

SiG Meeting, Houston, TX, USA

**New Developments for the Simulation of
Galvanic and Induction Logging Measurements
in Deviated and Eccentric Wells**

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June 24, 2008



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GEOPHYSICAL RESISTIVITY APPLICATIONS

Type of Problems We Can Solve with our FE Software

Applications	Borehole Logging Controlled Source EM		
Spatial Dimensions	2D 3D		
Well Type	Vertical Well	Deviated Well	Eccentered Well
Logging Instruments	LWD/MWD	Normal/Laterolog	Dual Laterolog
	Triaxial Induction	Dielectric Instruments	Cross-Well
Frequency	0-1 GHz		
Materials	Isotropic Anisotropic		
Physical Devices	Magnetic Buffers	Insulators	Casing
	Casing Imperfections	Displacement Currents	Combination of All
Sources	Finite Size Antennas	Dipoles in Any Direction	Electrodes
	Solenoidal Antennas	Toroidal Antennas	Combination of All
Invasion	Water Oil etc.		

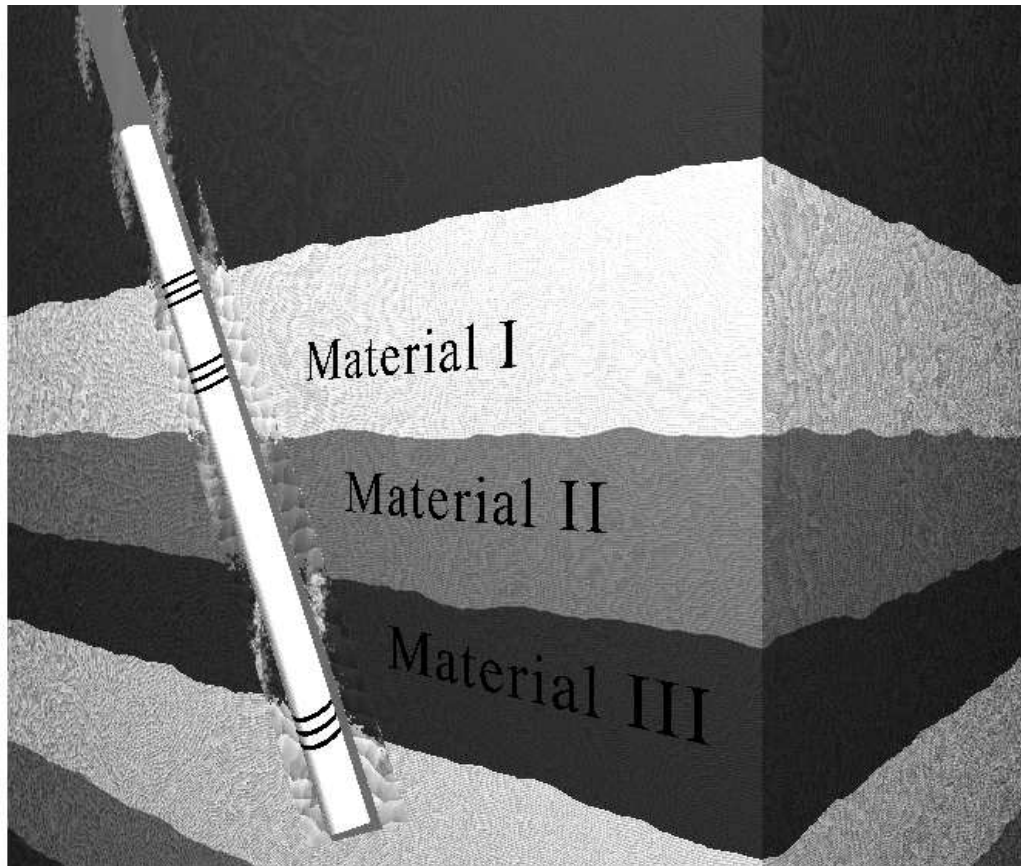
MOST (OIL-INDUSTRY) GEOPHYSICAL PROBLEMS

OVERVIEW

1. **Motivation:** Simulation of borehole logging measurements.
2. **Method:**
 - Fourier-finite-element formulation in a non-orthogonal system of coordinates.
 - Goal-oriented self-adaptive hp -FE method.
 - Parallel implementation.
 - Iterative solver.
3. **Numerical Simulations:**
 - Galvanic tools: Through-casing, dual laterolog, deviated wells, eccentric wells, anisotropy, etc.
 - Induction tools: LWD, Triaxial induction, deviated wells, borehole rugosity, anisotropy, etc.
4. **Conclusions and future work.**

MOTIVATION (BOREHOLE LOGGING)

Deviated Wells (Forward Problem)



Dip Angle
Invasion
Anisotropy
Triaxial Induction
Eccentricity
Laterolog
Through-Casing
Induction-LWD
Induction-Wireline
Inverse Problems
Multi-Physics

Objective: Find solution at the receiver antennas.

MATHEMATICAL FORMULATION (3D)

3D Variational Formulation

Time-Harmonic Maxwell's Equations

$$\nabla \times \mathbf{H} = \mathring{\sigma} \mathbf{E} + \mathbf{J}^{imp} \quad \text{Ampere's law } (\mathring{\sigma} = \sigma + j\omega\epsilon)$$

$$\nabla \times \mathbf{E} = \mathring{\mu} \mathbf{H} + \mathbf{M}^{imp} \quad \text{Faraday's law } (\mathring{\mu} = -j\omega\mu)$$

$$\nabla \cdot (\epsilon \mathbf{E}) = \rho \quad \text{Gauss' law of Electricity}$$

$$\nabla \cdot (\mu \mathbf{H}) = 0 \quad \text{Gauss' law of Magnetism}$$

E-VARIATIONAL FORMULATION:

$$\left\{ \begin{array}{l} \text{Find } \mathbf{E} \in \mathbf{E}_{\Gamma_E} + \mathbf{H}_{\Gamma_E}(\text{curl}; \Omega) \text{ such that:} \\ \langle \nabla \times \mathbf{F}, \mathring{\mu}^{-1} \nabla \times \mathbf{E} \rangle_{L^2(\Omega)} - \langle \mathbf{F}, \mathring{\sigma} \mathbf{E} \rangle_{L^2(\Omega)} = \langle \mathbf{F}, \mathbf{J}^{imp} \rangle_{L^2(\Omega)} \\ - \langle \mathbf{F}_t, \mathbf{J}_{\Gamma_H}^{imp} \rangle_{L^2(\Gamma_H)} + \langle \nabla \times \mathbf{F}, \mathring{\mu}^{-1} \mathbf{M}^{imp} \rangle_{L^2(\Omega)} \quad \forall \mathbf{F} \in \mathbf{H}_{\Gamma_E}(\text{curl}; \Omega) \end{array} \right.$$

FOURIER ANALYSIS

Dimensionality Reduction for Maxwell's Equations

Solving a 3D problem is CPU time and memory intensive. In some cases, we may reduce the complexity of the problem by using Fourier analysis.

