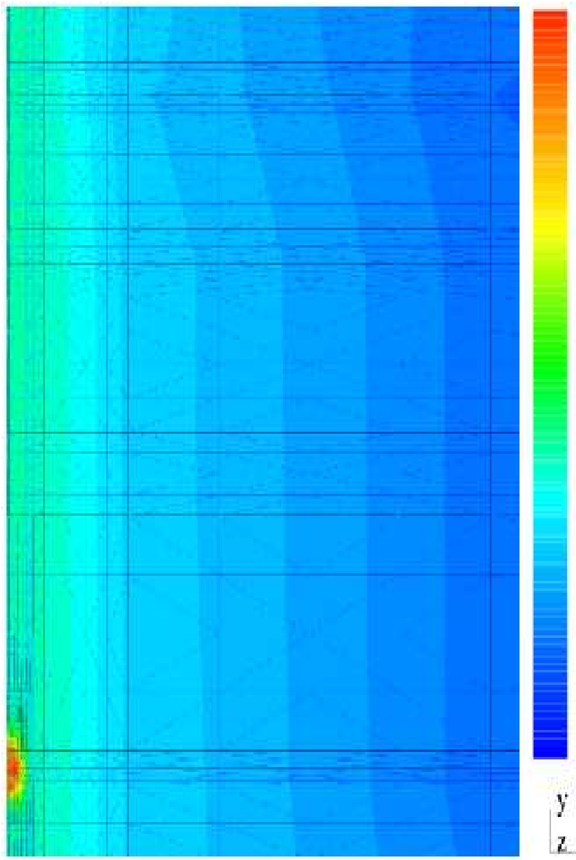
$$\begin{cases} \text{Find } G \in V \text{ such that :} \\ b(\Psi,G) = L(\Psi) \quad \forall \Psi \in V \; . \end{cases}$$

Notice that $L(e) = b(e,G)$.

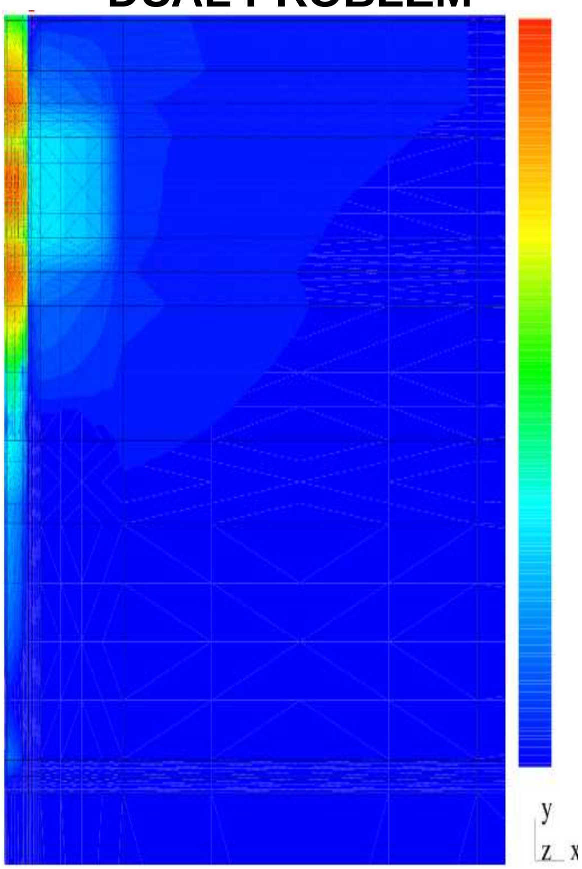
SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

Mathematical Formulation (Goal-Oriented Adaptivity)

DIRECT PROBLEM



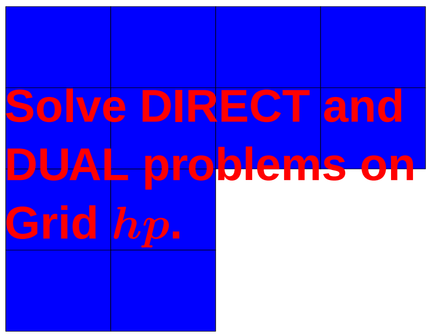
DUAL PROBLEM



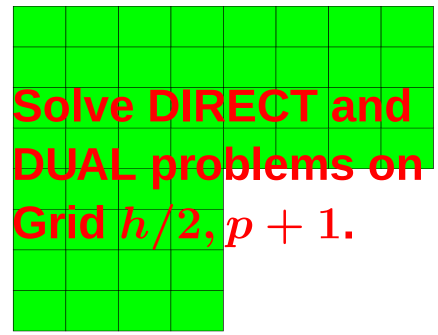
$$L(\Psi)=b(\Psi,G)$$

SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

Algorithm for Goal-Oriented Adaptivity



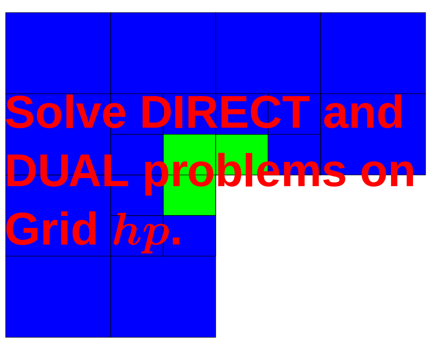
$\longrightarrow$



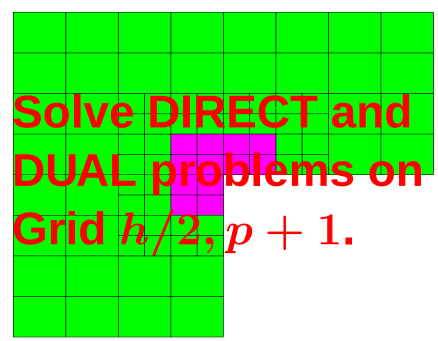
Compute $e = e_{h/2,p+1} - e_{hp}$**, and** $\epsilon = G_{h/2,p+1} - G_{hp}$**.**

