Shell Presentation, 11:00 am

Simulation of Resistivity Logging Instruments with Mandrel Using a Self-Adaptive Goal-Oriented hp-Finite Element Method

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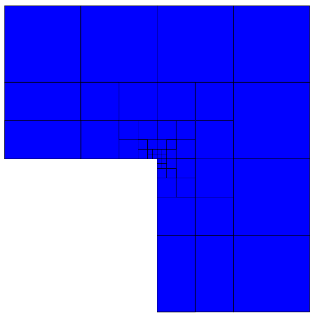
Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

OVERVIEW

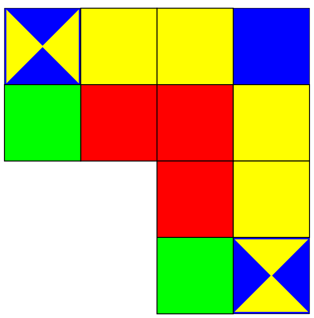
1. Overview
2. Main Idea of hp Goal-Oriented Adaptivity
3. Current Stage of the 2D High Performance FE Software
   - Flexibility
   - Reliability
   - Accuracy
   - Performance
4. Simulation of Resistivity Logging Instruments with Mandrel
5. Conclusions and Future Work (3D Problems)

THE hp-FINITE ELEMENT METHOD



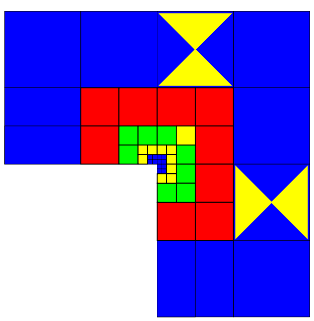
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal *h*-grids do NOT converge exponentially in real applications.
3. They may "lock" (100$\%$ error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.

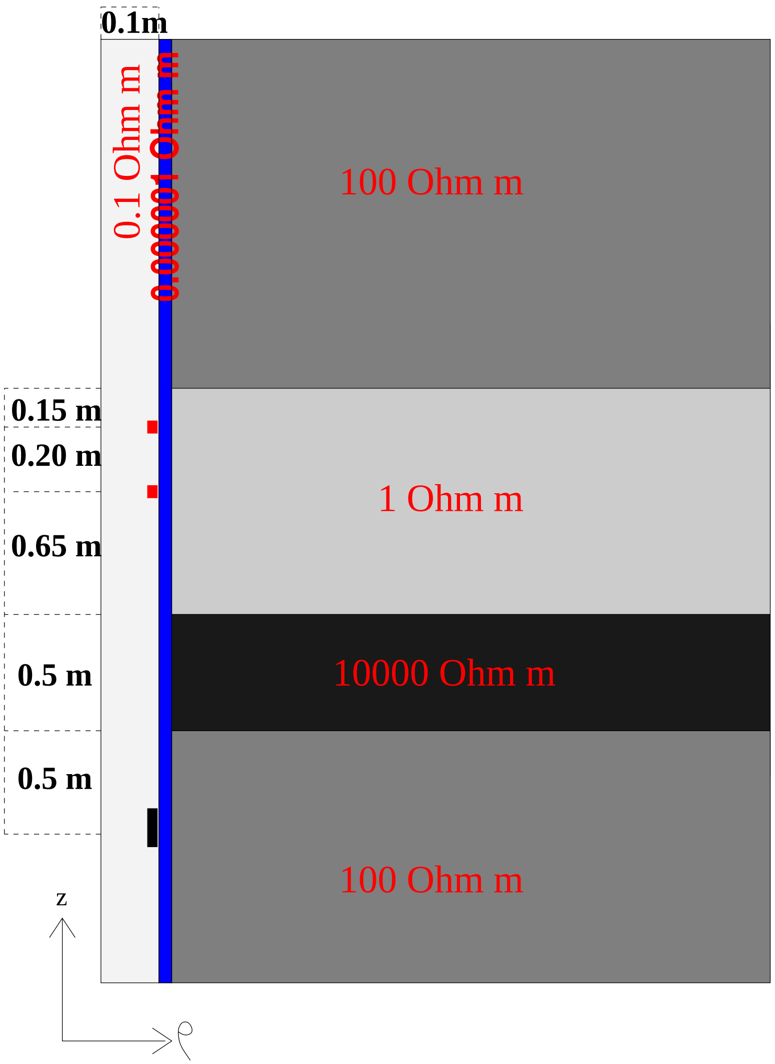


The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

THE hp-FINITE ELEMENT METHOD

Model Problem with Steel Casing



Frequency: 10 Hz.

Casing resistivity: 10^{-6} Ohm · m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 %

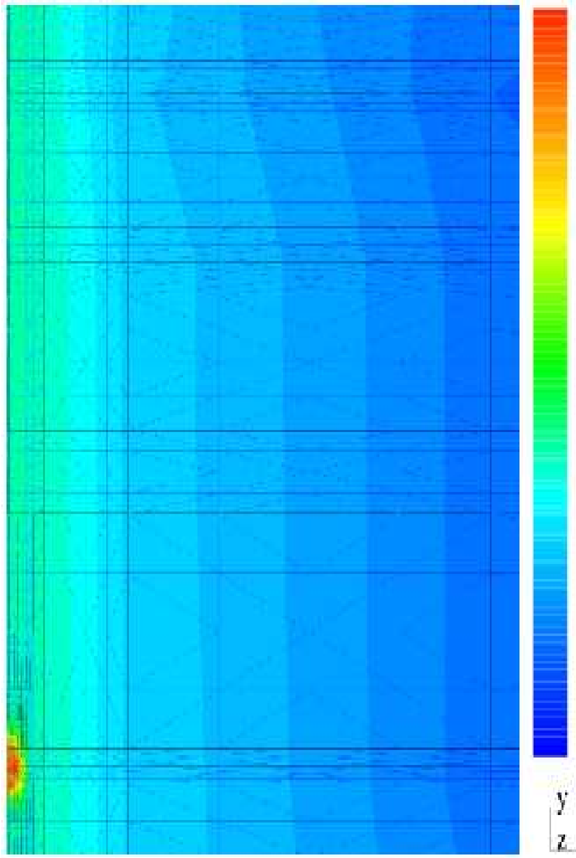
Toroidal antennas

Size (domain): 500m x 4000m

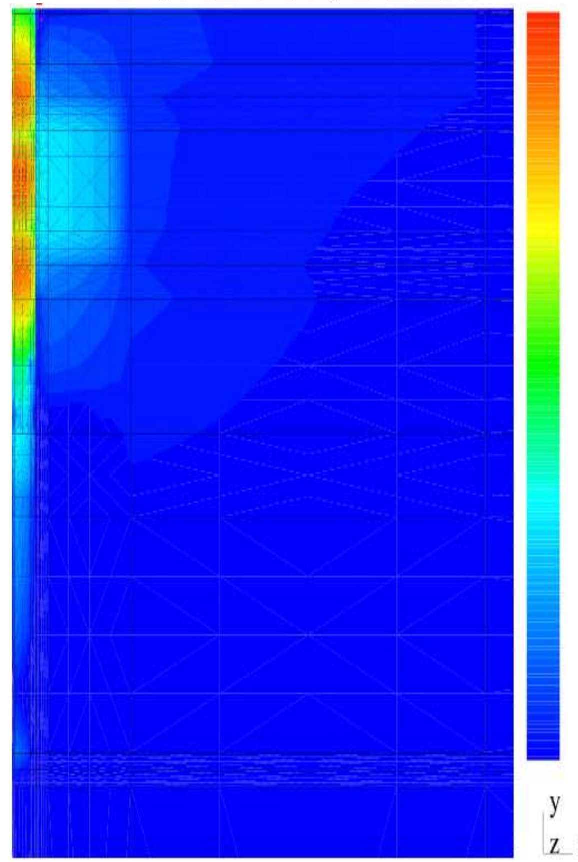
GOAL-ORIENTED ADAPTIVITY

Representation Formula

DIRECT PROBLEM



DUAL PROBLEM

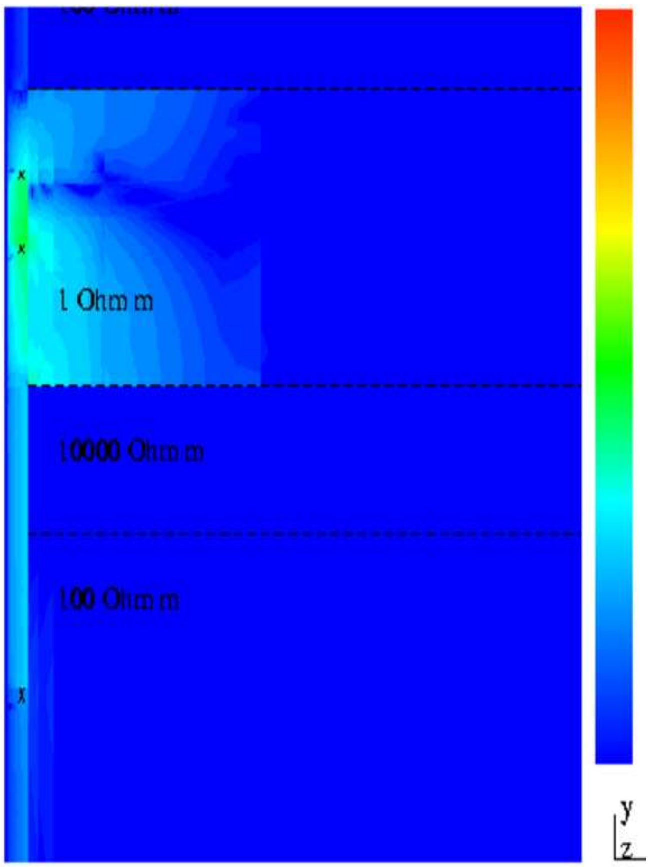


$$V = L(\Psi) = \int \bar{\bar{\sigma}} \nabla \Psi \cdot \nabla G$$

GOAL-ORIENTED ADAPTIVITY

Movie Presentation (Sensitivity Functions)

We want to study resolution and depth of investigation of a logging instrument.



We have: $|L(\Psi)| = |\int S\, dV| \leq \int |S|\, dV$.

In the next movies, we display: $\log_{10}|S|$.

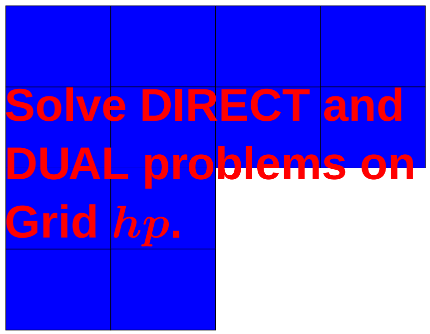
Scales:

- **Red** $\longrightarrow |S| = |L(\Psi)| * 10^4$.
- **Blue** $\longrightarrow |S| = |L(\Psi)| * 10^{-2}$.

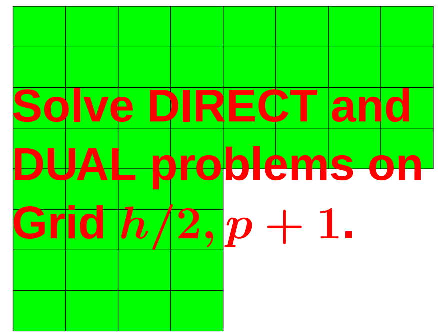
Direct Current

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Algorithm for Goal-Oriented Adaptivity



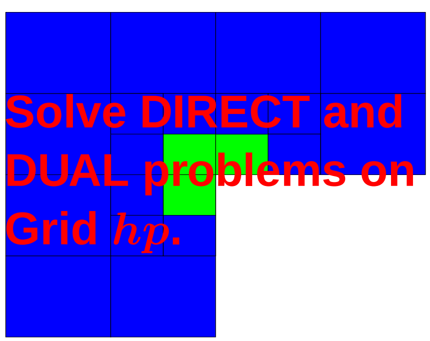
$\longrightarrow$



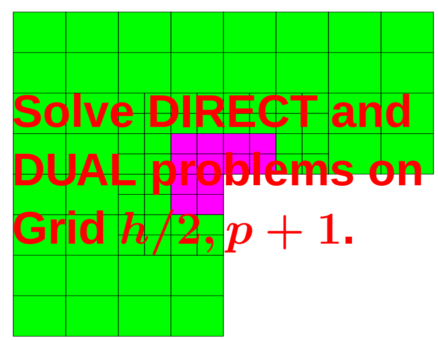
Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, **and** $\tilde{e} = \Psi_{h/2,p+1} - \Pi_{hp}\Psi_{h/2,p+1}$.
Compute $\epsilon = G_{h/2,p+1} - G_{hp}$, **and** $\tilde{\epsilon} = G_{h/2,p+1} - \Pi_{hp}G_{h/2,p+1}$.

$$|L(e)| = |b(e,\epsilon)| \sim |b(\tilde{e},\tilde{\epsilon})| \leq \sum_K |b_K(\tilde{e},\tilde{\epsilon})| \leq \sum_K \| \tilde{e} \|_{E,K} \| \tilde{\epsilon} \|_{E,K}.$$

Apply the fully automatic hp-adaptive algorithm.

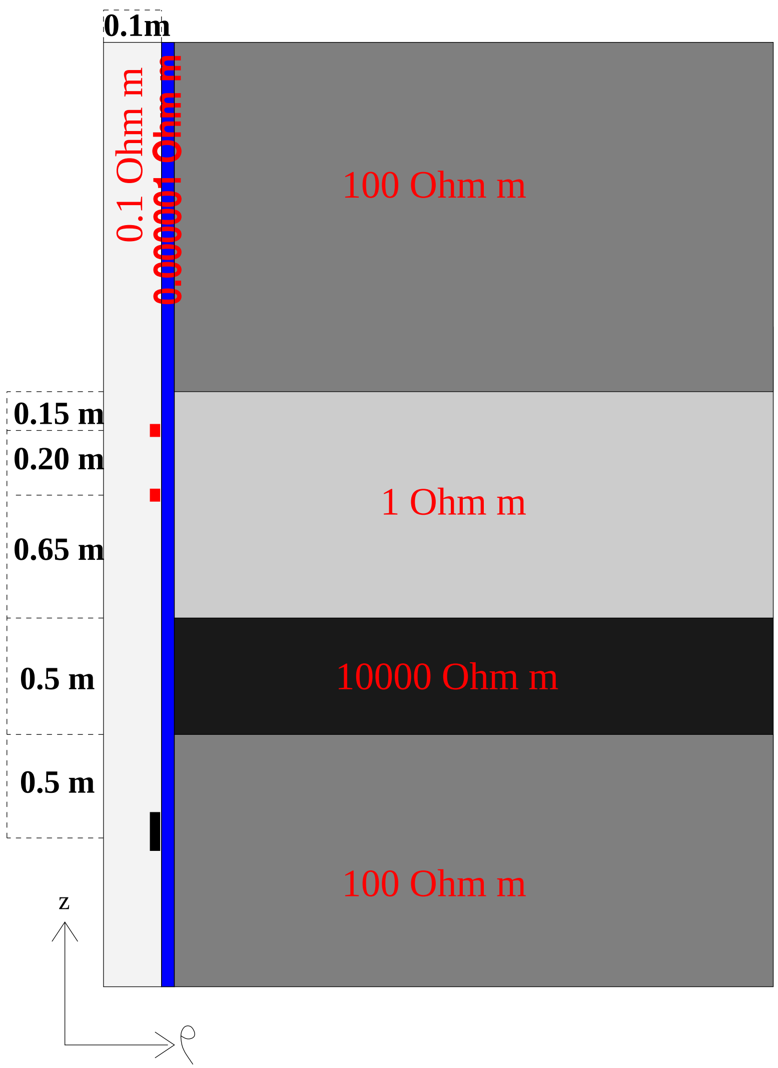


$\longrightarrow$



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Model Problem with Steel Casing



Frequency: 10 Hz.

