Summer Project. Part II.

Limitations of a fully automatic hp-adaptive procedure for the Finite Element (FE) method.

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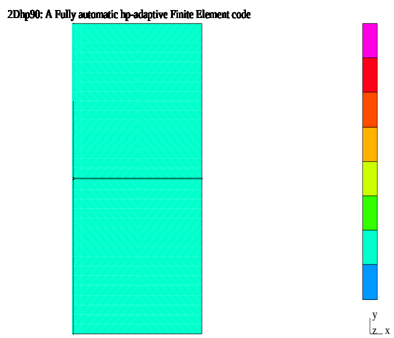
OVERVIEW

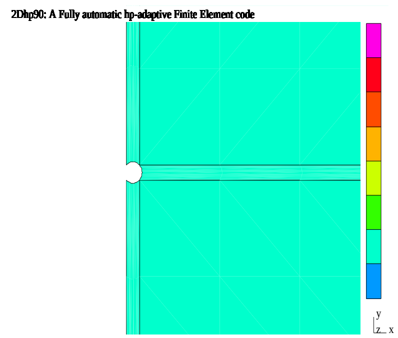
1. Overview.
2. Answer to questions.
3. Model problem.
4. Limitations of the fully automatic hp-adaptive procedure.
5. Conclusions and Future Work.

2. MODELING OF GEOMETRICAL DETAILS THROUGH HIGHER ORDER METHODS

Geometrical modeling of a circle



Zoom=1



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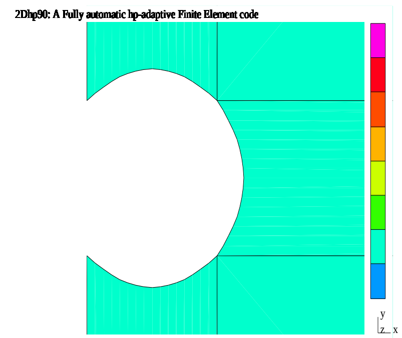
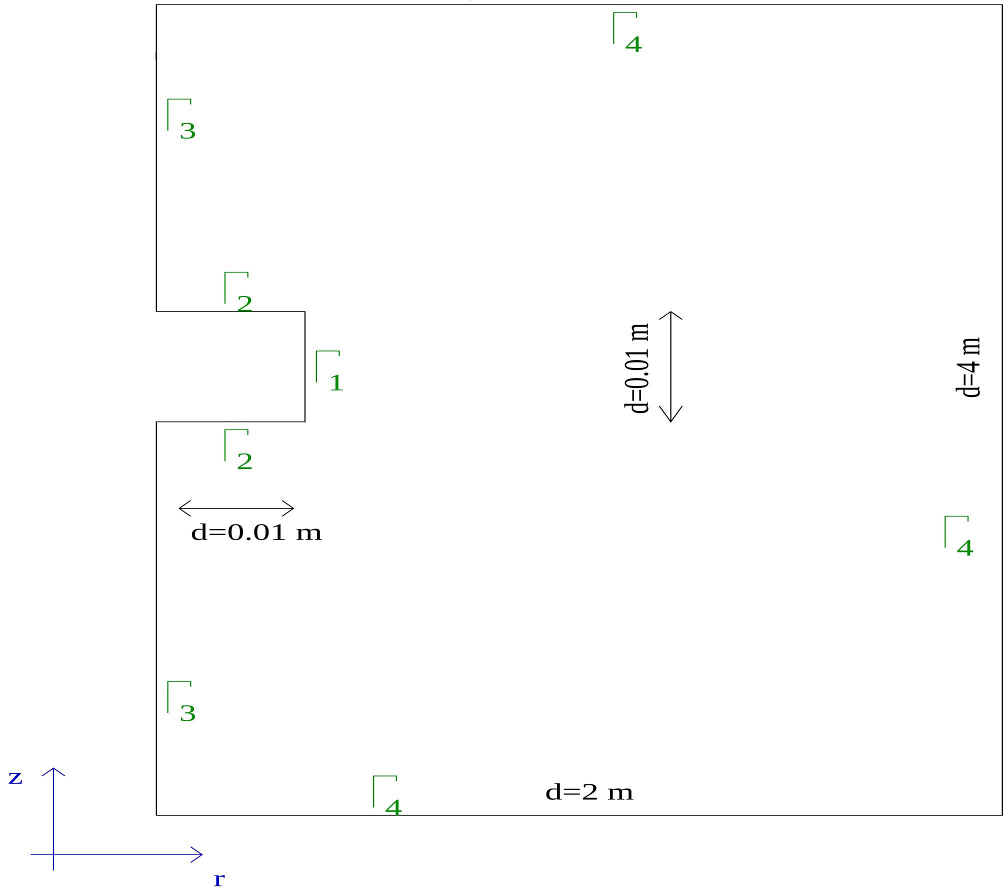


Zoom=100

Only five elements needed to model a small circle embedded into a large domain!!!