Use estimate $|L(e)| = |b(e,\epsilon)| \leq \sum_K |b_K(e,\epsilon)|$**.**

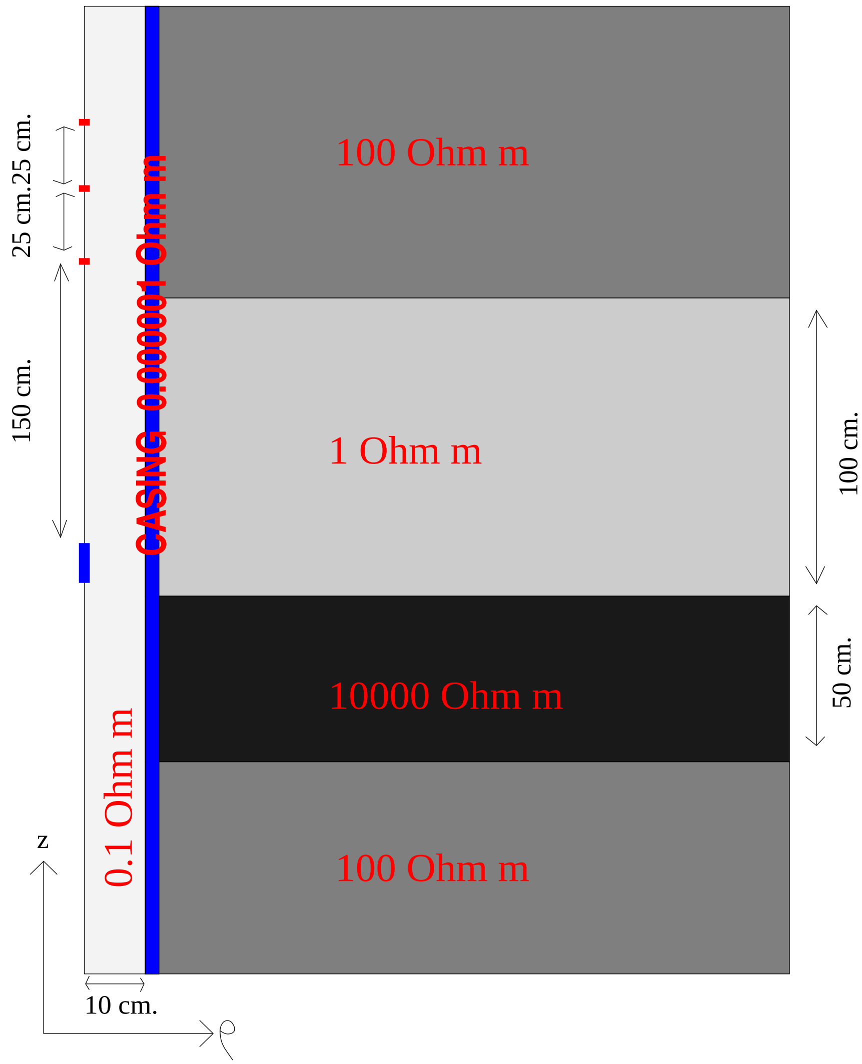
Apply the fully automatic hp**-adaptive algorithm.**



$\longrightarrow$



THROUGH CASING RESISTIVITY INSTRUMENTS



Axisymmetric 3D problem.

Five different materials.

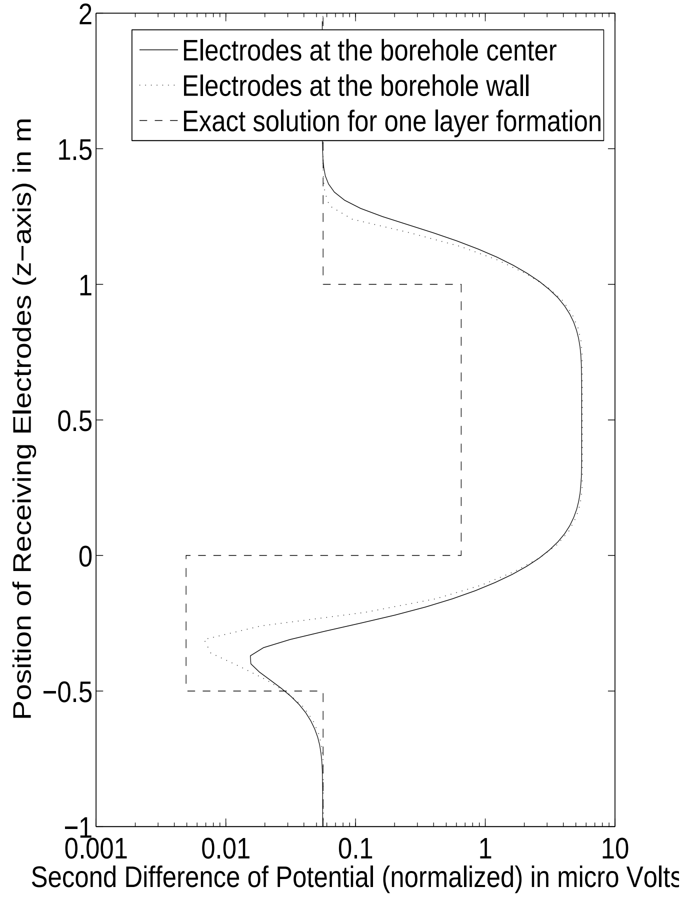
Size of computational domain: SEVERAL MILES.

Material properties varying by up to TEN orders of magnitude (10000000000!!!).

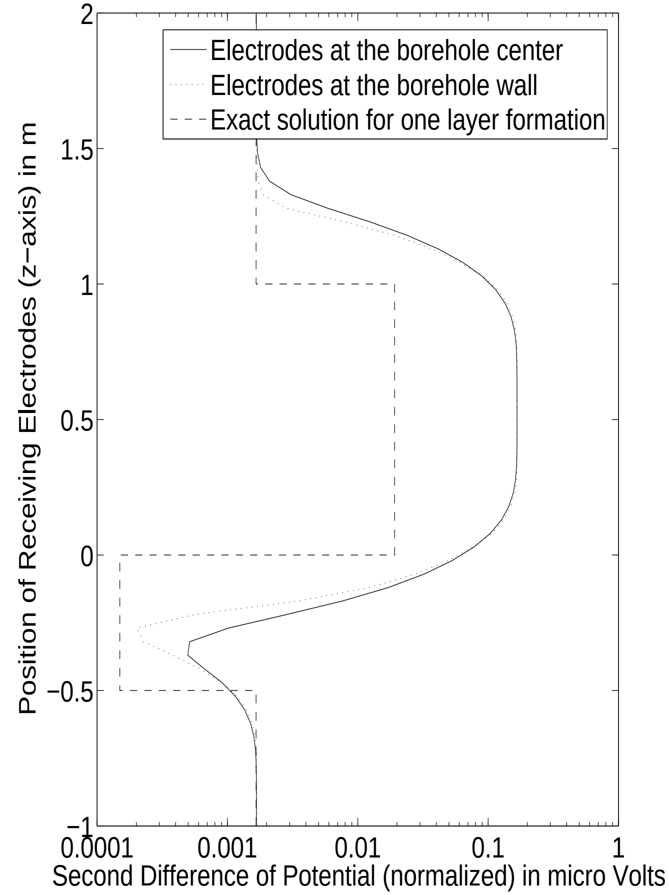
Objective: Determine Second Difference of Potential Receiving Electrodes.

