Summer Project

Analysis of edge singularities arising in electromagnetic computations.

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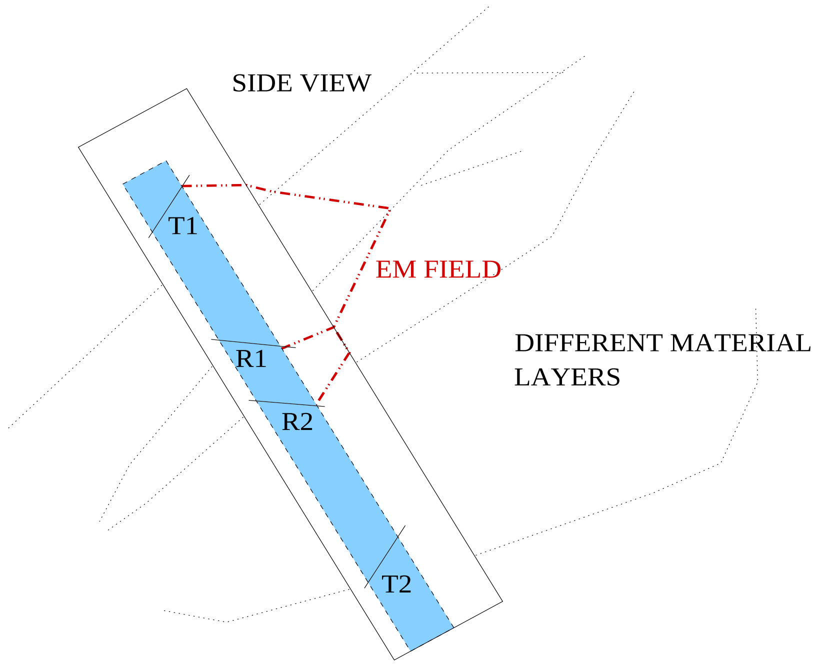
OVERVIEW

1. Overview.
2. Motivation.
3. Maxwell's Equations.
4. *hp*-Finite Elements.
5. Numerical results.
6. Conclusions and Future Work.

2. MOTIVATION

Modeling of a Logging While Drilling (LWD) electromagnetic measuring device



Goal: Determine EM field at the receiver antennas.

3. MAXWELL'S EQUATIONS

Time Harmonic Maxwell's Equations:

$$\nabla \times \mathbf{E} = -j\mu\omega\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \sigma\mathbf{E} + \mathbf{J}^{imp}$$

Reduced Wave Equation:

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega\boldsymbol{J}^{imp} \, ,$$

Boundary Conditions (BC):

- **Dirichlet BC at a PEC surface:**

$$\mathbf{n} \times \boldsymbol{E}^s = -\mathbf{n} \times \boldsymbol{E}^{inc} \qquad \mathbf{n} \times \boldsymbol{E} = 0$$

- **Neumann continuity BC at a material interface:**

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E}^s = -\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E}^{inc} \qquad \mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega\mathbf{J}_S^{imp}$$

- **Silver Müller radiation condition at ∞:**

$$\mathbf{e}_r \times (\nabla \times \boldsymbol{E}^s) - jk_0 \times \boldsymbol{E}^s = O(r^{-2})$$

3. MAXWELL'S EQUATIONS

Main difficulties in order to solve Maxwell's equations numerically:

Solve Laplace equation numerically

Solution: A suitable numerical method for solving PDE's

For zero time frequency, curl equations do NOT see gradients

Solution: Impose explicitly divergence free conditions into the formulation

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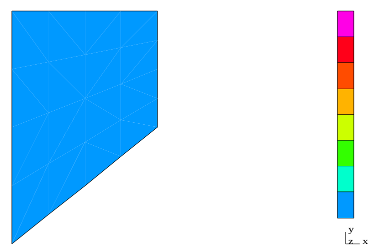
The problem is NOT positive definite

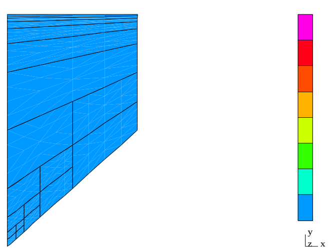
Solution: Use a minimum of about 10 nodes per wavelength

4. *hp*-FINITE ELEMENTS

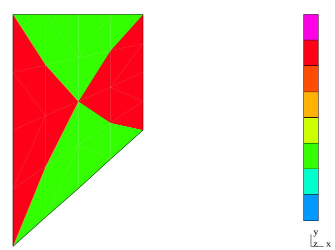
Different refinement strategies for finite elements:



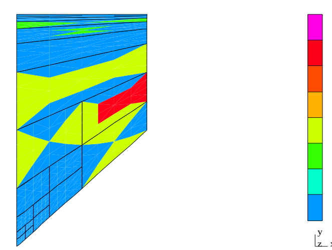
Given initial grid



***h*-refined grid**



***p*-refined grid**



***hp*-refined grid**

4. *hp*-FINITE ELEMENTS

Exponential convergence rates

for a number of regular and SINGULAR problems

for optimal hp-grids

in the asymptotic range (theoretical and numerical results), and

in the pre-asymptotic range (numerical results).

Smaller dispersion (pollution) error

as *p* increases.

More geometrical details captured

as *h* decreases.

4. *hp*-FINITE ELEMENTS

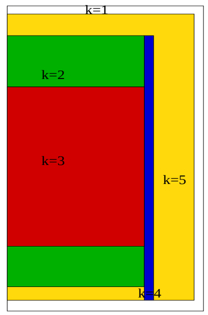
2Dhp90, 3Dhp90: main features

- **Isoparametric hexahedras.**
- **Isotropic and anisotropic mesh refinements.**
- **Geometrical Modeling Package (GMP).**
- **New data structure in Fortran 90.**
- **Constrained information reconstructed (not stored).**
- **Two levels of logical operations:**
  1. **operations for nodes - problem independent.**
  2. **operations for nodal dof - problem dependent.**
- **Fully automatic hp-adaptive strategy.**

—provides exponential convergence rates—

4. *hp*-FINITE ELEMENTS

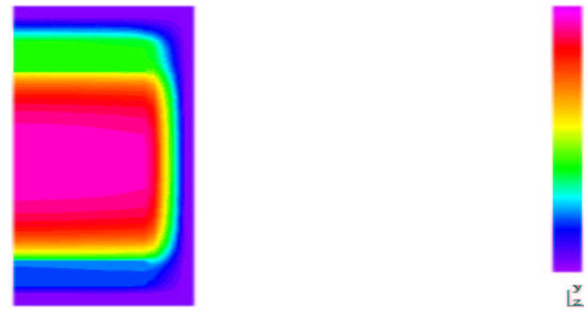
Orthotropic heat conduction example



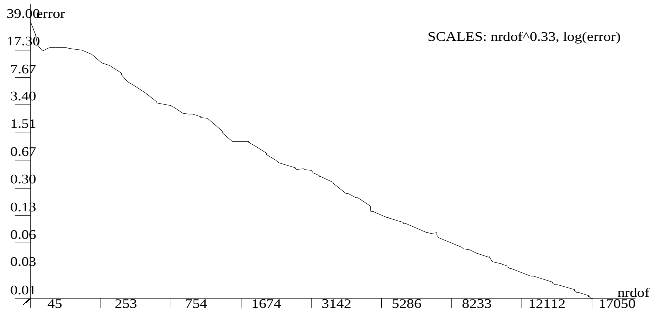
Equation: $\nabla(\mathrm{K}\nabla u) = f^{(k)}$

$$\mathrm{K} = \mathrm{K}^{(k)} = \begin{bmatrix} K_x^{(k)} & 0 \\ 0 & K_y^{(k)} \end{bmatrix}$$

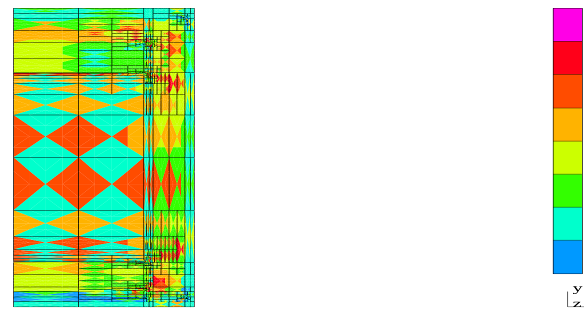
$$K_x^{(k)} = (25,\ 7,\ 5,\ 0.2,\ 0.05)$$
$$K_y^{(k)} = (25,\ 0.8,\ 0.0001,\ 0.2,\ 0.05)$$



Solution: unknown
Boundary Conditions:
$$K^{(i)} \nabla u \cdot n = g^{(i)} - \alpha^{(i)} u$$



Convergence history
(tolerance error = 0.1 %)

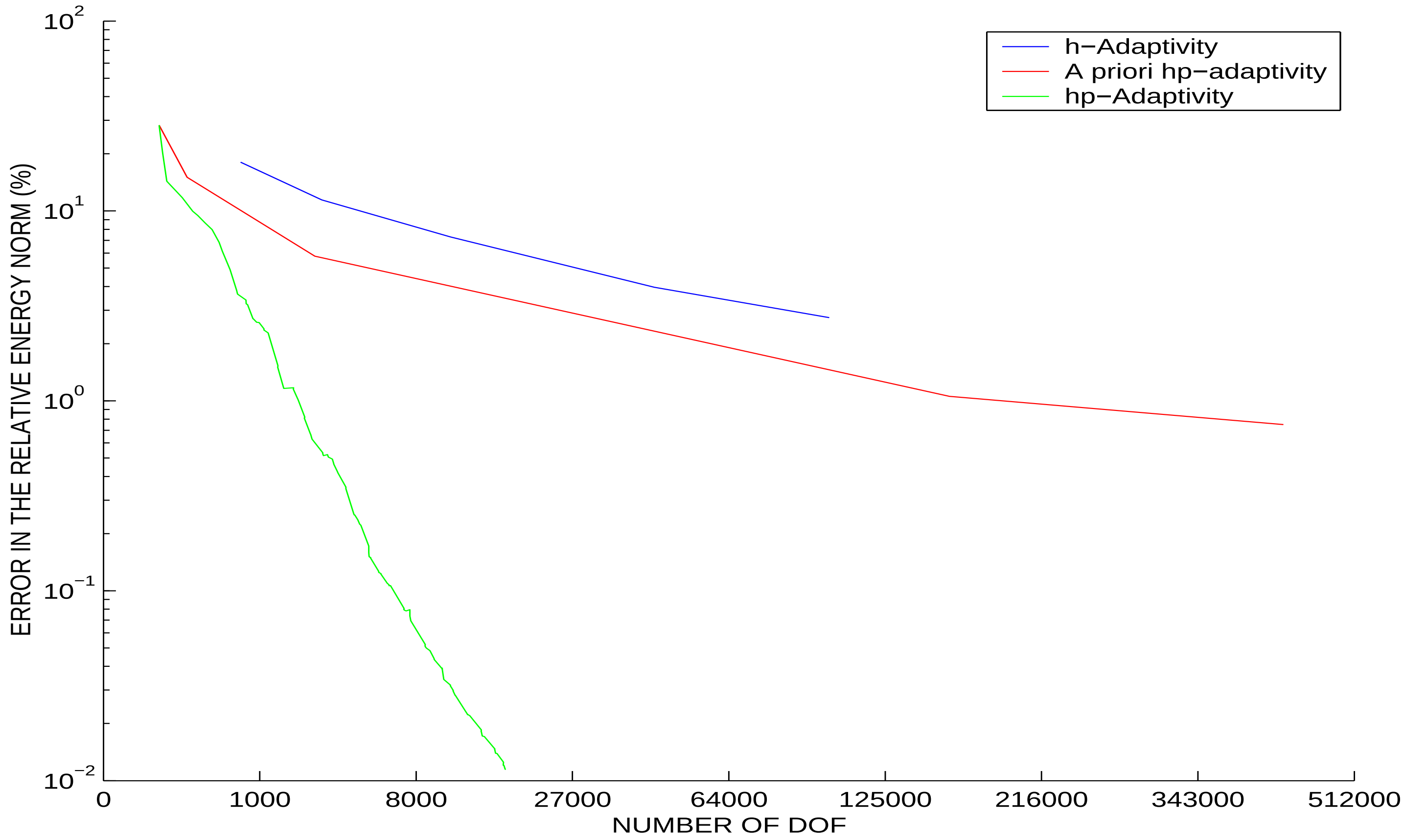


Final hp grid

4. *hp*-FINITE ELEMENTS

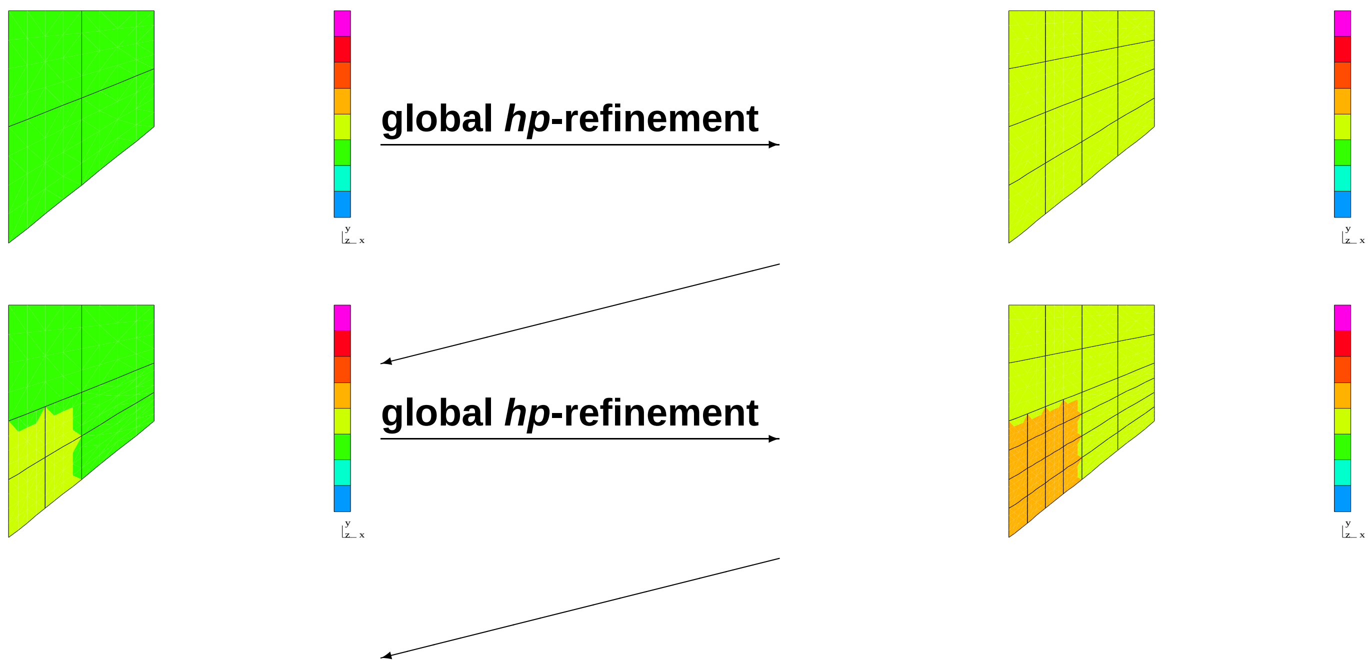
Convergence comparison

Orthotropic heat conduction example



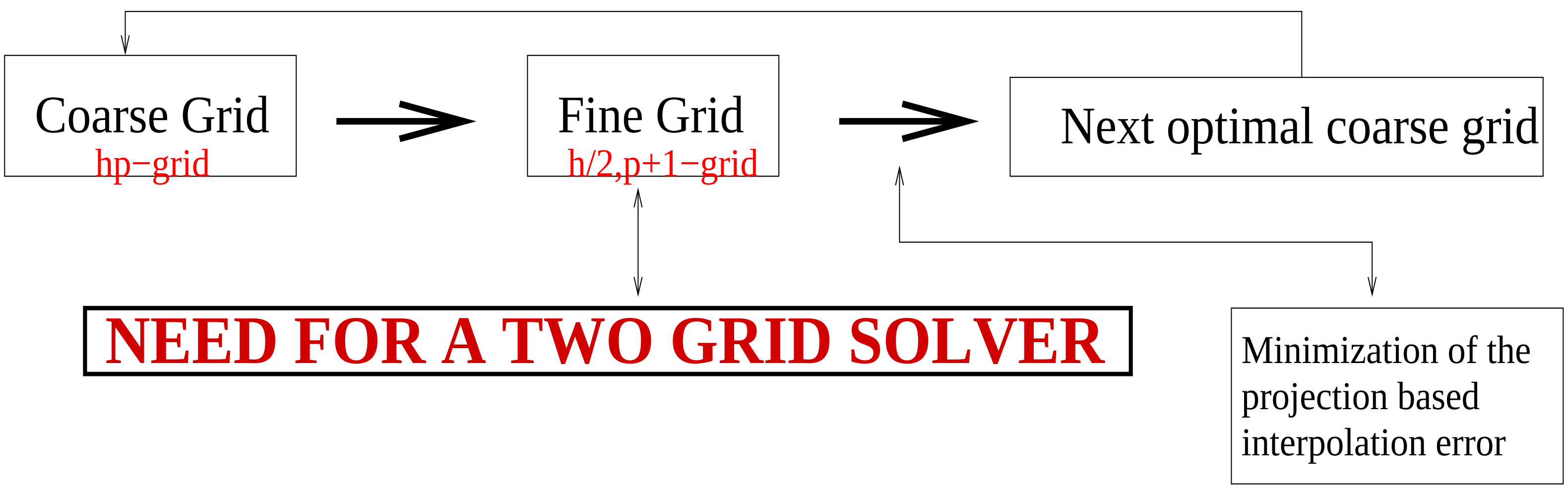
4. *hp*-FINITE ELEMENTS

Fully automatic *hp*-adaptive strategy



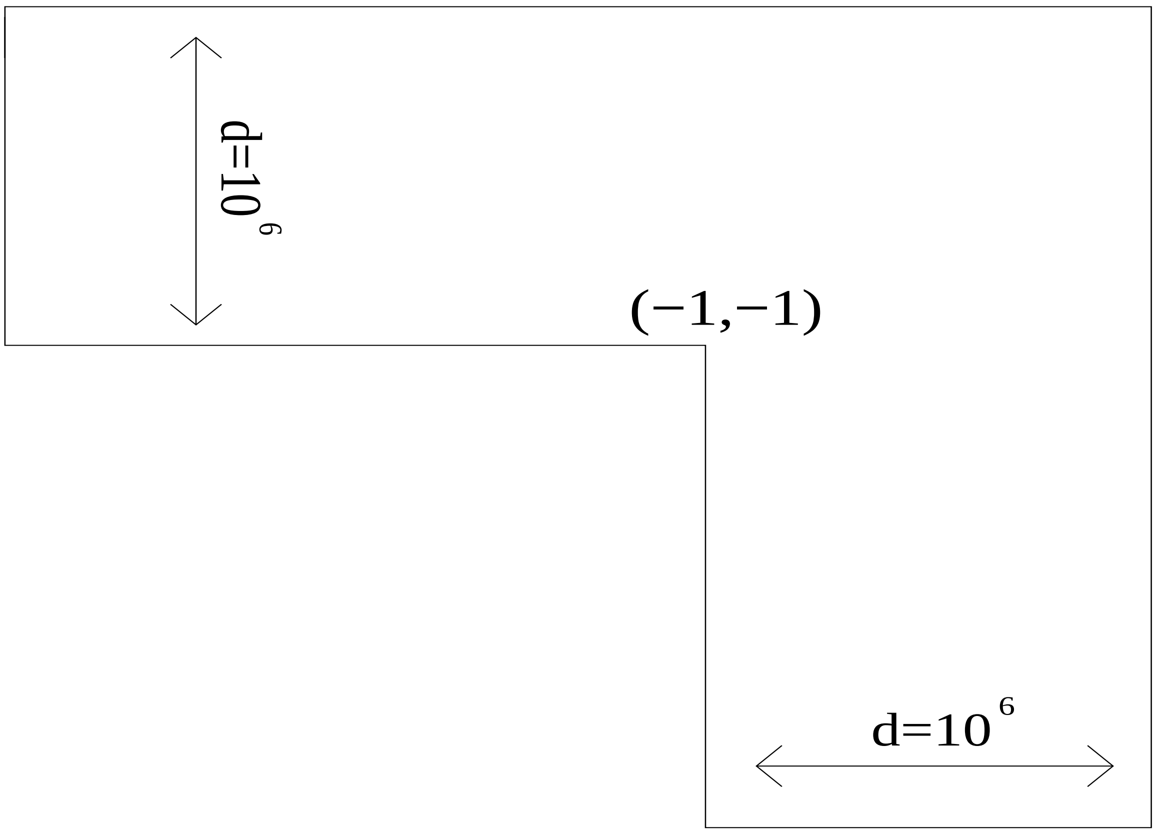
4. *hp*-FINITE ELEMENTS

Automatic *hp*-adaptivity delivers exponential convergence and enables solution of challenging EM problems