2. MODEL PROBLEM

Modeling of a battery

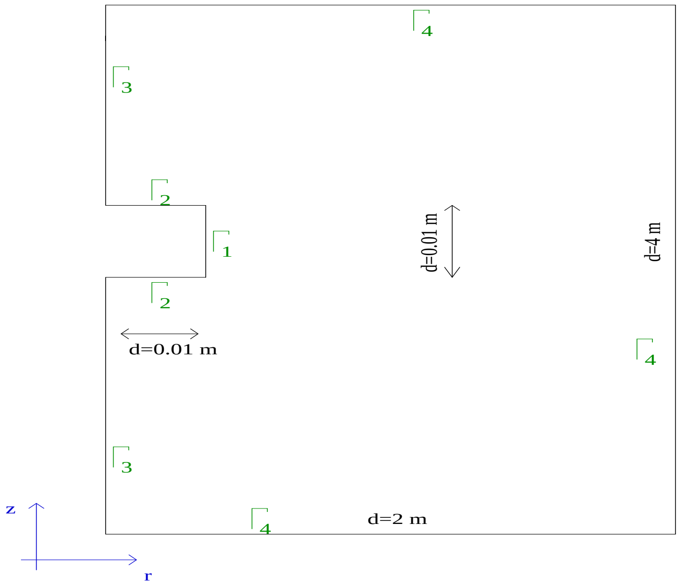


Goal: Determine EM field far from the battery (receiver antennas).

2. MODEL PROBLEM

Time Harmonic Maxwell's Equations

$$\nabla \times \mathbf{E} = -j\mu\omega\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \sigma\mathbf{E}$$



Reduced Wave Equation:

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega J^{imp}$$

Boundary Conditions (BC):

Dirichlet BC at a PEC surface:

$$\mathbf{n} \times \boldsymbol{E} = 0 \;\; on \;\; \Gamma_2 \cup \Gamma_4$$

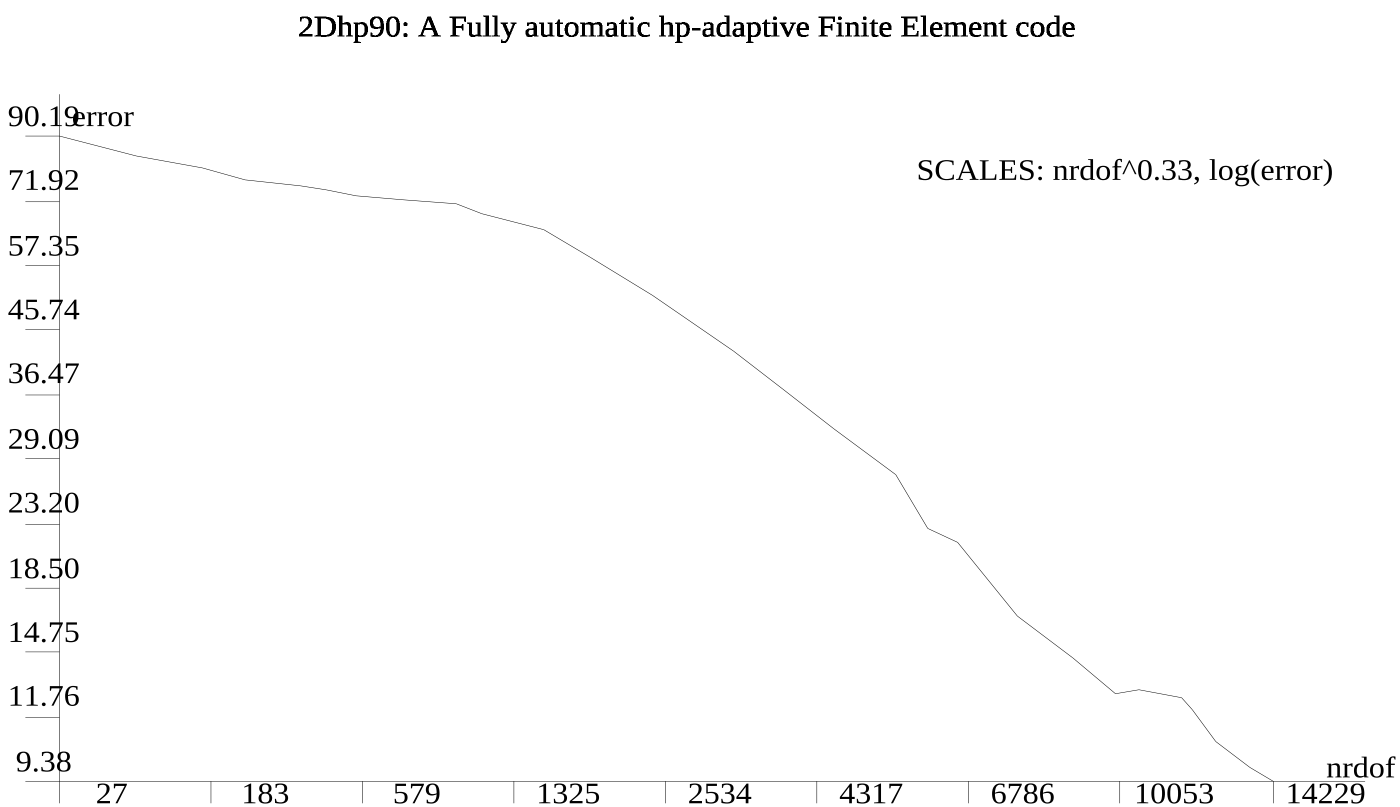
Neumann BC's:

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega \;\; on \;\; \Gamma_1$$

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = 0 \;\; on \;\; \Gamma_3$$

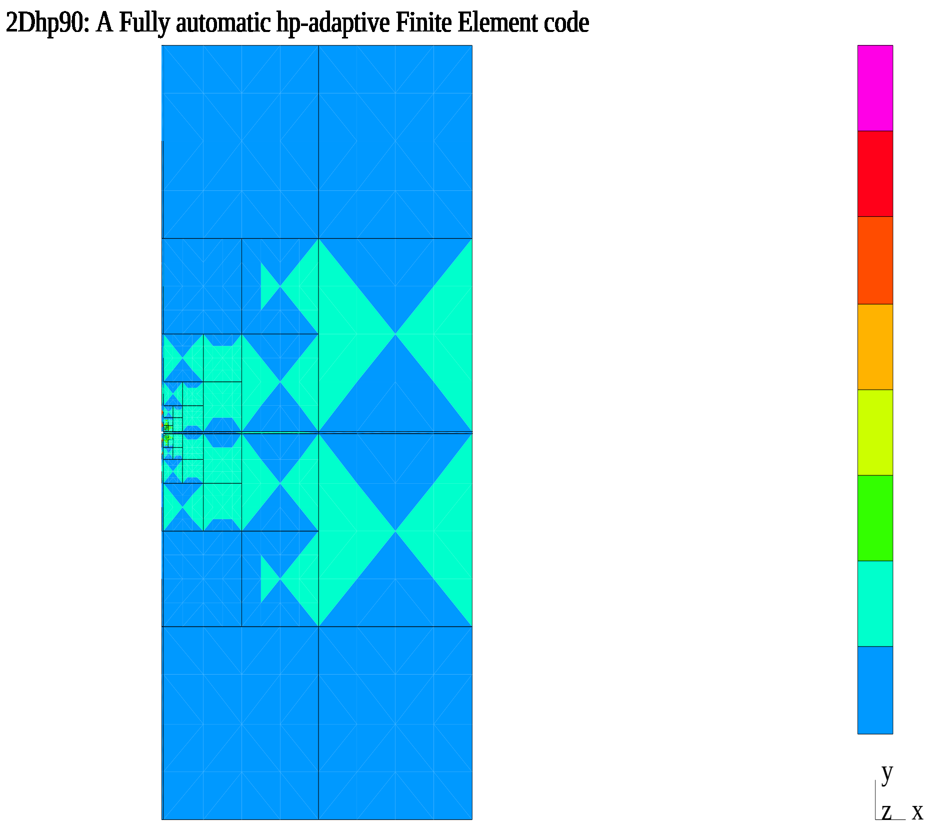
5. NUMERICAL RESULTS