THROUGH CASING RESISTIVITY INSTRUMENTS

Final Log Obtained by Our Finite Element Software



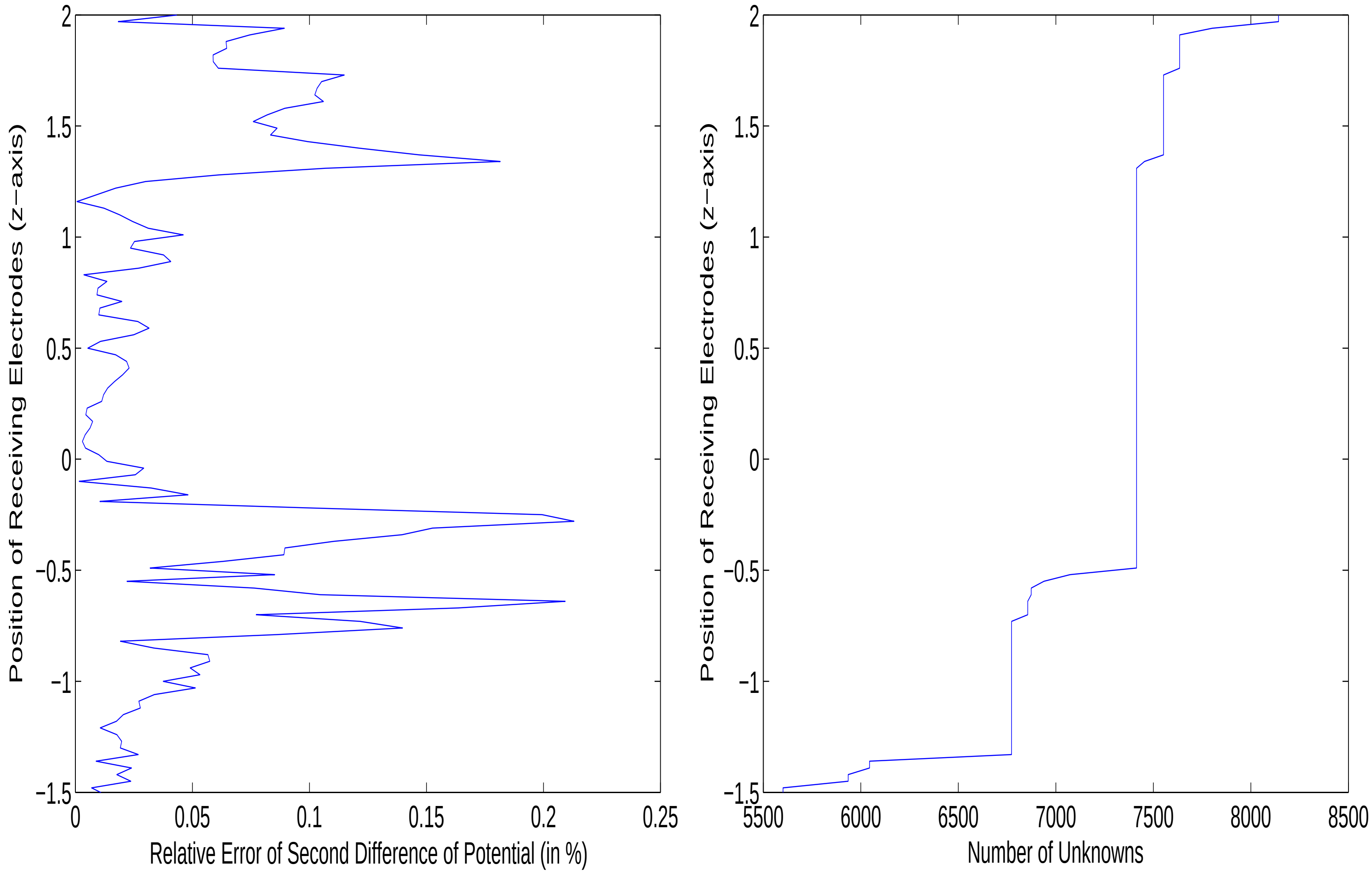
Resistivity of casing $= 10^{-6}$



Resistivity of casing $= 10^{-7}$

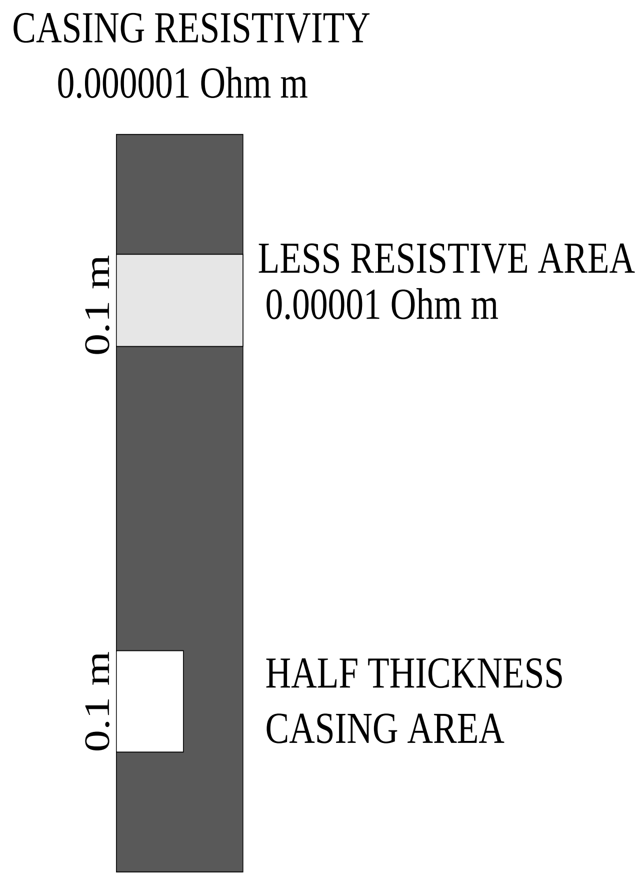
THROUGH CASING RESISTIVITY INSTRUMENTS

Approximation Error

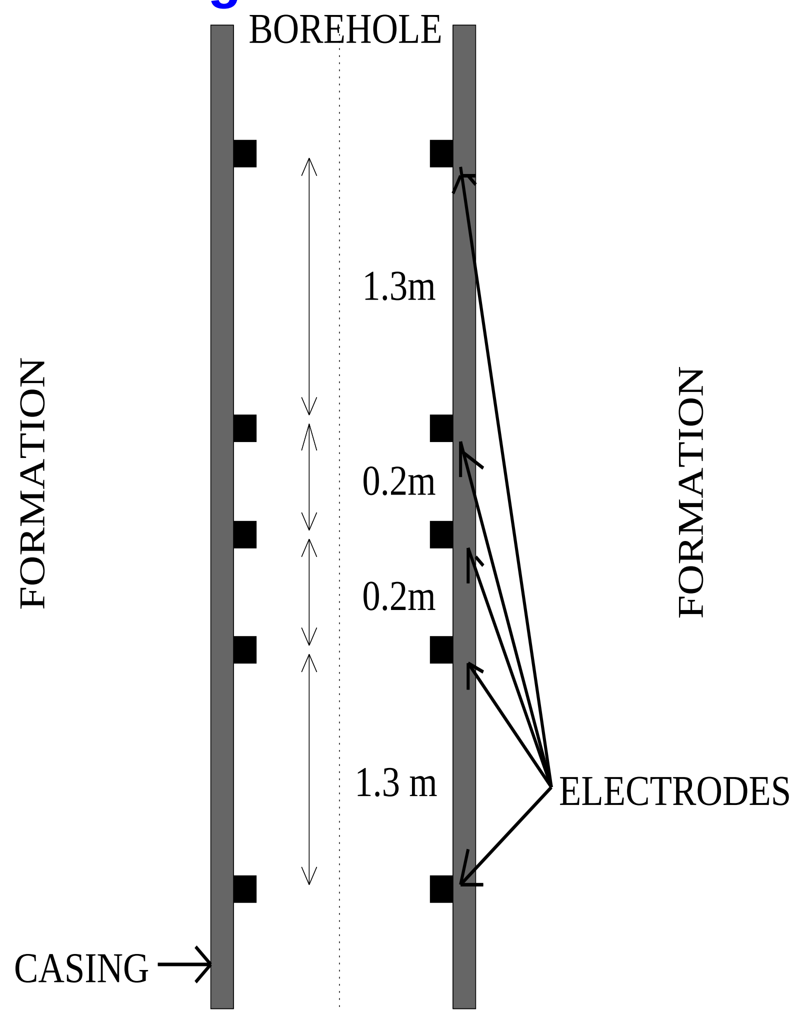


THROUGH CASING RESISTIVITY INSTRUMENTS