Borehole Problems

Cylindrical Coordinates

Fourier Series Expansion

$$\mathbf{E}(\phi) := \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathbf{E}) e^{jn\phi}$$

X-Well, CSEM Problems

Cartesian Coordinates

Fourier Transform

$$\mathbf{E}(x_1) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathcal{F}_r(\mathbf{E}) e^{jrx_1} dx_1$$

FOURIER ANALYSIS

Fourier Series Expansion

Fourier series expansion

$$\mathcal{F}_n(\mathbf{E}) := \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \mathbf{E}(\phi) e^{-jn\phi} d\phi$$

Inverse Fourier series expansion

$$\mathbf{E}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathbf{E}) e^{jn\phi}.$$

Main properties

- Compatibility with differentiation $\mathcal{F}_n\left(\frac{\partial \mathbf{E}}{\partial \phi}\right) = jn\mathcal{F}_n(\mathbf{E})$:

$$\mathcal{F}_n(\nabla \times \mathbf{E}) = \nabla^n \times (\mathcal{F}_n(\mathbf{E})),$$

where

$$\nabla^n \times \mathbf{E} := \left(\frac{jnE_z}{\rho} - \frac{\partial E_\phi}{\partial z}, \frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}, \frac{1}{\rho} \frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{jnE_\rho}{\rho} \right),$$

- L_2 -Orthogonality:

$$\frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{jn\phi} e^{-jm\phi} d\phi = \sqrt{2\pi} \delta_{nm}.$$

FOURIER ANALYSIS

Fourier Transform

Fourier transform

Inverse Fourier transform

$$\mathcal{F}_r(\mathbf{E}) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathbf{E}(x) e^{-jrx} dx \quad ; \quad \mathbf{E}(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathcal{F}_r(\mathbf{E}) e^{jrx} dr.$$

Main properties

- Compatibility with differentiation $\mathcal{F}_r\left(\frac{\partial \mathbf{E}}{\partial x}\right) = jr \mathcal{F}_r(\mathbf{E})$:

$$\mathcal{F}_r(\nabla \times \mathbf{E}) = \nabla^r \times (\mathcal{F}_r(\mathbf{E})),$$

where

$$\nabla^r \times \mathbf{E} := \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - jr E_z, jr E_y - \frac{\partial E_x}{\partial y} \right),$$

- L_2 -Orthogonality:

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{jrx} e^{-jsx} = \sqrt{2\pi} \delta_{sr}.$$

FOURIER ANALYSIS

E-Variational Formulations (Cylindrical Coordinates)

FINITE ELEMENT —3D—:

$$\left\{ \begin{array}{l} \text{Find } \mathbf{E} \in \mathbf{E}_{\Gamma_E} + \mathbf{H}_{\Gamma_E}(\text{curl}; \Omega) \text{ such that:} \\ \langle \nabla \times \mathbf{F}, \dot{\mu}^{-1} \nabla \times \mathbf{E} \rangle_{L^2(\Omega)} - \langle \mathbf{F}, \dot{\sigma} \mathbf{E} \rangle_{L^2(\Omega)} = \langle \mathbf{F}, \mathbf{J}^{imp} \rangle_{L^2(\Omega)} \\ - \langle \mathbf{F}_t, \mathbf{J}_{\Gamma_H}^{imp} \rangle_{L^2(\Gamma_H)} + \langle \nabla \times \mathbf{F}, \dot{\mu}^{-1} \mathbf{M}^{imp} \rangle_{L^2(\Omega)} \quad \forall \mathbf{F} \in \mathbf{H}_{\Gamma_E}(\text{curl}; \Omega) \end{array} \right.$$

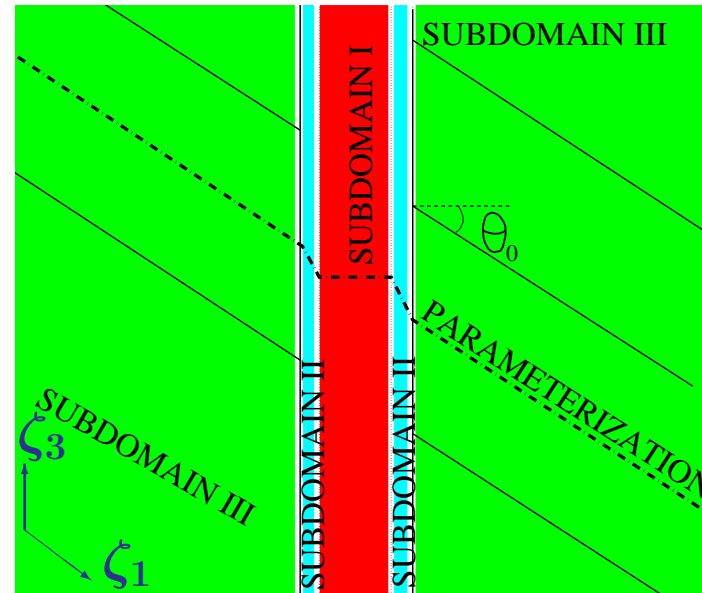
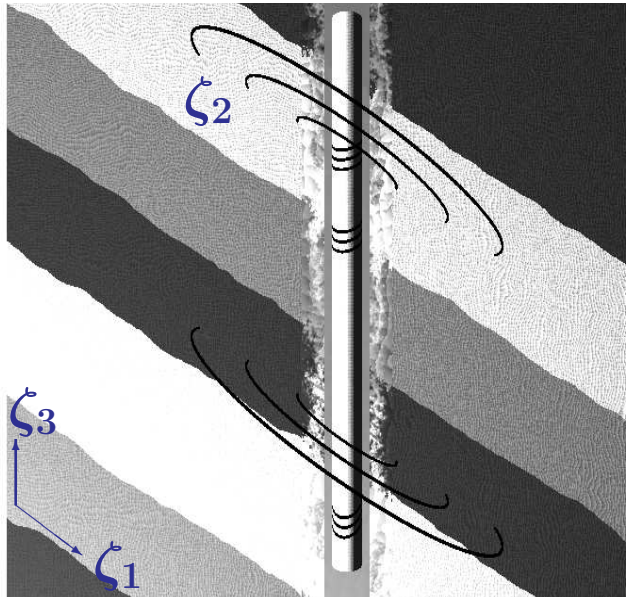
FOURIER FINITE ELEMENT —3D = Sequence of **Coupled** 2D Problems—:

$$\left\{ \begin{array}{l} \text{Find } \mathbf{E} = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathbf{E}) e^{jn\phi}, \text{ where for each } n: \\ \mathcal{F}_n(\mathbf{E}) \in \mathcal{F}_n(\mathbf{E}_{\Gamma_{E,1D}}) + \mathbf{H}_{\Gamma_{E,1D}}(\text{curl}^n; \Omega_{2D}), \text{ and} \\ \sum_{m=-\infty}^{\infty} \langle \nabla^n \times \mathcal{F}_n(\mathbf{F}), \mathcal{F}_{n-m}(\dot{\mu}^{-1}) \nabla^m \times \mathcal{F}_m(\mathbf{E}) \rangle_{L^2(\Omega_{2D})} - \langle \mathcal{F}_n(\mathbf{F}), \mathcal{F}_{n-m}(\dot{\sigma}) \mathcal{F}_m(\mathbf{E}) \rangle_{L^2(\Omega_{2D})} \\ = \langle \mathcal{F}_n(\mathbf{F}), \mathcal{F}_n(\mathbf{J}^{imp}) \rangle_{L^2(\Omega_{2D})} - \langle \mathcal{F}_n(\mathbf{F}_t), \mathcal{F}_n(\mathbf{J}_S^{imp}) \rangle_{L^2(\Gamma_{H,1D})} \\ + \sum_{m=-\infty}^{\infty} \langle \nabla^n \times \mathcal{F}_n(\mathbf{F}), \mathcal{F}_{n-m}(\dot{\mu}^{-1}) \mathcal{F}_m(\mathbf{M}^{imp}) \rangle_{L^2(\Omega_{2D})} \quad \forall \mathcal{F}_n(\mathbf{F}) \in \mathbf{H}_{\Gamma_{E,1D}}(\text{curl}^n; \Omega_{2D}) \end{array} \right.$$

FOURIER ANALYSIS

Cartesian system of coordinates: $\mathbf{x} = (x, y, z)$.