Battery example: Convergence history



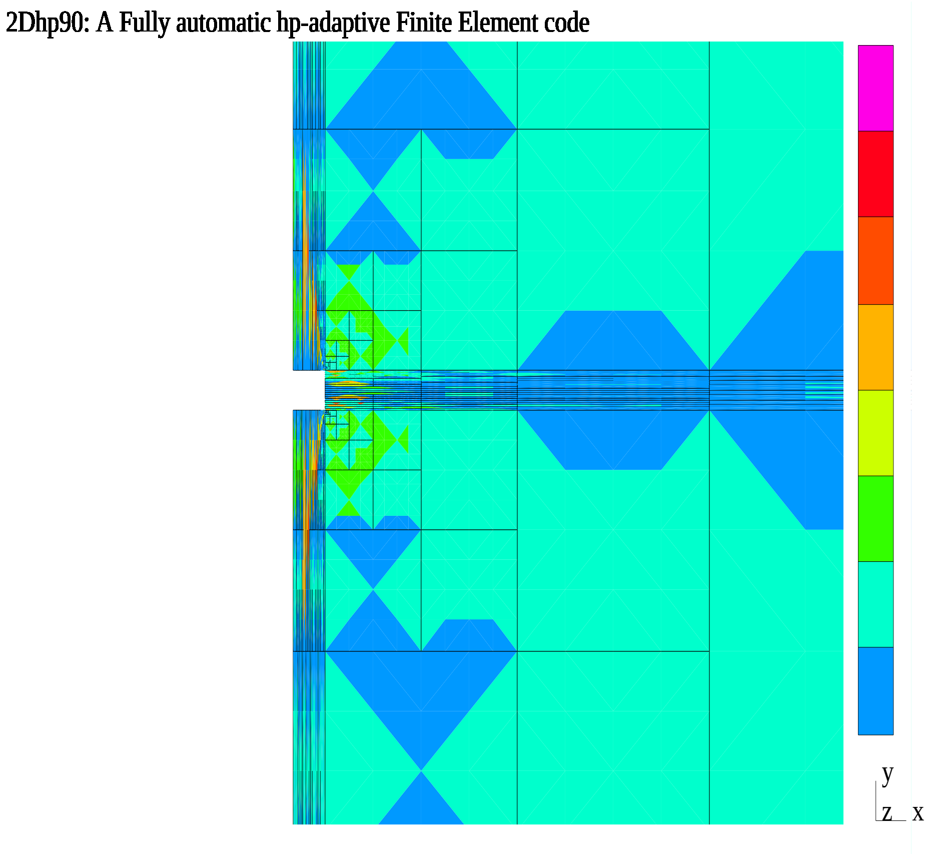
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 1



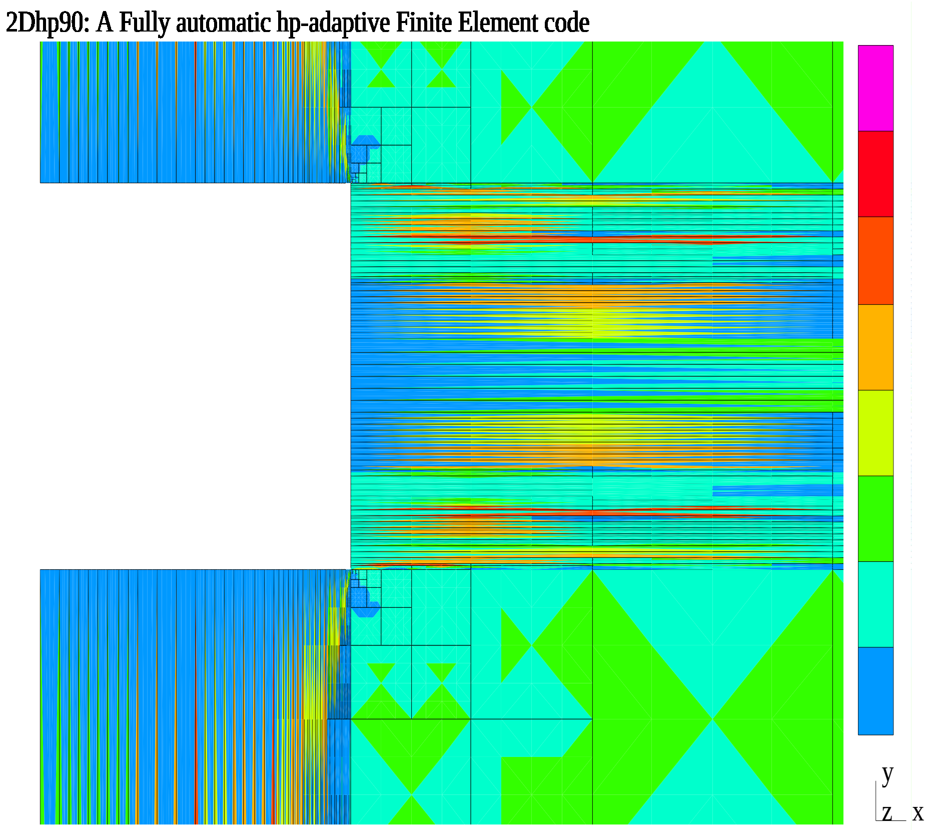
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 10