Damaged Casing

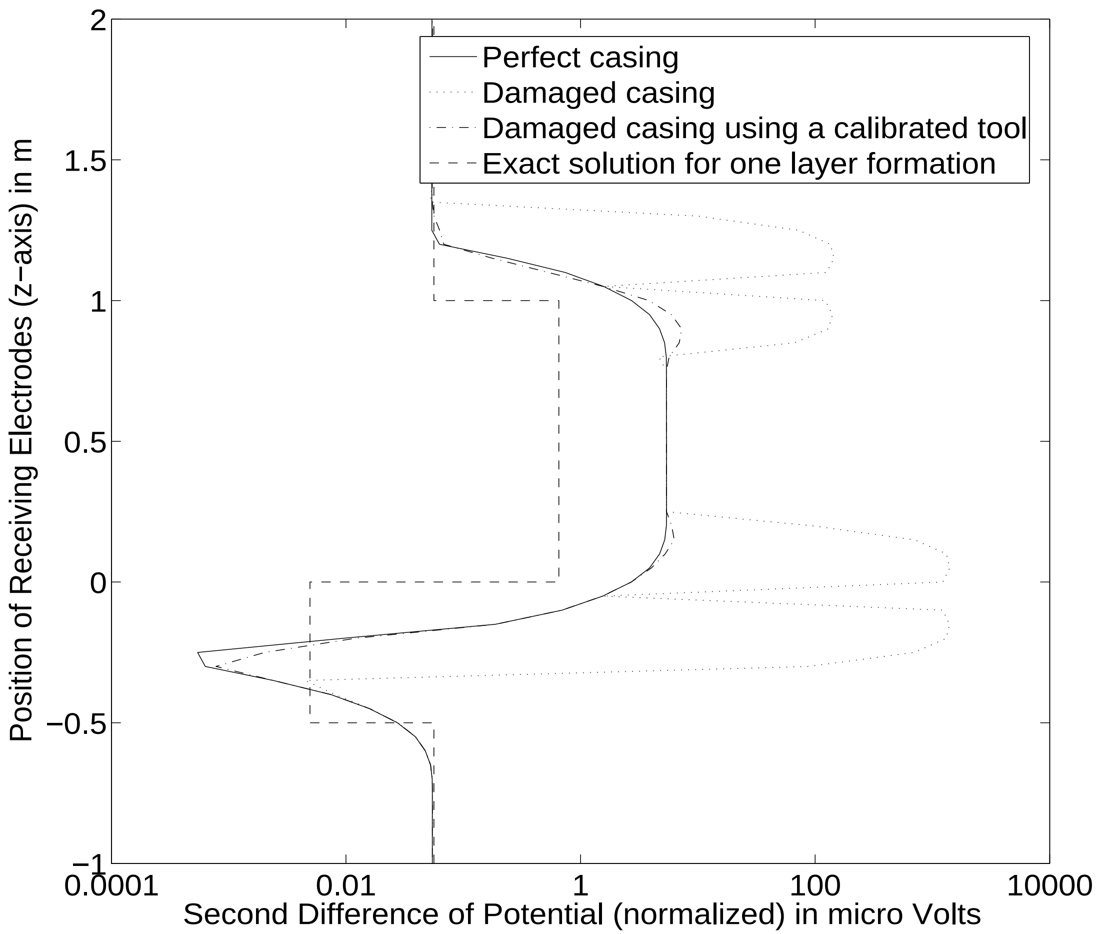


Imperfect Casing



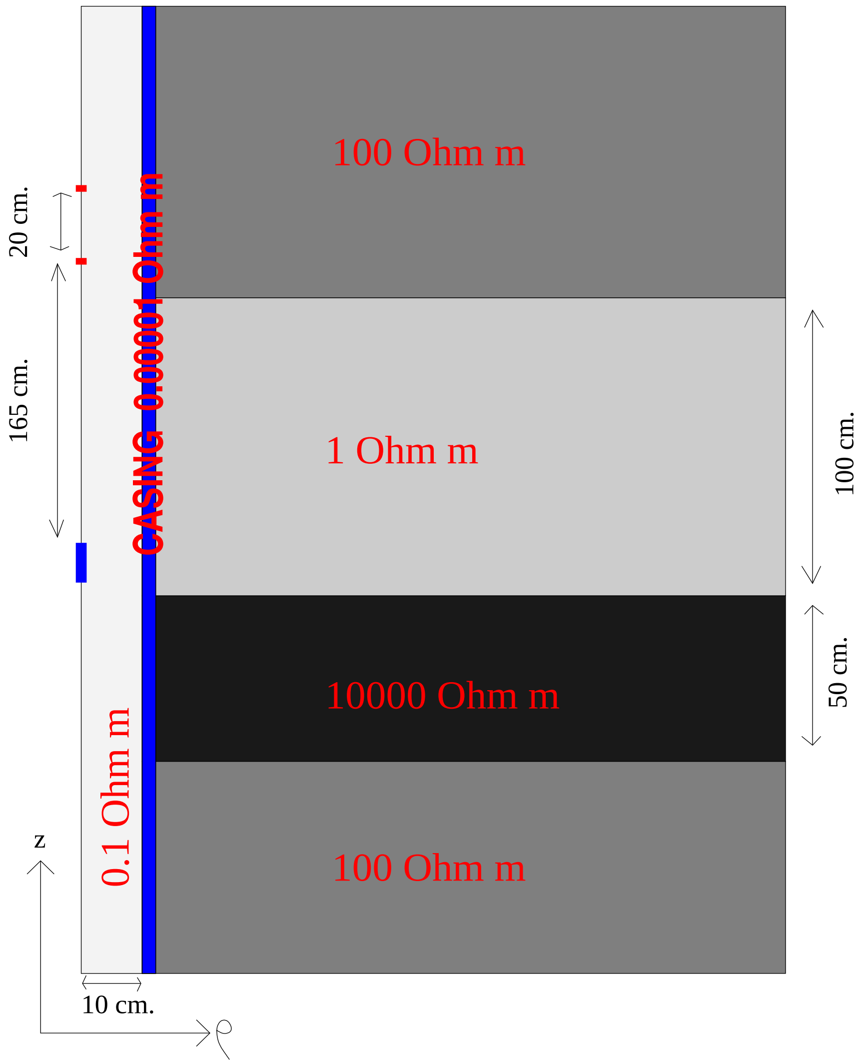
THROUGH CASING RESISTIVITY INSTRUMENTS

Damaged Casing



In the presence of damaged casing, the use of calibrated instruments is essential.

THROUGH CASING RESISTIVITY INSTRUMENTS



Axisymmetric 3D problem.

Toroid Antennas.

Size of computational domain: SEVERAL MILES.