Casing resistivity: 10^{-6} Ohm $\cdot$ m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 $\%$

Toroidal antennas

Size (domain): 500m x 4000m

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

Time-Harmonic Maxwell's Equations

$\nabla \times \mathrm{H} = (\bar{\bar{\sigma}} + j\omega\bar{\bar{\epsilon}})\mathrm{E} + \mathrm{J}^{imp}$	**Ampere's law**
$\nabla \times \mathrm{E} = -j\omega\bar{\bar{\mu}}\mathrm{H} - \mathrm{M}^{imp}$	**Faraday's law**
$\nabla \cdot (\bar{\bar{\epsilon}}\mathrm{E}) = \rho$	**Gauss' law of Electricity**
$\nabla \cdot (\bar{\bar{\mu}}\mathrm{H}) = 0$	**Gauss' law of Magnetism**

E-VARIATIONAL FORMULATION:

$$\left\{ \begin{array}{l} \text{Find } \mathrm{E} \in \mathrm{E}_D + H_D(\mathrm{curl};\Omega) \text{ such that:} \\ \displaystyle\int_\Omega (\bar{\bar{\mu}}^{-1}\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\bar{\bar{k}}^2\mathrm{E}) \cdot \bar{\mathrm{F}}\, dV = -j\omega \int_\Omega \mathrm{J}^{imp} \cdot \bar{\mathrm{F}}\, dV \\ \displaystyle +j\omega \int_{\Gamma_N} \mathrm{J}^{imp}_{\Gamma_N} \cdot \bar{\mathrm{F}}_t\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}\mathrm{M}^{imp}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV \quad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega) \end{array} \right.$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

AXISYMMETRIC PROBLEMS

E_ϕ **-Variational Formulation (Azimuthal)**

$$\begin{cases} \text{Find } E_\phi \in E_{\phi,D} + \tilde{H}^1_D(\Omega) \text{ such that:} \\ \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \nabla \times \mathrm{E}_\phi) \cdot (\nabla \times \bar{\mathrm{F}}_\phi)\, dV - \int_\Omega (\bar{\bar{k}}^2_\phi \mathrm{E}_\phi) \cdot \bar{\mathrm{F}}_\phi \, dV = -j\omega \int_\Omega J^{imp}_\phi \, \bar{F}_\phi \, dV \\ \quad + j\omega \int_{\Gamma_N} J^{imp}_{\phi,\Gamma_N} \, \bar{F}_\phi \, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \mathrm{M}^{imp}_{\rho,z}) \cdot \bar{\mathrm{F}}_\phi \, dV \qquad \forall \, F_\phi \in \tilde{H}^1_D(\Omega) \end{cases}$$

$\mathrm{E}_{\rho,z}$ **-Variational Formulation (Meridian)**

$$\begin{cases} \text{Find } (E_\rho, E_z) \in \mathrm{E}_D + \tilde{H}_D(\mathrm{curl};\Omega) \text{ such that:} \\ \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \nabla \times \mathrm{E}_{\rho,z}) \cdot (\nabla \times \bar{\mathrm{F}}_{\rho,z})\, dV - \int_\Omega (\bar{\bar{k}}^2_{\rho,z} \mathrm{E}_{\rho,z}) \cdot \bar{\mathrm{F}}_{\rho,z} \, dV = \\ \quad -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z \, dV + j\omega \int_{\Gamma_N} J^{imp}_{\rho,\Gamma_N} \bar{F}_\rho + J^{imp}_{z,\Gamma_N} \bar{F}_z \, dS \\ \quad - \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \mathrm{M}^{imp}_\phi) \cdot \bar{\mathrm{F}}_{\rho,z} \, dV \qquad \forall \, (F_\rho, F_z) \in \tilde{H}_D(\mathrm{curl};\Omega) \end{cases}$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

- **Physical Devices: Casing, Casing Imperfections, Mandrel, Magnetic Buffers, Insulators, Displacement Currents, Combination of All, etc.**
- **Materials: Isotropic, Anisotropic*.**
- **Sources: Toroidal Antennas, Solenoidal Antennas, Dipoles in Any Direction, Electrodes, Finite Size Antennas, Combination of All, etc.**
- **Logging Instruments: Logging While Drilling (LWD), Laterolog, Normal, Induction, Dielectric Instruments, Cross-well, etc.**
- **Any Frequency (0-10 Ghz).**

ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
  2. Exact solution in a homogeneous media with a mandrel.
  3. Exact solution in a homogeneous media with casing.
- **Verification of Physical Properties.**
  1. Reciprocity principle (Gregory Itskovich).
  2. Discrete divergence free approximation for edge elements.
- **Numerical Verifications.**
  1. Different size of domain and antennas.
  2. Comparison against other numerical software (Yang Wei).
  3. Error control provided by the fine grid solution.
  4. Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

Continuous Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516098429E-08	-0.1456374493E-08
FINE GRID	0.1516094029E-08	-0.1456390824E-08

Edge Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516060872E-08	-0.1456337248E-08
FINE GRID	0.1516093804E-08	-0.1456390864E-08

Error control provided by the fine grid solution.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

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Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

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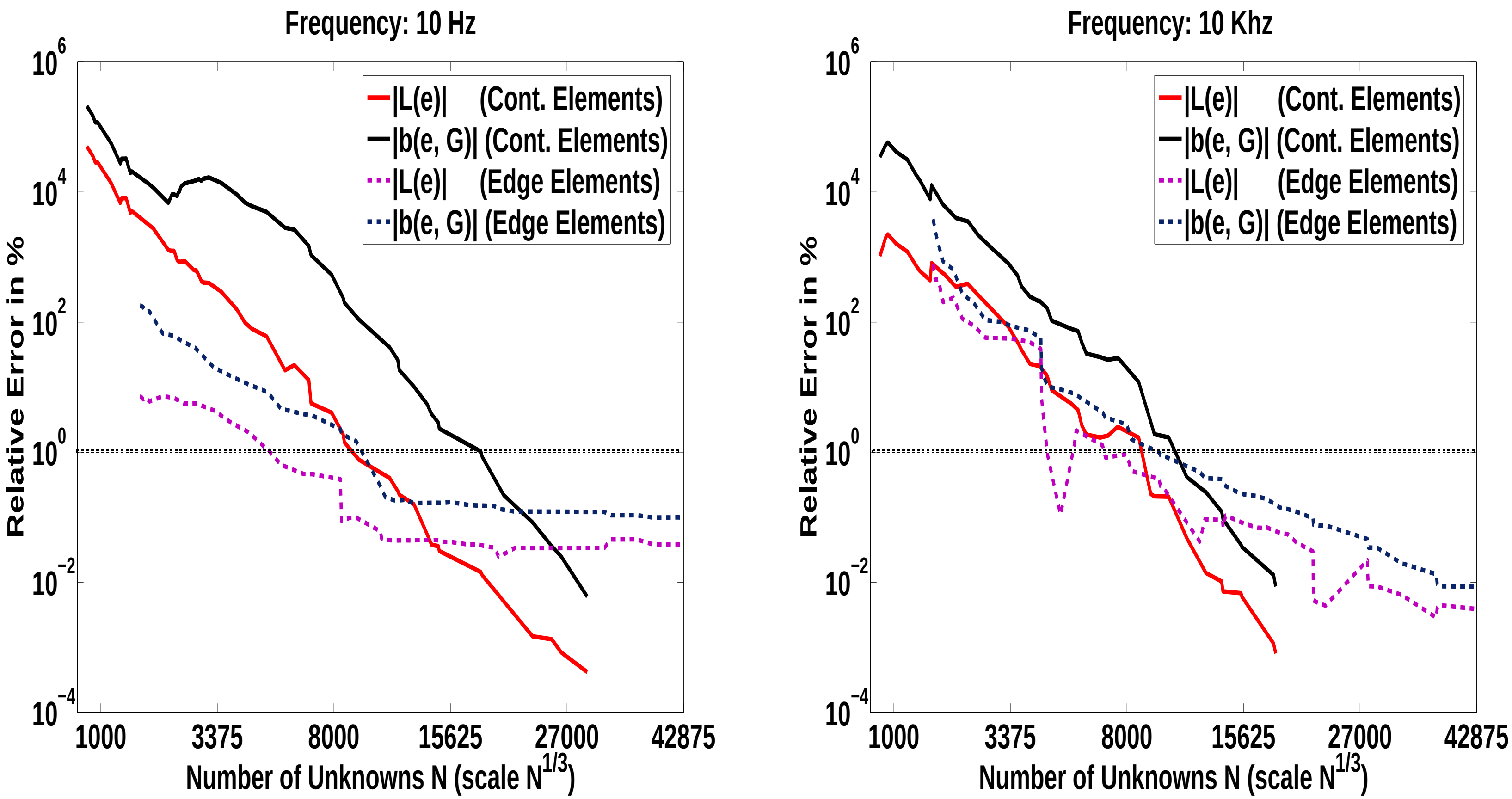
HIGHLY RELIABLE SOFTWARE

CURRENT STAGE OF THE 2D *hp*-FE SOFTWARE

Accuracy (Are the Solutions Accurate?)

Problem with Casing (Convergence Curve)



EXTREMELY ACCURATE SOFTWARE

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Performance (How Fast Can We Solve the Problems?)

80 Vert. Pos.	$10^{-6}\Omega \cdot m$	$10^{-5}\Omega \cdot m$
Toroid (10 Khz)	19’ 46”	16’ 28”
Ring of Vert. Dipoles (10 Khz)	22’ 47”	17’ 02”
Ring of Horiz. Dipoles (10 Khz)	19’ 25”	13’ 25”
Electrodes (0 Hz)	10’ 10”	8’ 35”

IBM Power 4 compiler 1.3 Ghz (4 years old)

Possible improvements in performance:

- **To use a 3.4 Ghz processor.**
- **To execute the code in 8 processors (10 positions per processor).**
- **To improve implementation.**

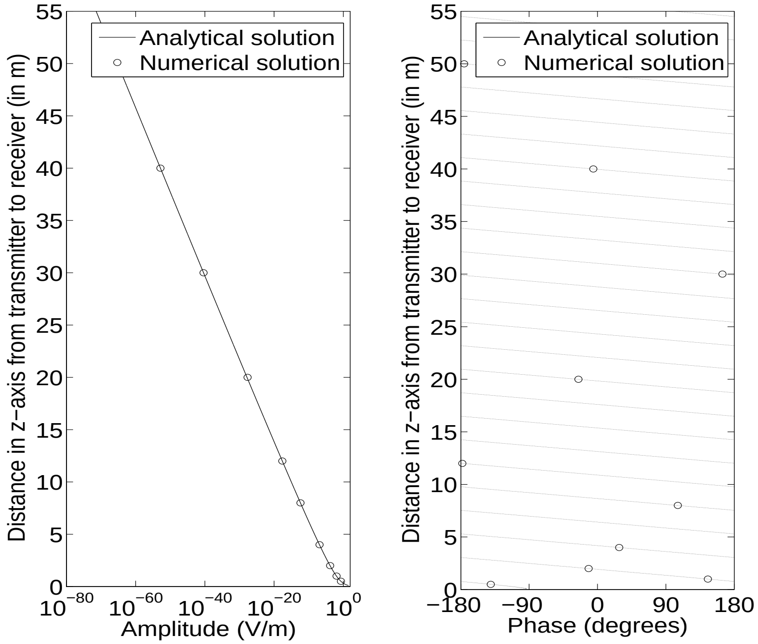
HIGH PERFORMANCE SOFTWARE

SIMULATION OF LOGGING INSTRUMENTS

Comparison Against Analytical Solutions

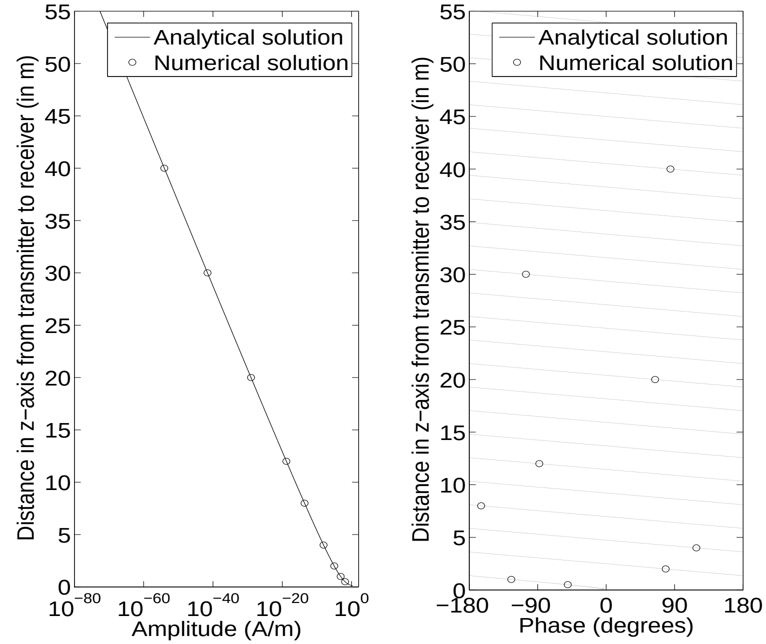
Solutions in a Homogeneous Lossy ($1\ \Omega\ m$) Media (2 Mhz)