5. NUMERICAL RESULTS

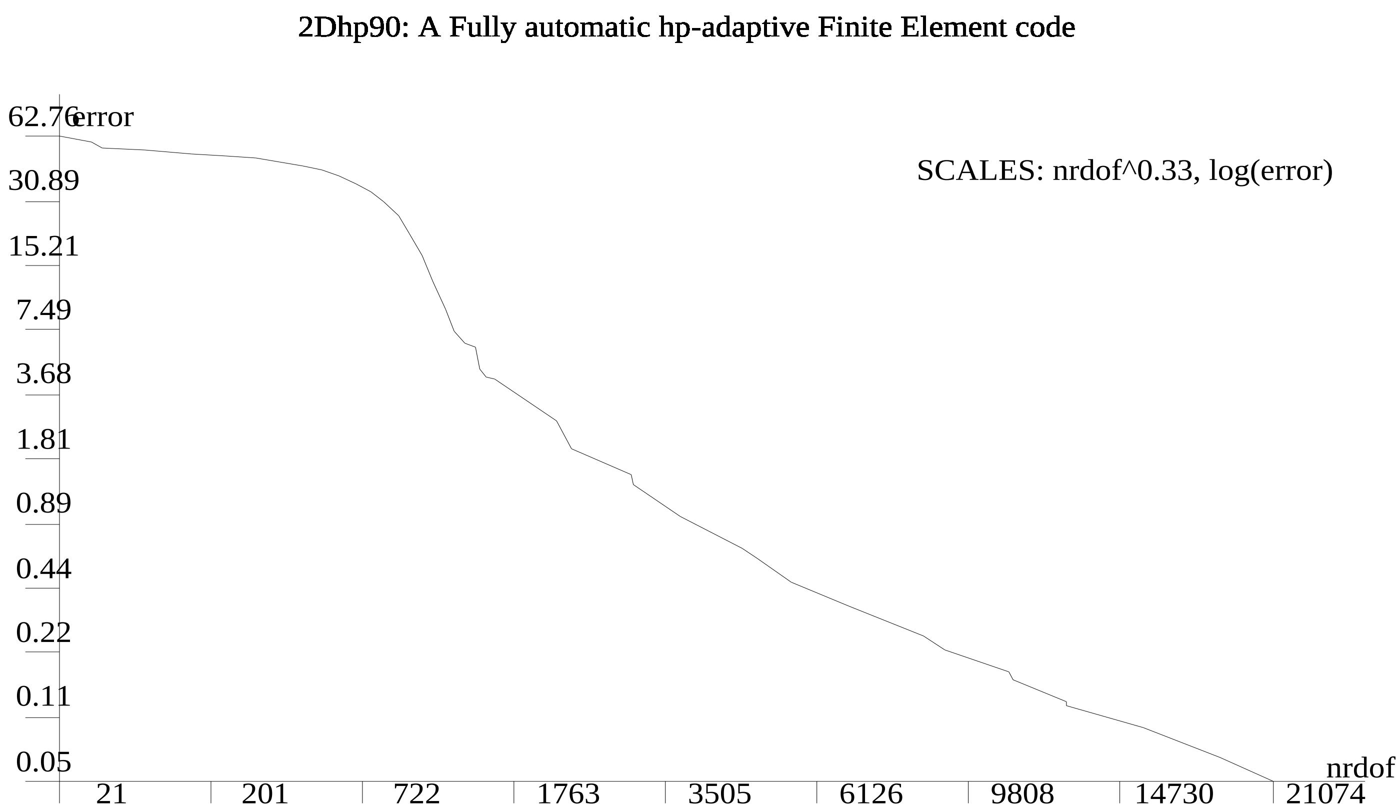
Edge diffraction example: Laplace equation over L-shape domain



Dirichlet Boundary Conditions
u(boundary)=−ln r, r=sqrt (x*x+y*y)

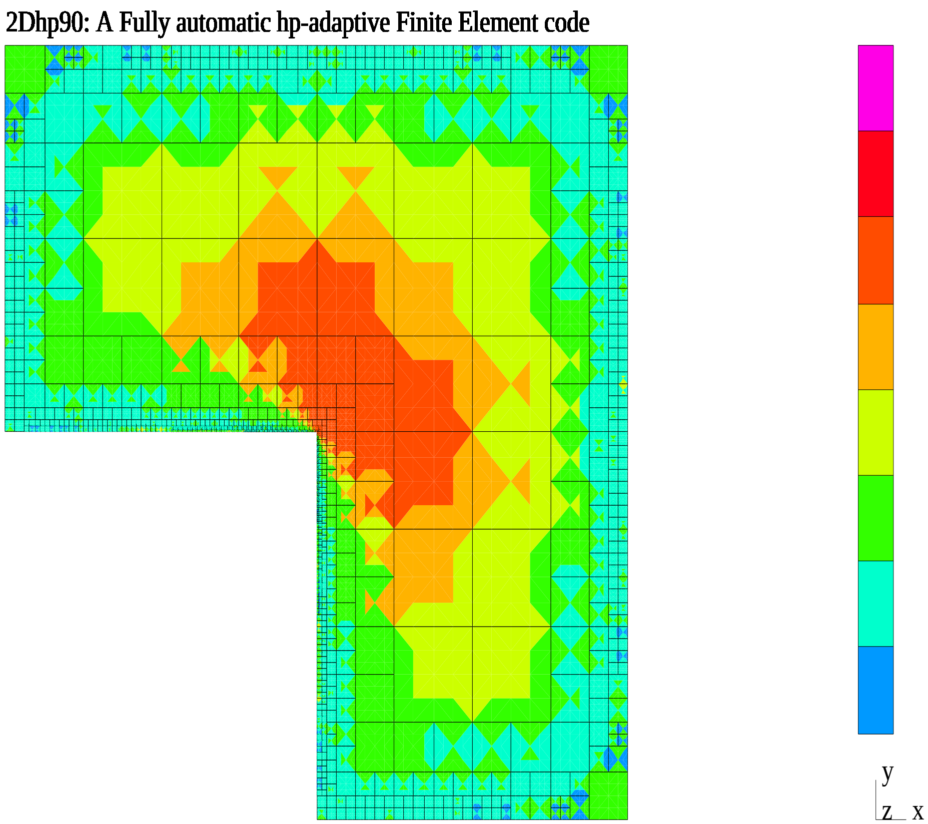
5. NUMERICAL RESULTS

Edge diffraction example: Convergence history



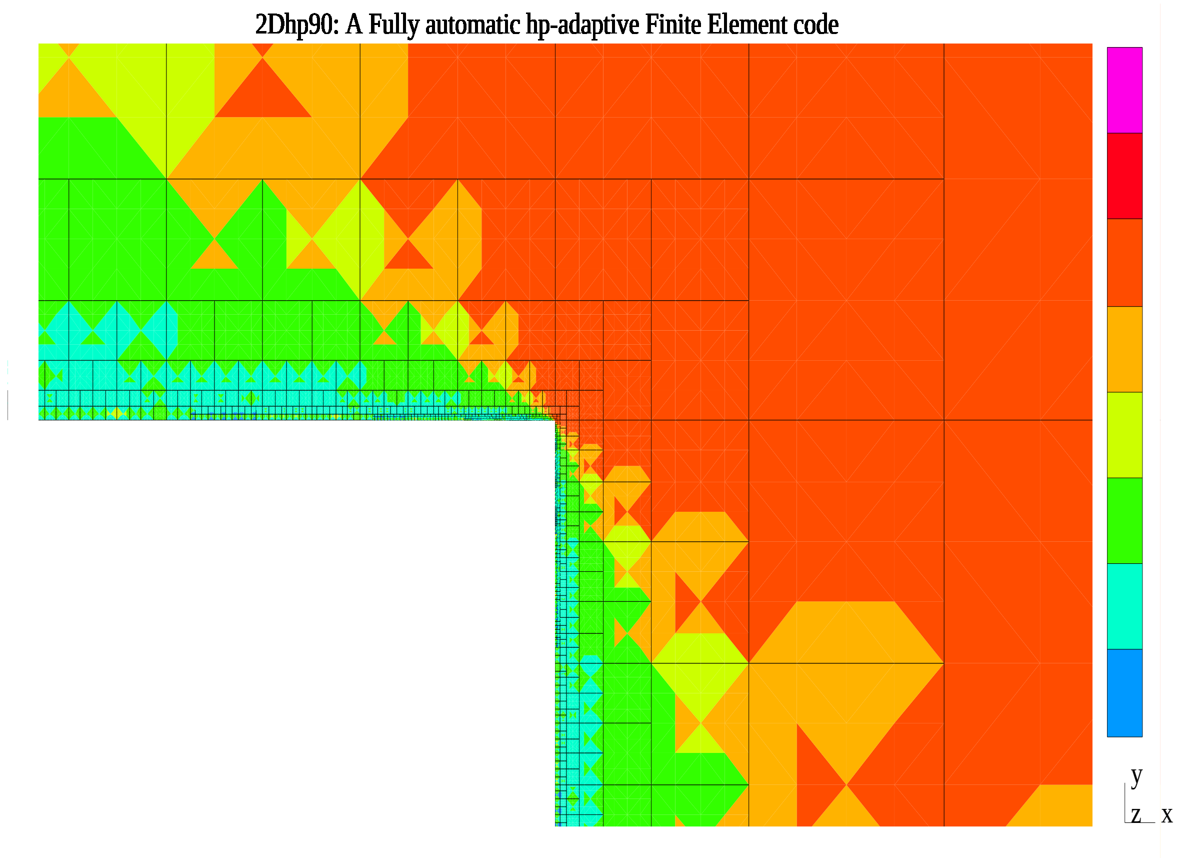
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1



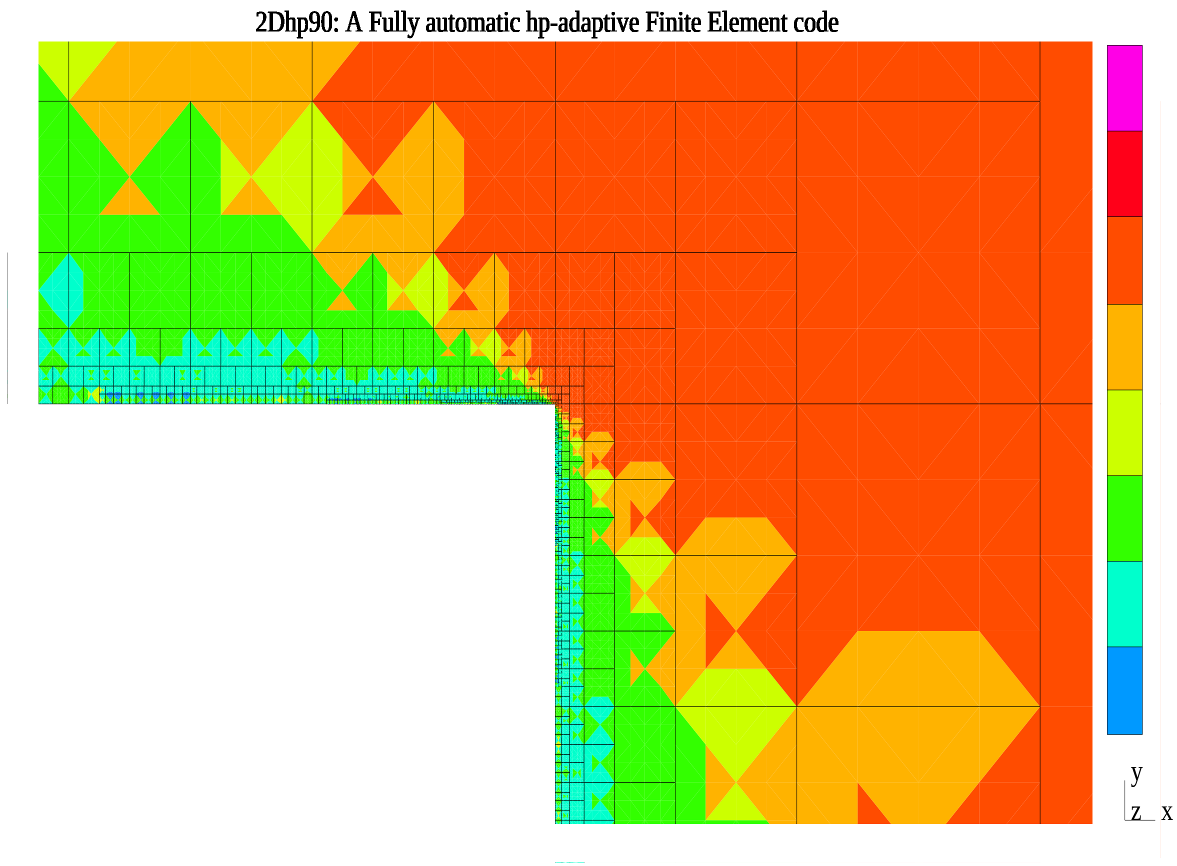
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10



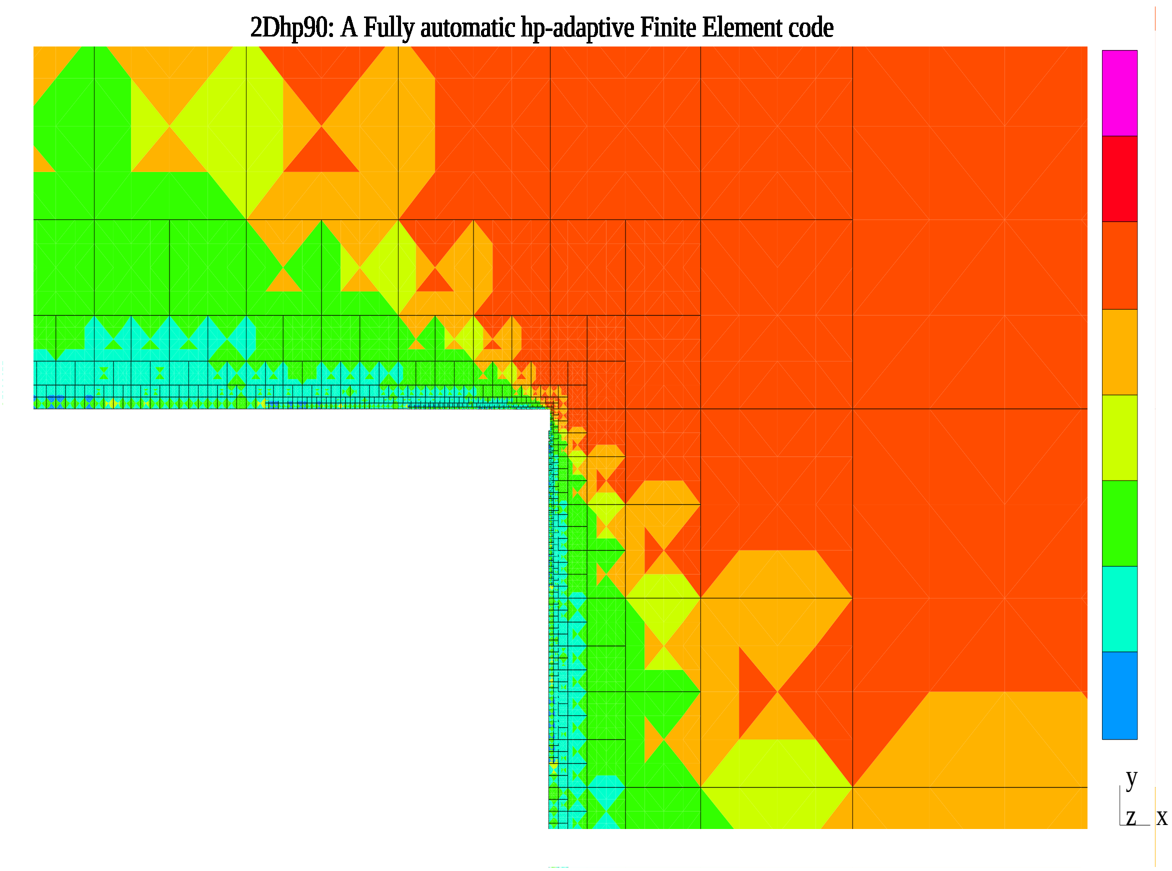
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100



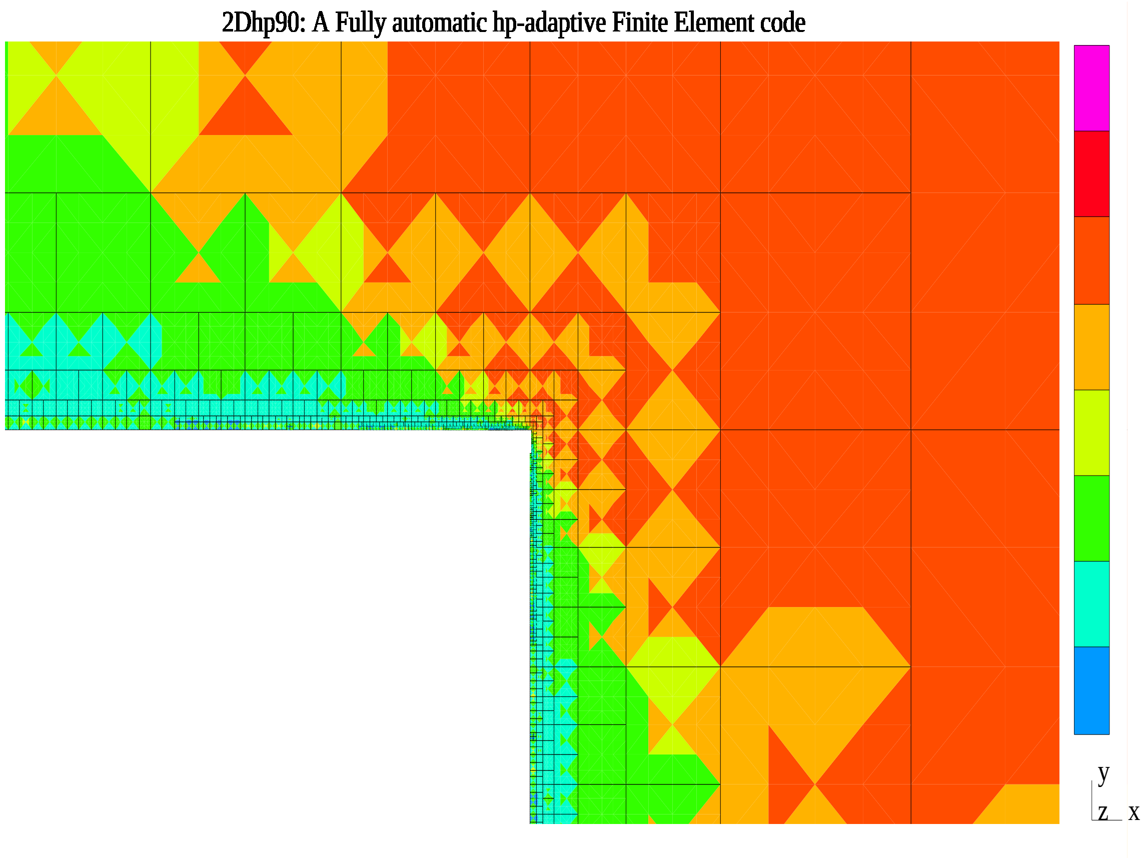
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1000



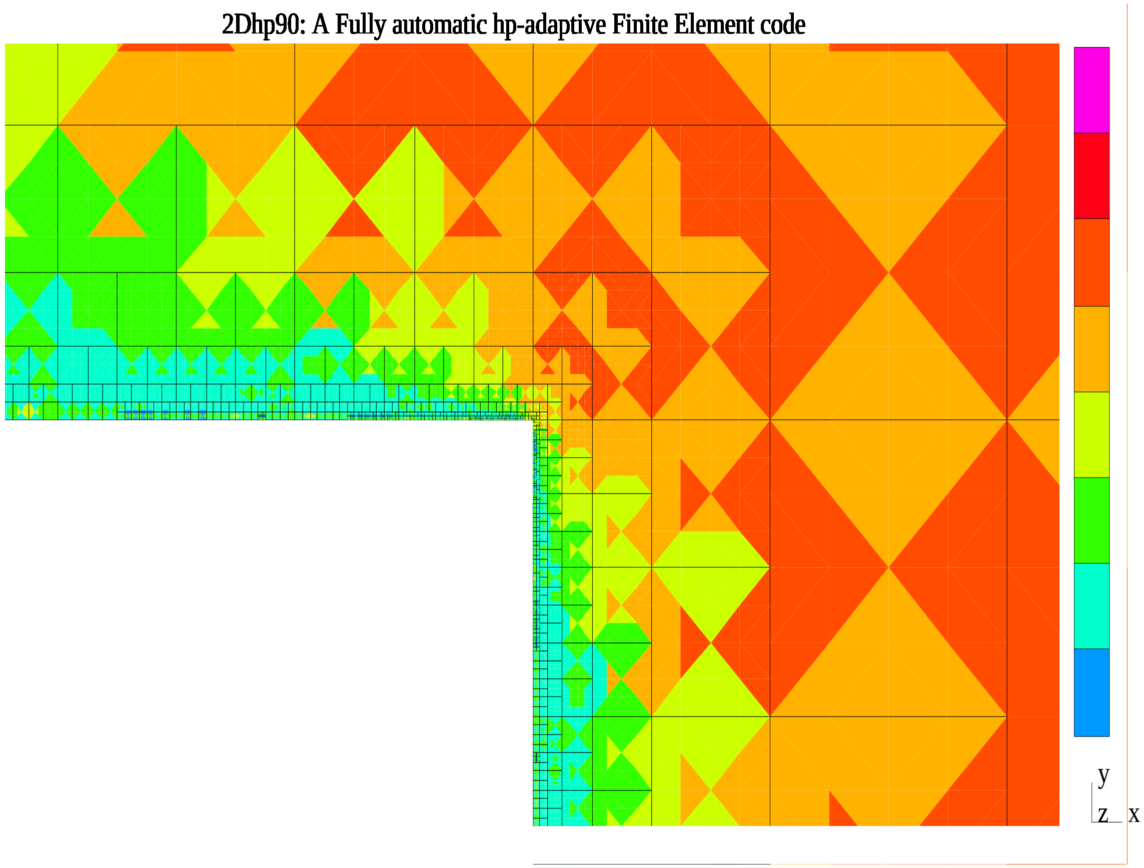
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000