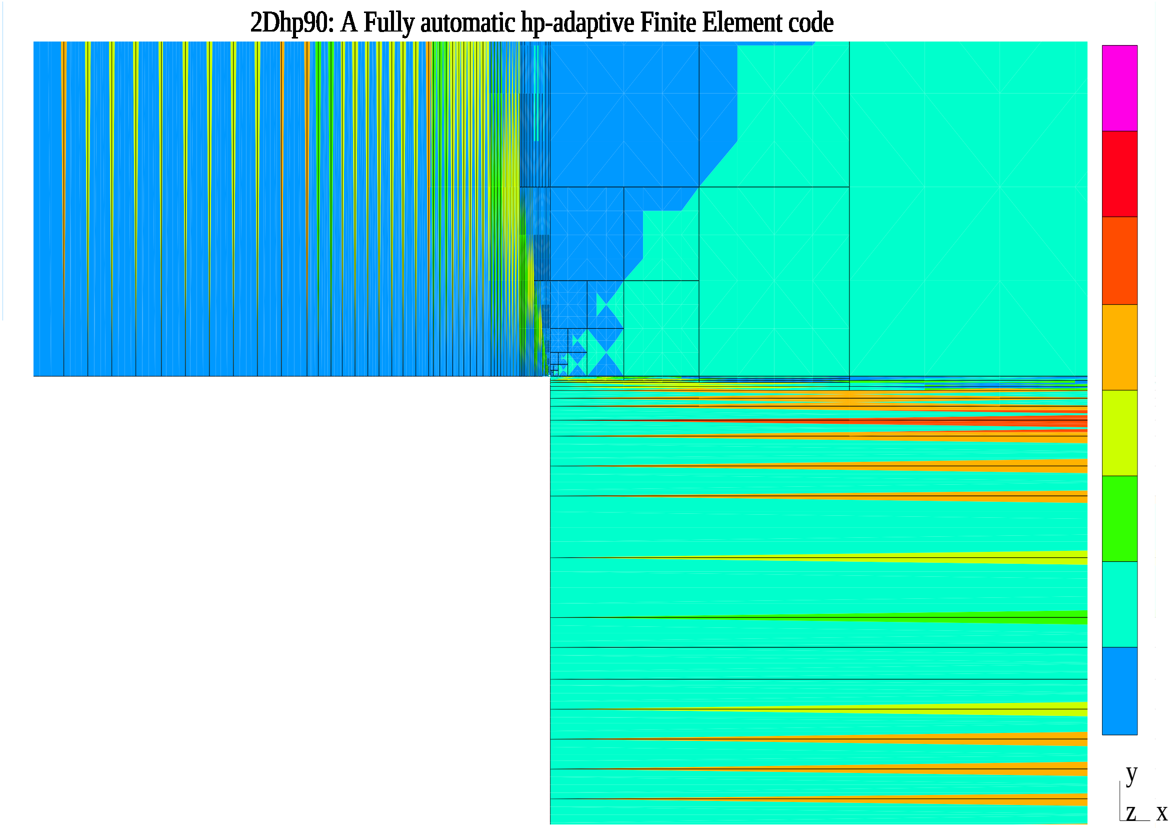
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 100



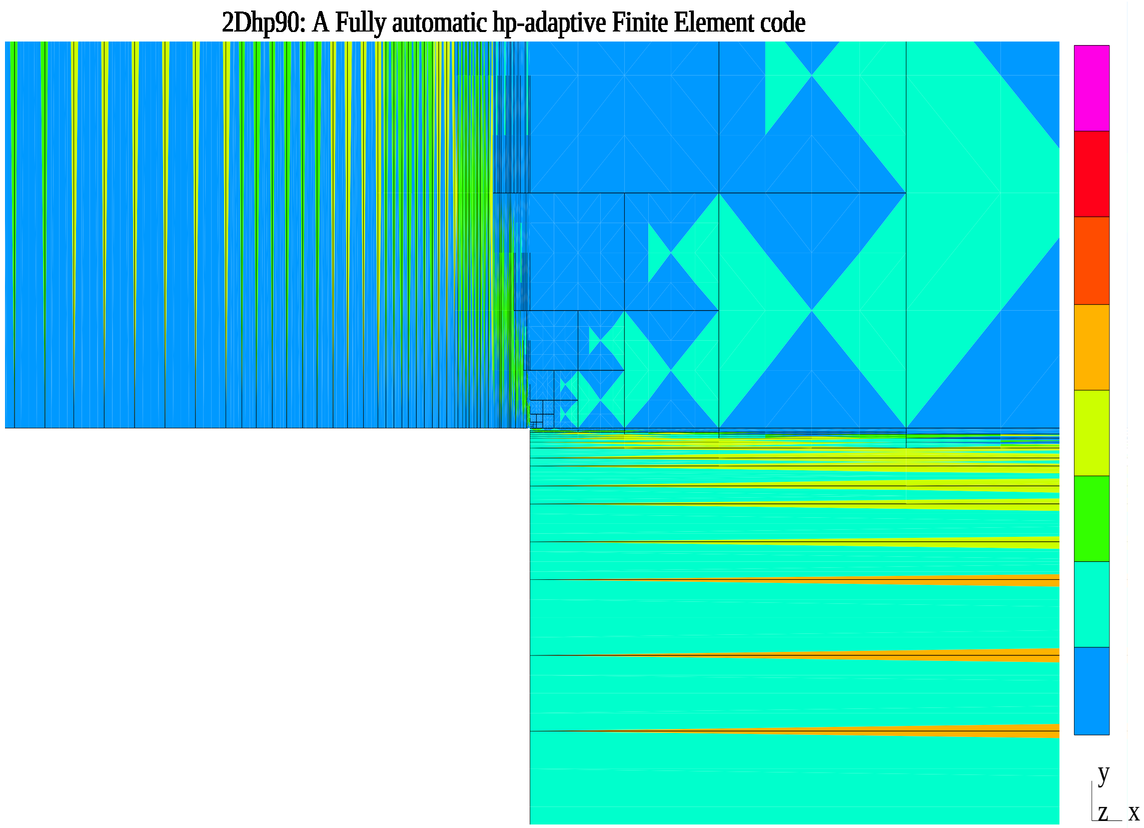
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 1000