New non-orthogonal system of coordinates: $\zeta = (\zeta_1, \zeta_2, \zeta_3)$.



Subdomain I

;

Subdomain II

;

Subdomain III

$$\left\{ \begin{array}{l} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 \end{array} \right. ; \left\{ \begin{array}{l} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 + \tan \theta_0 \frac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \end{array} \right. ; \left\{ \begin{array}{l} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 + \tan \theta_0 \zeta_1 \end{array} \right.$$

FOURIER ANALYSIS

E-Variational Formulation in the New System of Coordinates ζ

In the new system of coordinates, we obtain:

3D FOURIER FINITE ELEMENT FORMULATION
 — Sequence of “Weakly” Coupled 2D Problems —

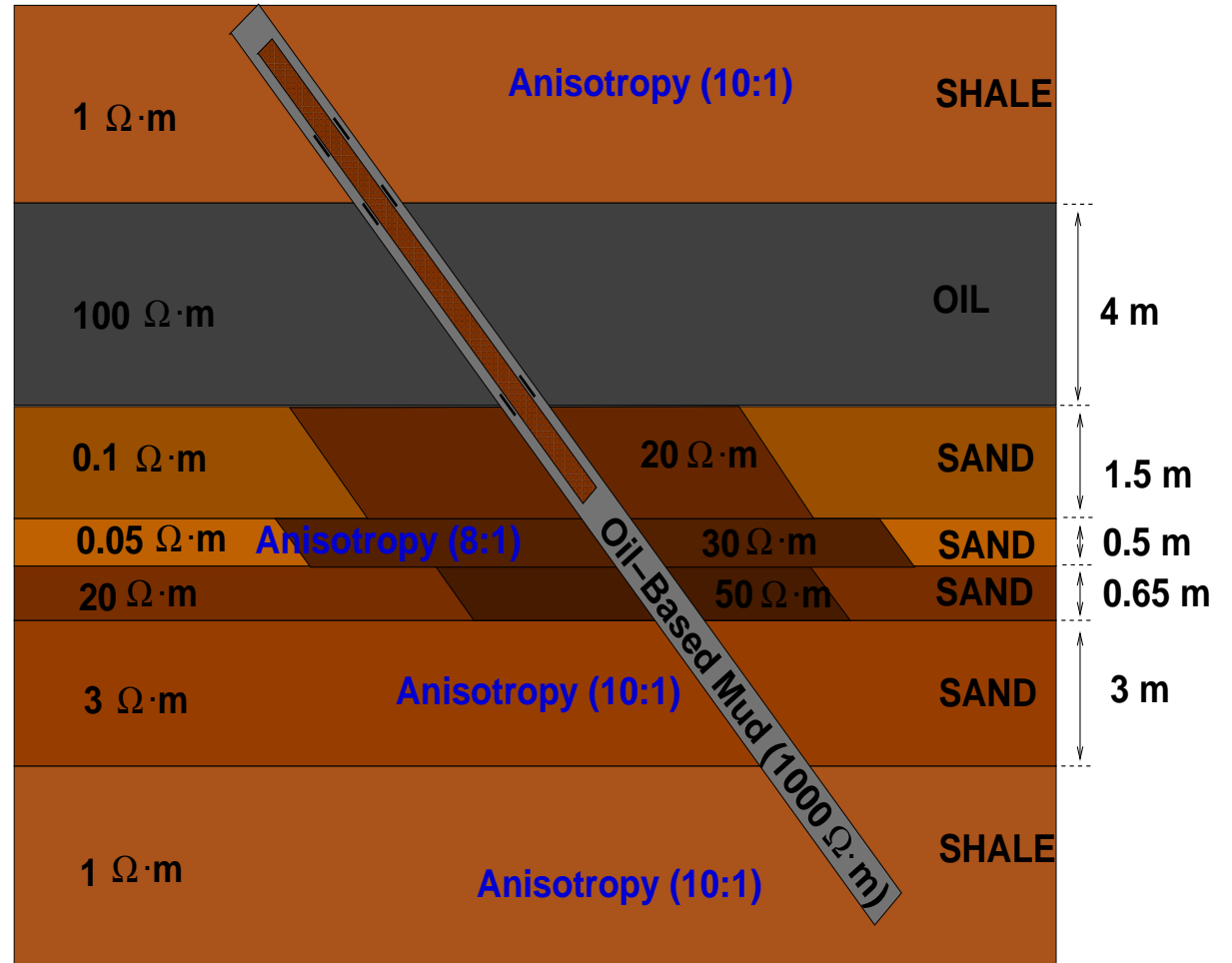
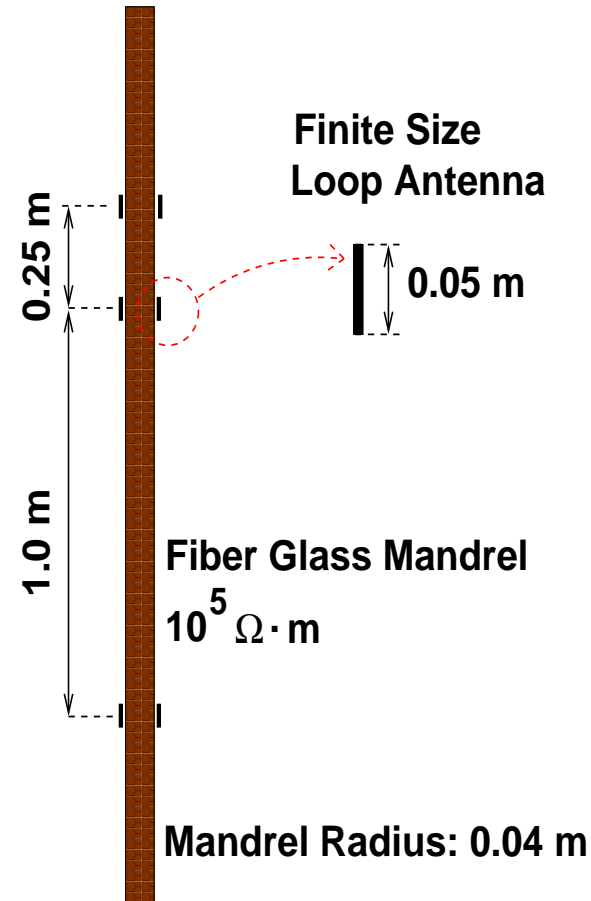
$$\left\{ \begin{array}{l} \text{Find } \mathbf{E} = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathbf{E}) e^{jn\zeta_2}, \text{ where for each } n: \\ \mathcal{F}_n(\mathbf{E}) \in \mathcal{F}_n(\mathbf{E}_{\Gamma_{E,1D}}) + H_{\Gamma_{E,1D}}(\text{curl}^n; \Omega_{2D}), \text{ and} \\ \sum_{m=-2}^2 \langle \nabla^n \times \mathcal{F}_n(\mathbf{F}), \mathcal{F}_{n-m}(\dot{\mu}_{mod}^{-1}) \nabla^m \times \mathcal{F}_m(\mathbf{E}) \rangle_{L^2(\Omega_{2D})} - \langle \mathcal{F}_n(\mathbf{F}), \mathcal{F}_{n-m}(\dot{\sigma}_{mod}) \mathcal{F}_m(\mathbf{E}) \rangle_{L^2(\Omega_{2D})} \\ = \langle \mathcal{F}_n(\mathbf{F}), \mathcal{F}_n(\mathbf{J}^{imp}) \rangle_{L^2(\Omega_{2D})} - \langle \mathcal{F}_n(\mathbf{F}_t), \mathcal{F}_n(\mathbf{J}_S^{imp}) \rangle_{L^2(\Gamma_{H,1D})} \\ + \sum_{m=-2}^2 \langle \nabla^n \times \mathcal{F}_n(\mathbf{F}), \mathcal{F}_{n-m}(\dot{\mu}_{mod}^{-1}) \mathcal{F}_m(\mathbf{M}^{imp}) \rangle_{L^2(\Omega_{2D})} \quad \forall \mathcal{F}_n(\mathbf{F}) \in H_{\Gamma_{E,1D}}(\text{curl}^n; \Omega_{2D}) \end{array} \right.$$