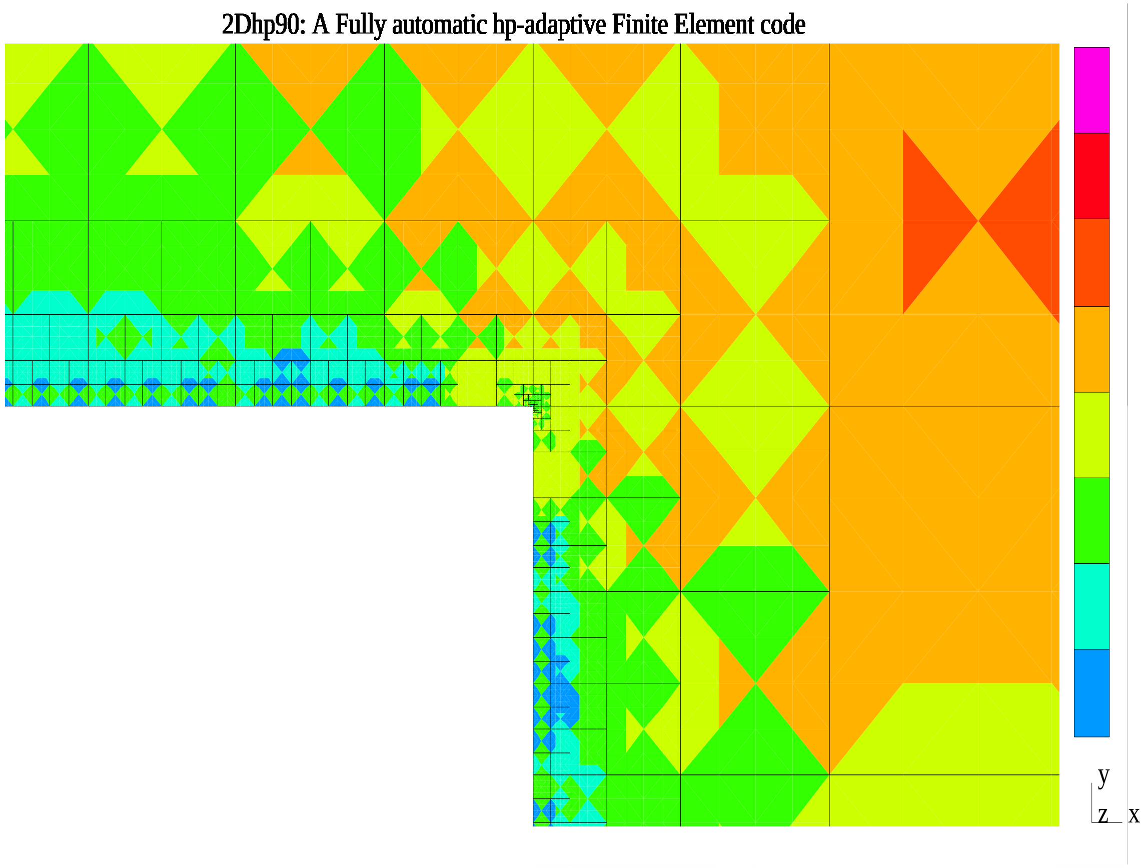
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100000



5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1000000



5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000000

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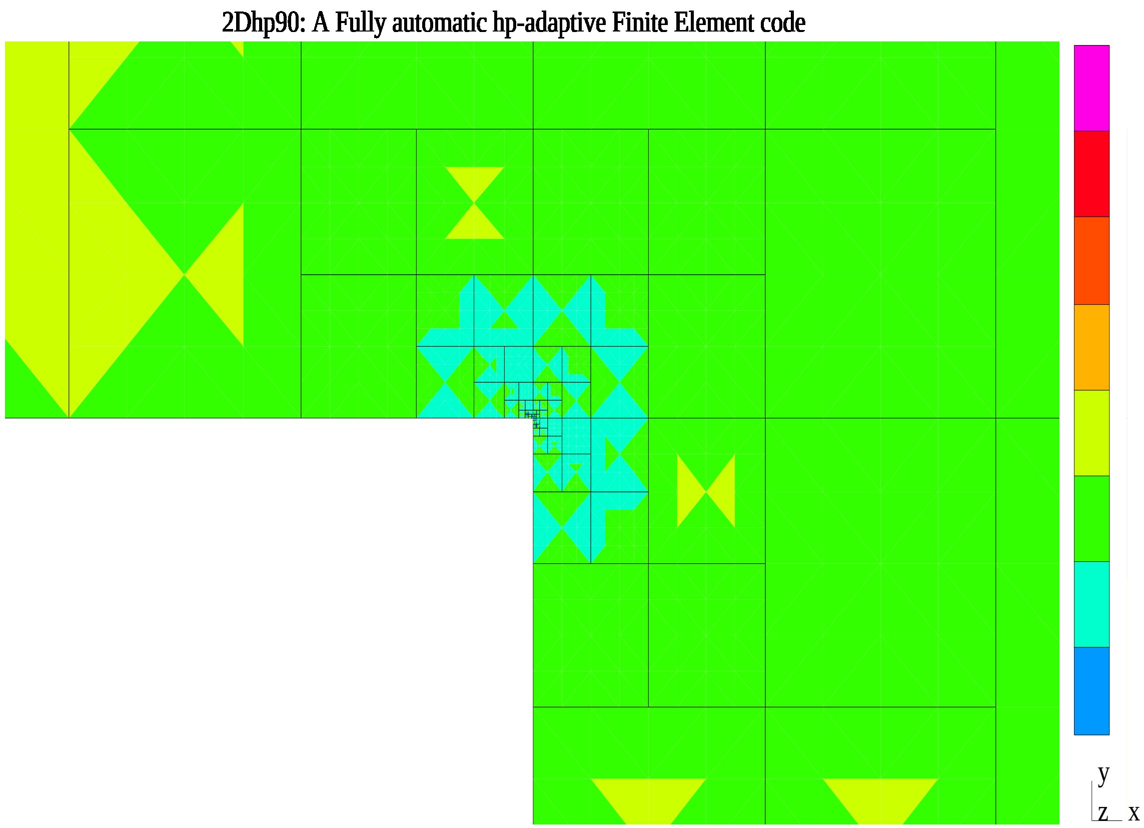
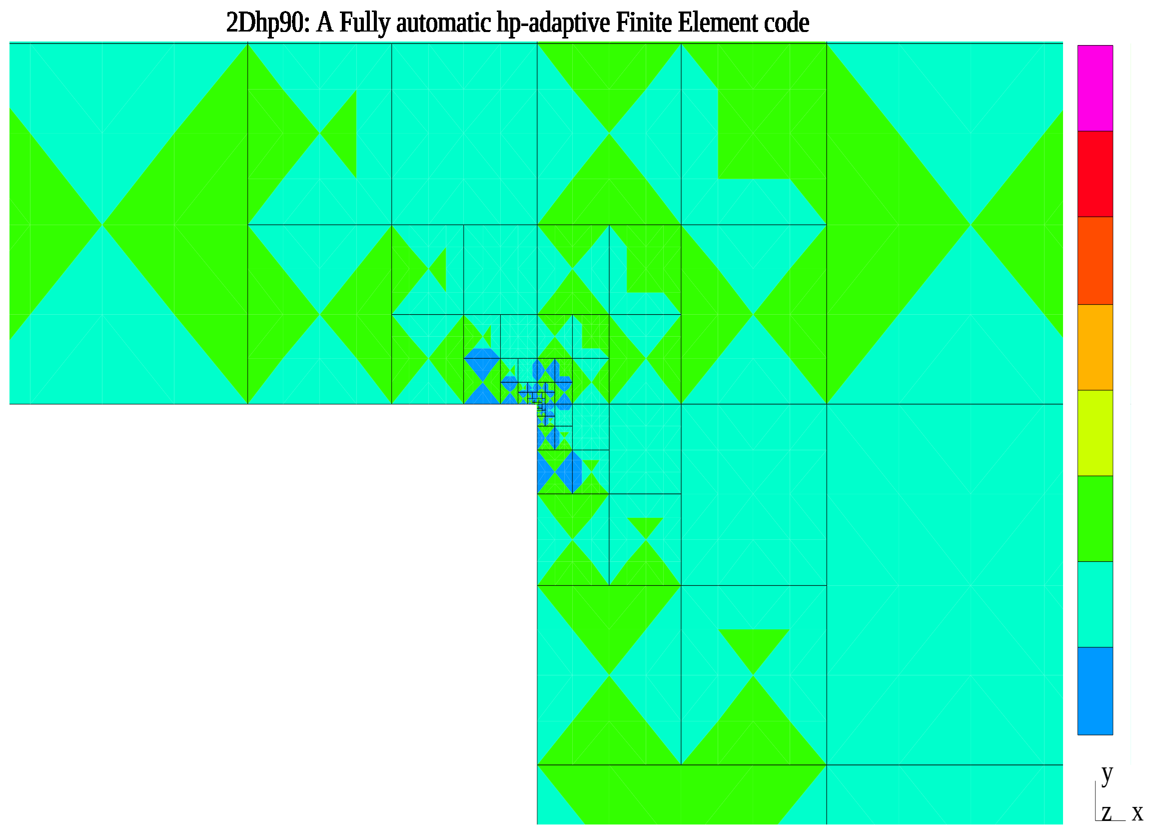
Edge diffraction example: final *hp*-grid, Zoom = 100000000



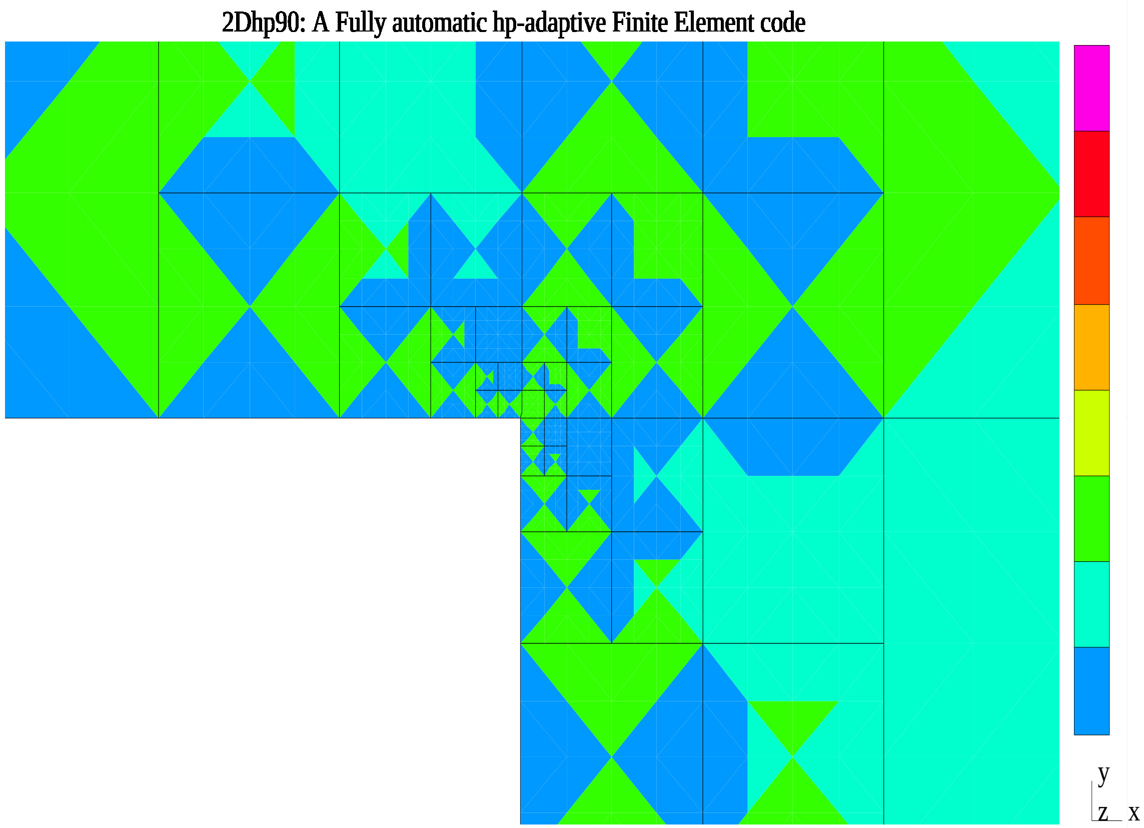
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1000000000



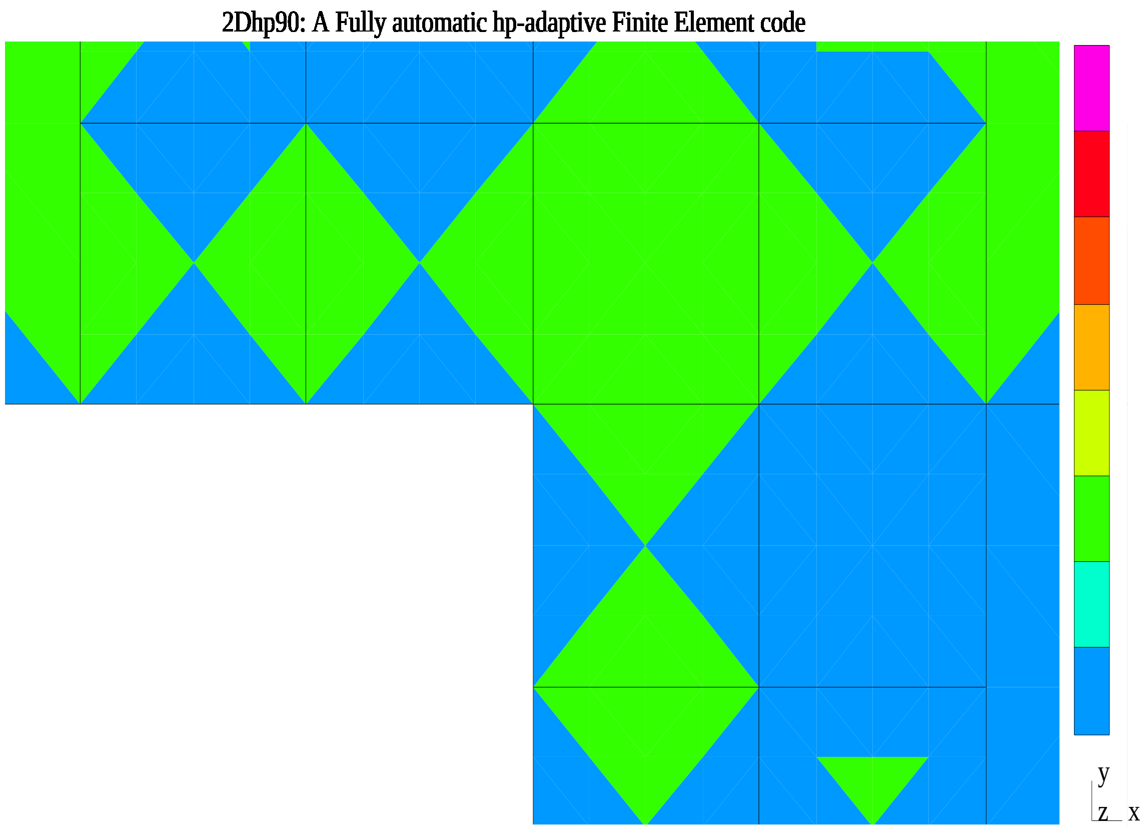
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000000000



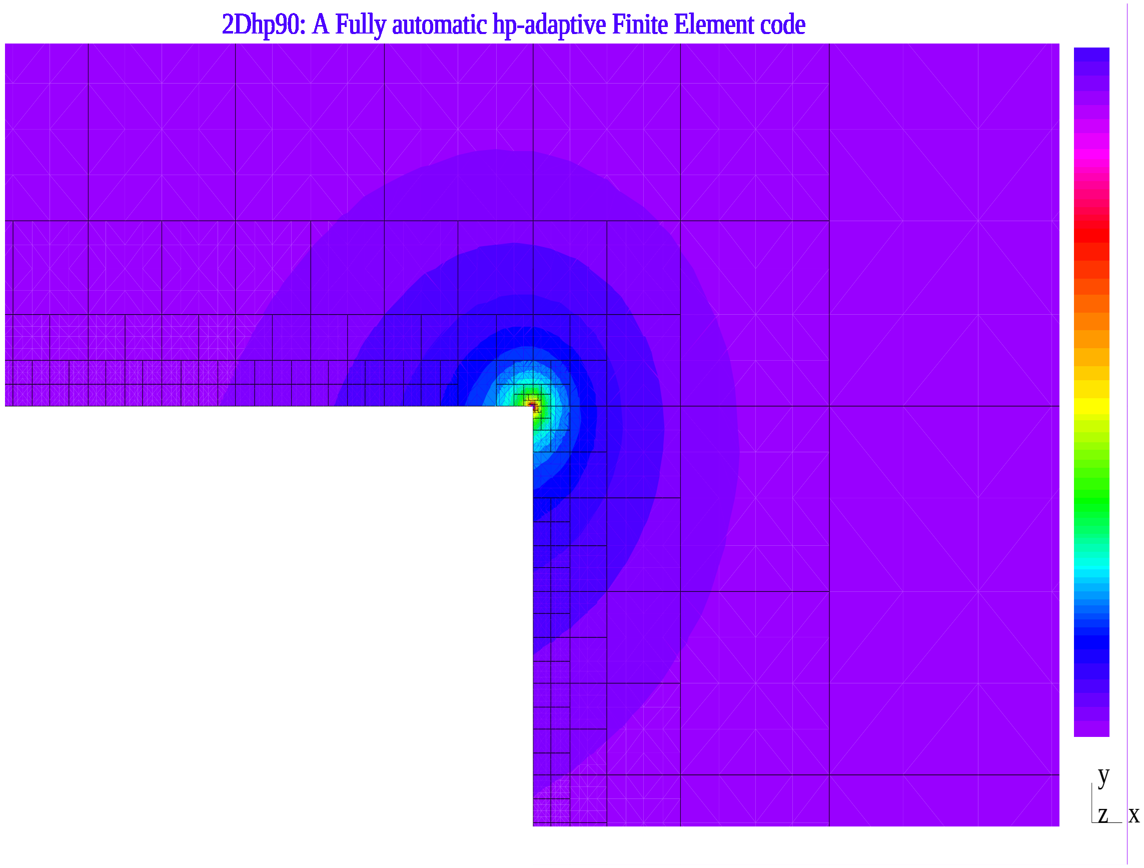
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100000000000