Solenoid Antenna



Electric Field

Toroid Antenna



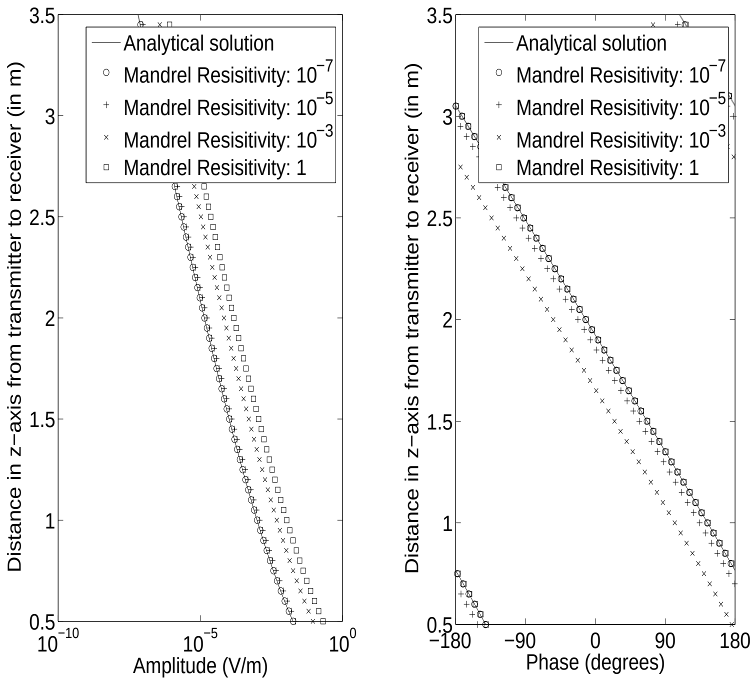
Magnetic Field

SIMULATION OF LOGGING INSTRUMENTS

Comparison Against Analytical Solutions

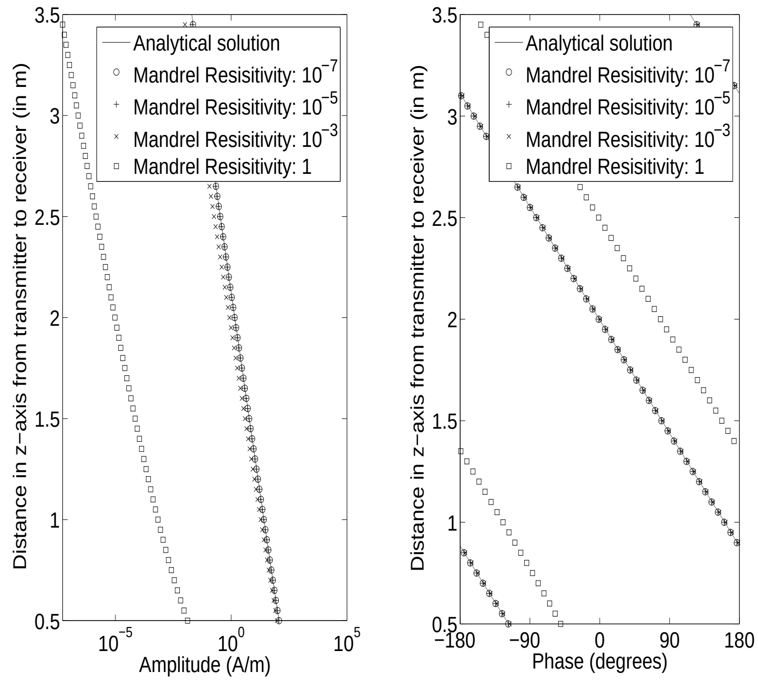
Solutions in a Homogeneous Lossy ($1\ \Omega\ m$) Media (2 Mhz) in Presence of a Conductive Mandrel

Solenoid Antenna



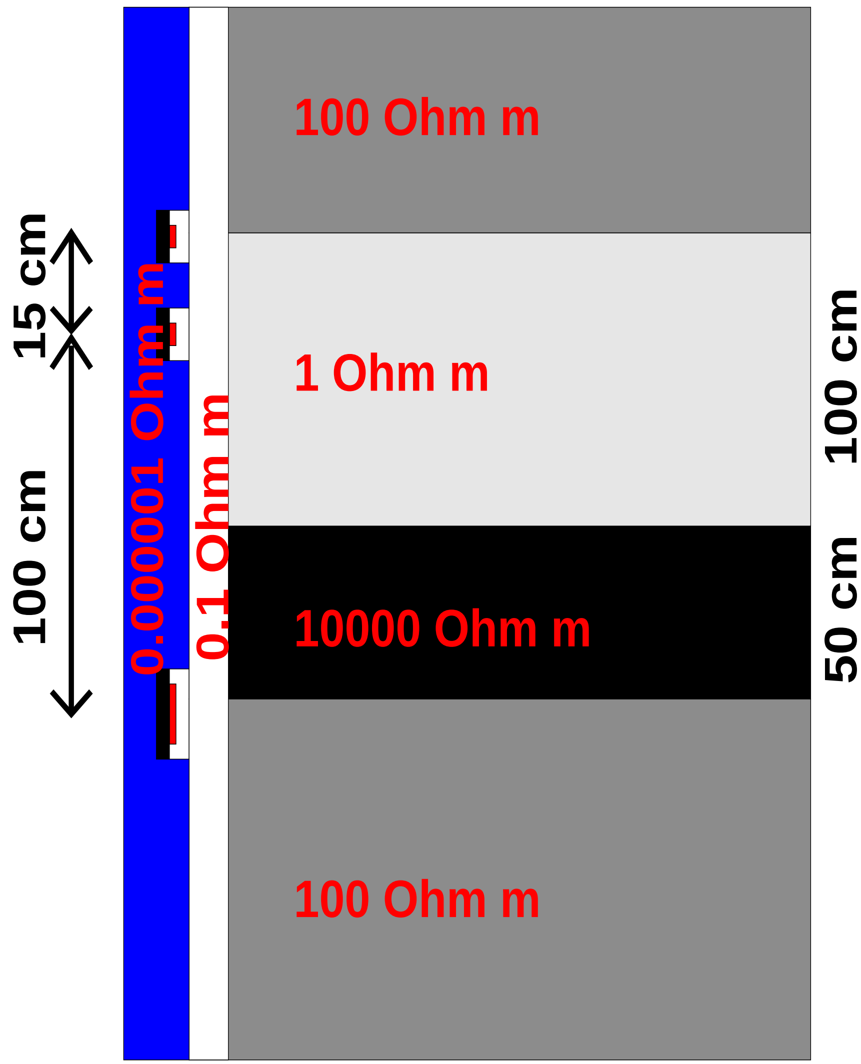
Electric Field

Toroid Antenna

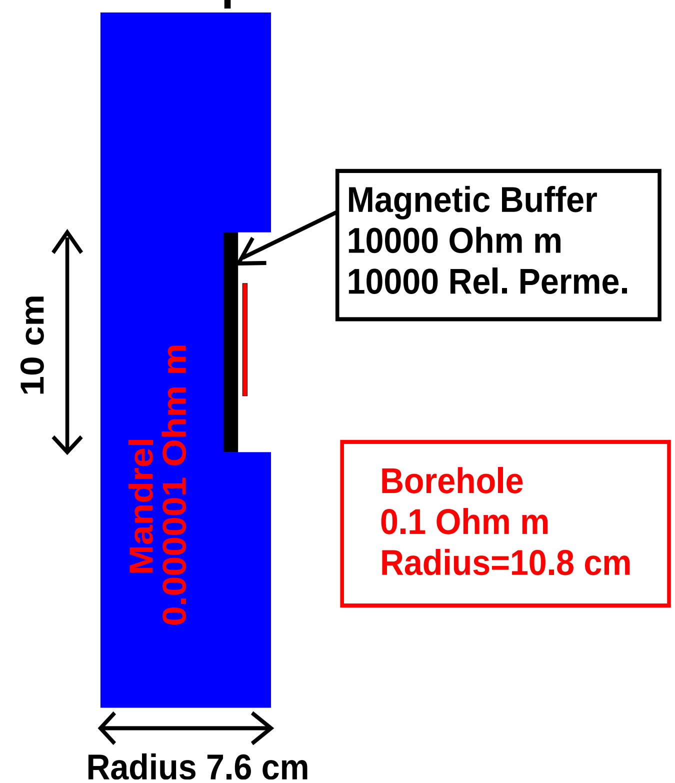


Magnetic Field

SIMULATION OF LOGGING INSTRUMENTS



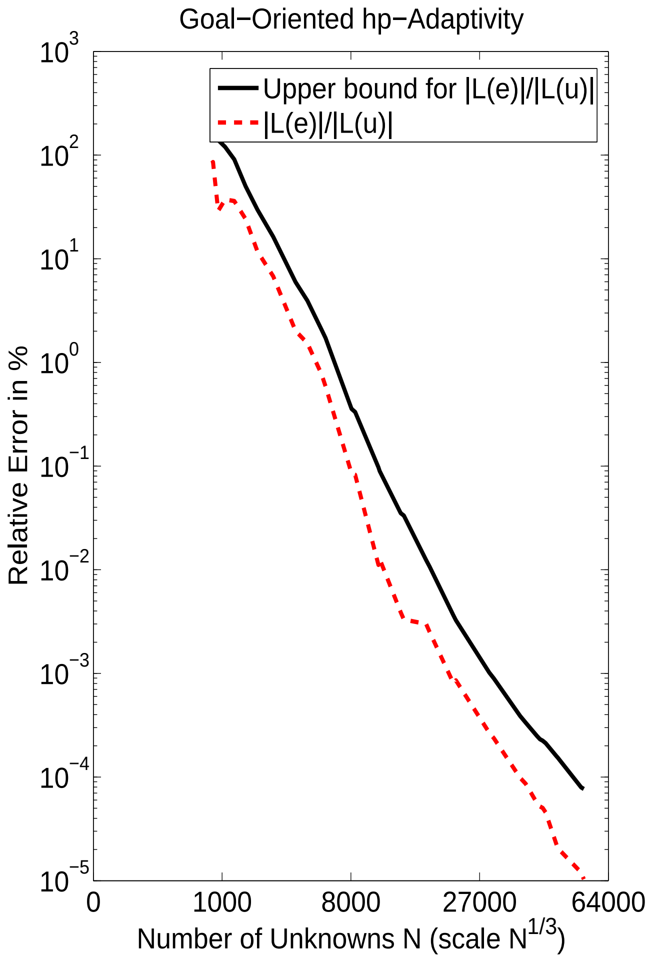
Description of Antennas

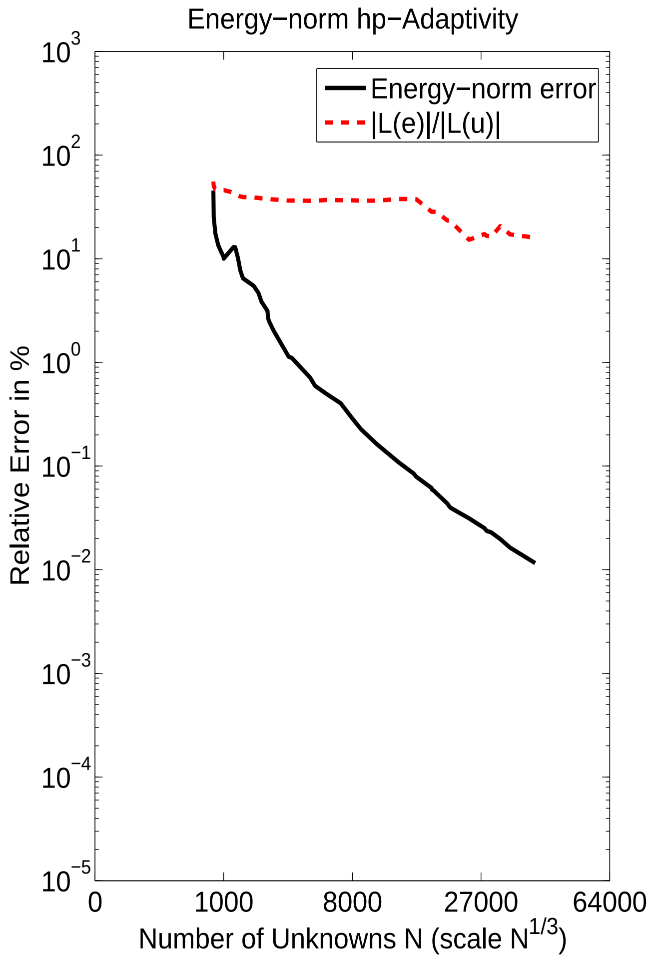


Goal: To Compute First Difference of Potential on Receiving Antennas

SIMULATION OF LOGGING INSTRUMENTS

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m





SIMULATION OF LOGGING INSTRUMENTS

Goal-Oriented vs. Energy-norm hp-Adaptivity

Problem with Mandrel at 2 Mhz.