Five Fourier modes are sufficient to represent EXACTLY the new material coefficients resulting from incorporating the change of coordinates.

FOURIER-FINITE-ELEMENT: ILLUSTRATION

Model Problem

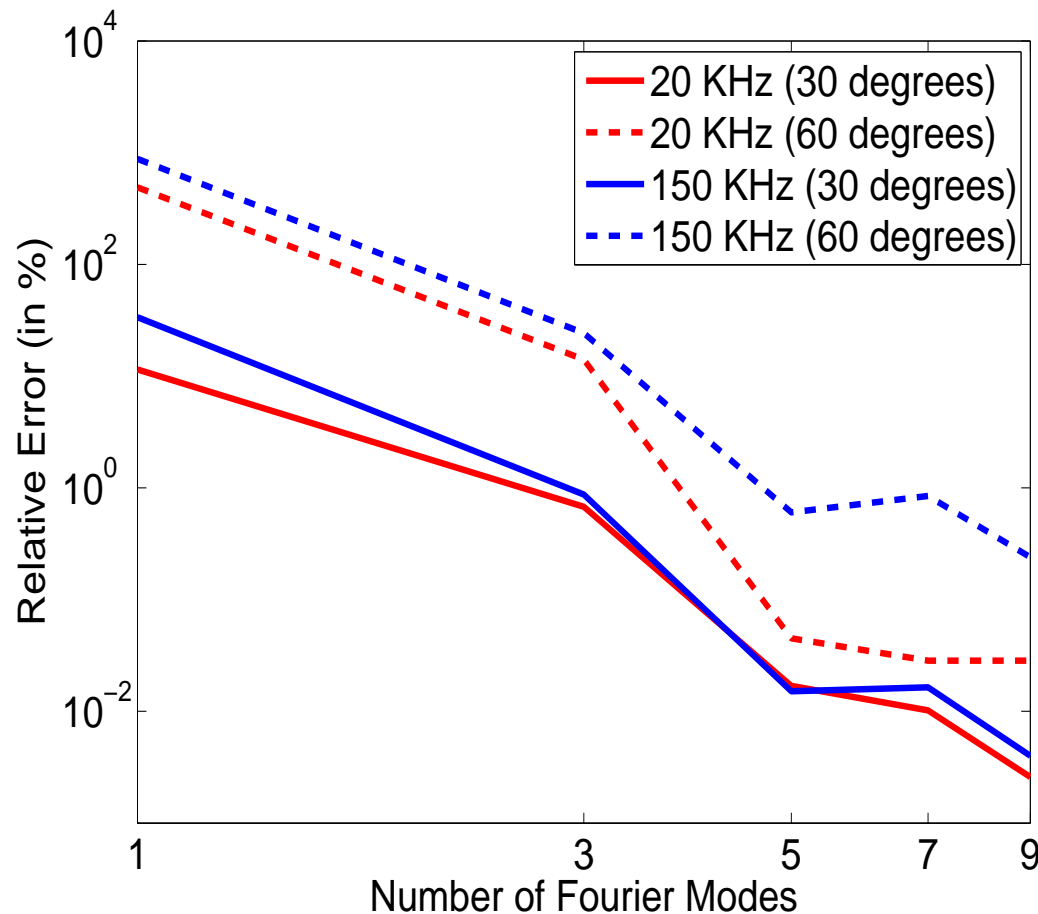
150 kHz (Wireline)