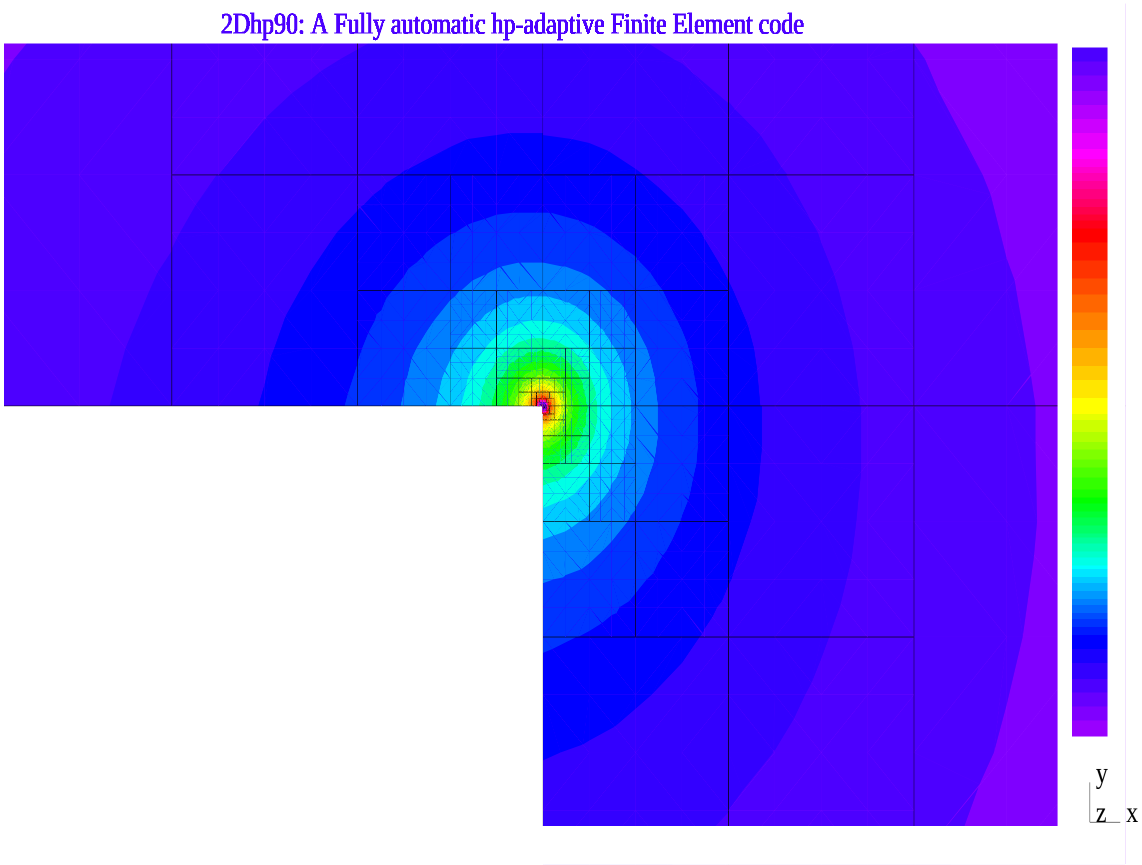
5. NUMERICAL RESULTS

Edge diffraction: final gradient of solution, Zoom = 1000000



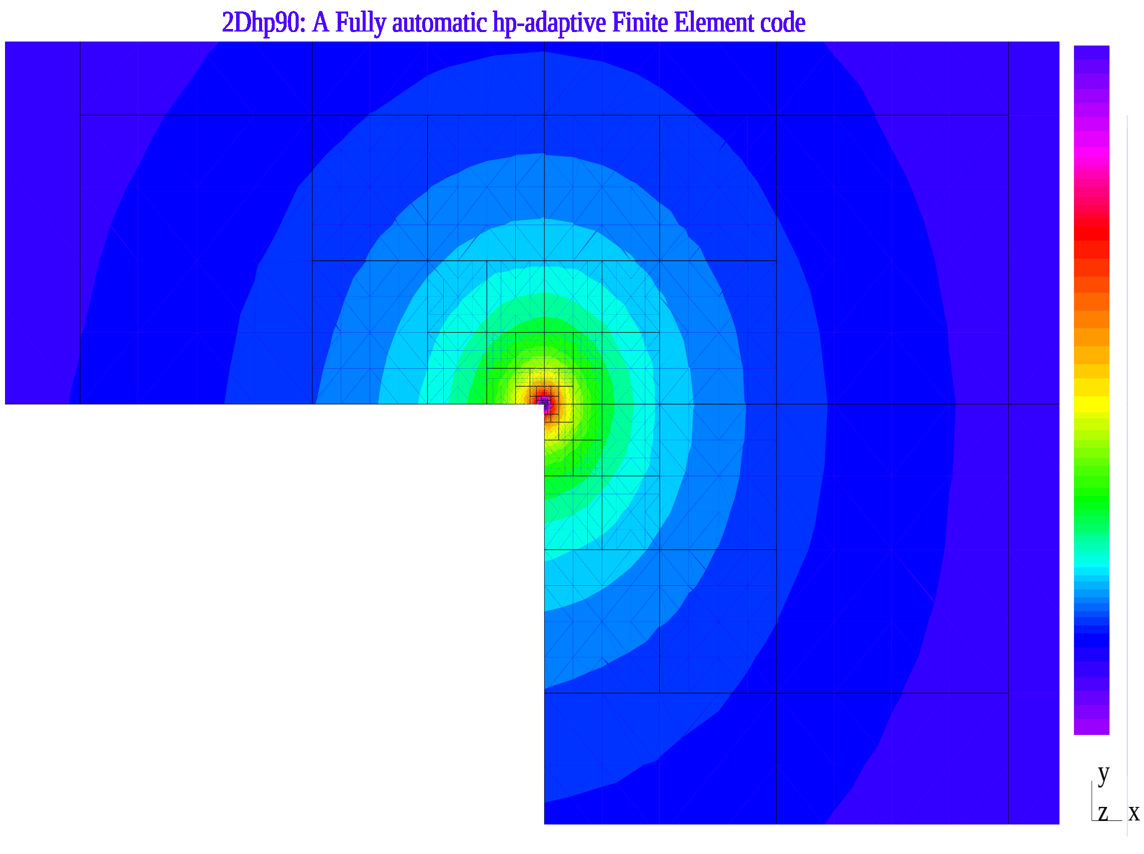
5. NUMERICAL RESULTS

Edge diffraction: final gradient of solution, Zoom = 10000000



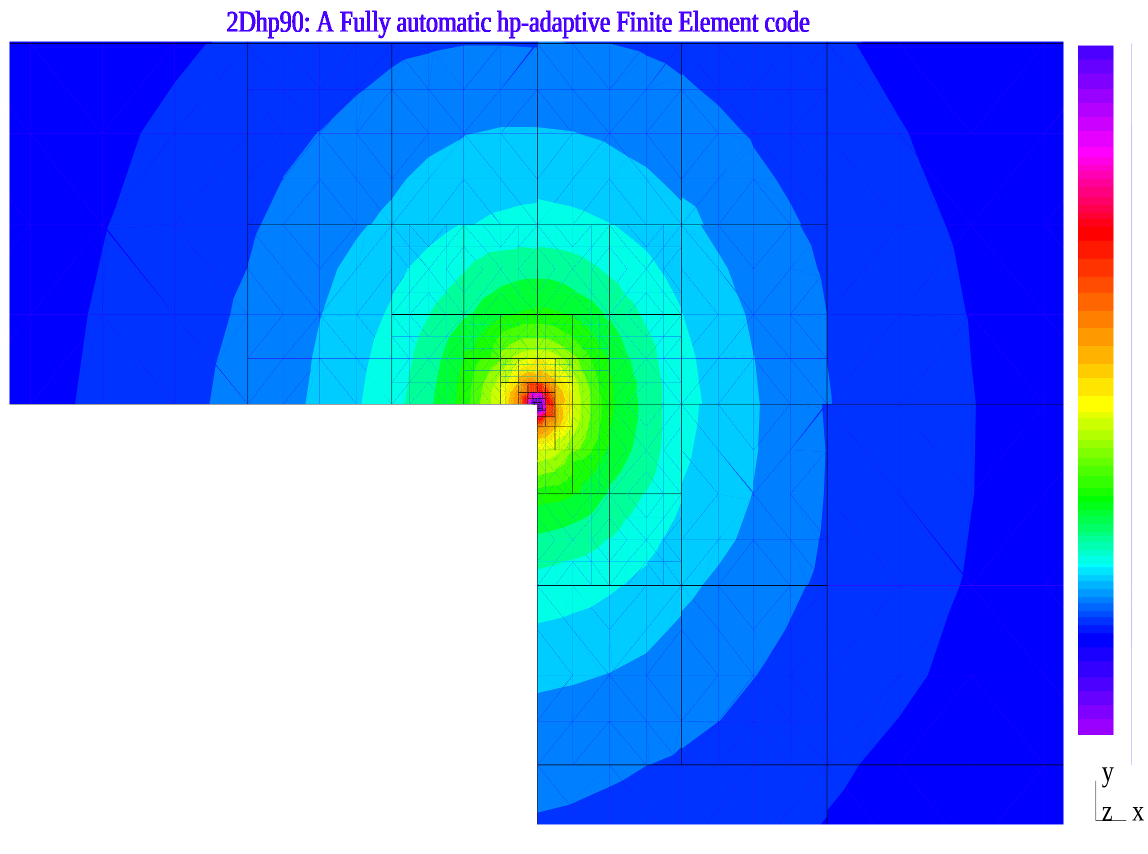
5. NUMERICAL RESULTS

Edge diffraction: final gradient of solution, Zoom = 100000000



5. NUMERICAL RESULTS

Edge diffraction: final gradient of solution, Zoom = 1000000000



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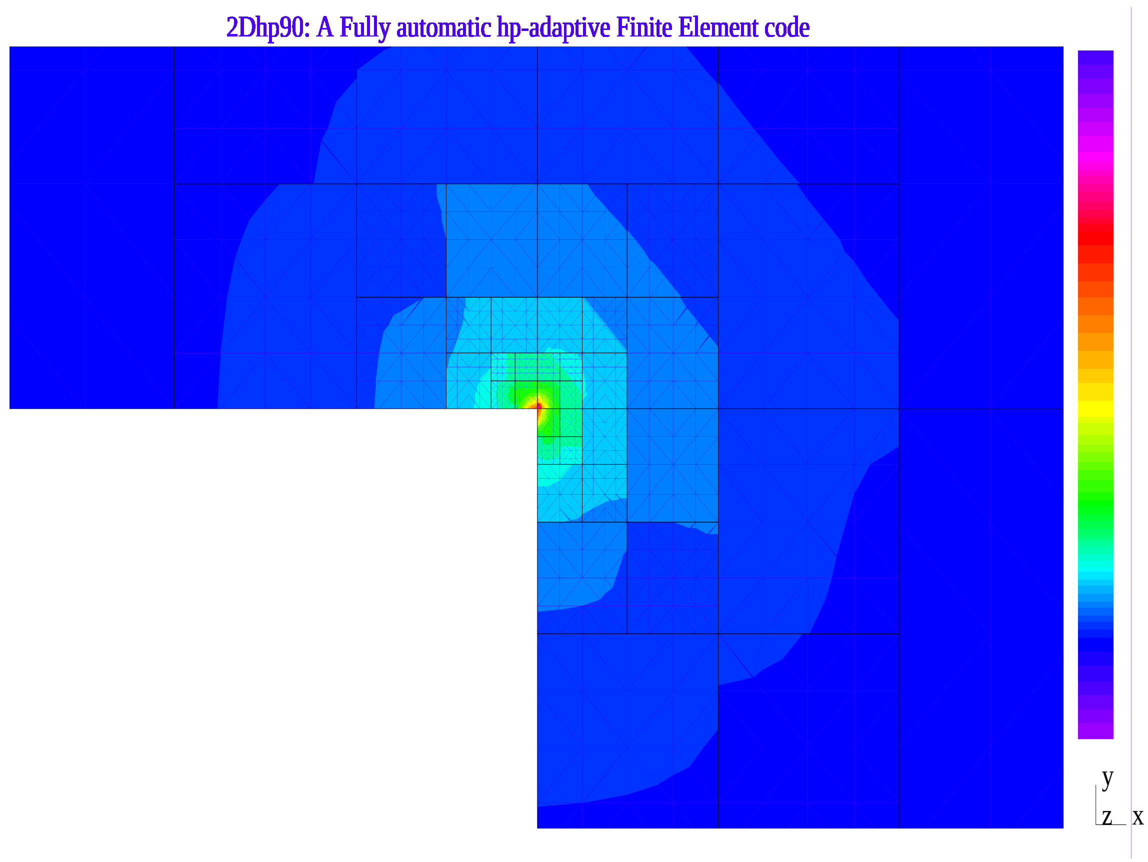
Edge diffraction: final gradient of solution, Zoom = 1000000000



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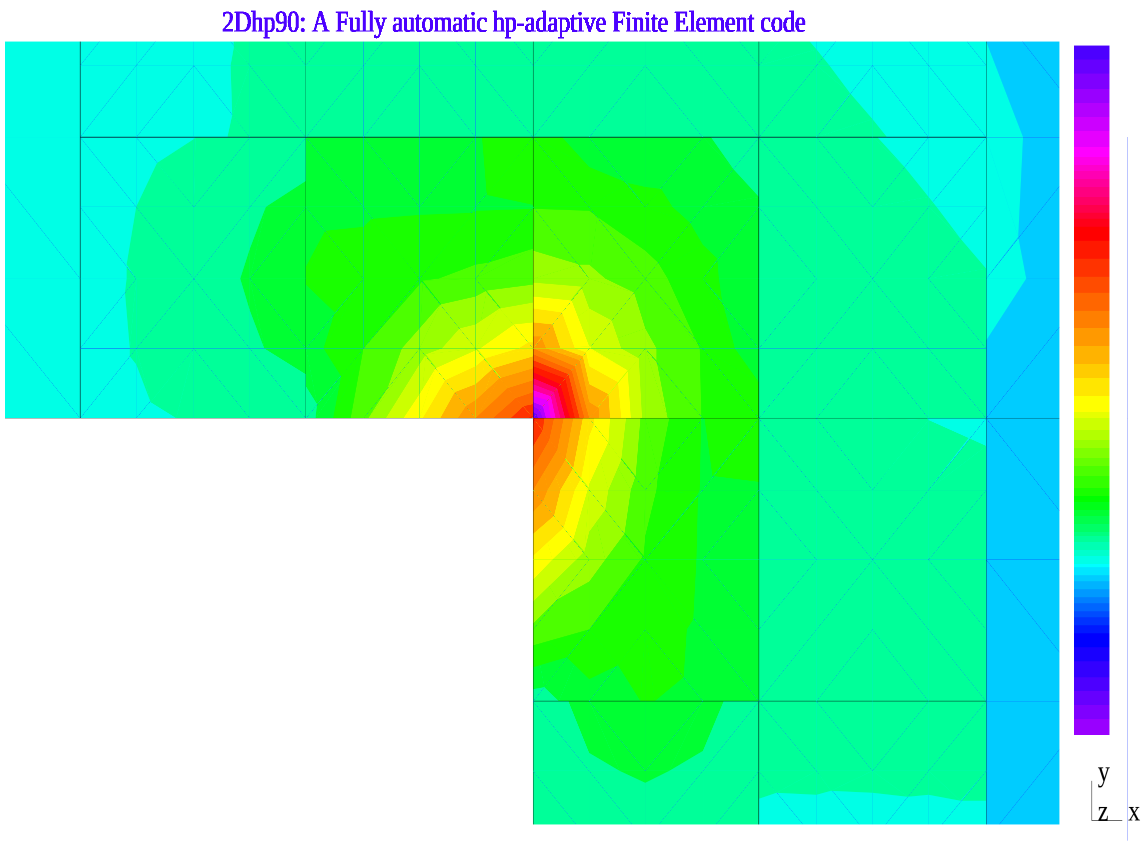
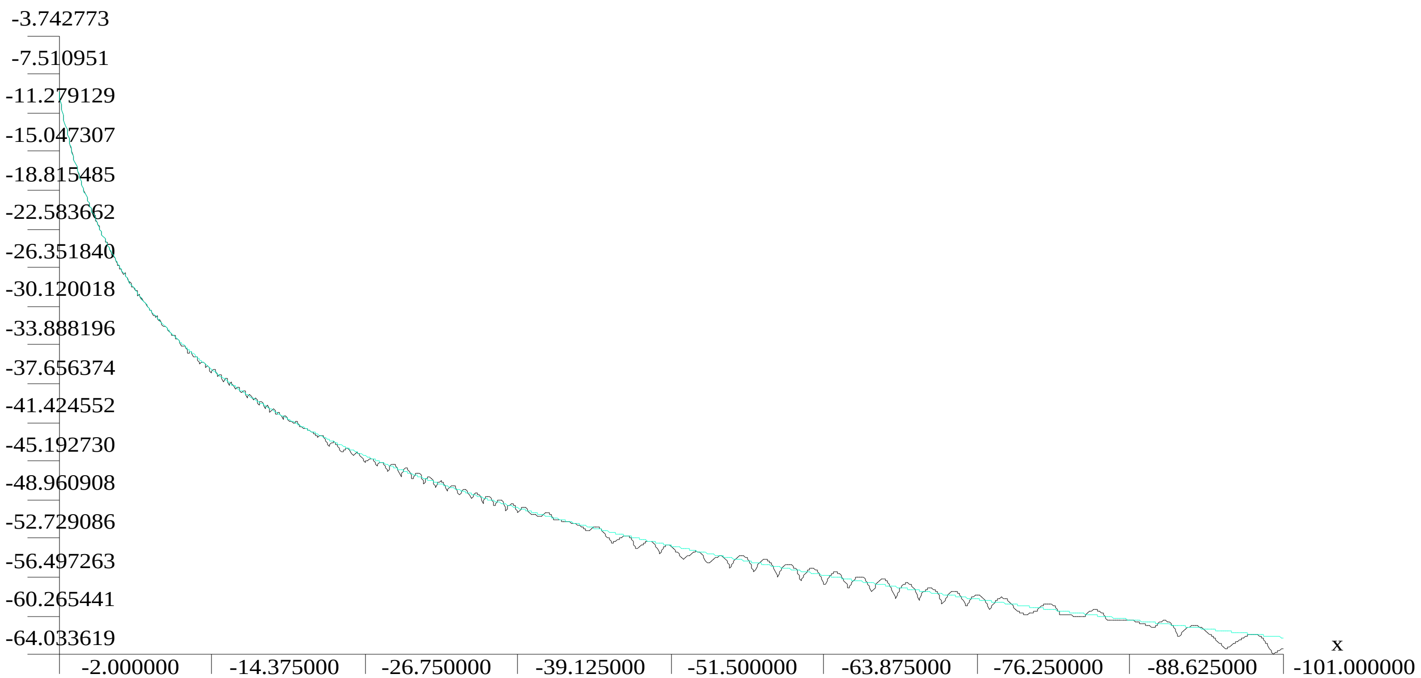
Edge diffraction: final gradient of solution, Zoom = 100000000000



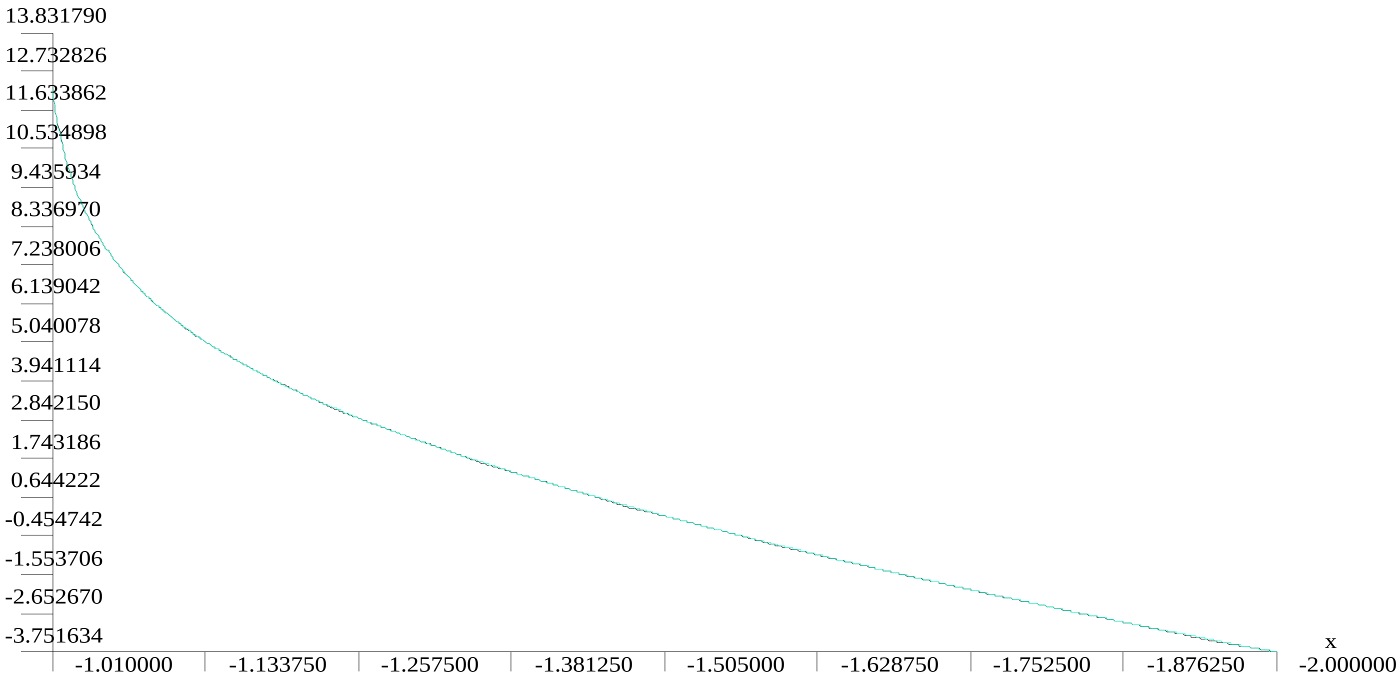
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 1-100 from the singularity