Continuous Elements (Goal-Oriented Adaptivity)

Quantity of Interest	Real Part	Imag Part
COARSE GRID	-0.1629862203E-01	-0.4016944732E-02
FINE GRID	-0.1629862347E-01	-0.4016944223E-02

Continuous Elements (Energy-norm Adaptivity)

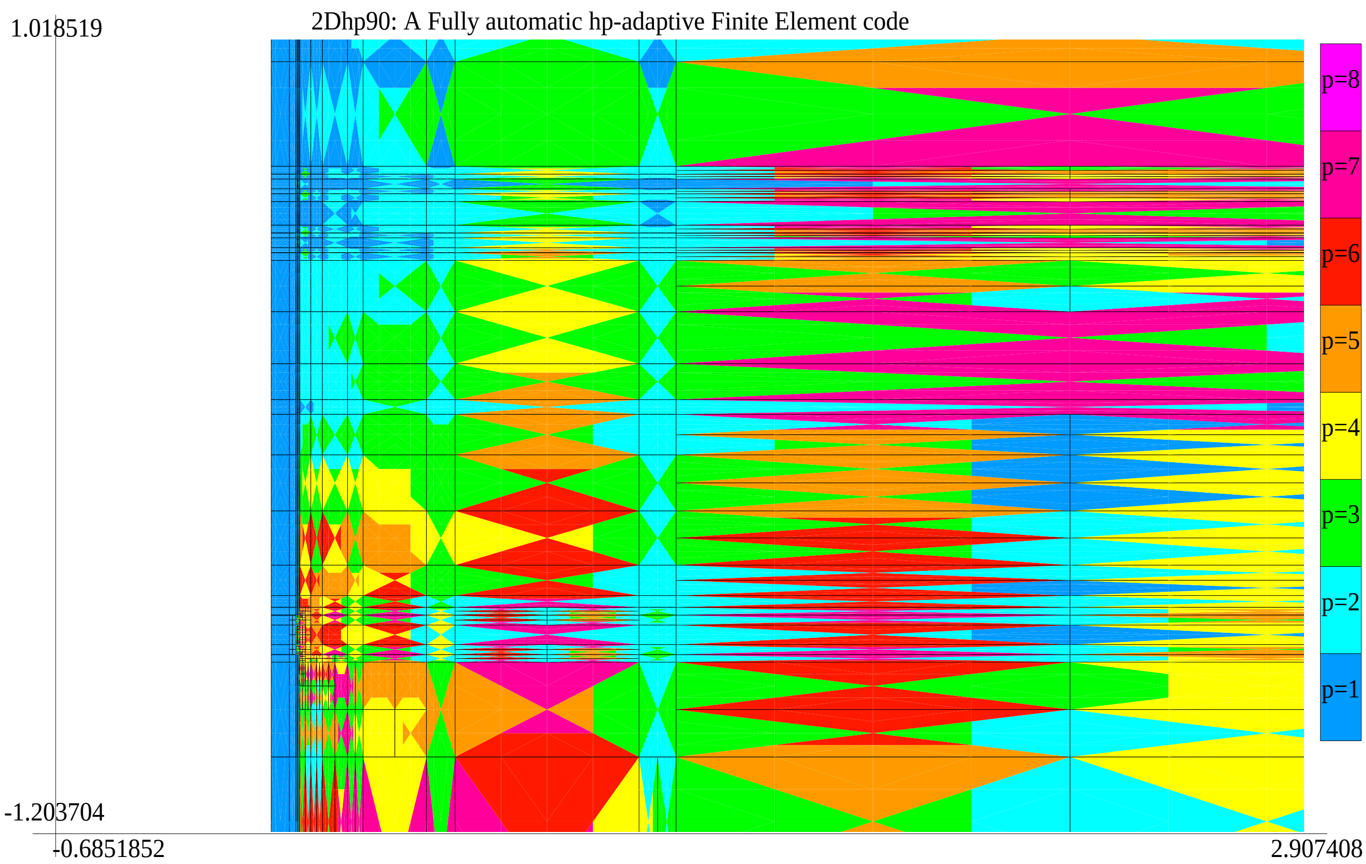
Quantity of Interest	Real Part	Imag Part
0.01% ENERGY ERROR	-0.1382759158E-01	-0.2989492851E-02

It is critical to use GOAL-ORIENTED adaptivity.

SIMULATION OF LOGGING INSTRUMENTS

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

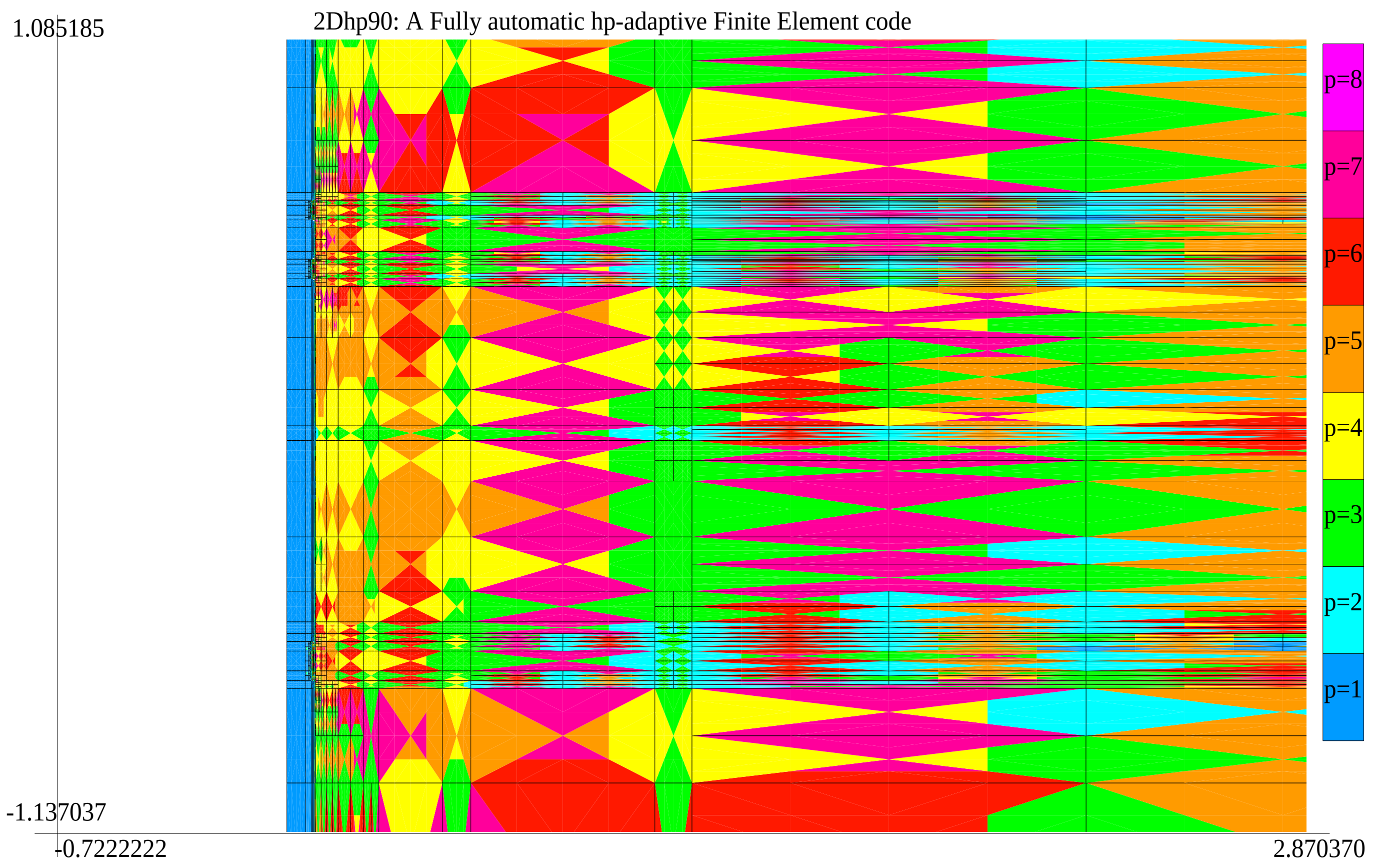
ENERGY-NORM HP-ADAPTIVITY



SIMULATION OF LOGGING INSTRUMENTS

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

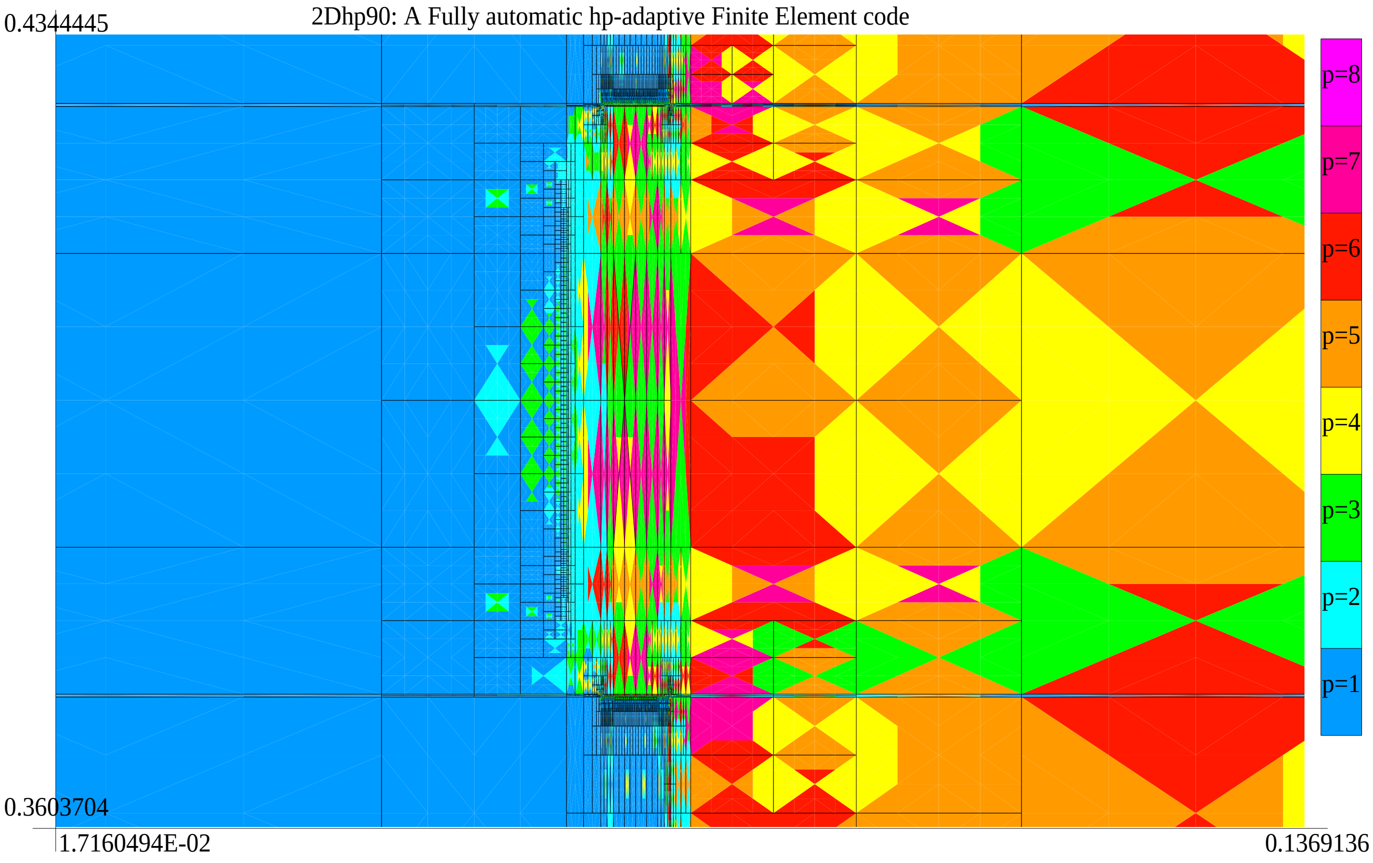
GOAL-ORIENTED HP-ADAPTIVITY



SIMULATION OF LOGGING INSTRUMENTS

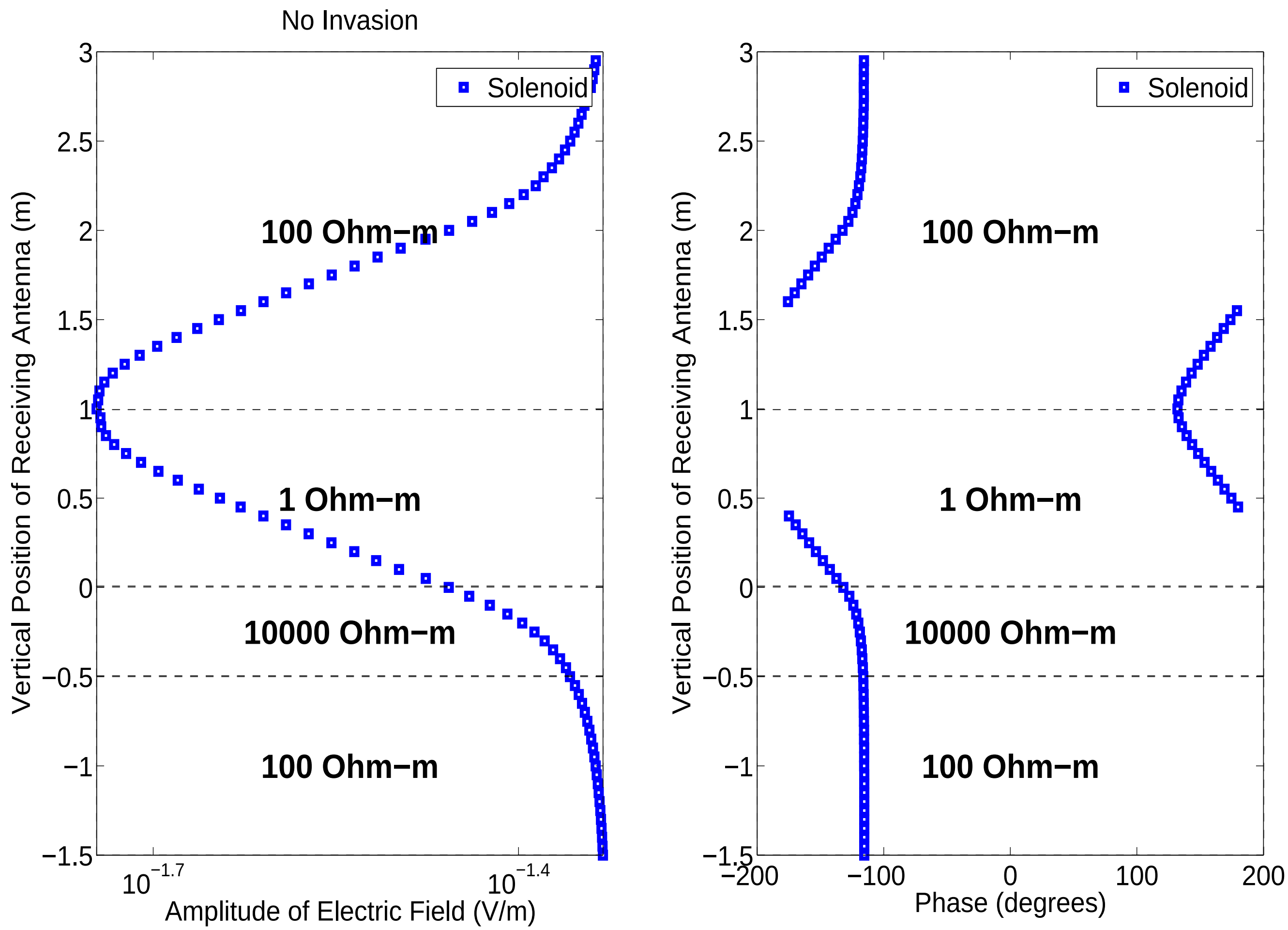
First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

GOAL-ORIENTED HP-ADAPTIVITY (ZOOM TOWARDS FIRST RECEIVER ANTENNA)