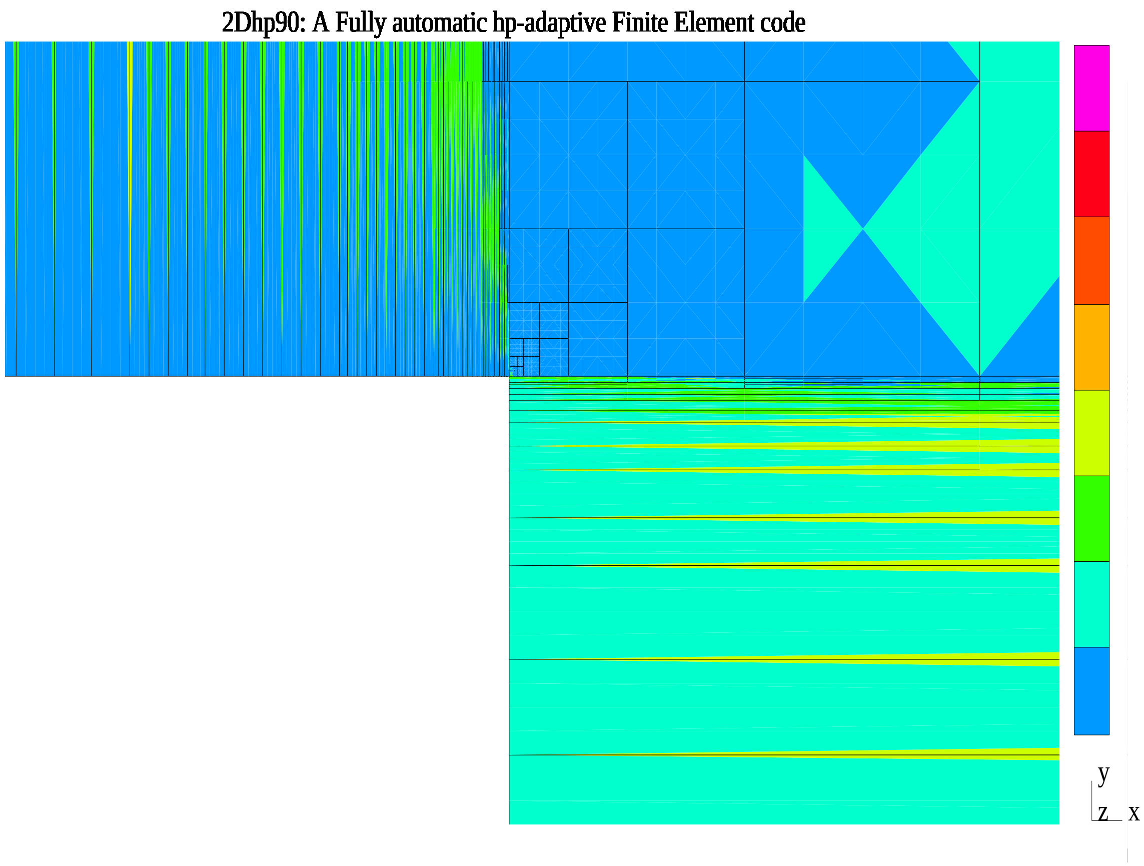
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 10000