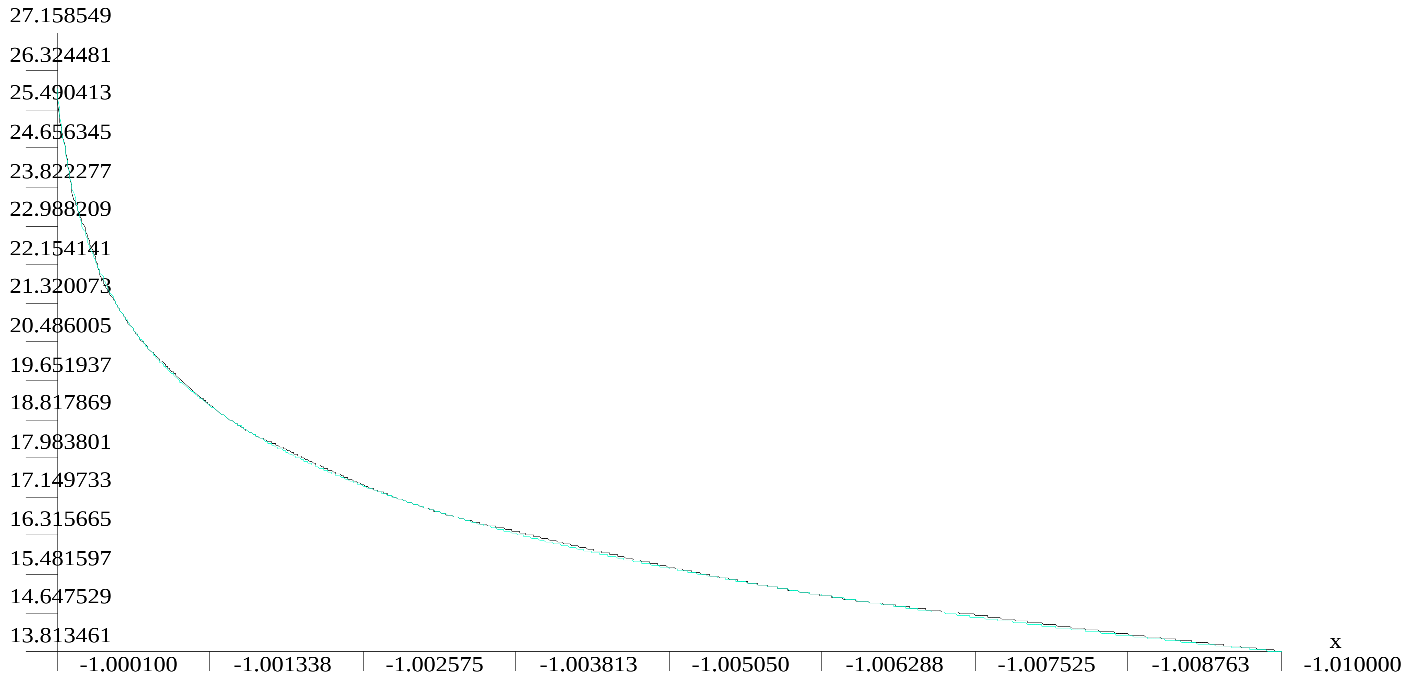
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.01-1 from the singularity



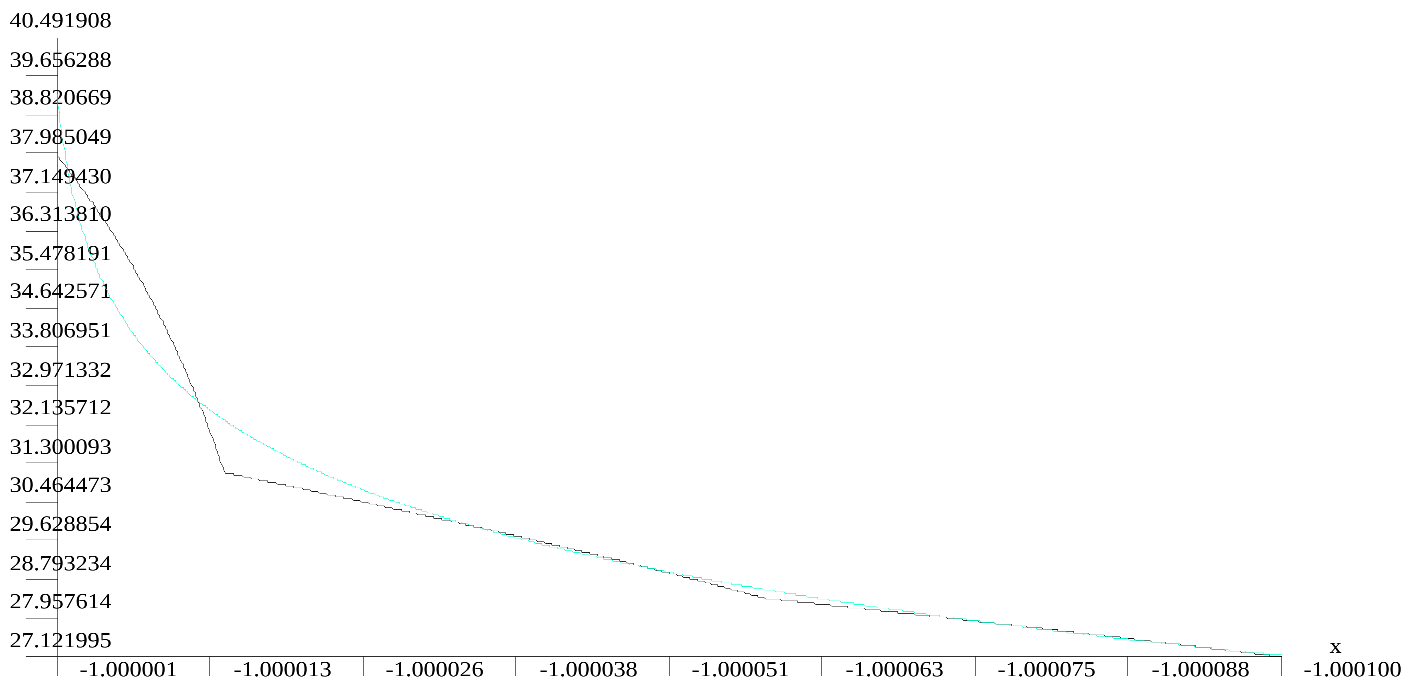
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.0001-0.01 from the singularity



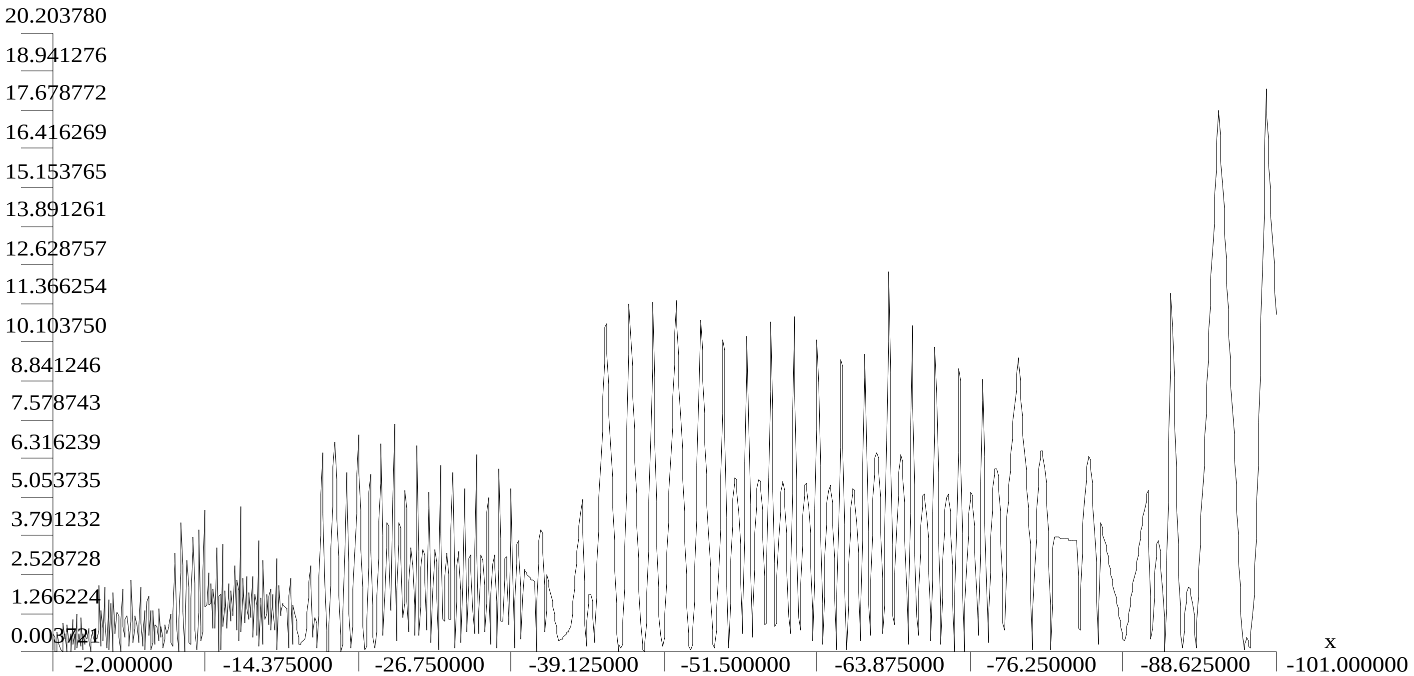
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.000001-0.0001 from the singularity



5. NUMERICAL RESULTS

Edge diffraction example: Relative error in percentage of the approximate solution at distances 1-100 from the singularity



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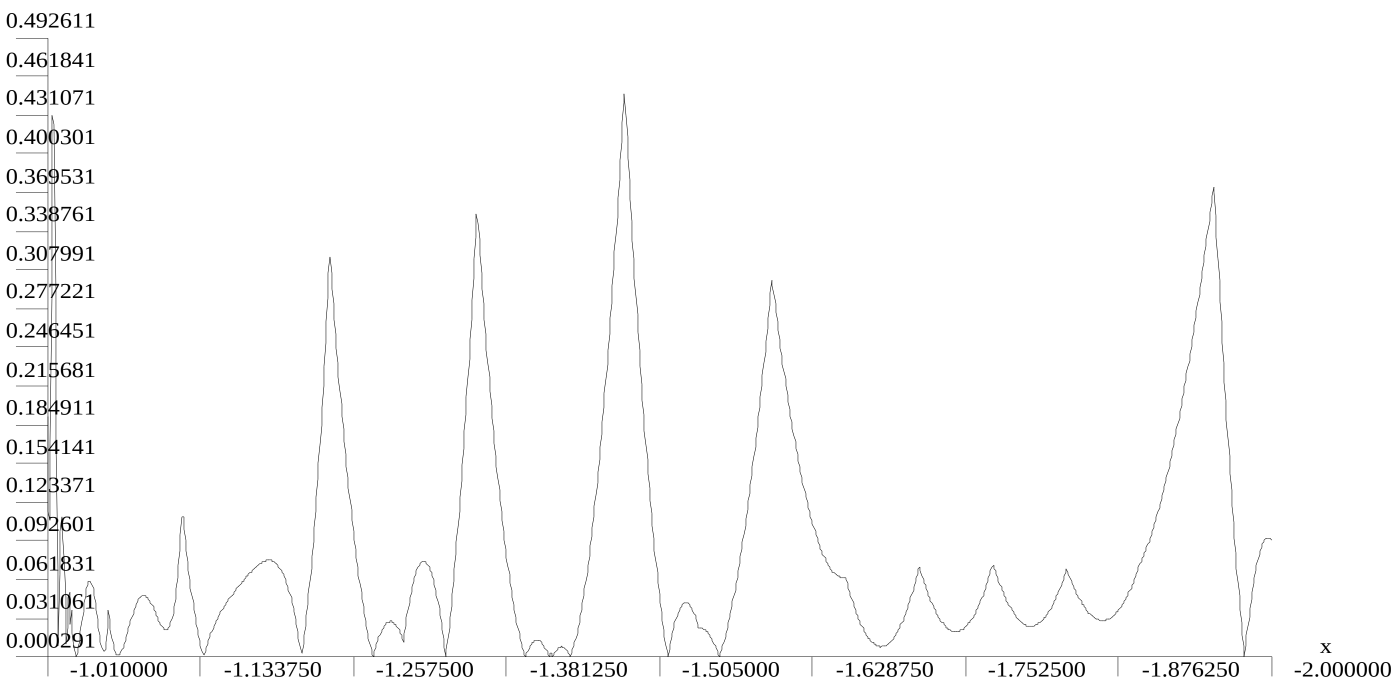

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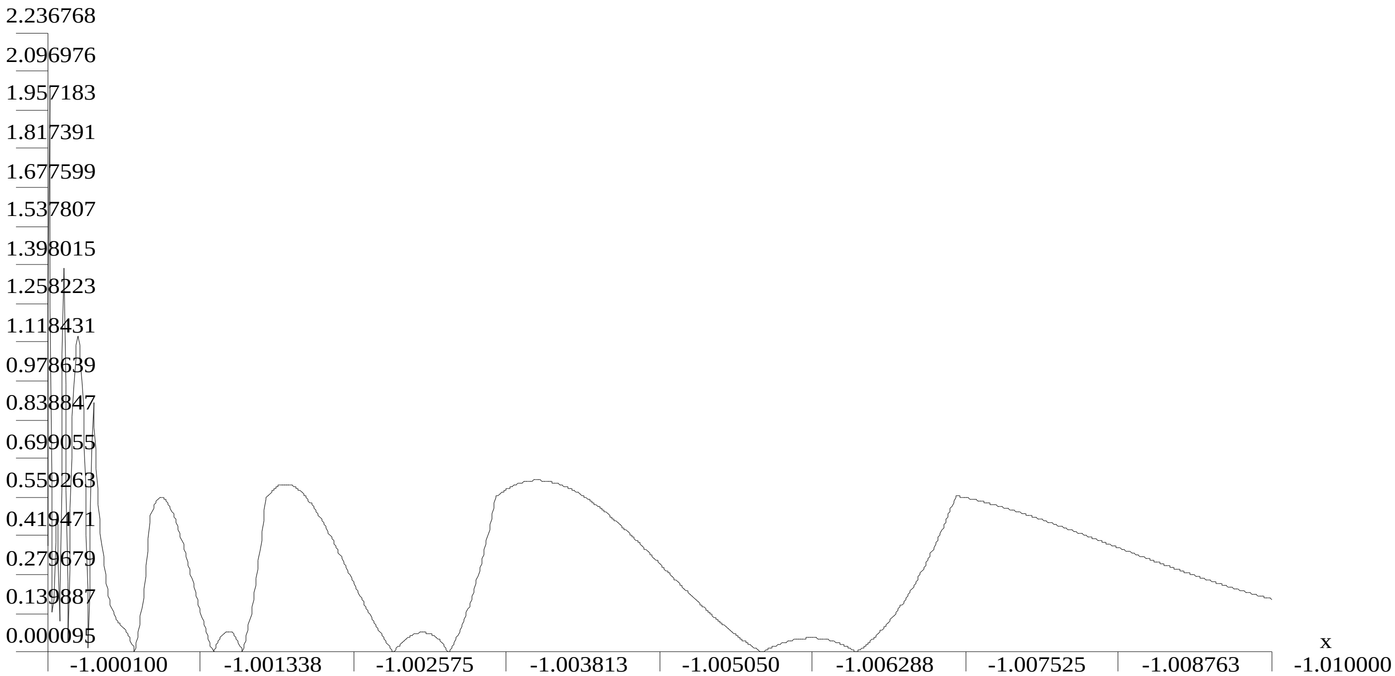
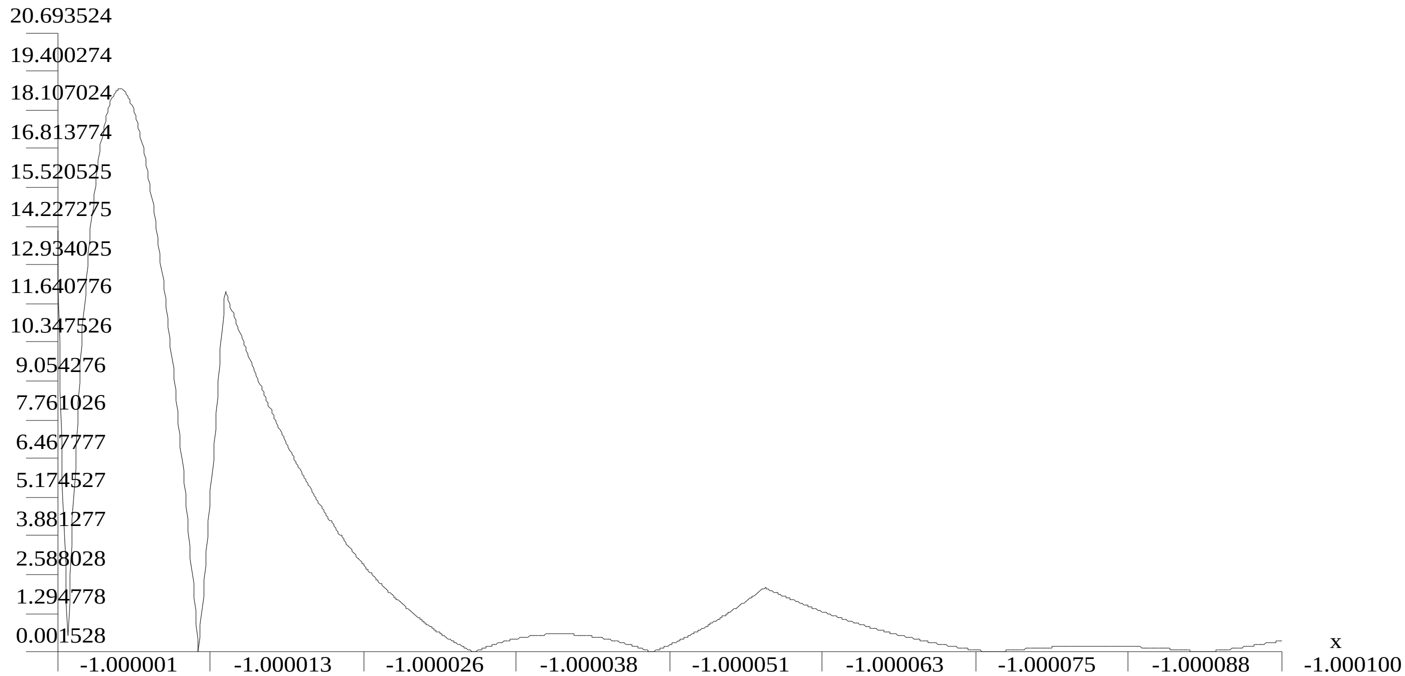
Edge diffraction example: Relative error in percentage of the approximate solution at distances 0.0001-0.01 from the singularity



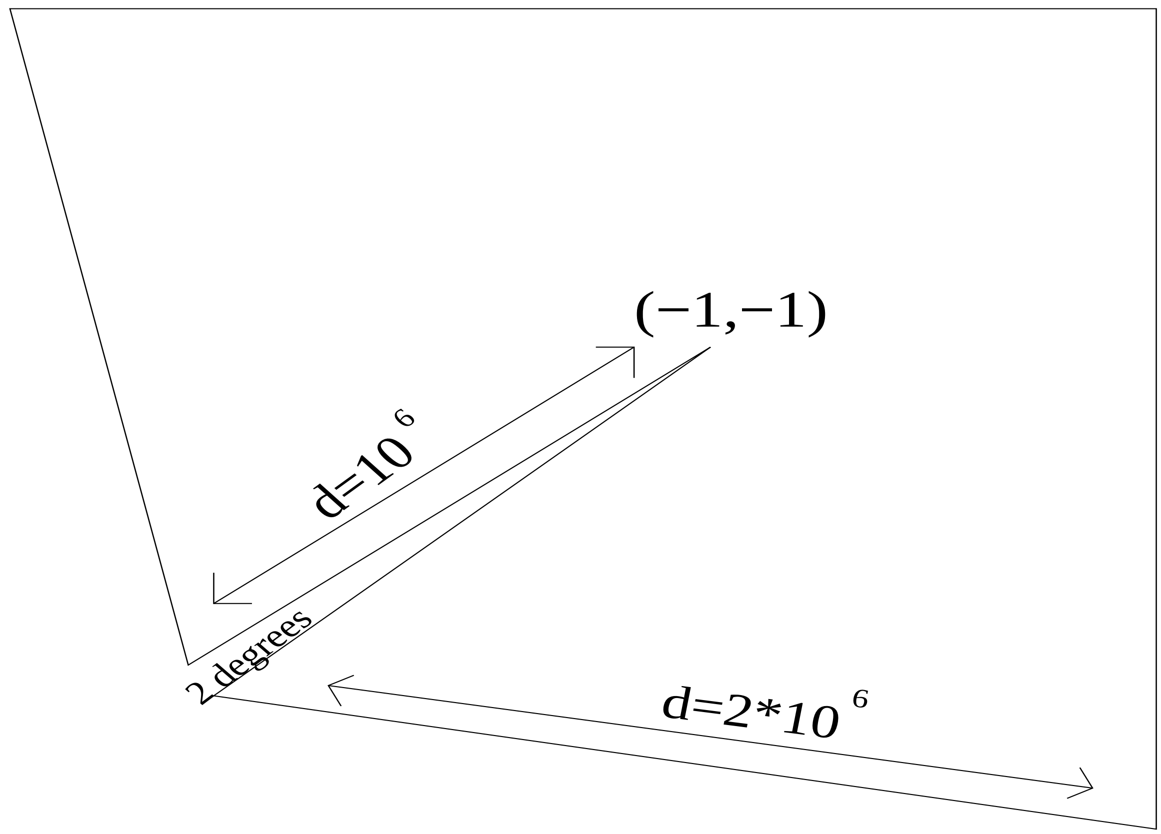
5. NUMERICAL RESULTS

Edge diffraction example: Relative error in percentage of the approximate solution at distances 0.000001-0.0001 from the singularity