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

Logging Instrument in a Homogeneous Formation

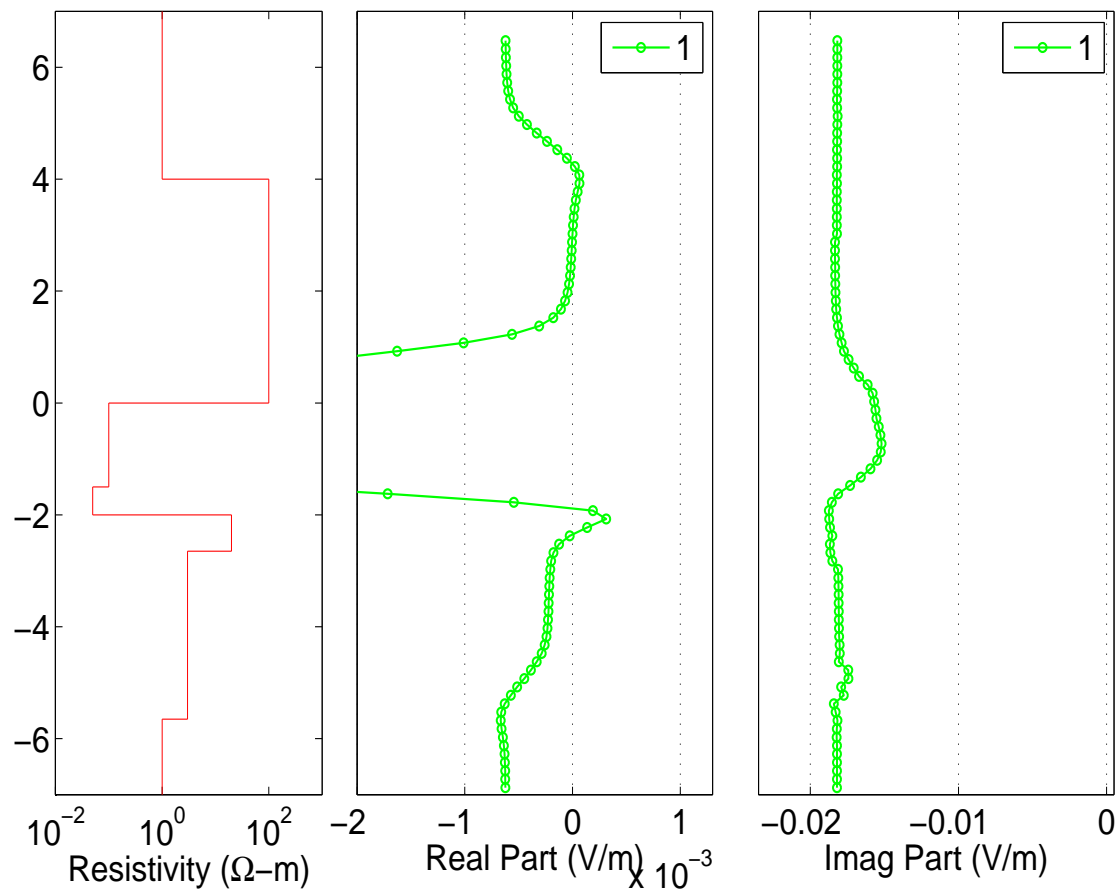


FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

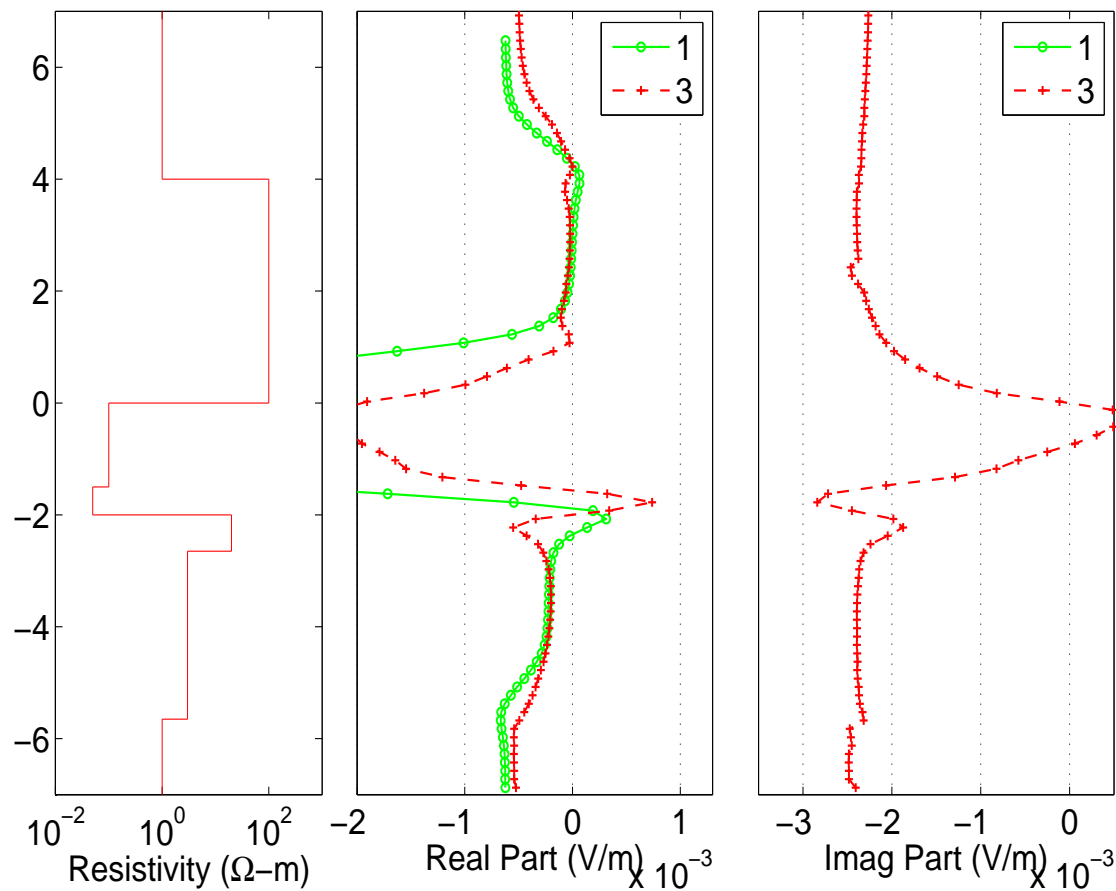


FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

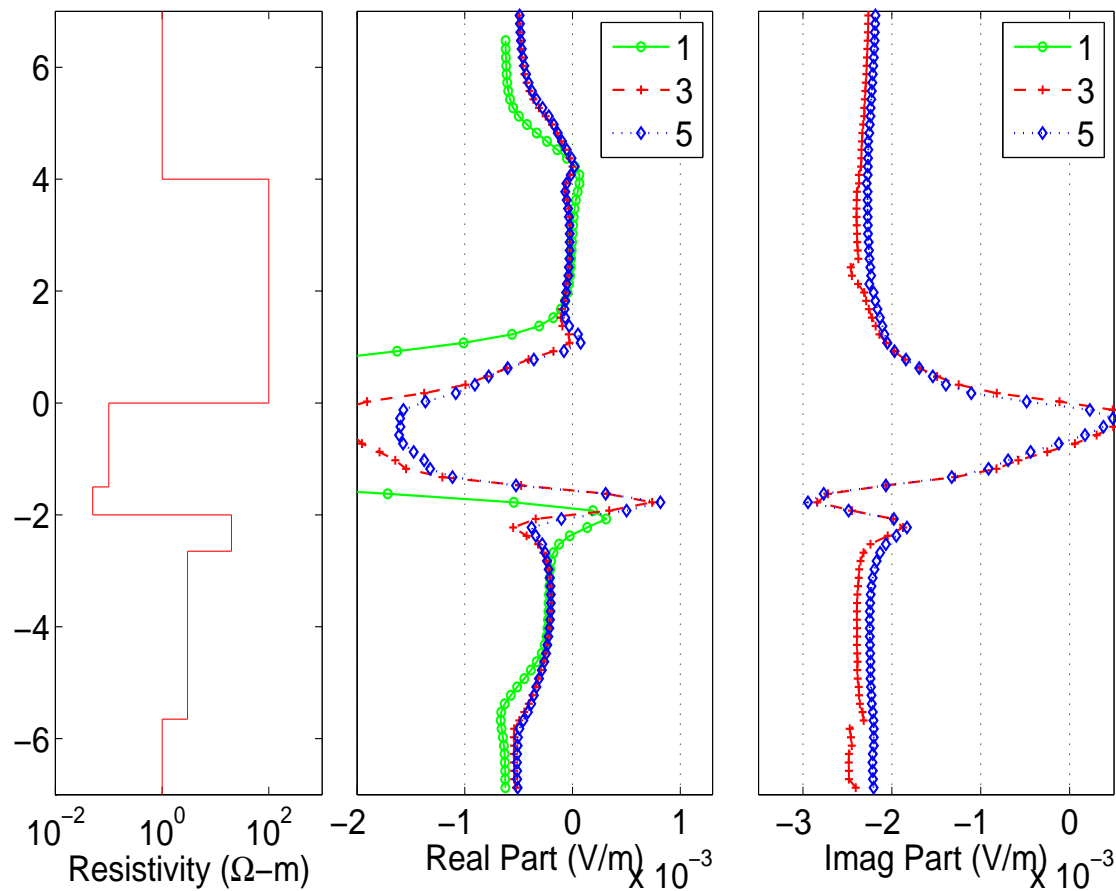


FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

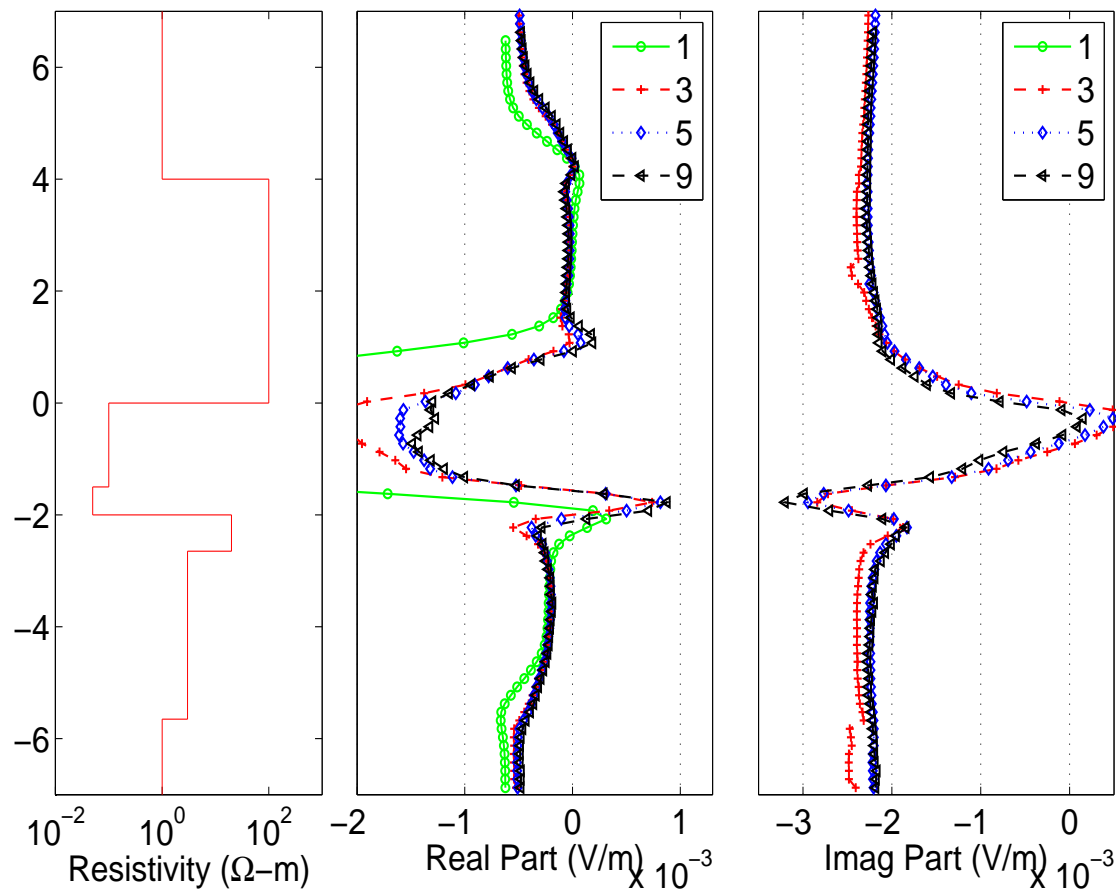


FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

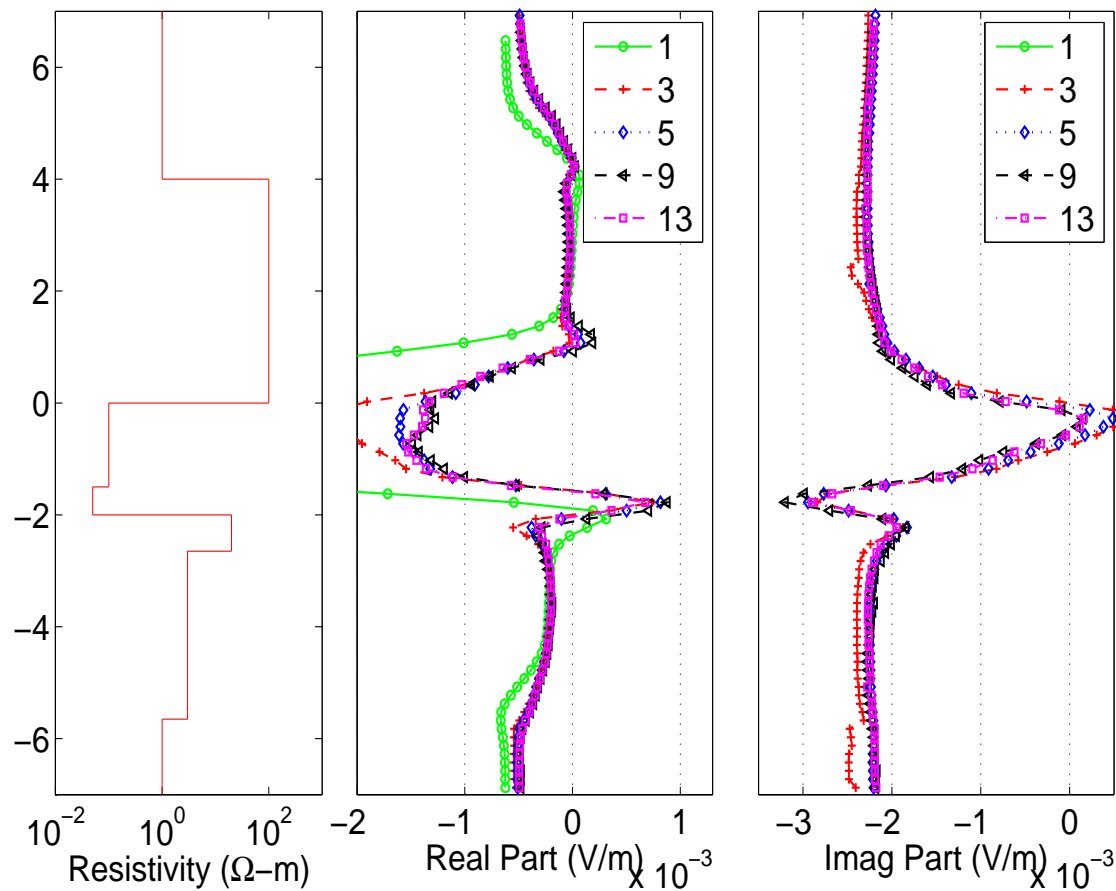


FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

Logging Instrument in a Homogeneous Formation

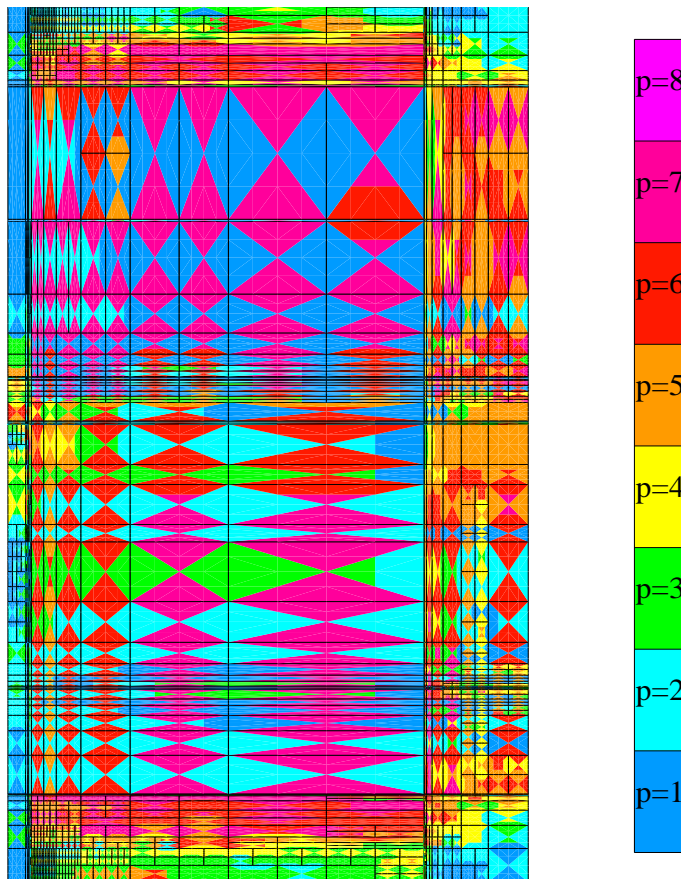
Wireline, 150 Khz



SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

A Self-Adaptive Goal-Oriented hp -FEM

Optimal 2D Grid
(Through Casing Resistivity Problem)



We vary locally the element size h and the polynomial order of approximation p throughout the grid.

Optimal grids are **automatically generated** by the computer.

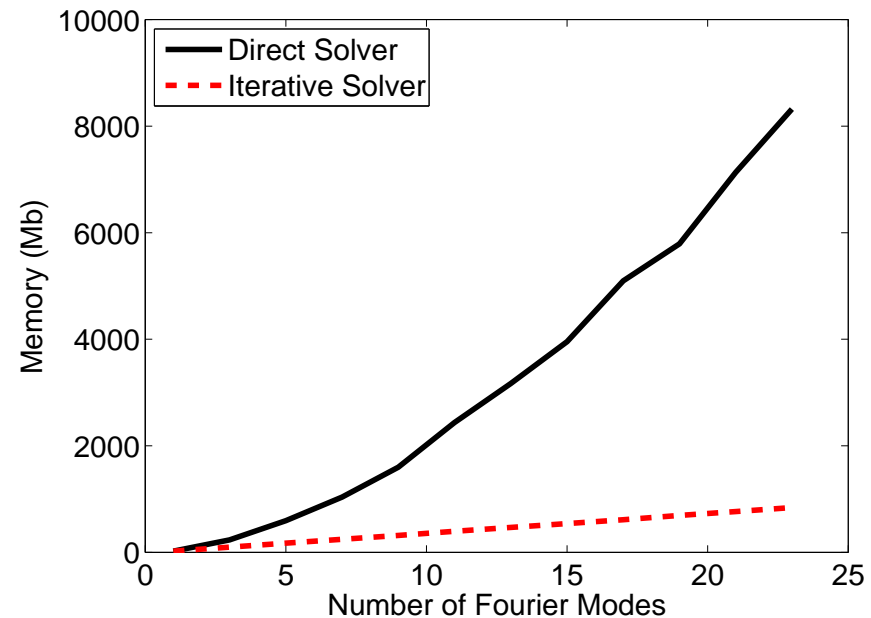
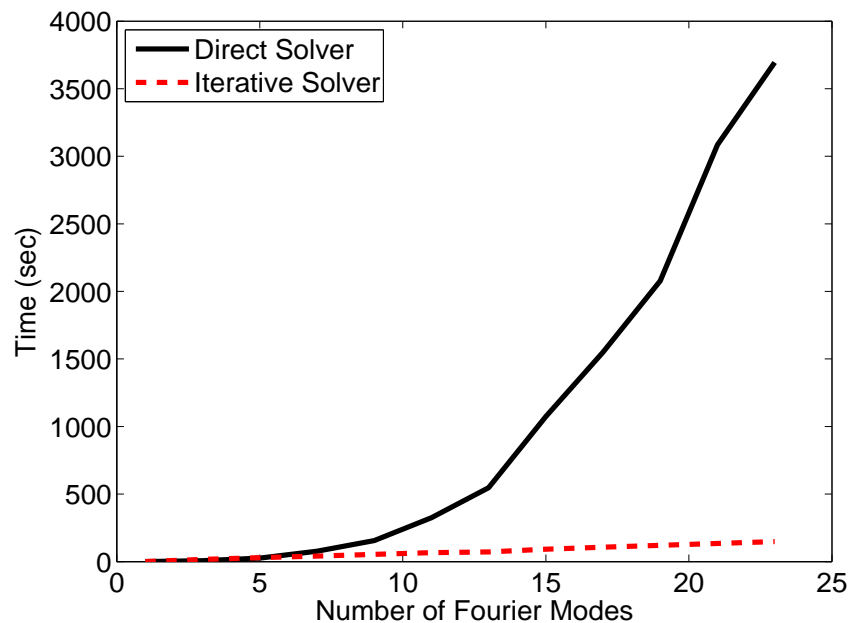
The self-adaptive goal-oriented hp -FEM provides **exponential convergence** rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.

ITERATIVE SOLVER

Description and Performance of the Iterative Solver

Block-Jacobi Preconditioner + Krylov subspace optimization method (CG or GMRES).

The block Jacobi preconditioner consists of a 2.5D problem defined by ignoring the couplings occurring between the different Fourier basis functions in the original problem.

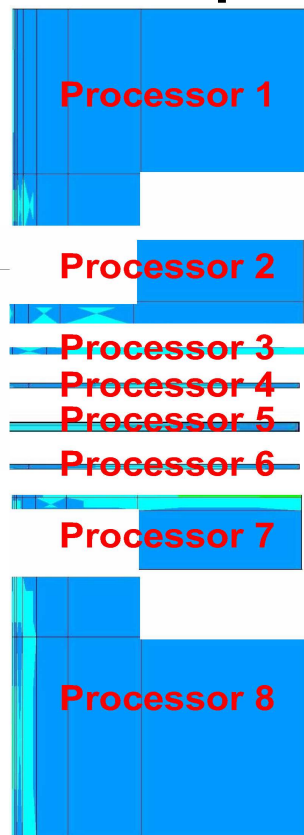


This simple iterative solver enables fast computations.