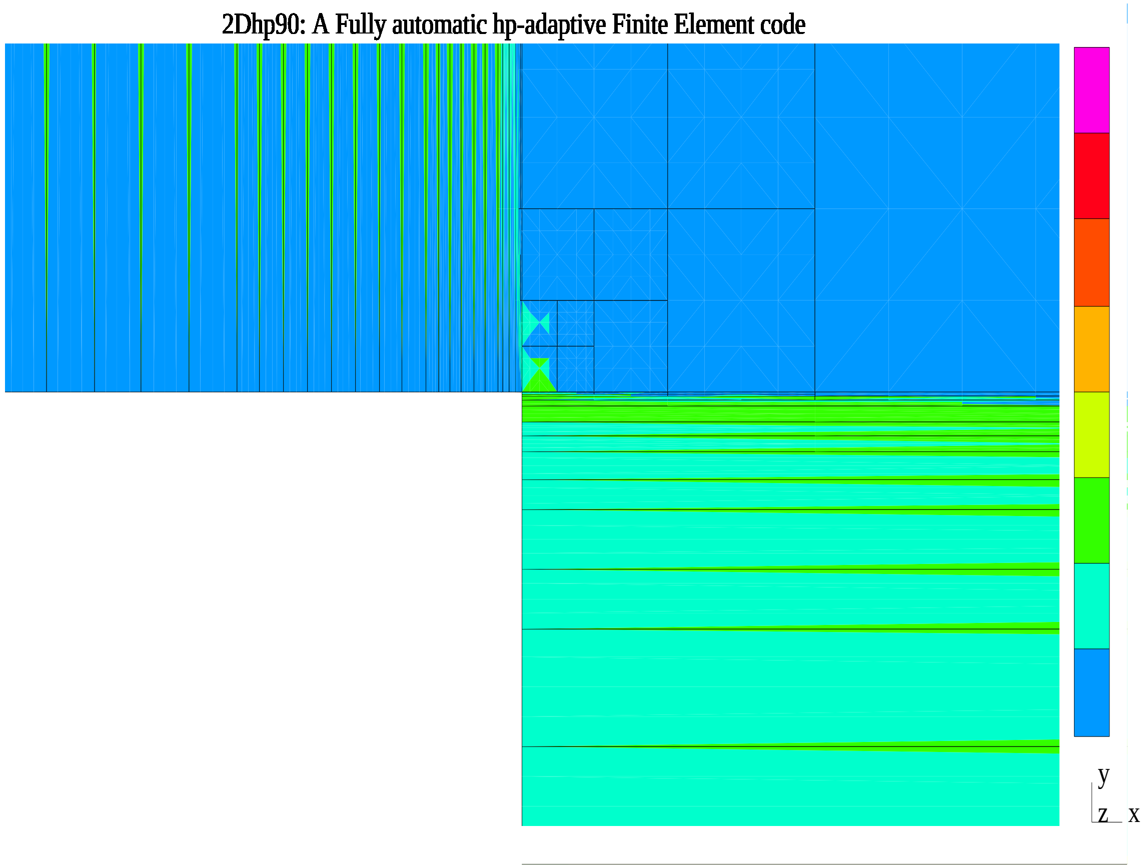
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 100000



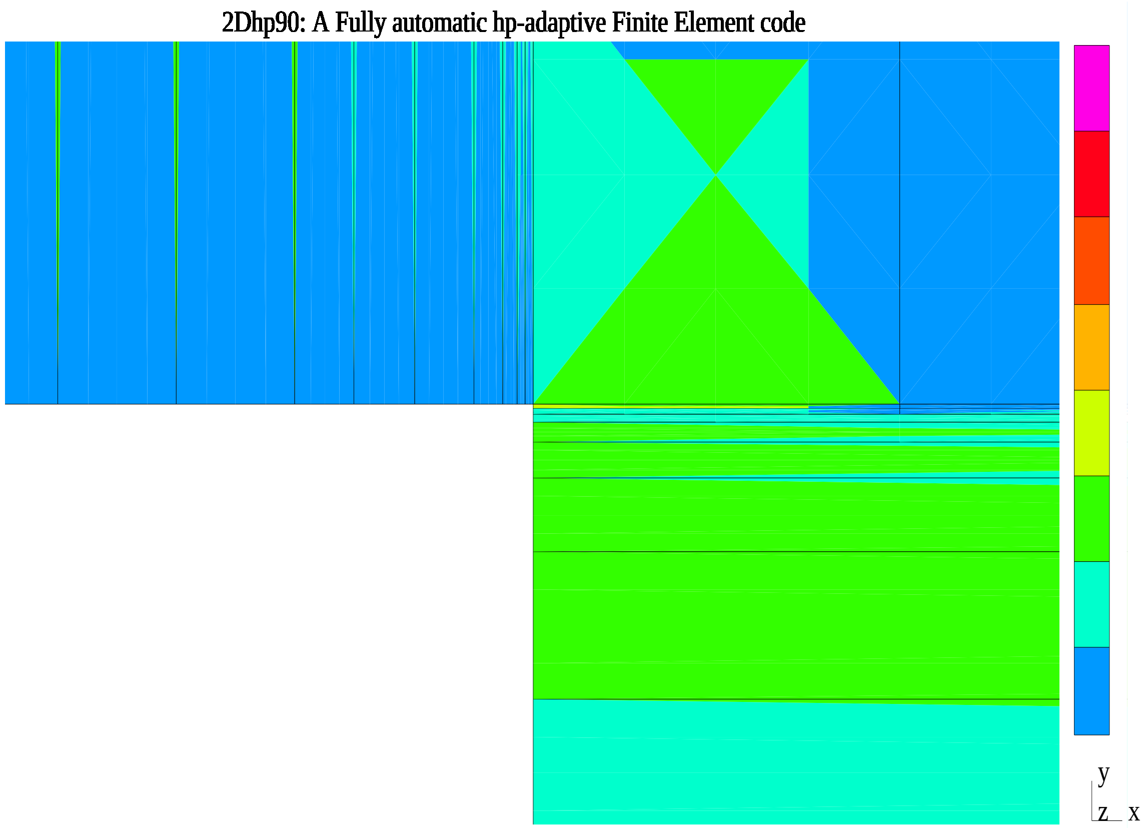
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 1000000