5. NUMERICAL RESULTS

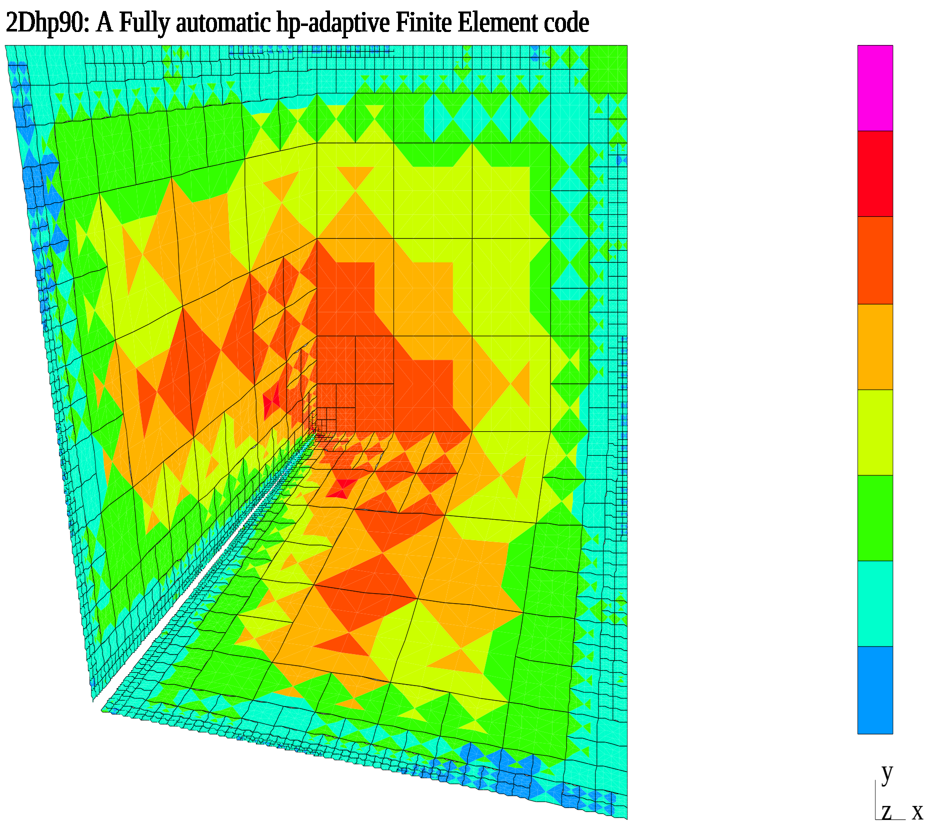
Edge diffraction example: Laplace equation



Dirichlet Boundary Conditions
u(boundary)=−ln r, r=sqrt (x*x+y*y)

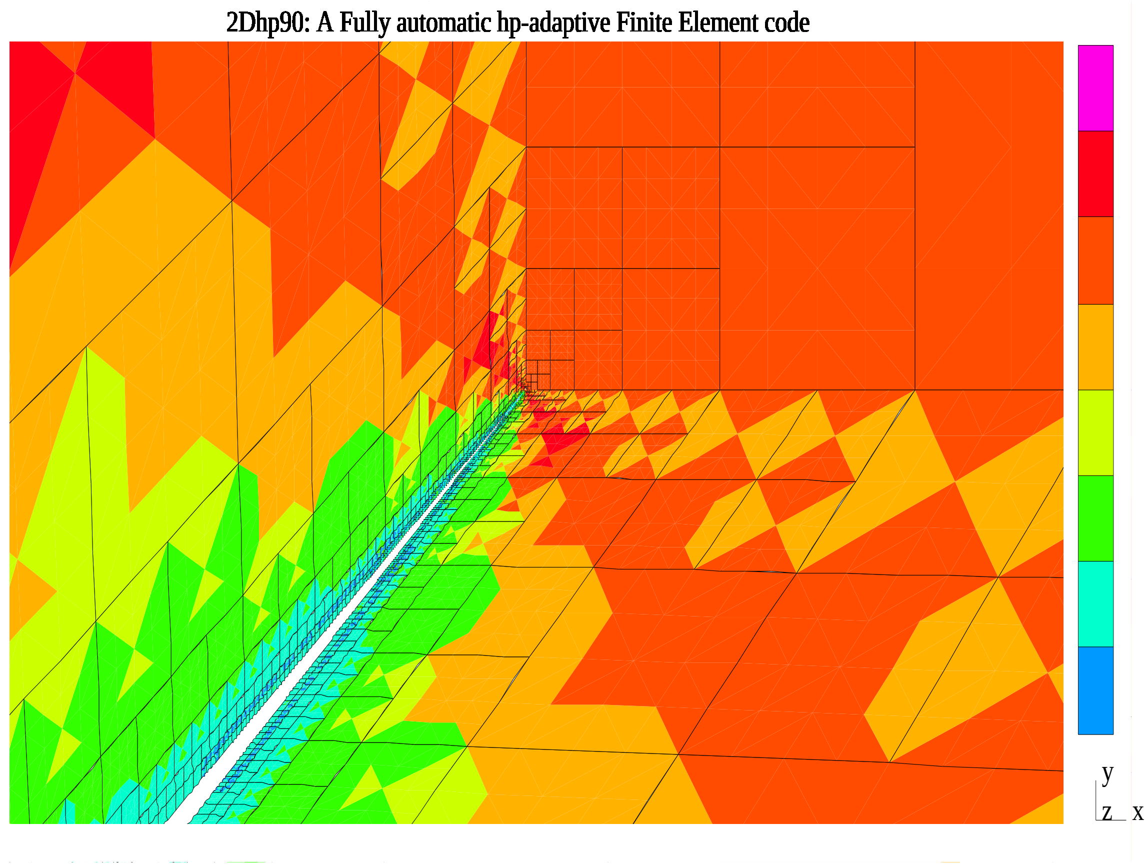
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1



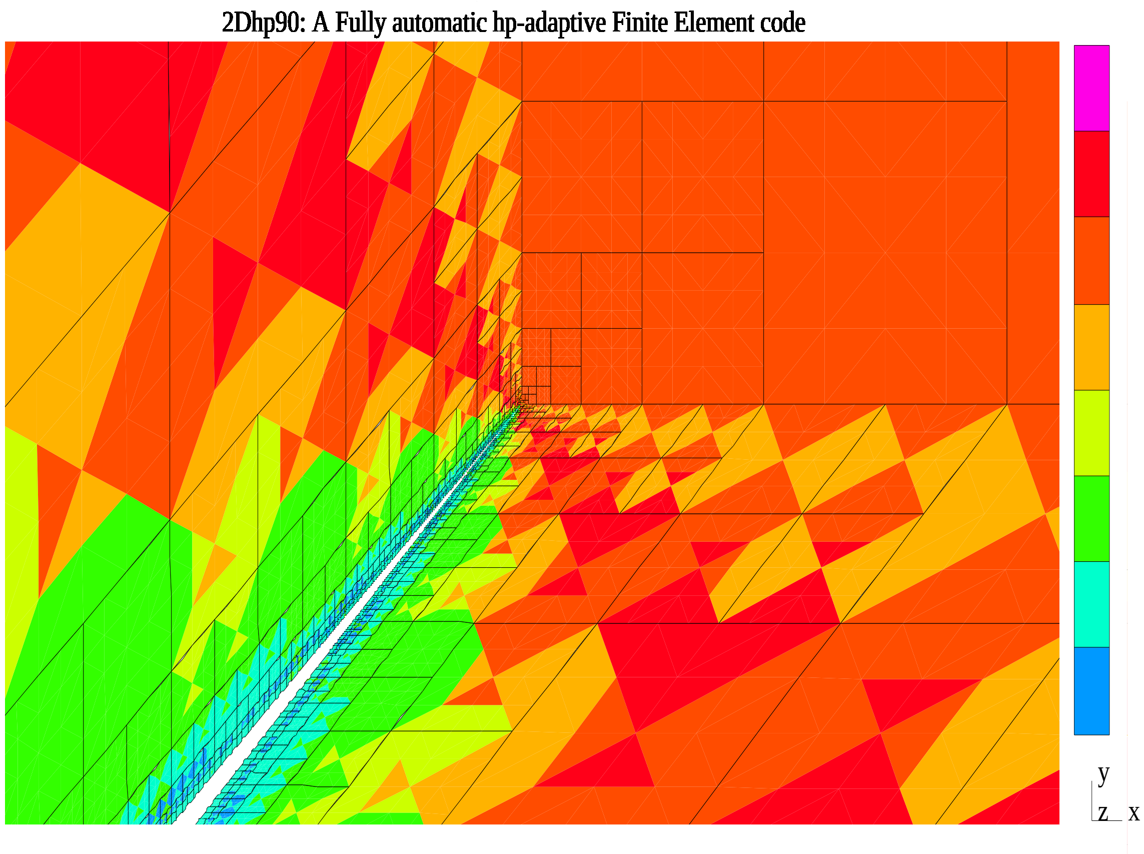
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10



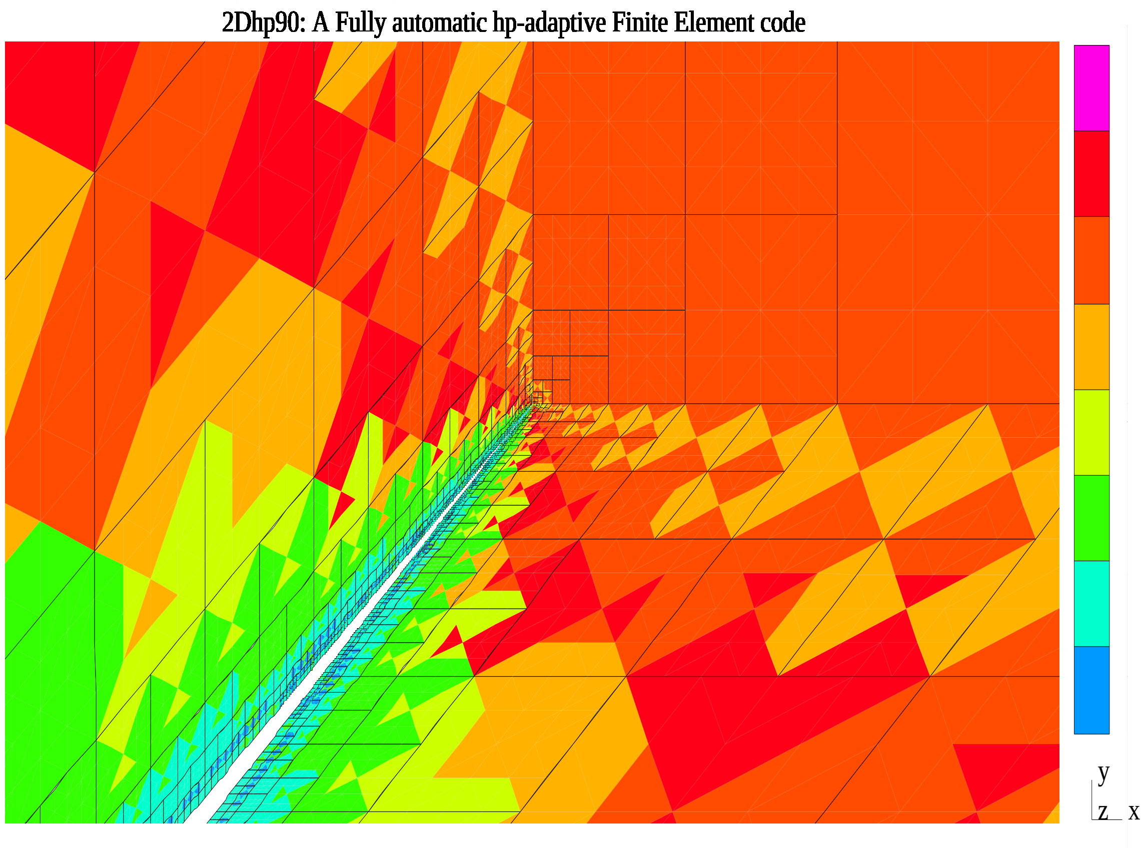
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100



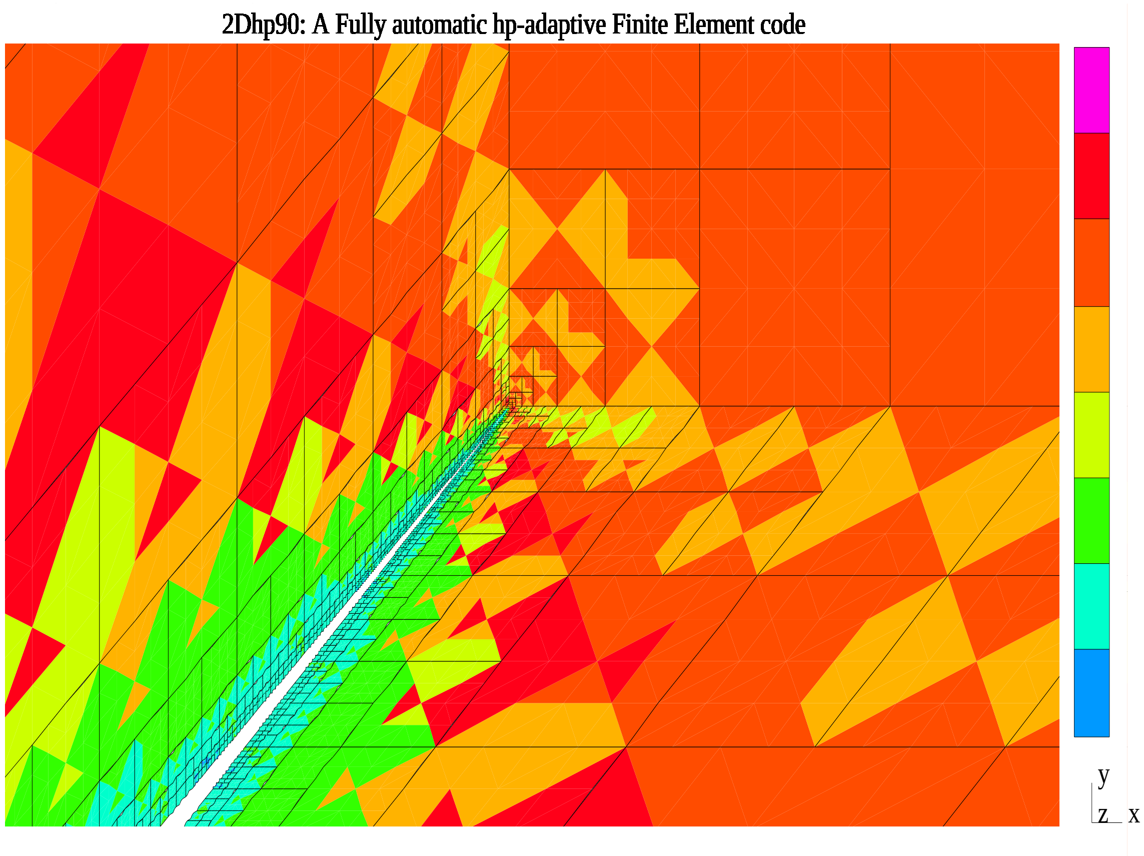
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1000



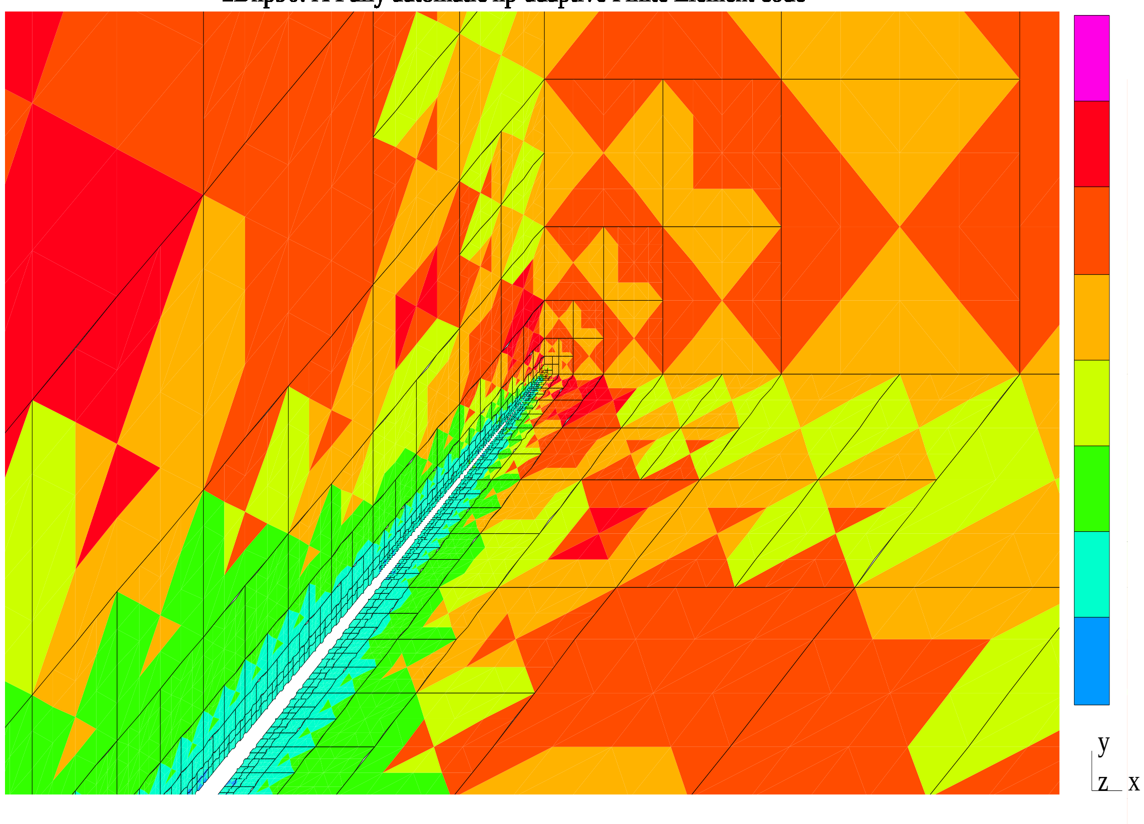
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000



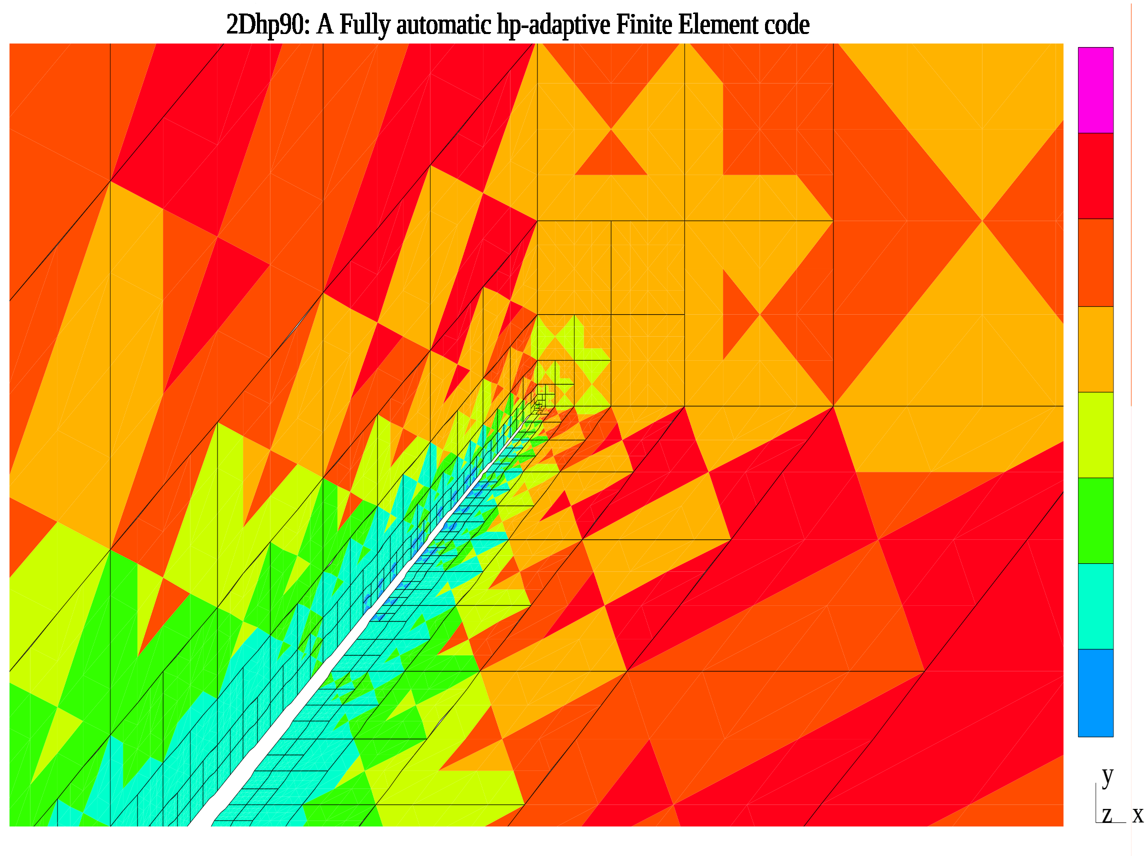
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100000