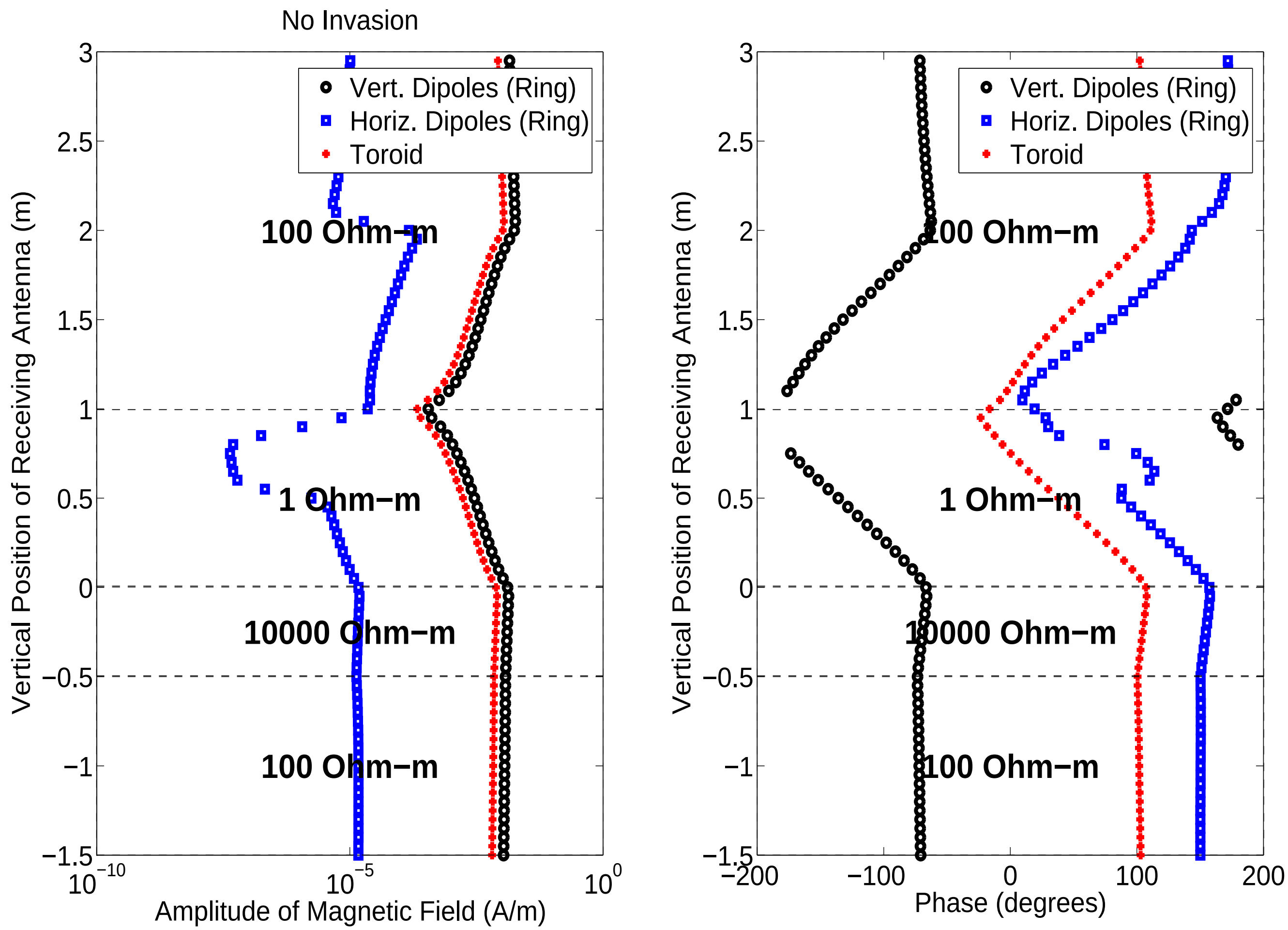
SIMULATION OF LOGGING INSTRUMENTS

E_ϕ for a solenoid antenna



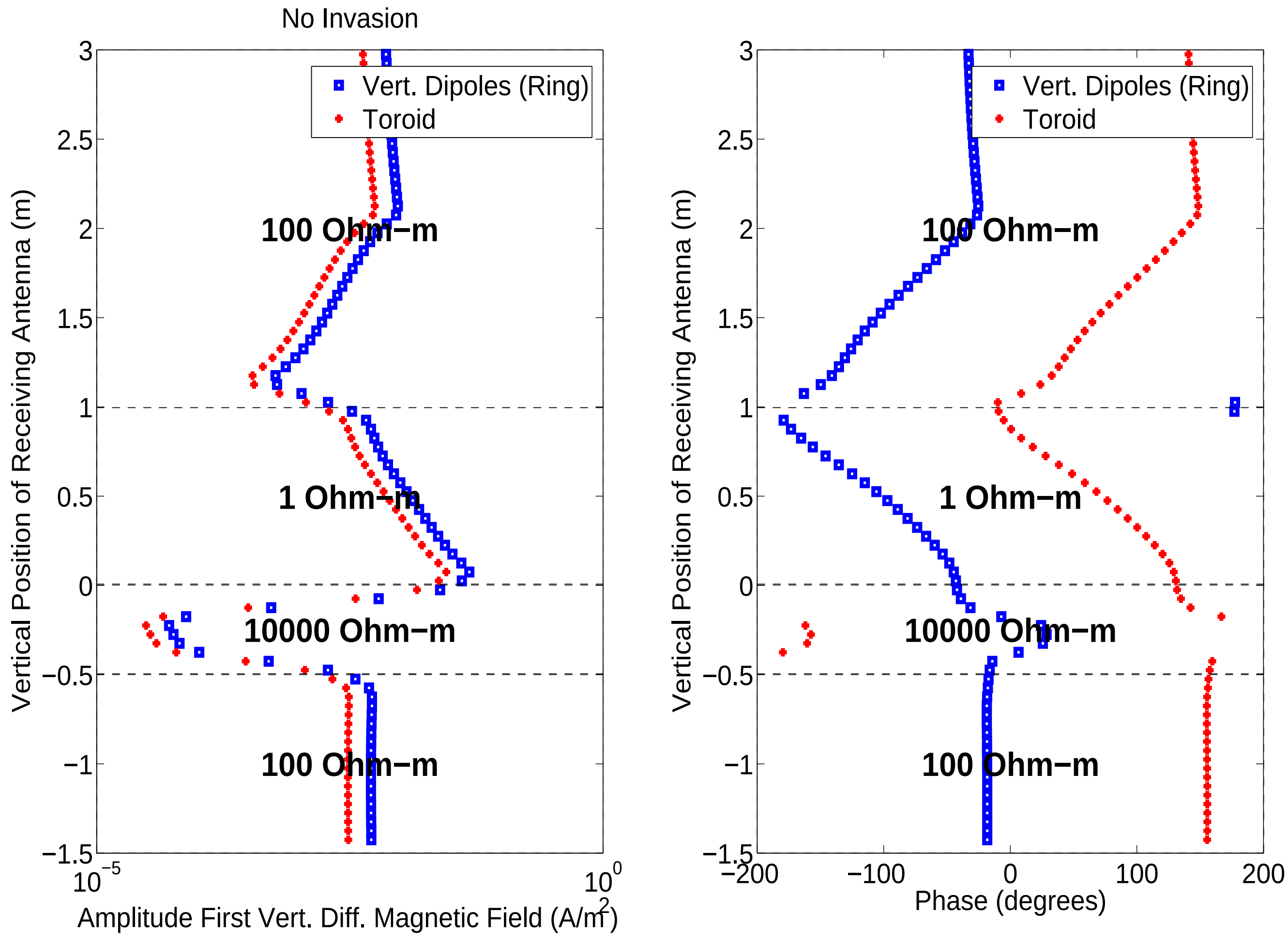
SIMULATION OF LOGGING INSTRUMENTS

H_ϕ for different antennas



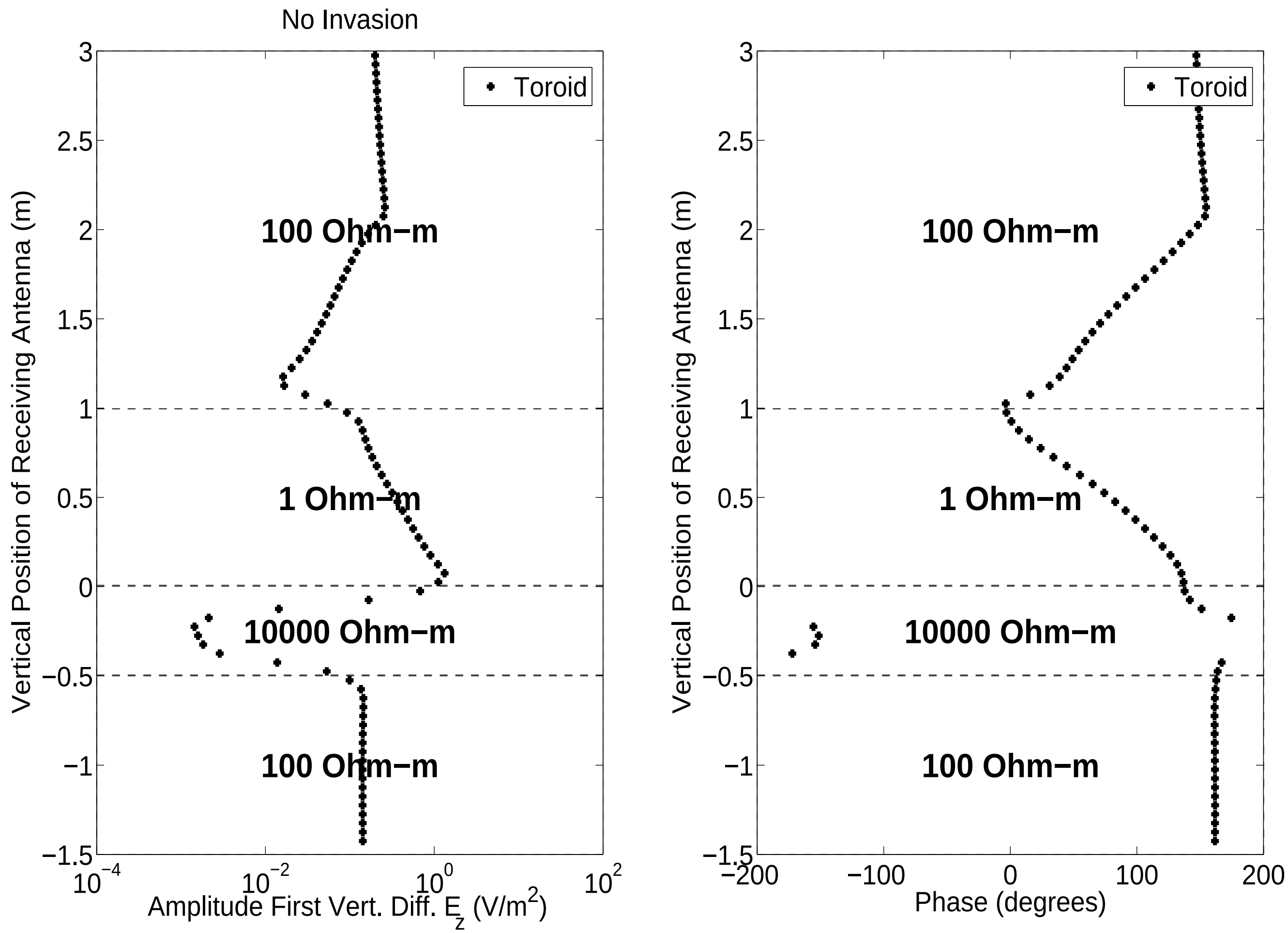
SIMULATION OF LOGGING INSTRUMENTS

First Vert. Diff. H_ϕ for different antennas



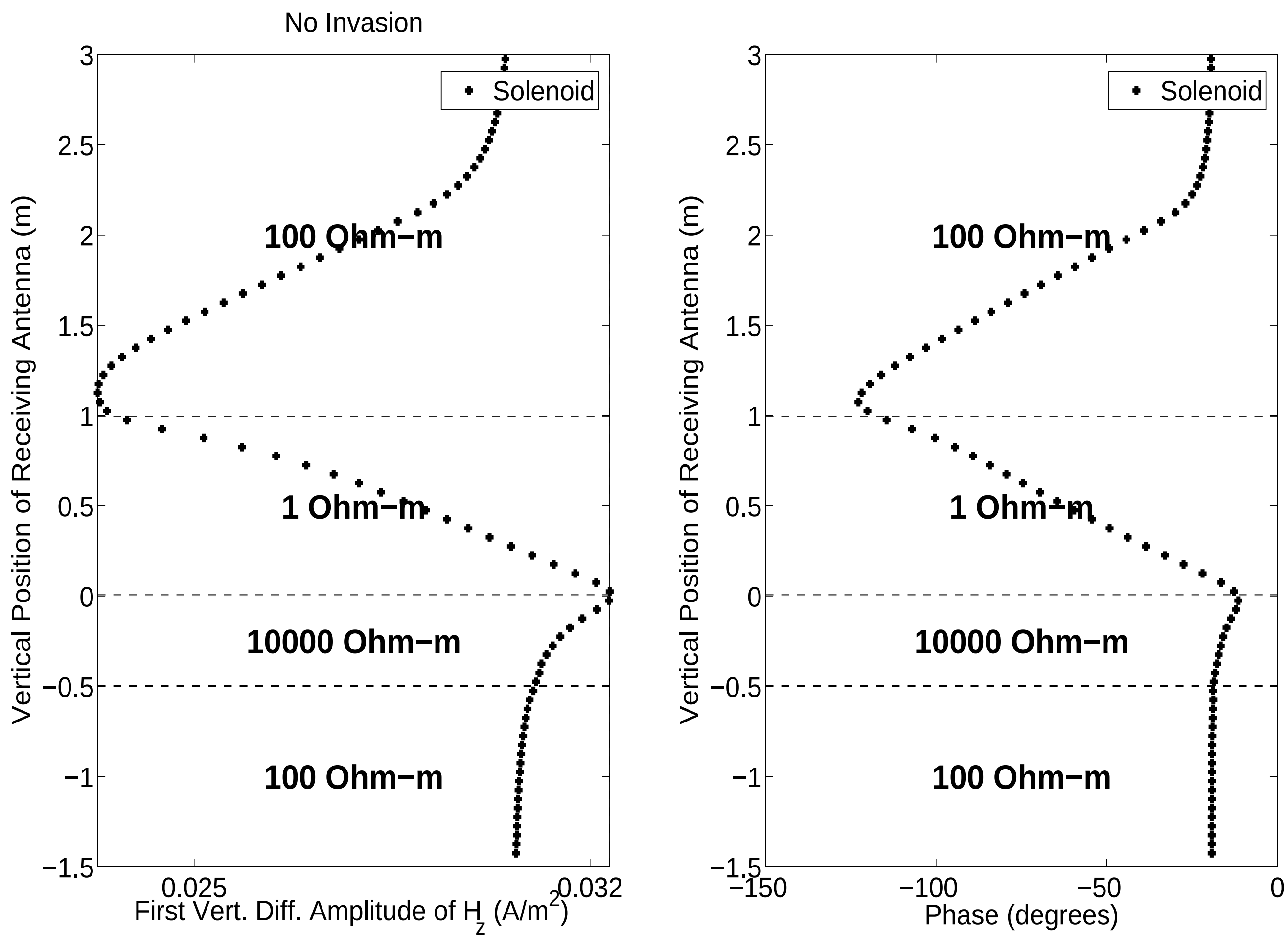
SIMULATION OF LOGGING INSTRUMENTS