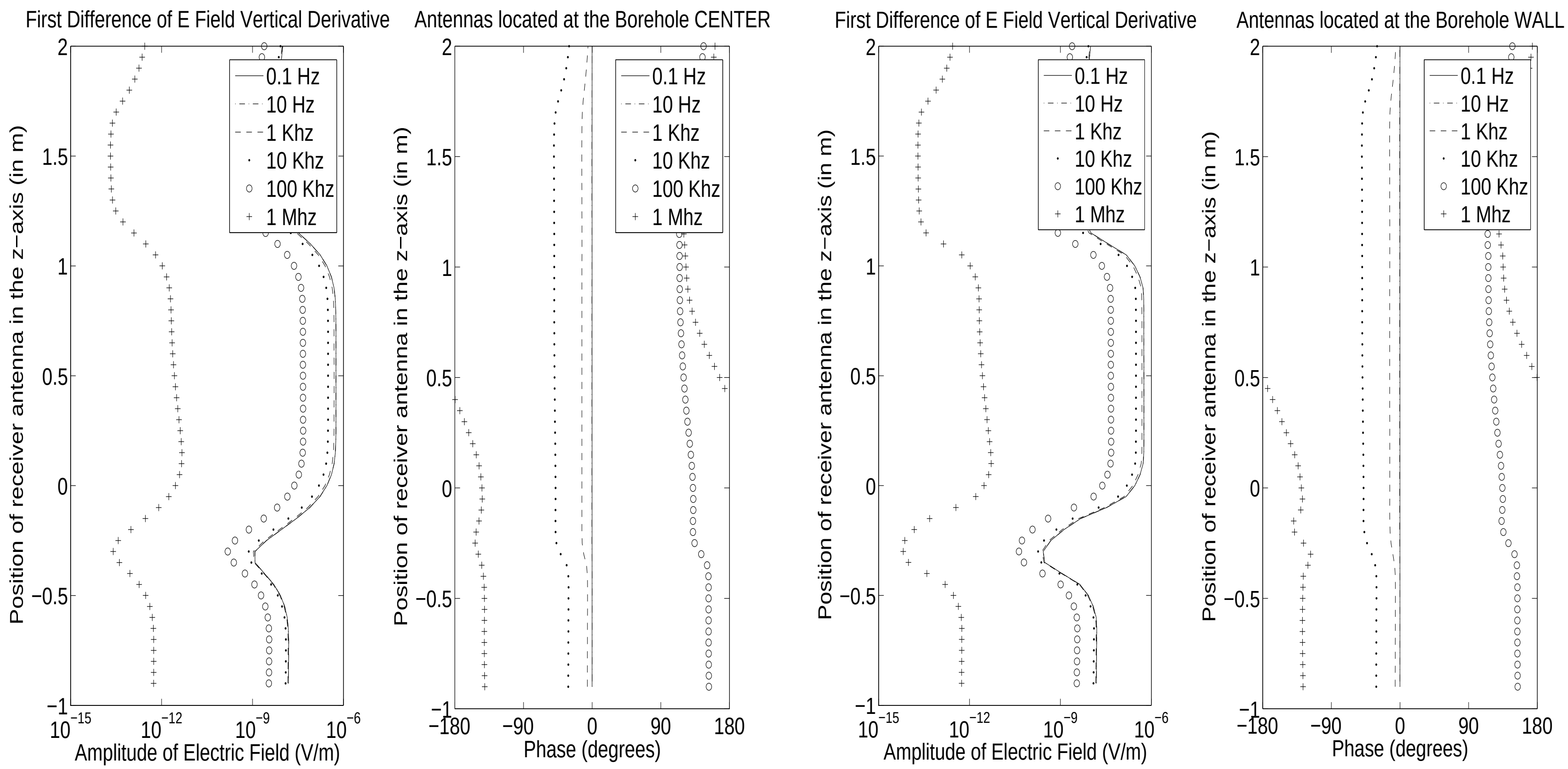
Different frequencies.

Material properties varying by up to NINE orders of magnitude (1000000000!!!).

Objective: Determine First Difference of Electric and Magnetic Fields.

THROUGH CASING RESISTIVITY INSTRUMENTS

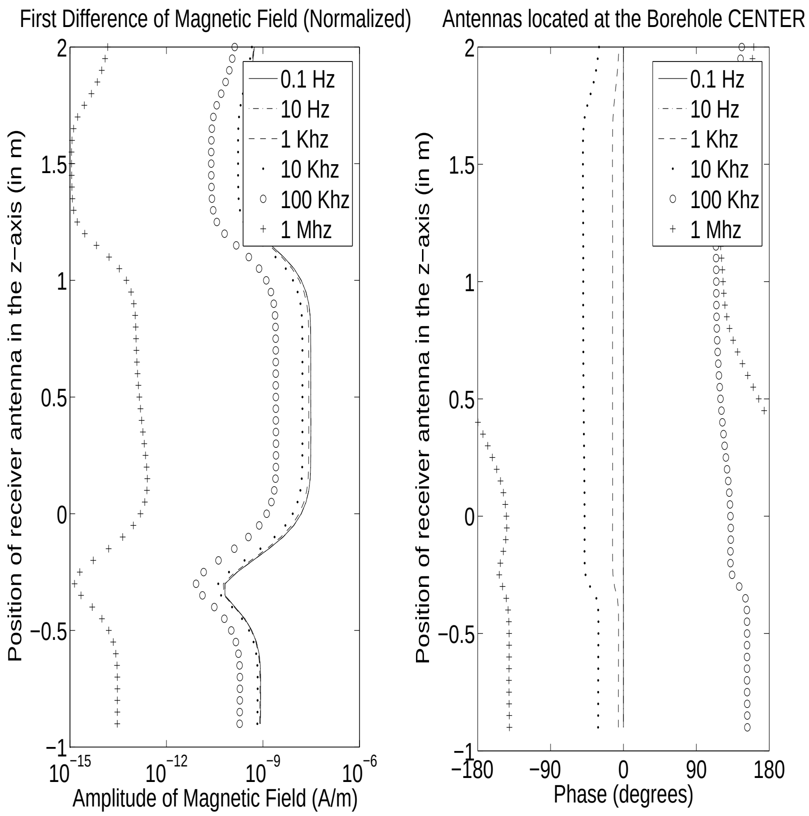
First Difference of Electric Field at Different Frequencies

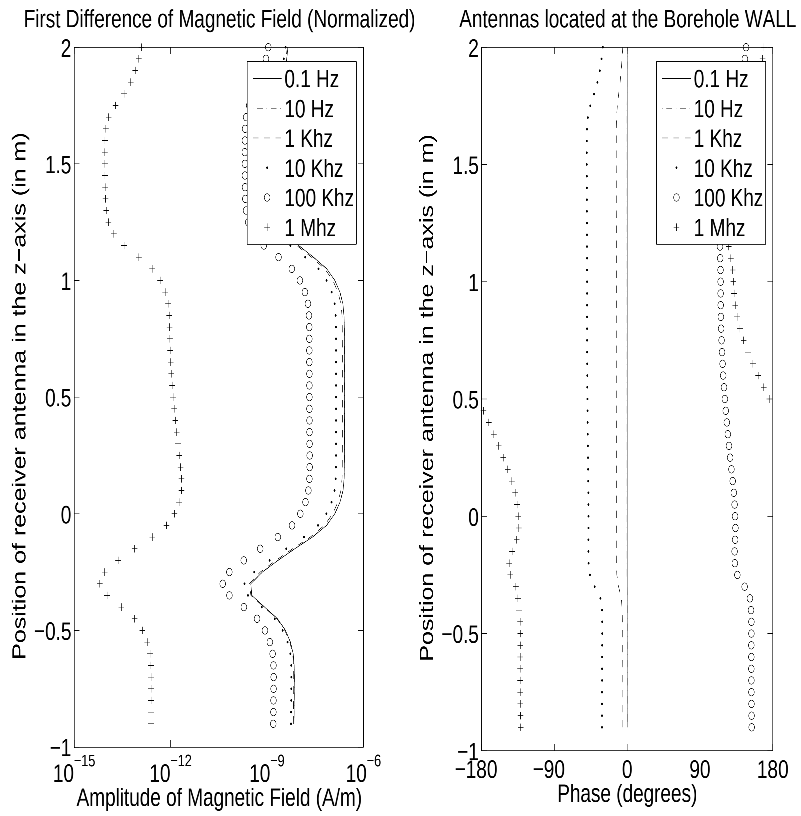


Toroid antennas are more sensitive to the rock formation resistivity when located on the borehole's wall