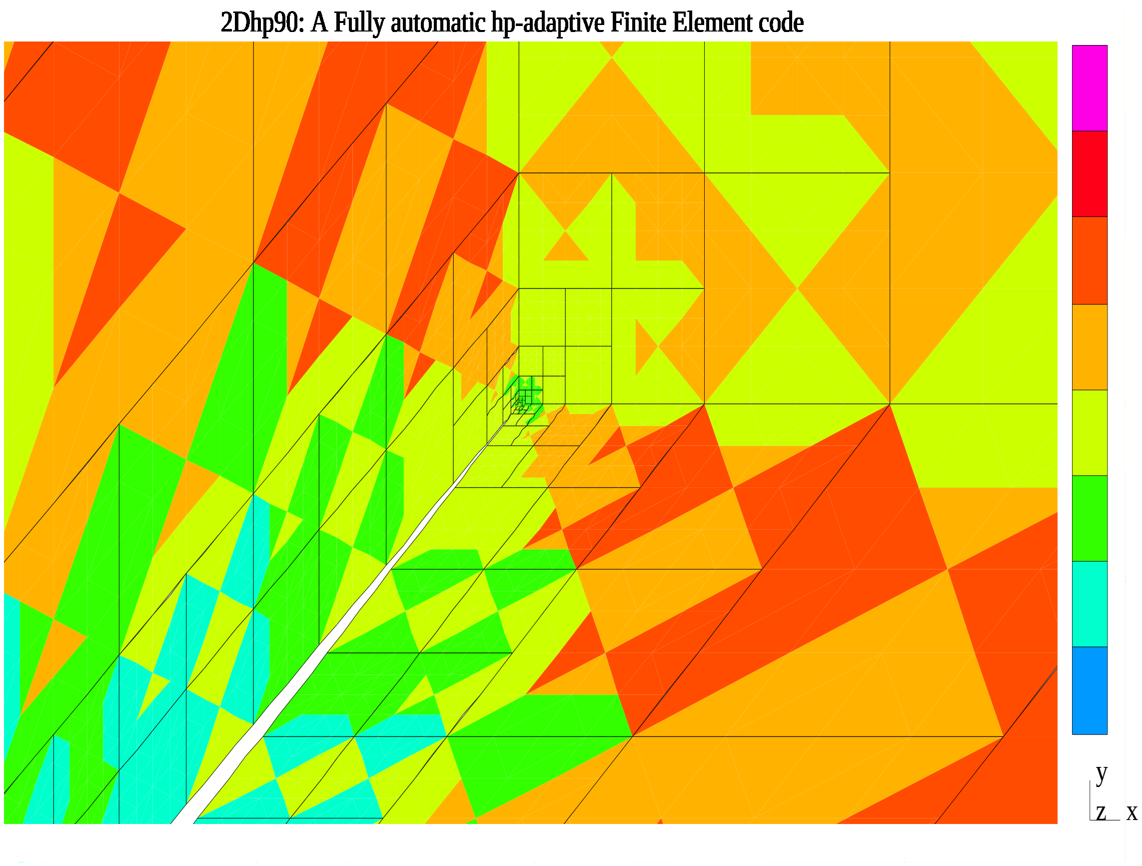
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1000000



5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000000

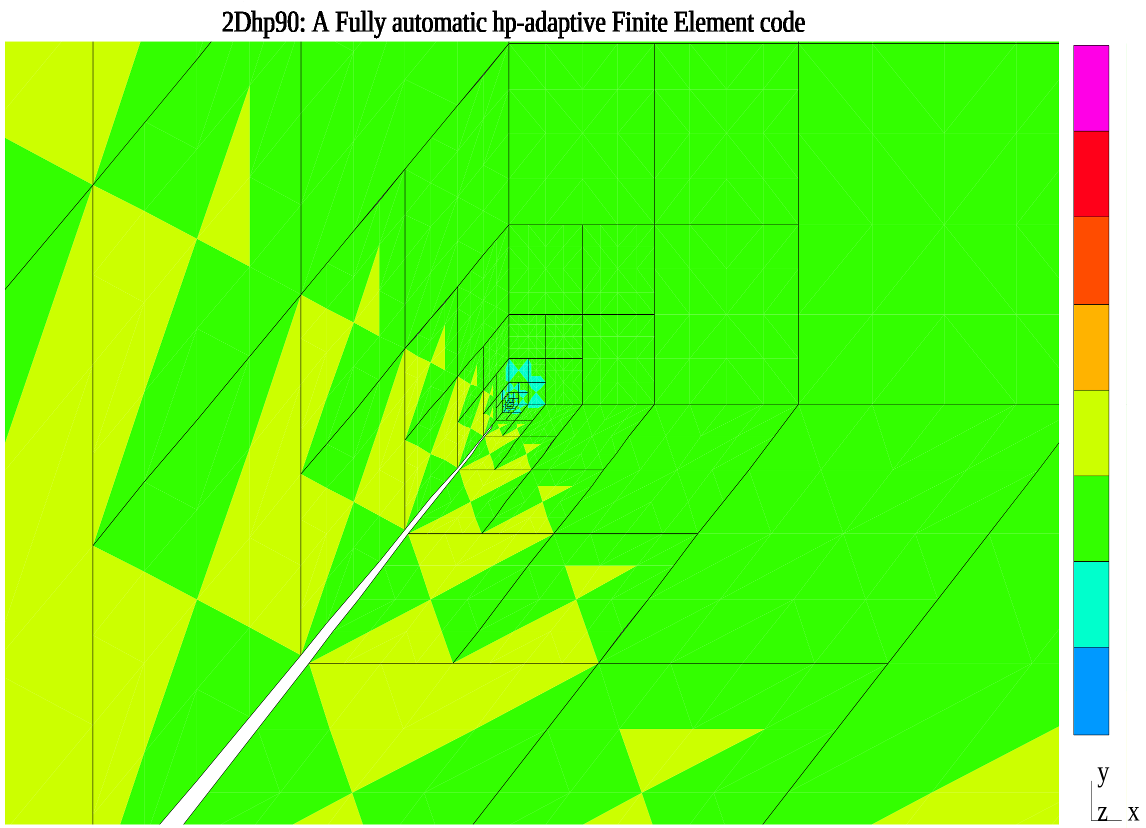


5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100000000

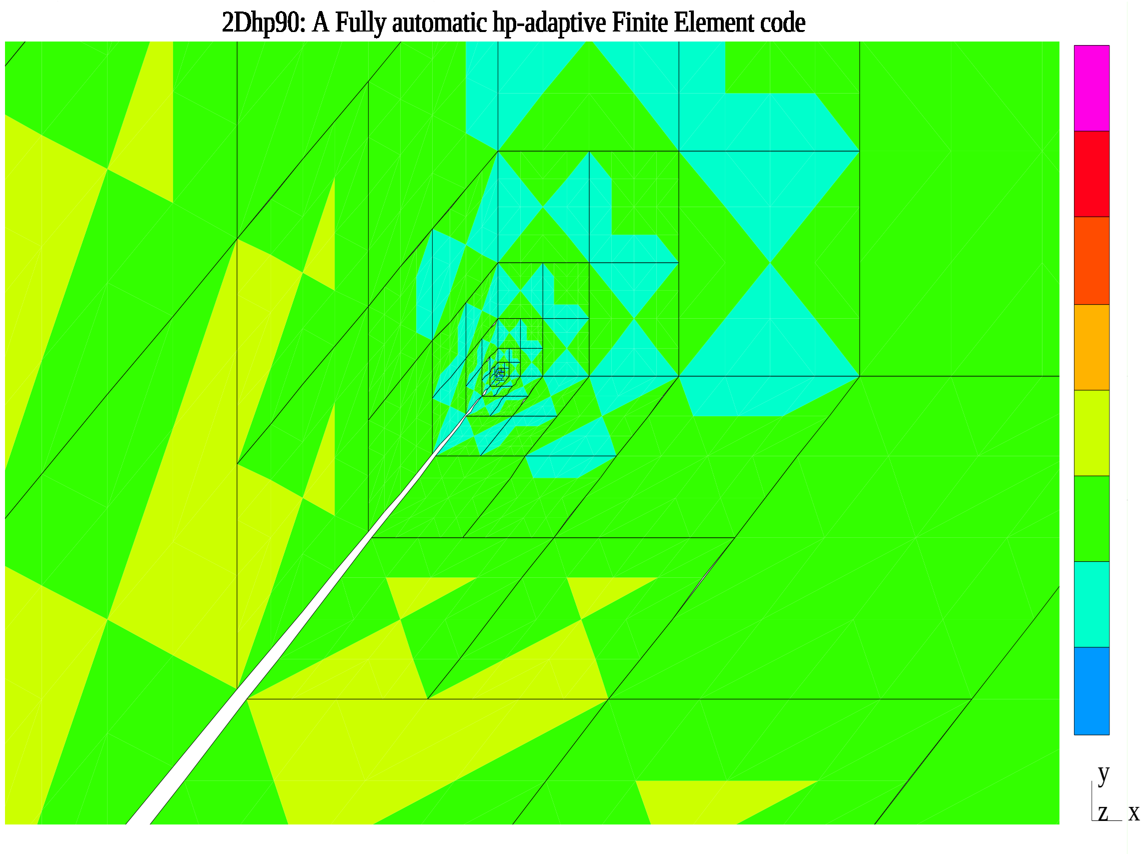
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1000000000



5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000000000



5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 100000000000

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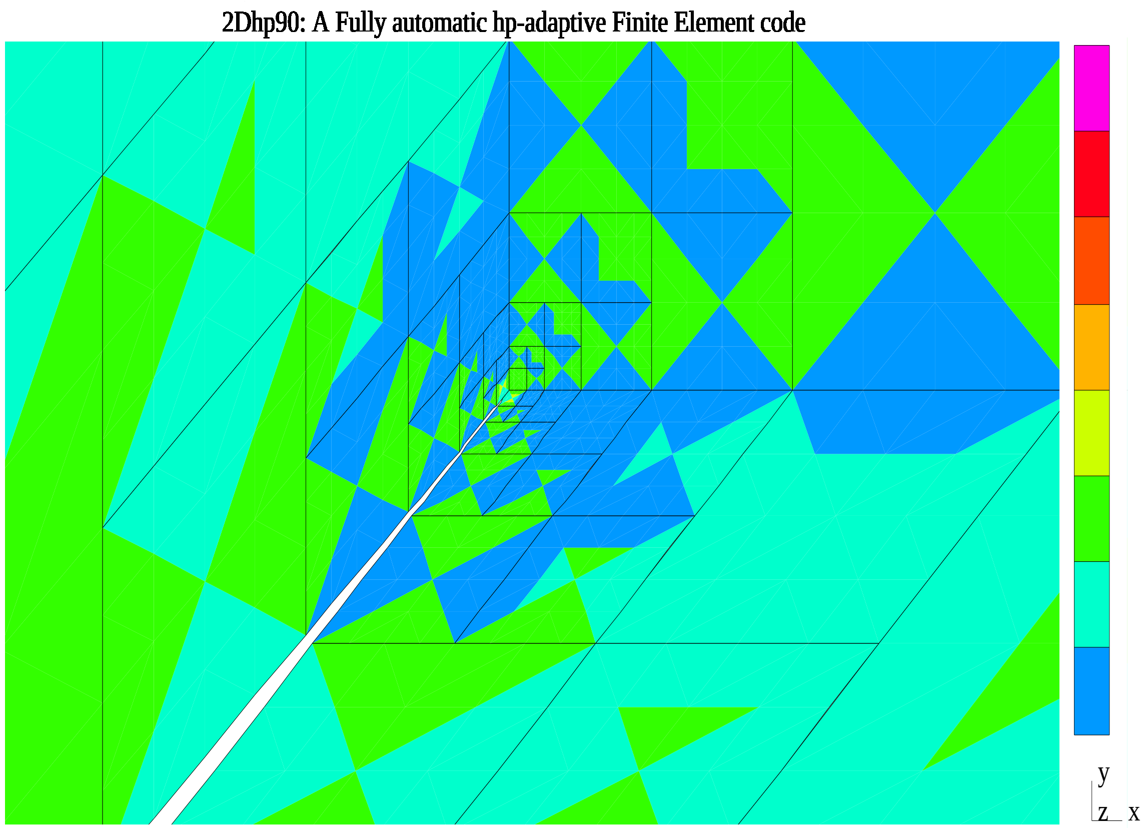
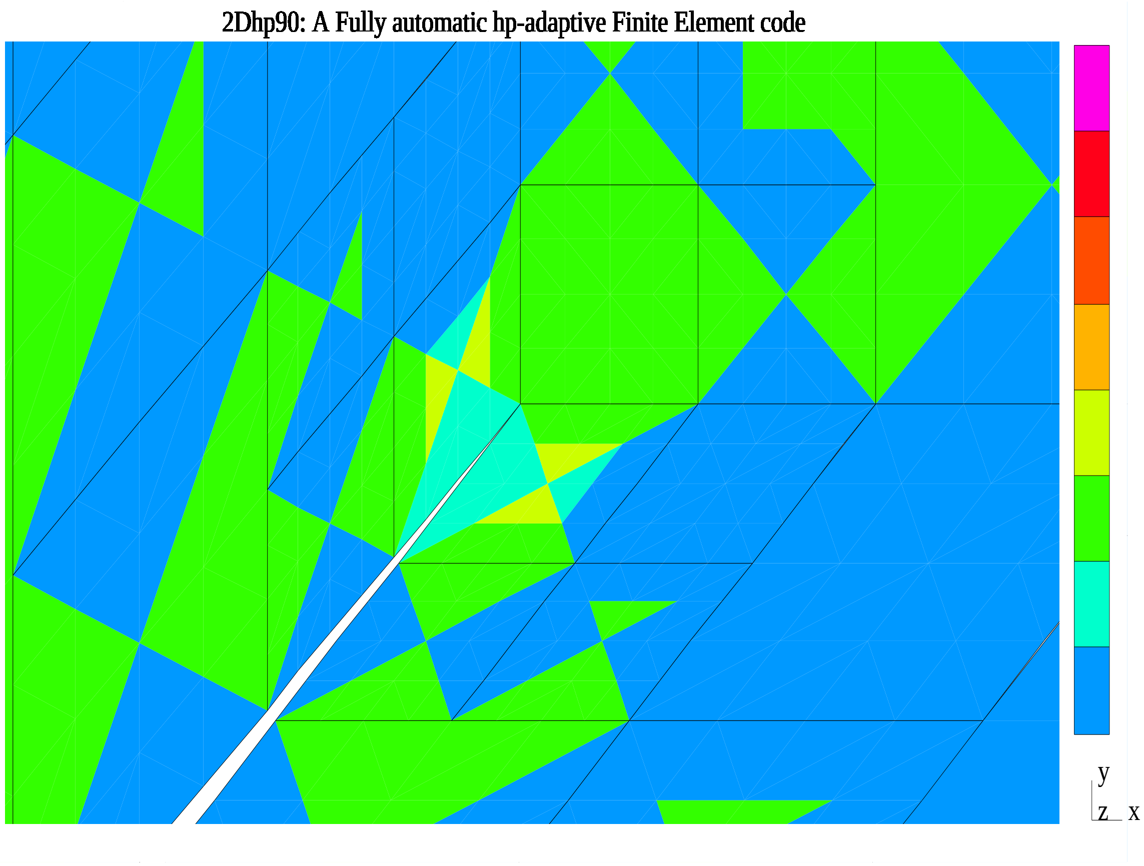


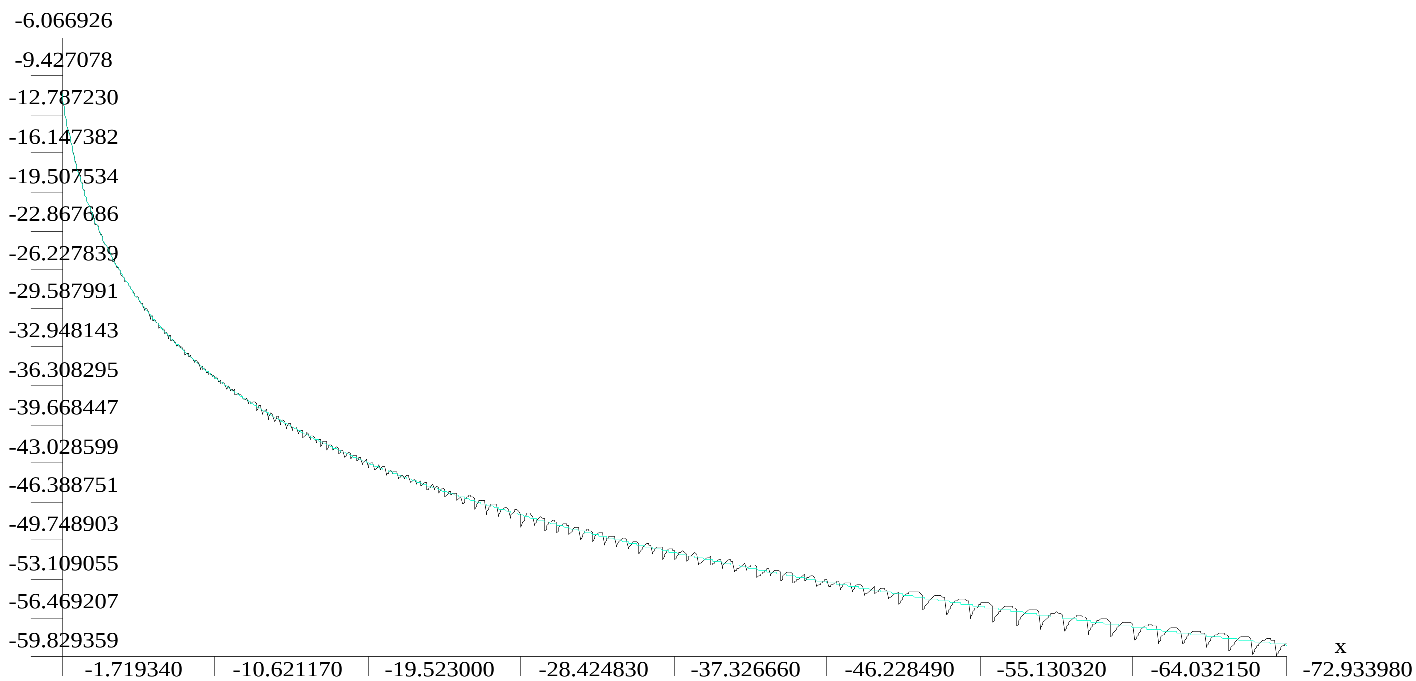
5. NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 10000000000000



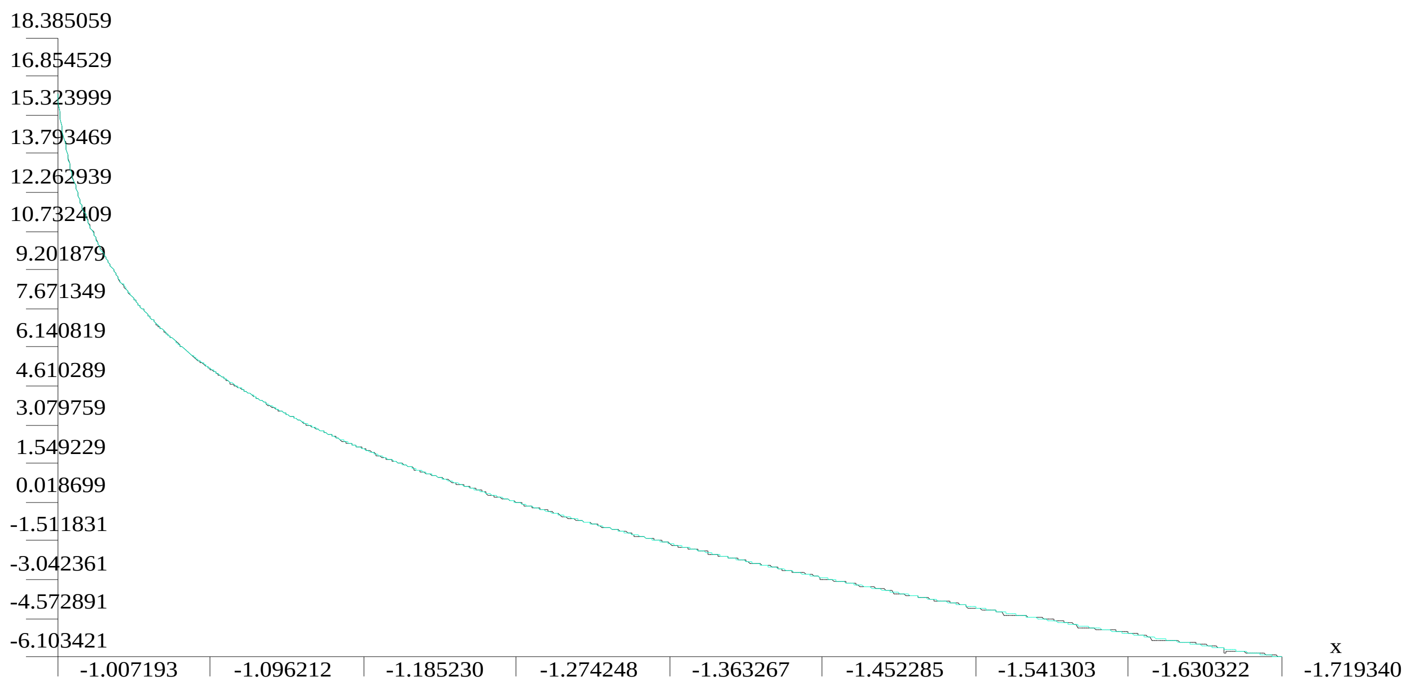
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 1-100 from the singularity