First Vert. Diff. E_z for a toroid antenna



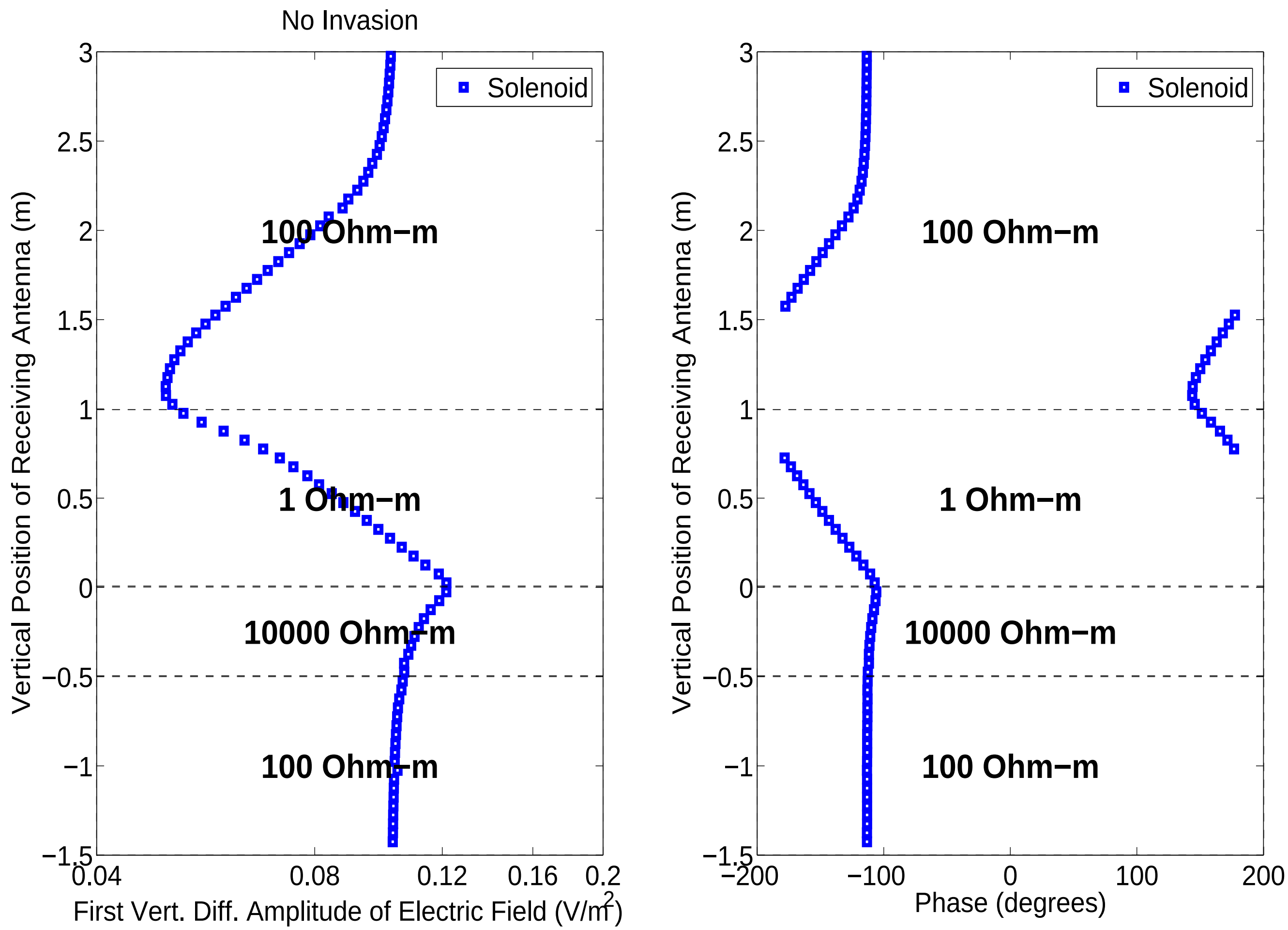
SIMULATION OF LOGGING INSTRUMENTS

First Vert. Diff. H_z for a solenoid antenna



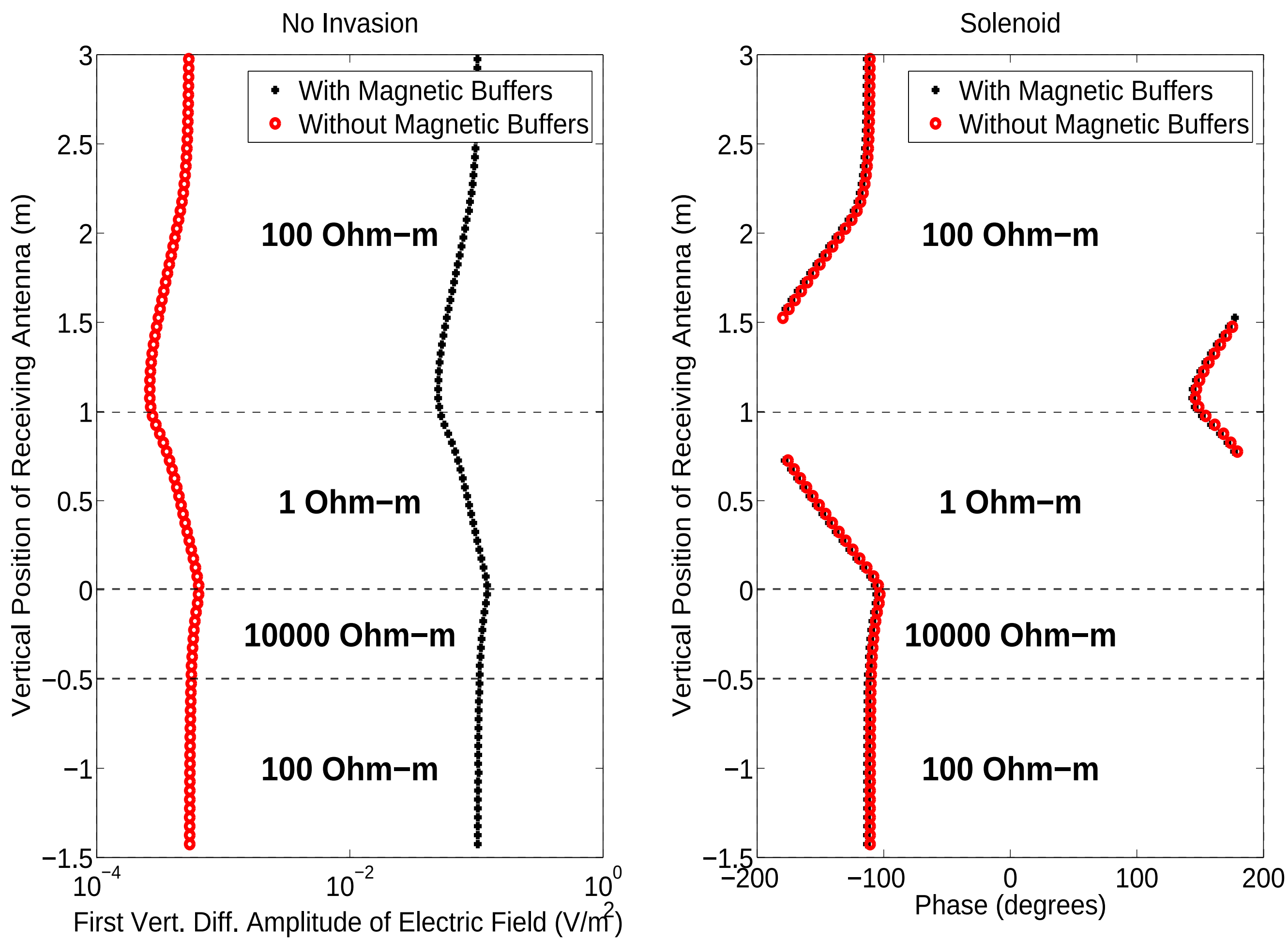
SIMULATION OF LOGGING INSTRUMENTS

First Vert. Diff. E_ϕ for a solenoid antenna



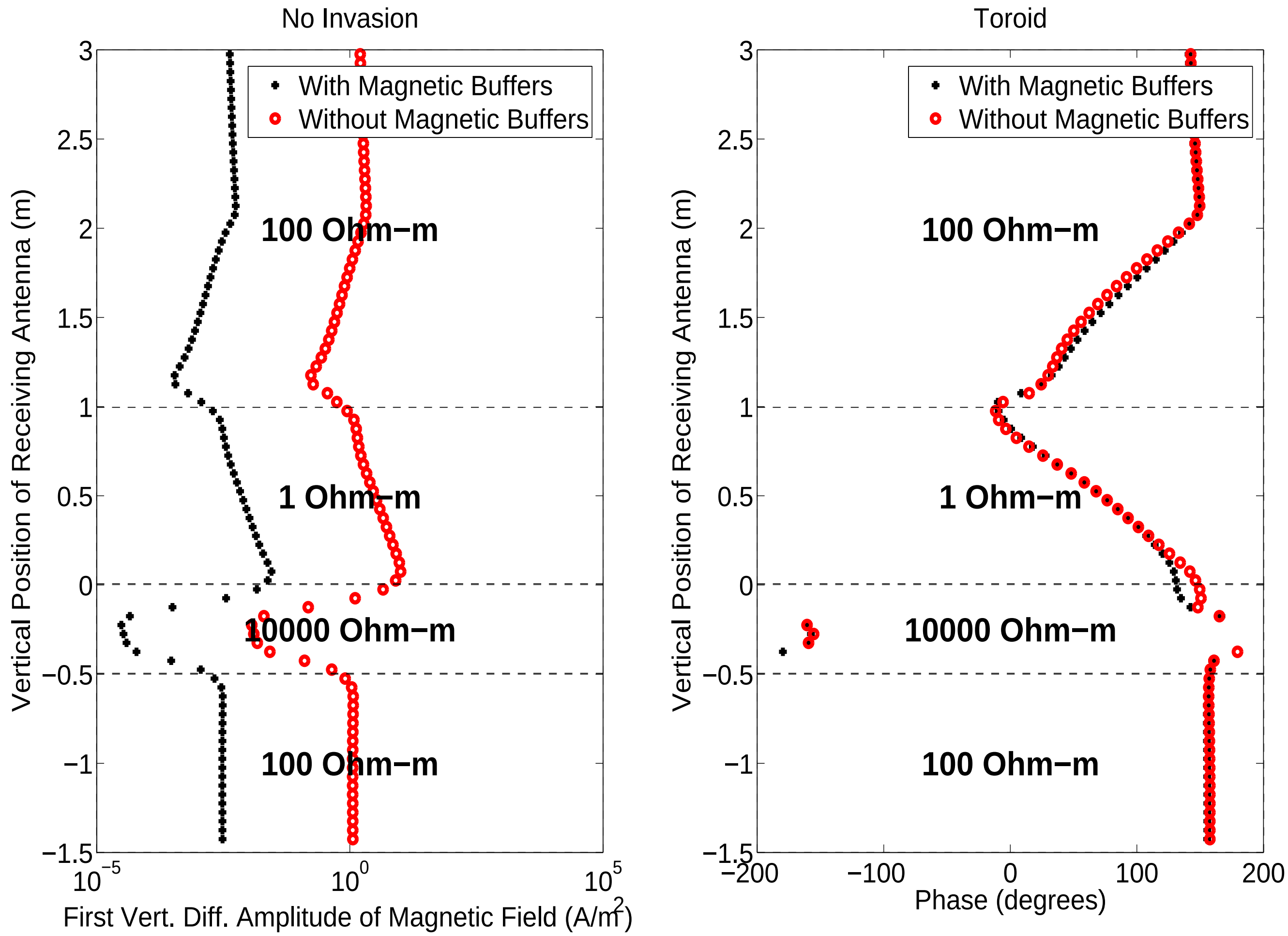
SIMULATION OF LOGGING INSTRUMENTS

Use of Magnetic Buffers (E_ϕ for a solenoid)