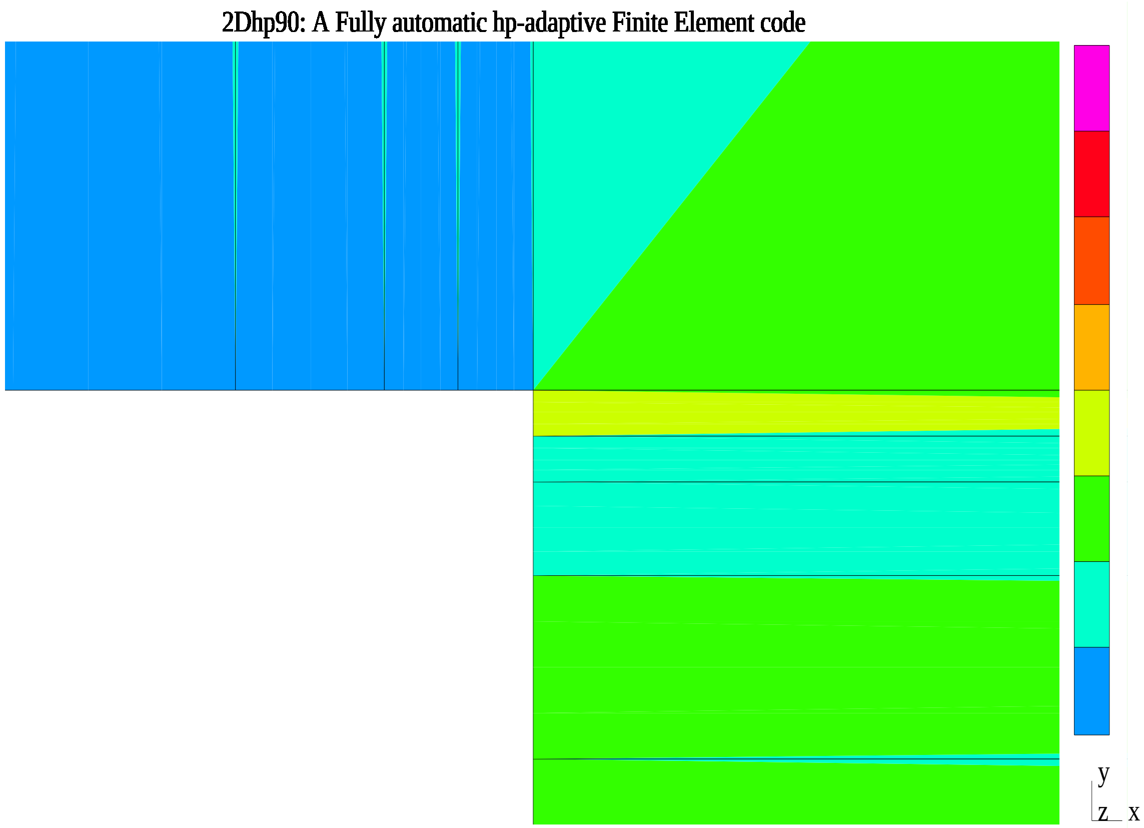
5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 10000000



5. NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 100000000



5. NUMERICAL RESULTS

Why the optimal grid is so bad?

Optimization is based on minimization of the ENERGY NORM of the error, given by:

$$\| error \| = \int | error | + \int | \nabla \times error |$$

Interpretation of results:

- **The grid is optimal for the selected refinement criteria,**
- **but our refinement criteria is inadequate for our pourposes.**

6. SUMMARY AND FUTURE WORK

SUMMARY:

- **The hp-finite element method allows for introducing geometrical details.**
- **The hp-adaptive algorithm is robust.**
- **Minimization of the error in energy norm is NOT an adequate criteria for solving many EM problems.**

FUTURE WORK:

- **Study the potential of a fully automatic goal-oriented hp-adaptive strategy for Petroleum Engineering EM applications.**
- **Improve performance of the adaptive strategy by creating a two grid solver.**