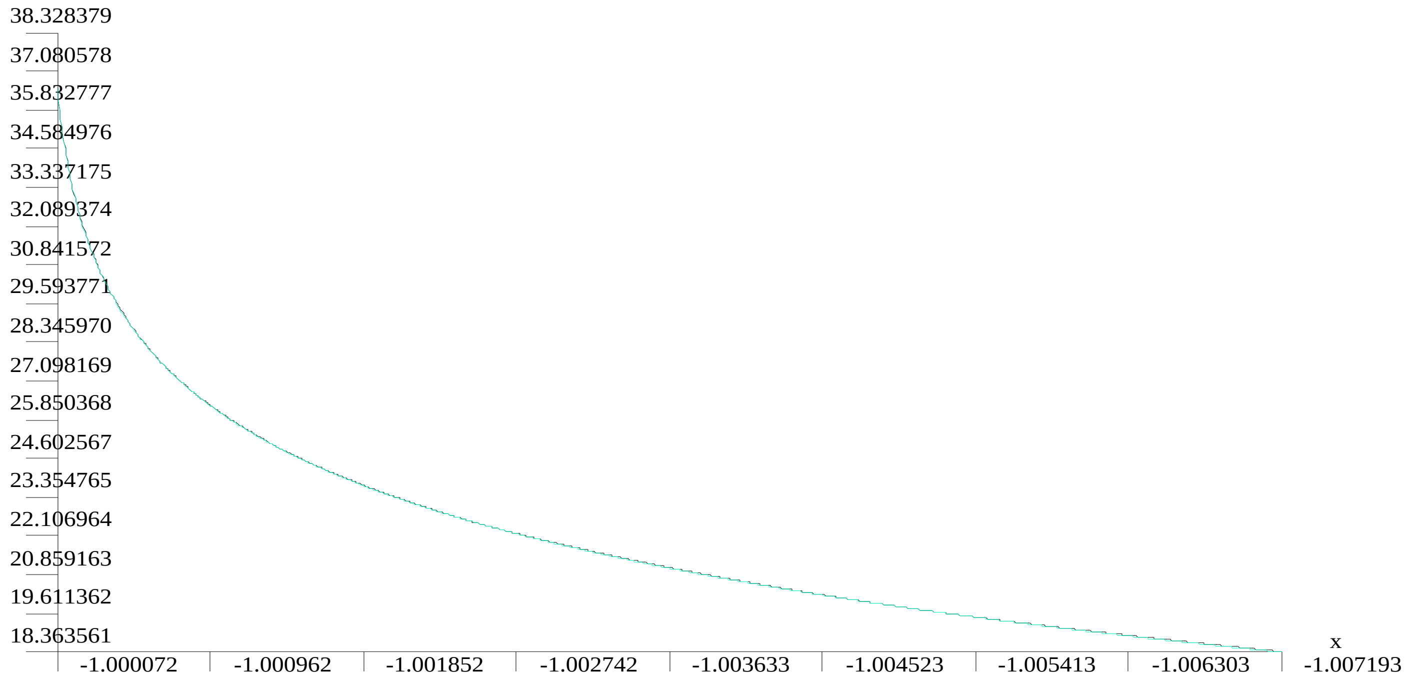
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.01-1 from the singularity



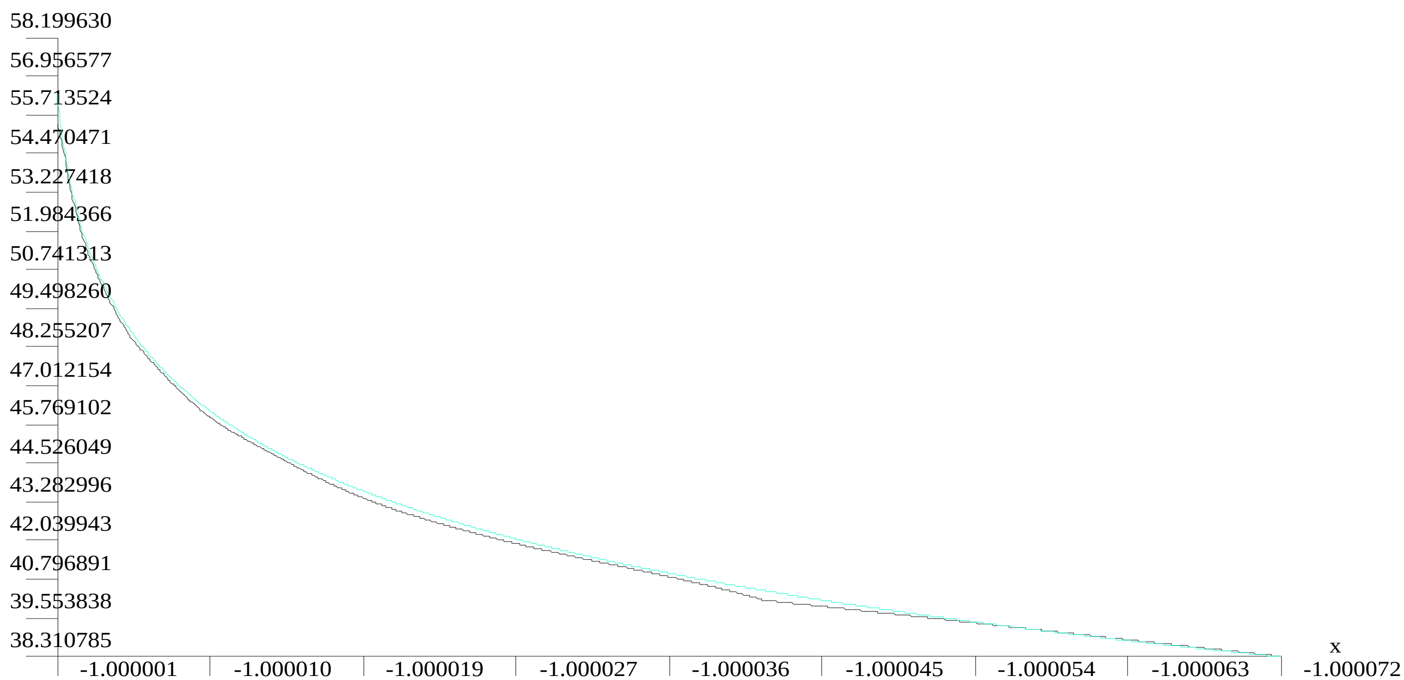
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.0001-0.01 from the singularity



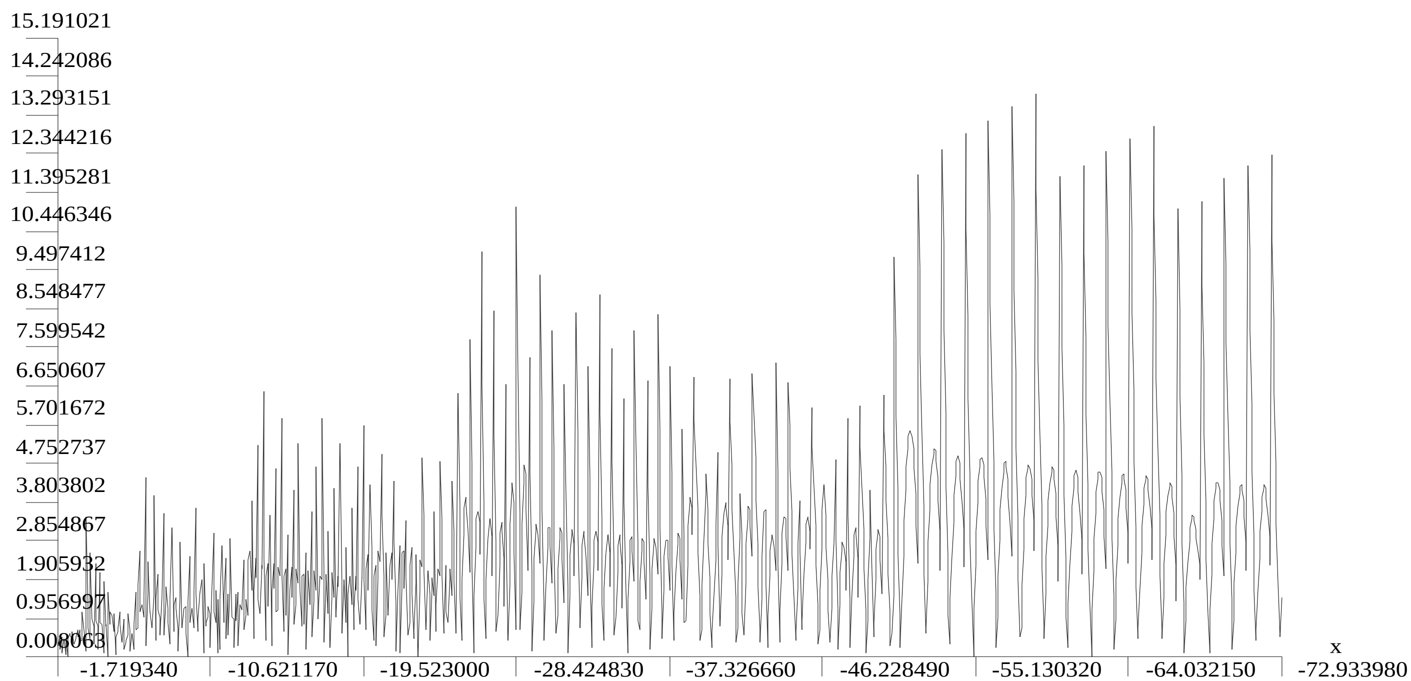
5. NUMERICAL RESULTS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.000001-0.0001 from the singularity



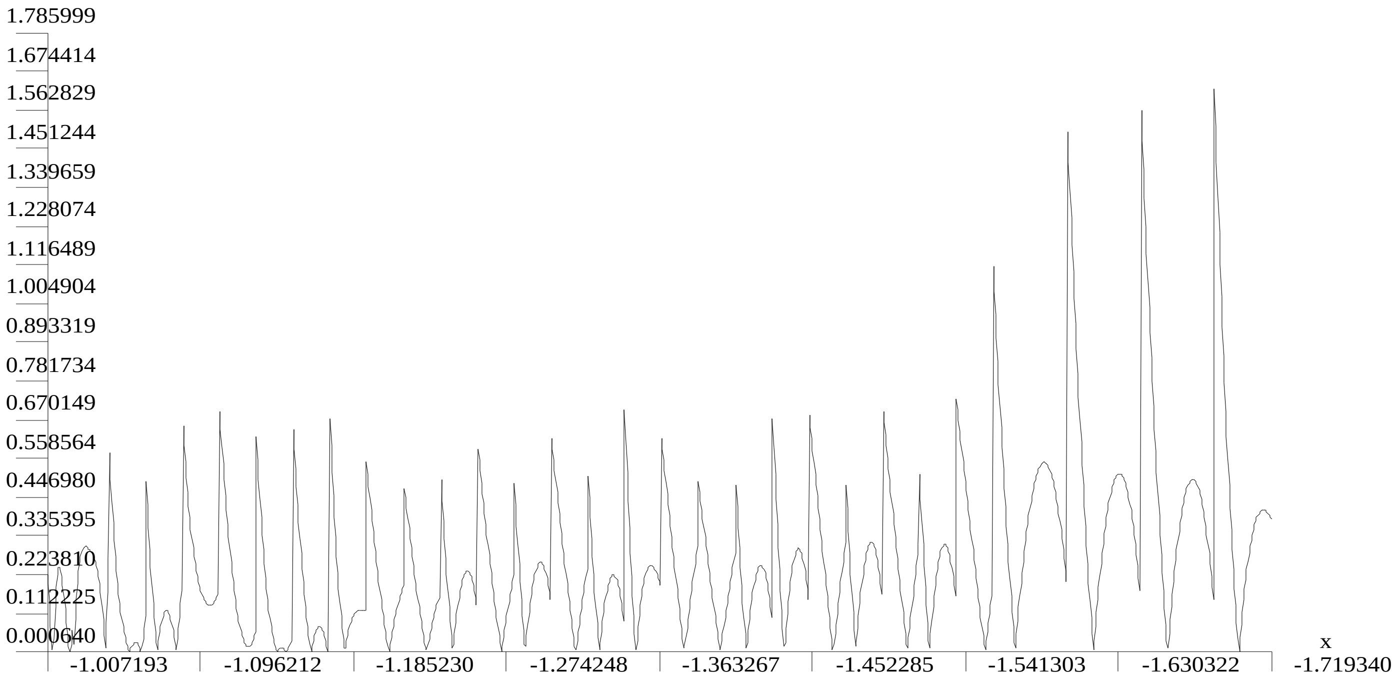
5. NUMERICAL RESULTS

Edge diffraction example: Relative error in percentage of the approximate solution at distances 1-100 from the singularity



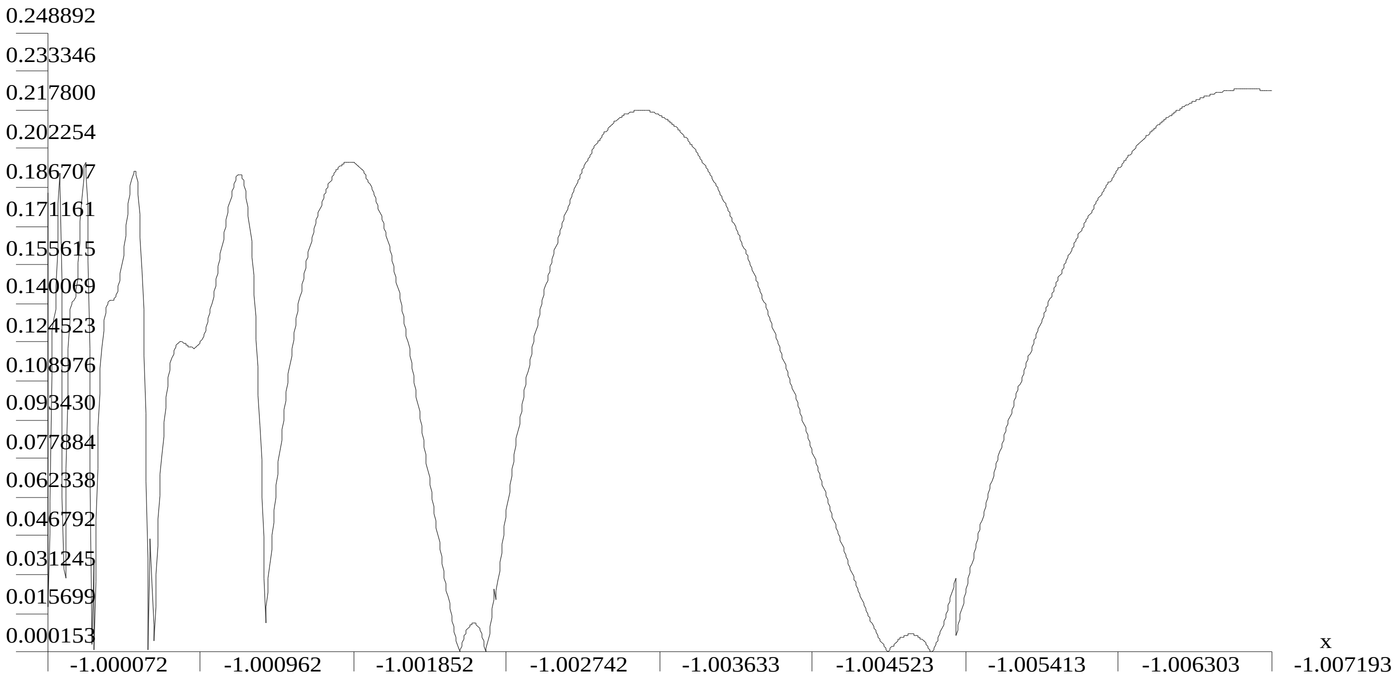
5. NUMERICAL RESULTS

Edge diffraction example: Relative error in percentage of the approximate solution at distances 0.01-1 from the singularity



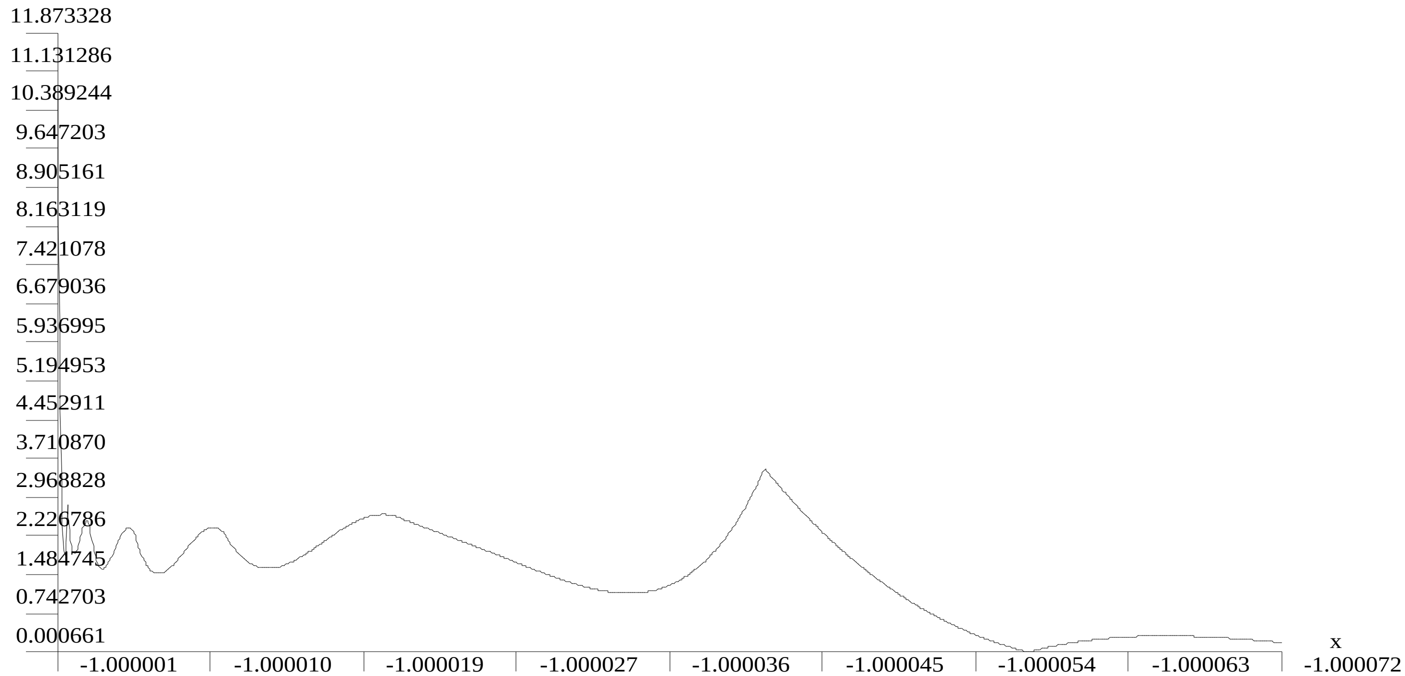
5. NUMERICAL RESULTS

Edge diffraction example: Relative error in percentage of the approximate solution at distances 0.0001-0.01 from the singularity



5. NUMERICAL RESULTS

Edge diffraction example: Relative error in percentage of the approximate solution at distances 0.000001-0.0001 from the singularity



6. SUMMARY AND FUTURE WORK

SUMMARY:

- **The finite element method is suitable for solving EM problems with edge singularities.**
- **In order to obtain high accuracy approximations, automatic hp-adaptivity is needed.**

FUTURE WORK:

- **Compare performance of the fully automatic hp-adaptive code against other commercial codes for *real life* Petroleum Engineering EM problems.**
- **Study the potencial of a fully automatic goal-oriented hp-adaptive strategy for Petroleum Engineering EM problems.**

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