THROUGH CASING RESISTIVITY INSTRUMENTS

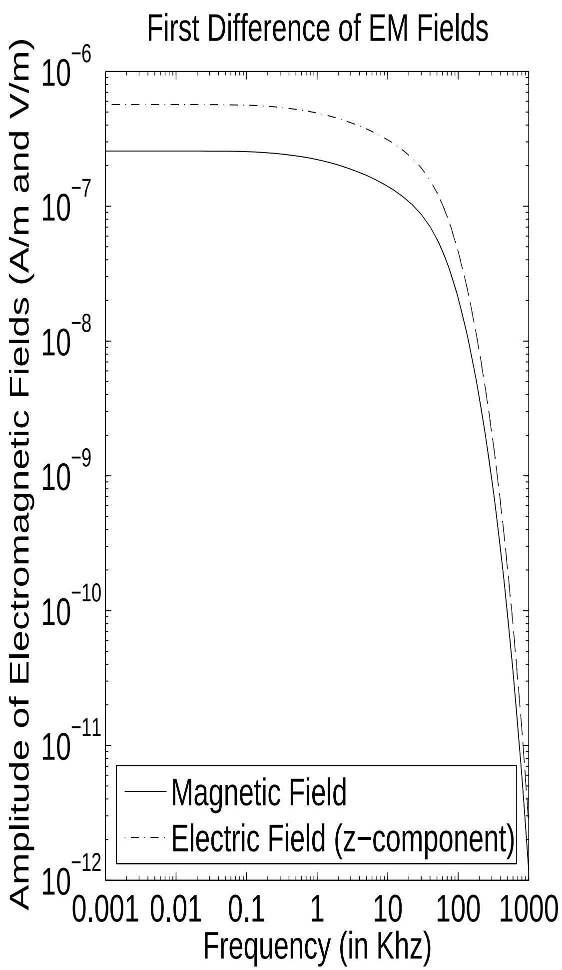
First Difference of Magnetic Field at Different Frequencies

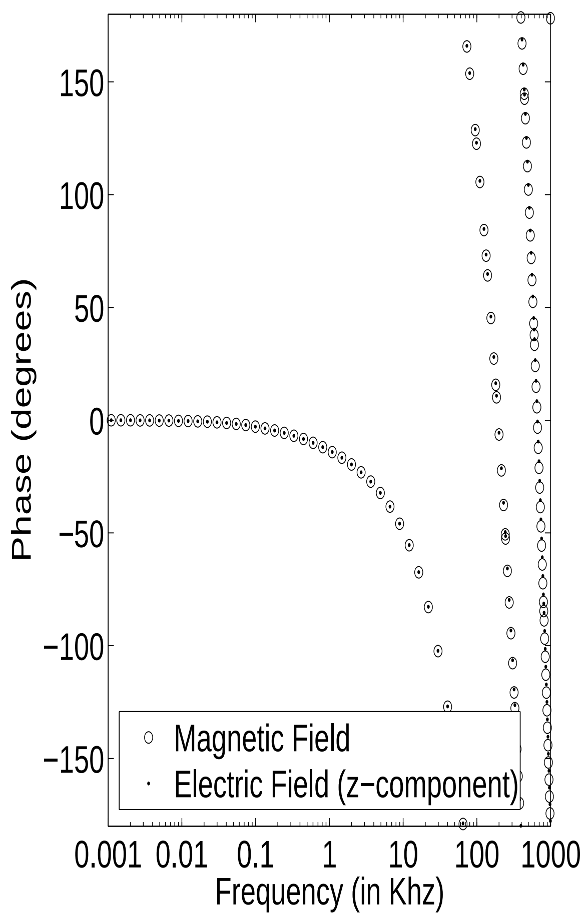




THROUGH CASING RESISTIVITY INSTRUMENTS

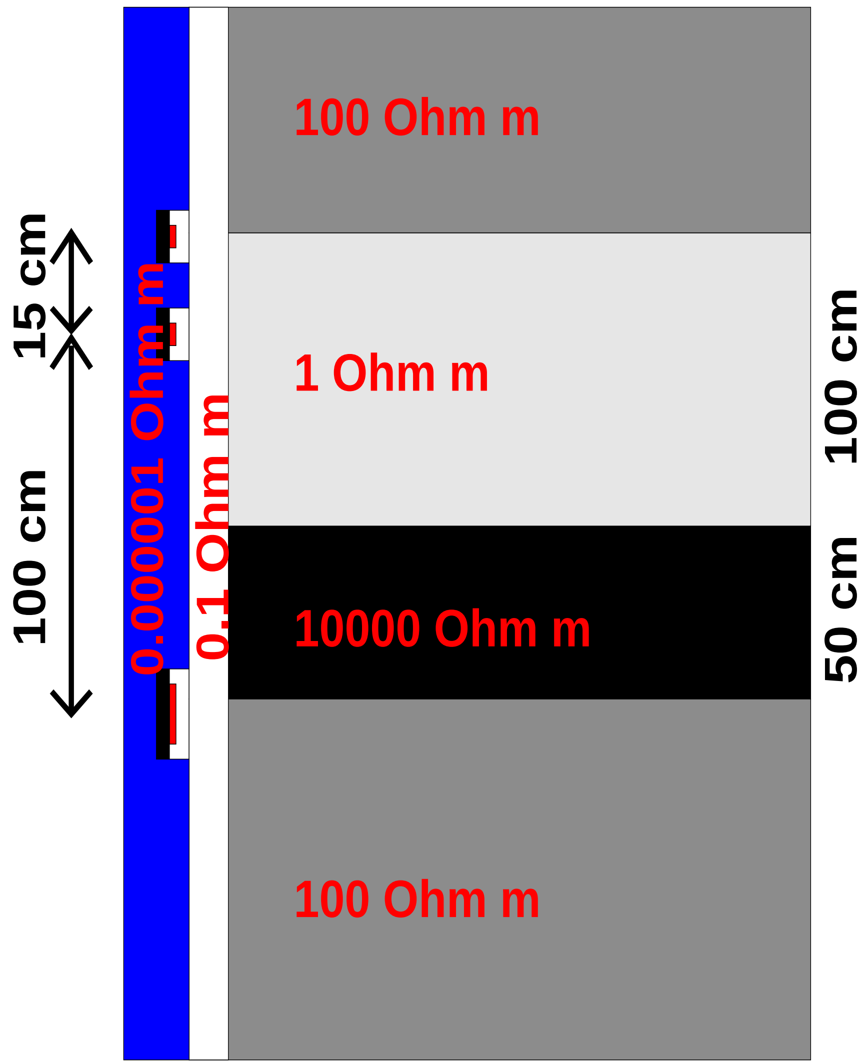
Electromagnetic Fields at Different Frequencies