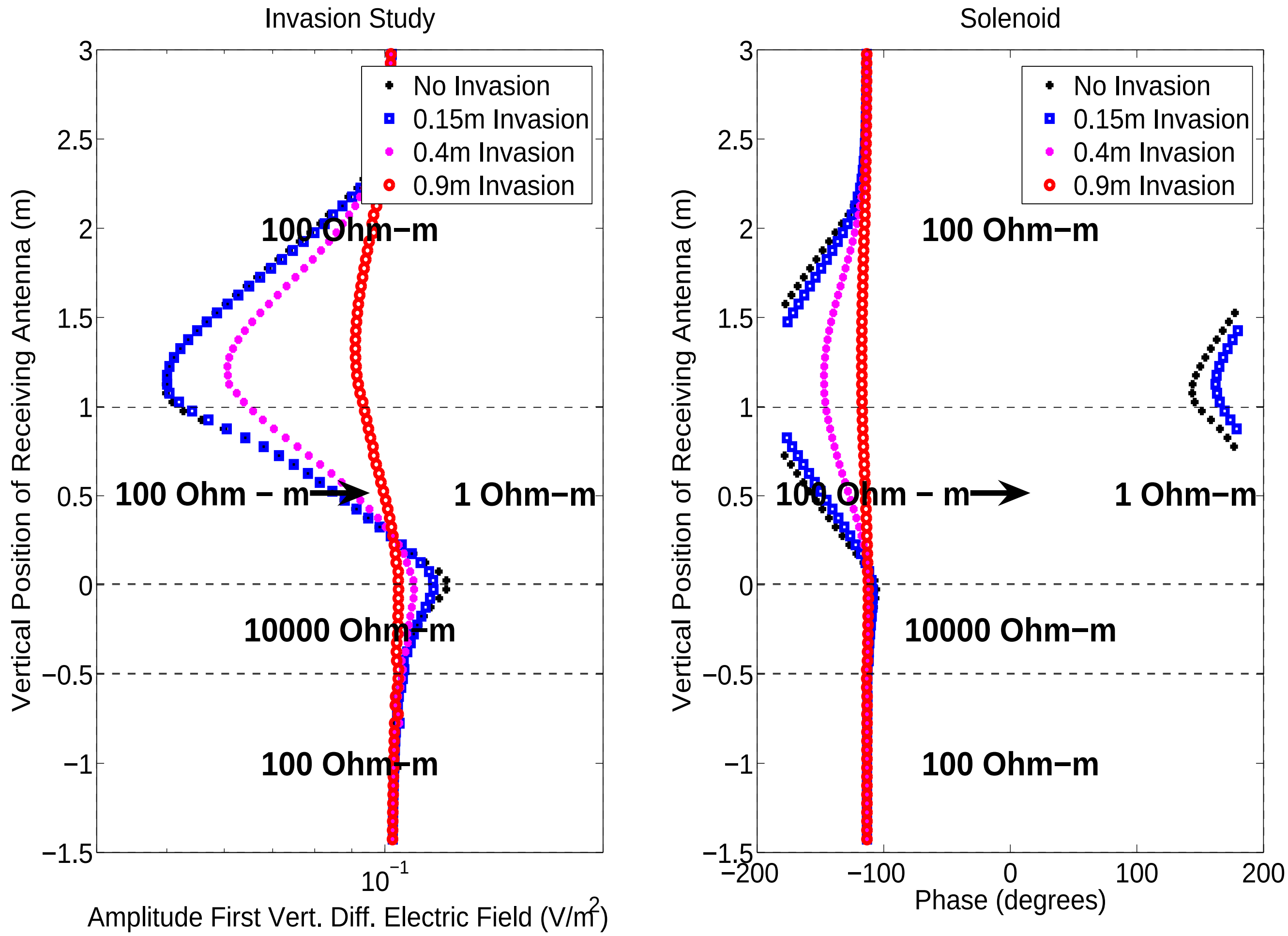
SIMULATION OF LOGGING INSTRUMENTS

Use of Magnetic Buffers (H_ϕ for a toroid)



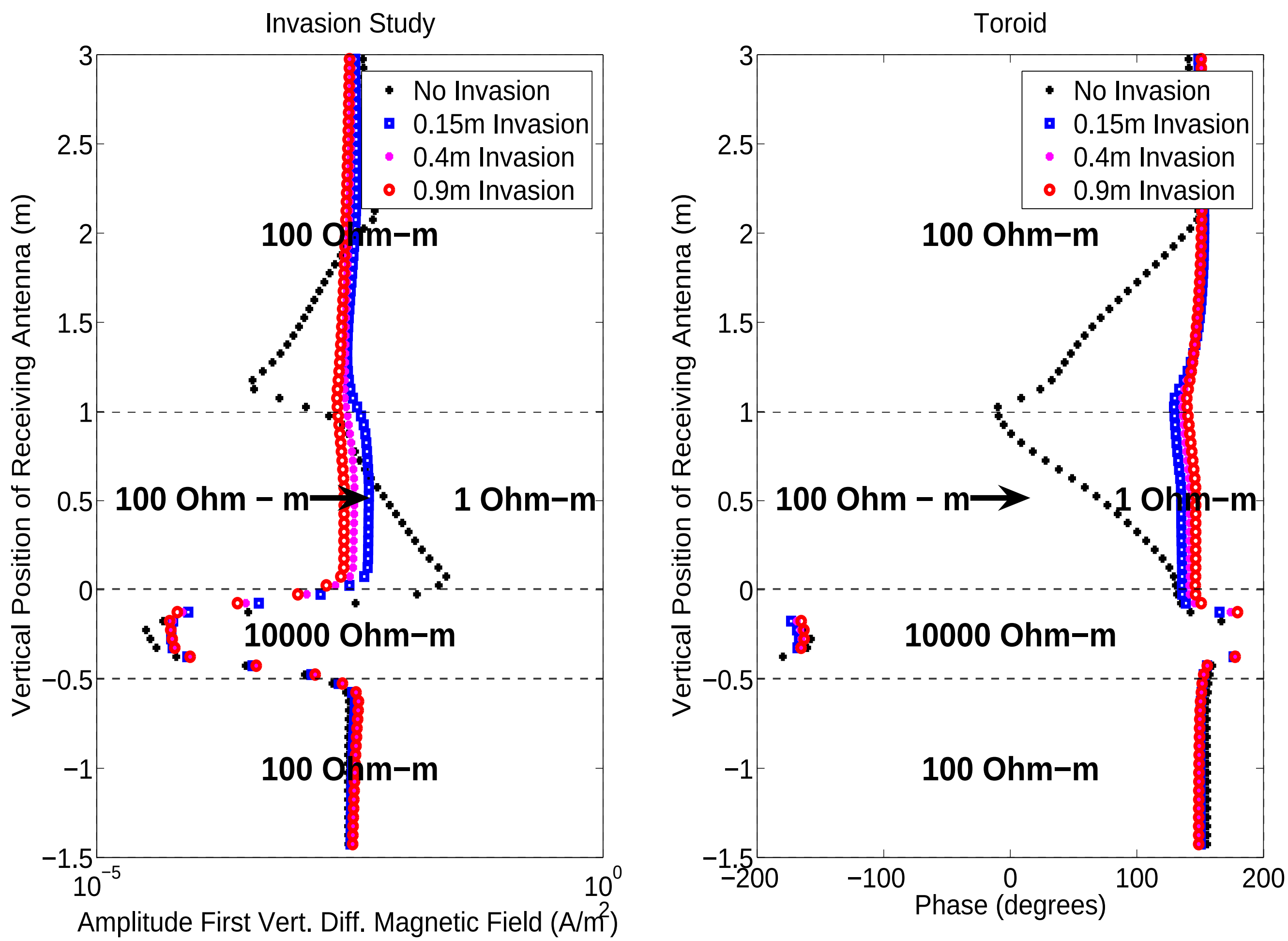
SIMULATION OF LOGGING INSTRUMENTS

Invasion study (E_ϕ for a solenoid)



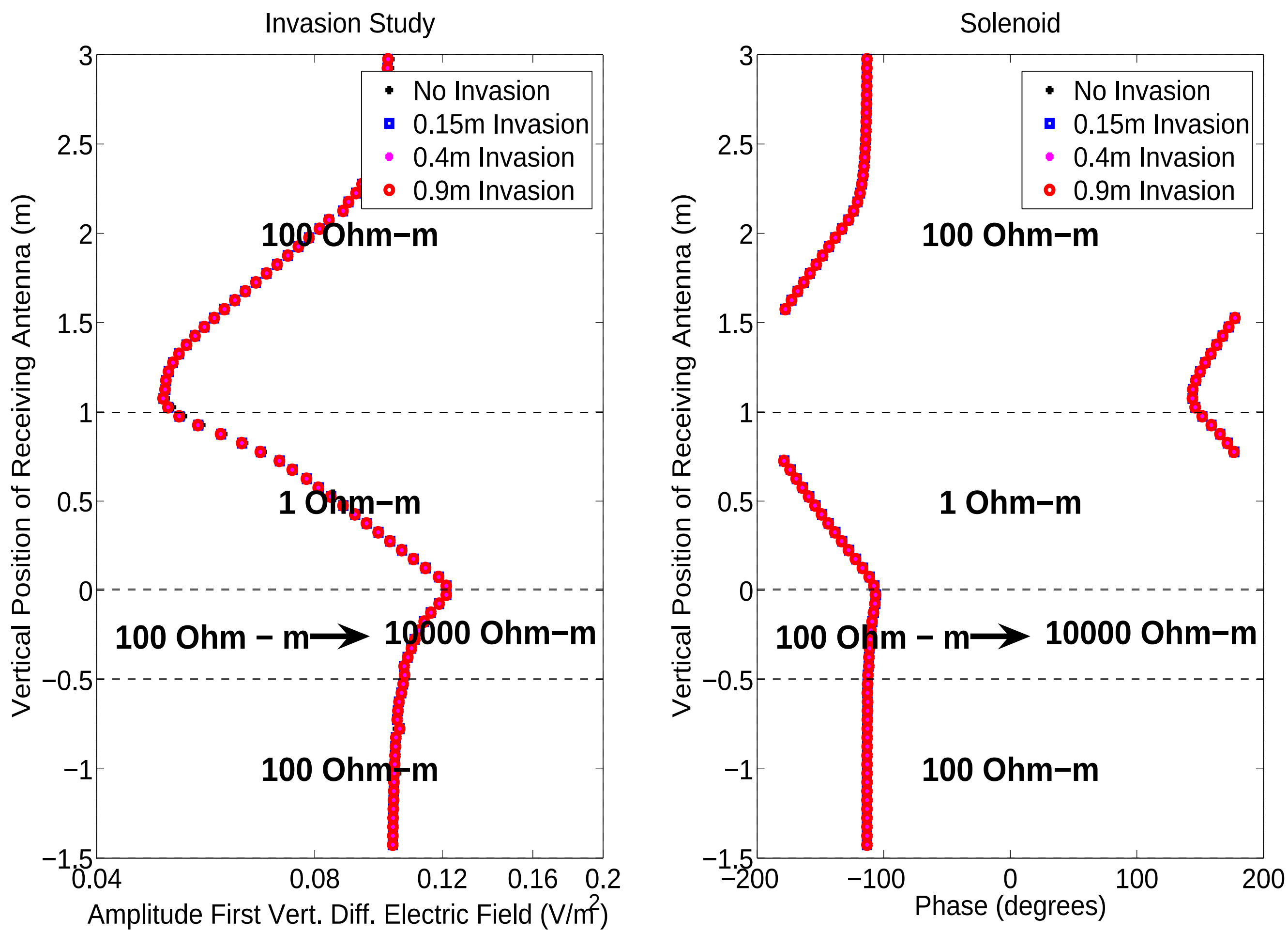
SIMULATION OF LOGGING INSTRUMENTS

Invasion study (H_ϕ for a toroid)



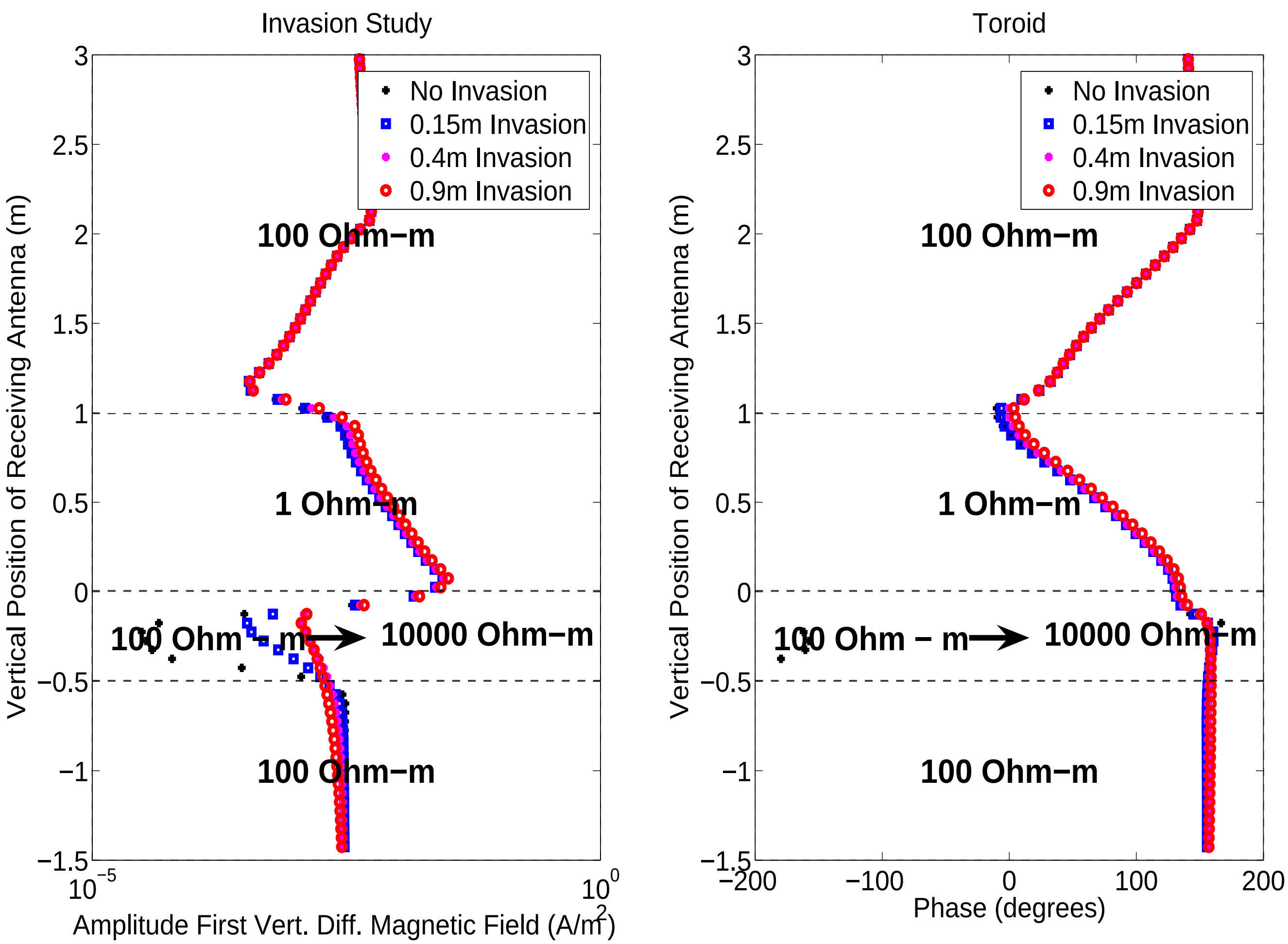
SIMULATION OF LOGGING INSTRUMENTS

Invasion study (E_ϕ for a solenoid)