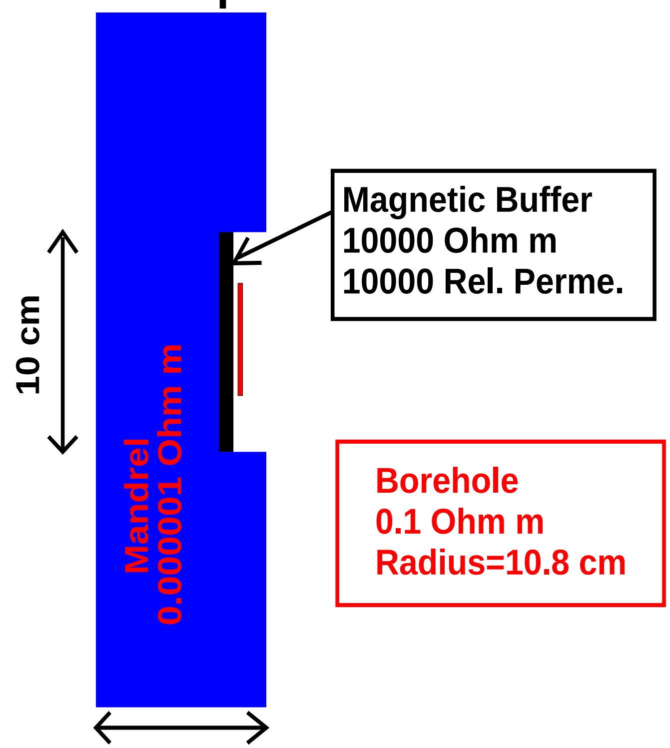


Electromagnetic Fields are almost constant for frequencies below 1 kHz. A sudden drop in the amplitude occurs at frequencies above 20 kHz.

LOGGING INSTRUMENTS WITH A MANDREL



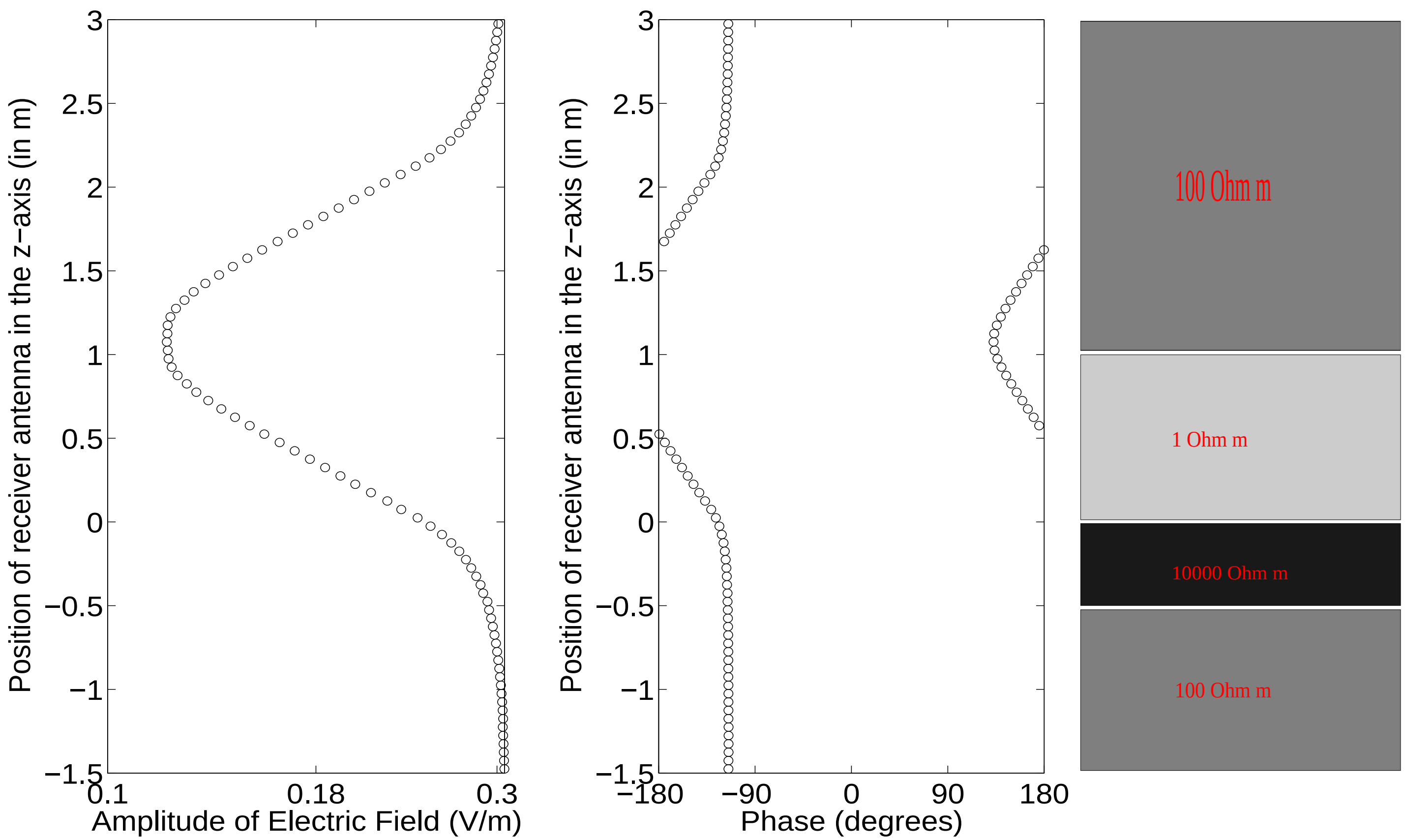
Description of Antennas



Goal: To Compute First Difference of Potential on Receiving Antennas

LOGGING INSTRUMENTS WITH A MANDREL

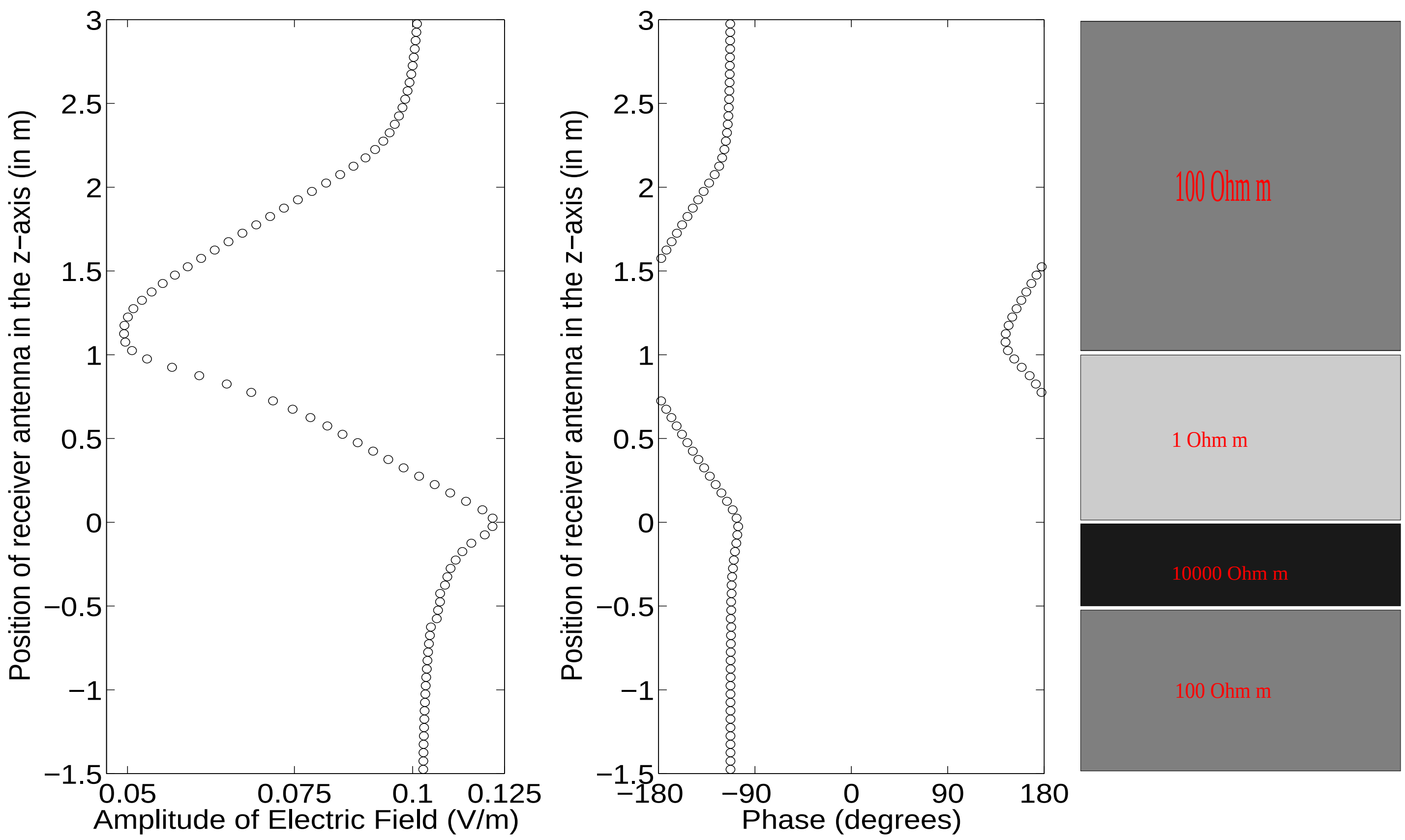
E_ϕ (normalized) for a solenoid antenna



Frequency: 2 Mhz

LOGGING INSTRUMENTS WITH A MANDREL

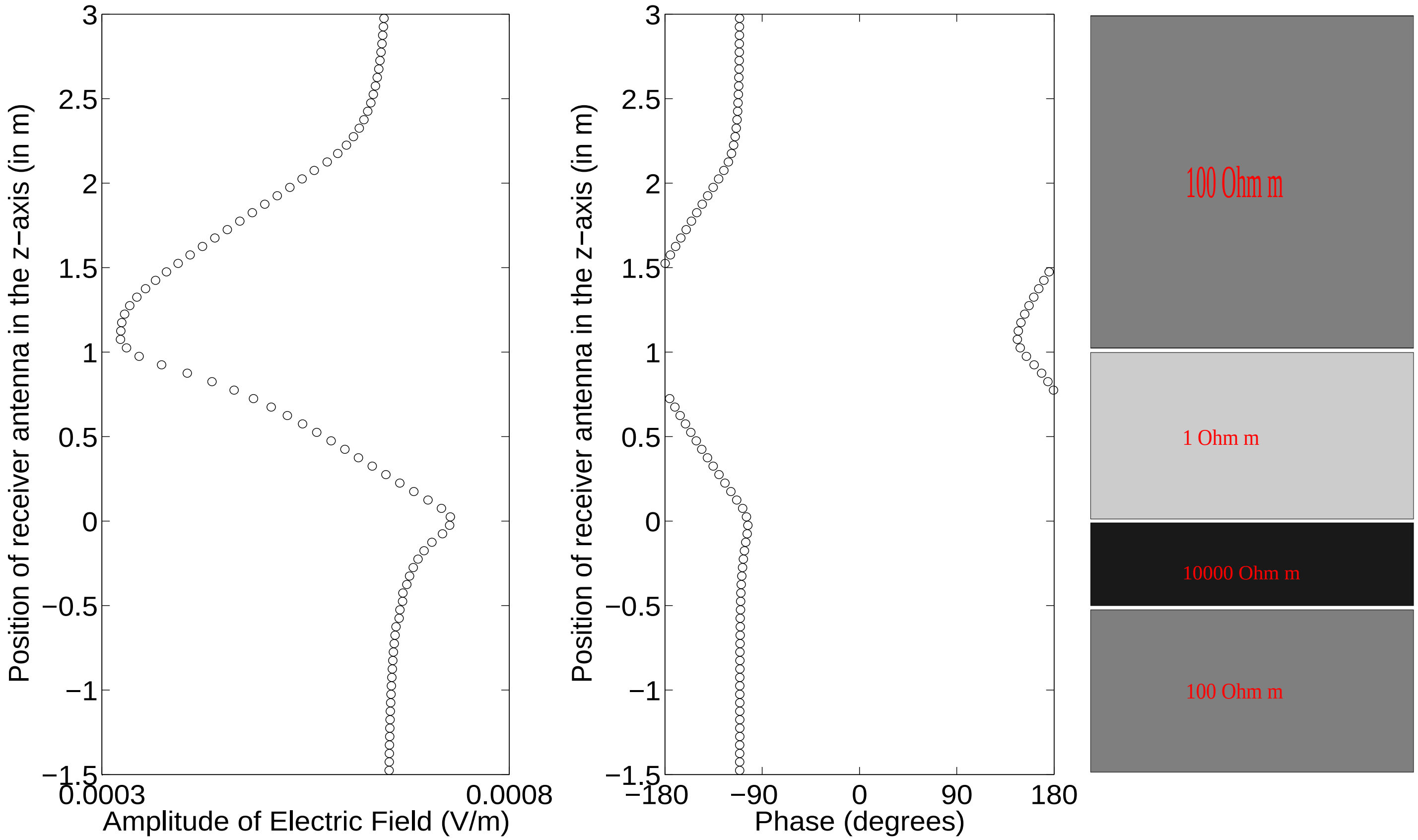
First Difference of E_ϕ (normalized) for a solenoid antenna



Frequency: 2 Mhz

LOGGING INSTRUMENTS WITH A MANDREL

First Difference of E_ϕ (normalized) for a solenoid antenna



Frequency: 2 Mhz, NO MAGNETIC BUFFER

CONCLUSIONS

Conclusions

It is possible to simulate a variety of resistivity logging instruments by using the self-adaptive goal-oriented hp-FEM.

Future Work

- **3D Implementation**
- **Multi-Physics (Ex.: Acoustics with Electromagnetics).**

Institute for Computational Engineering and Sciences
The University of Texas at Austin