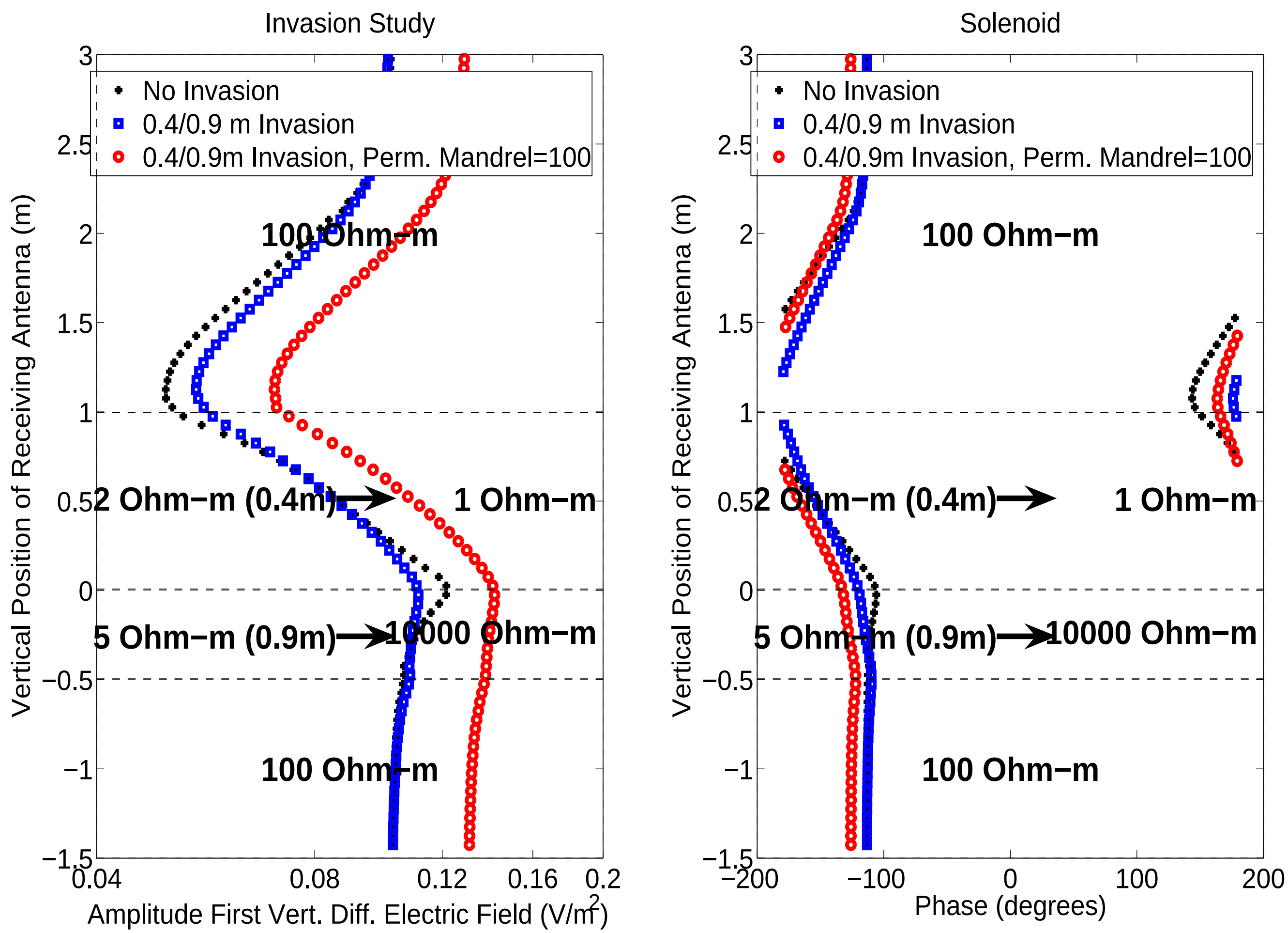
SIMULATION OF LOGGING INSTRUMENTS

Invasion study (H_ϕ for a toroid)



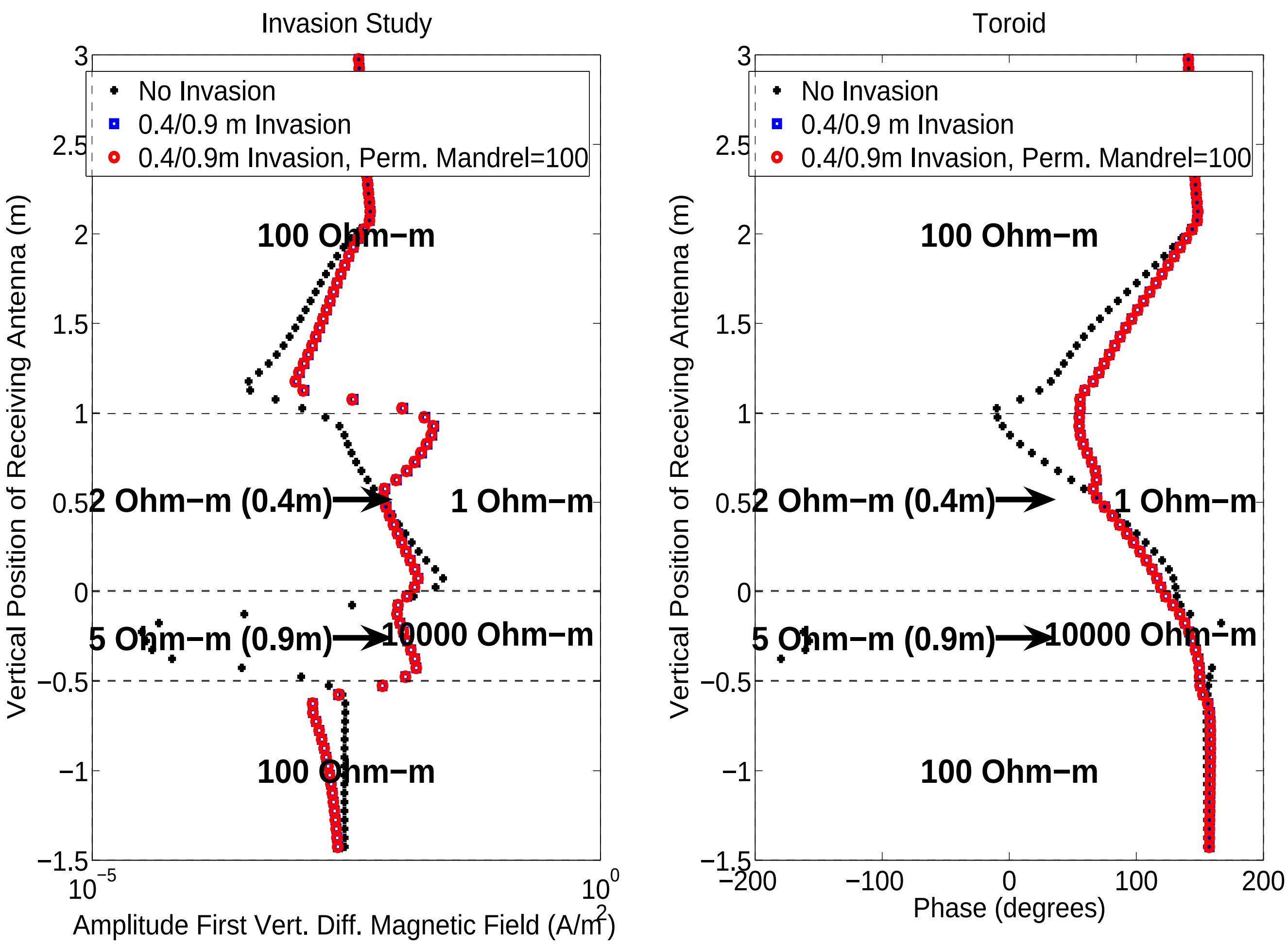
SIMULATION OF LOGGING INSTRUMENTS

Invasion and mandrel magnetic permeab. (E_ϕ)



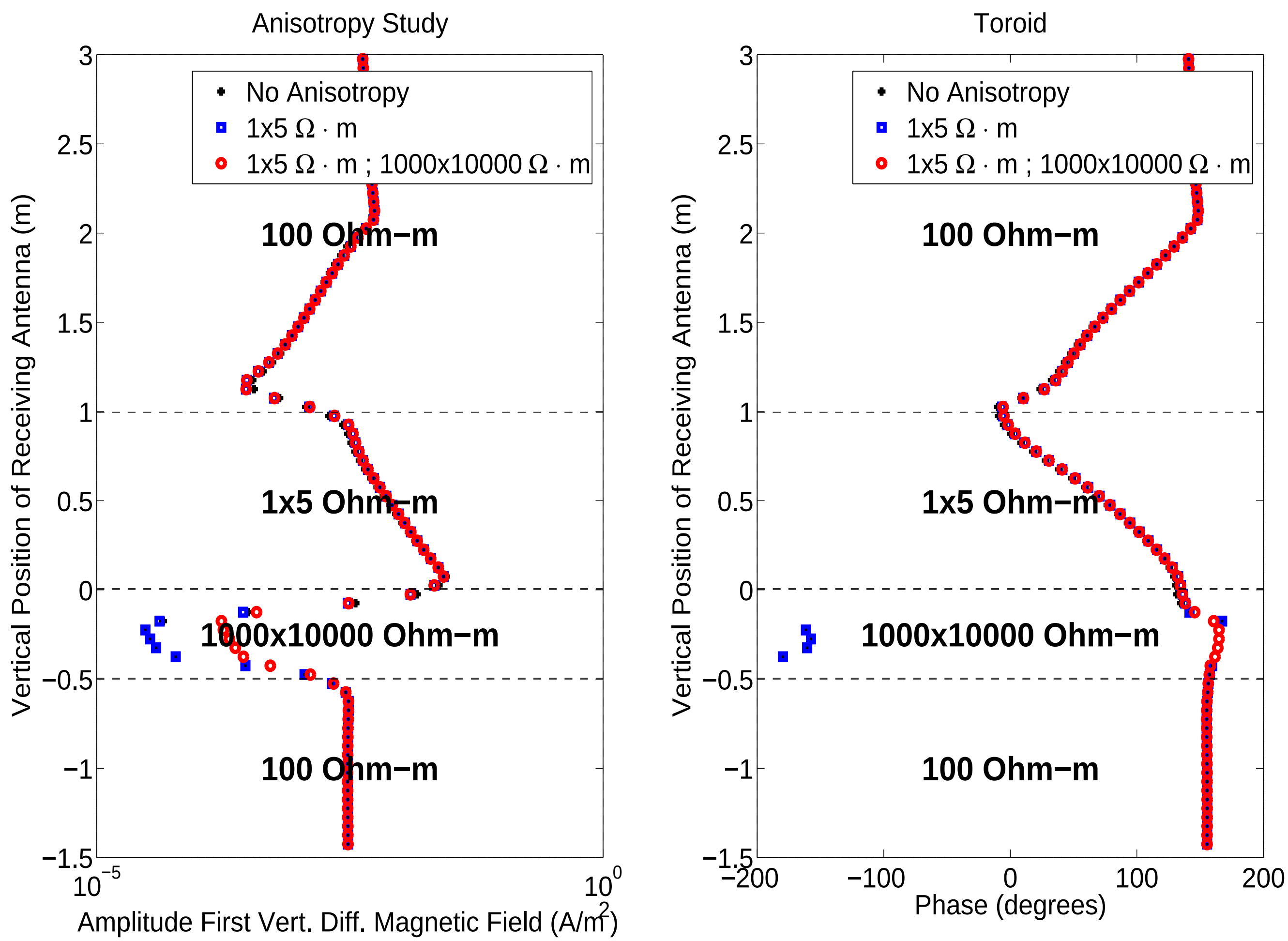
SIMULATION OF LOGGING INSTRUMENTS

Invasion and mandrel magnetic permeab. (H_ϕ)



SIMULATION OF LOGGING INSTRUMENTS

Anisotropy (H_ϕ)

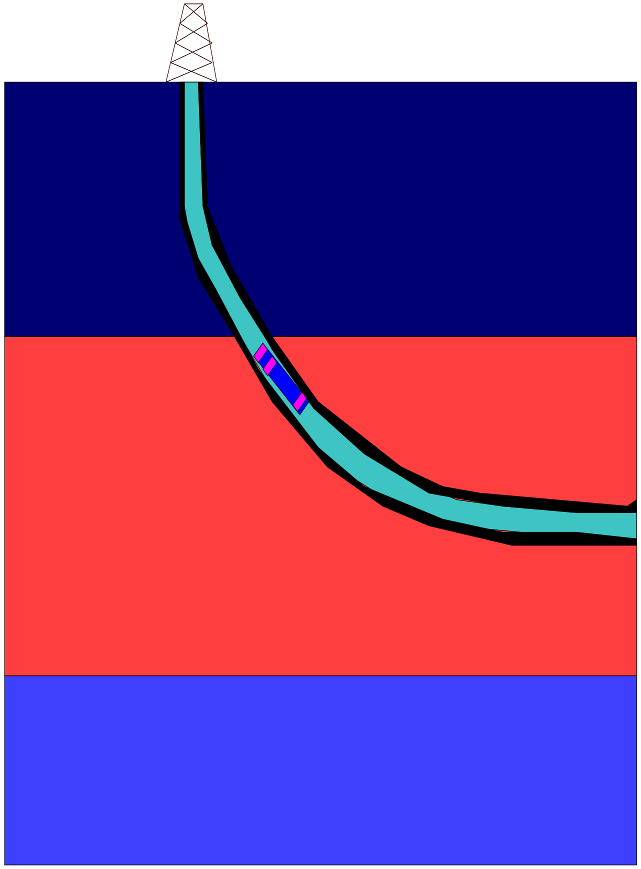


CONCLUSIONS AND FUTURE WORK

- **It is possible to simulate ANY axisymmetric resistivity logging instruments with mandrel (for example, LWD) by using the self-adaptive goal-oriented hp-FEM.**
- **We obtaine fast, reliable and accurate solutions.**
- **For the discussed LWD problem, numerical results suggest to:**
  1. **Measure first vertical differences of the EM fields.**
  2. **Use solenoids for formations with low resitivity, and toroids for highly resitive formations.**
  3. **Use magnetic buffers in combination with solenoids, not with toroids.**
  4. **Use solenoids for studying invasion in formations with low resitivity. Use toroids for studying invasion in highly resistive formations.**

FUTURE WORK

Simulation of 3D Resistivity Logging Problems



- **PART I: Simulate 3D DC Resistivity Logging Problems.**
  - **Estimated completion time: 8-10 months (40 hours/week).**
  - **Main challenge: Speed.**
  - **Expected results: Similar results as in 2D.**
- **PART II: Simulate 3D AC Resistivity Logging Problems.**
  - **Estimated completion time: 8-10 months (40 hours/week).**
  - **Main challenge: Speed and Implementation.**
  - **Expected results: Similar results as in 2D.**