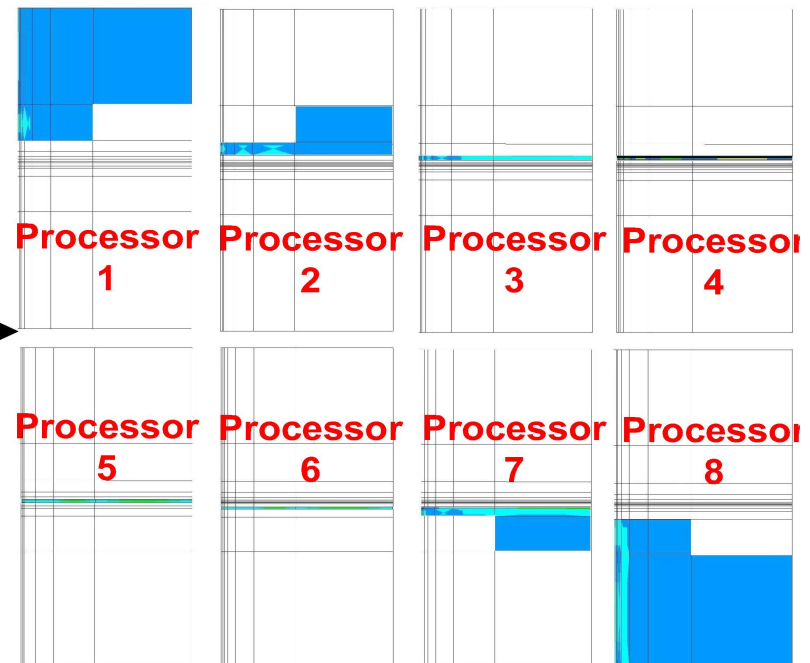
PARALLEL IMPLEMENTATION

We Use Shared Domain Decomposition

Distributed Domain Decomposition



Shared Domain Decomposition

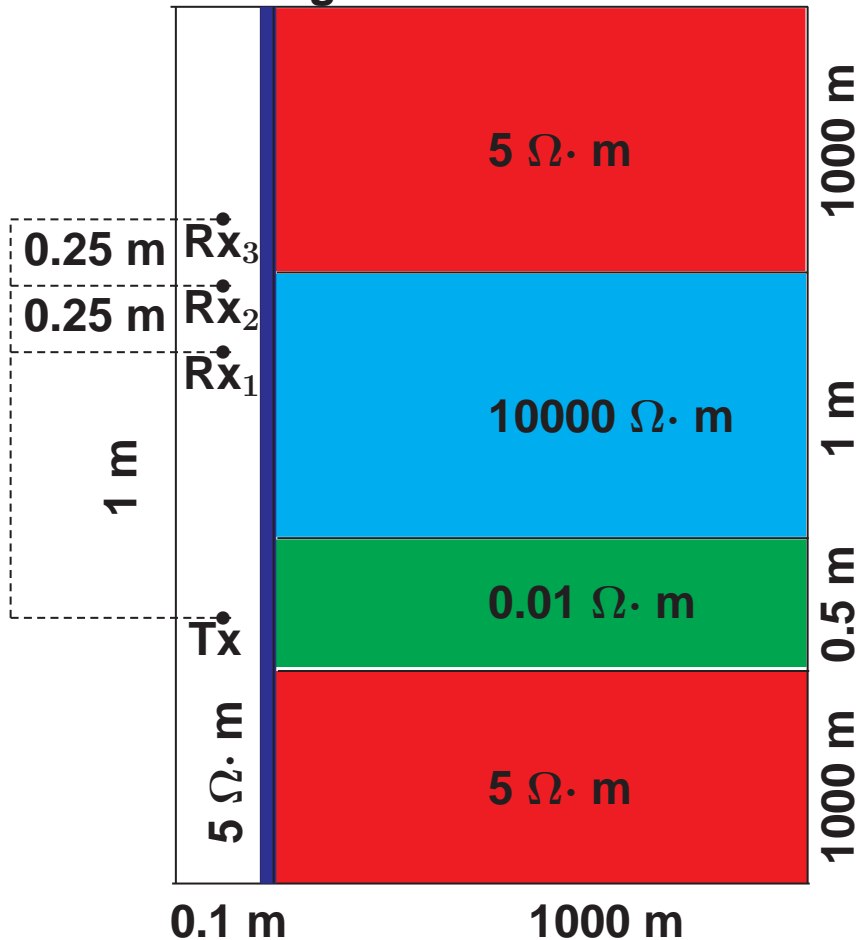


NUMERICAL RESULTS: DC RESULTS

Simulation of Through Casing Resistivity Measurements

Casing resistivity: $10^{-5} - 10^{-7} \Omega \cdot m$

Casing thickness: 0.0127 m



Left Figure:

Axial-symmetric model

One current electrode (emitter)

Three voltage electrodes (collectors)

Objective:

Compute second diff. of potential for various depth angles and possibly with water invasion

Method of solution:

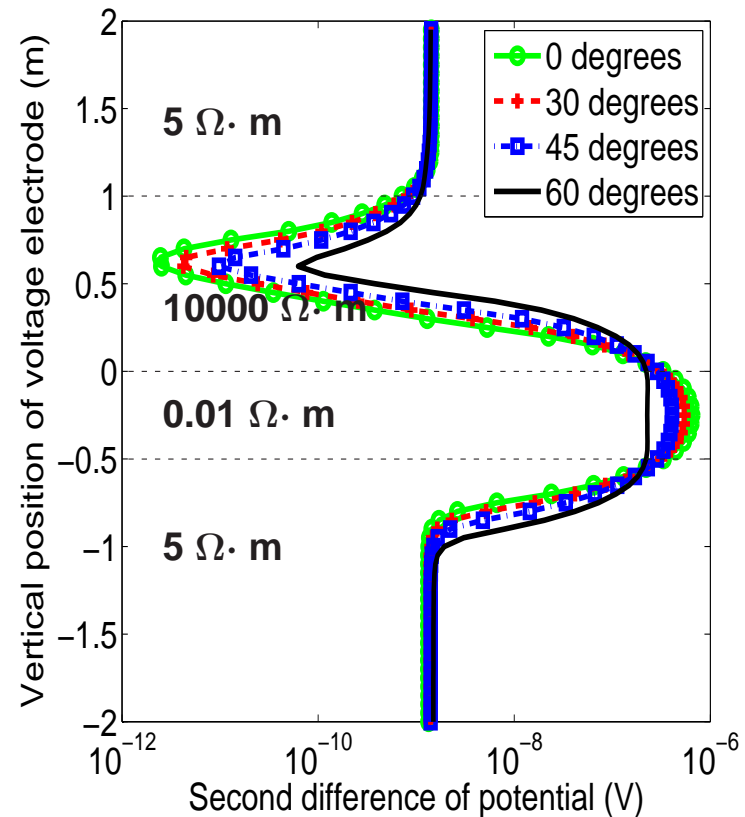
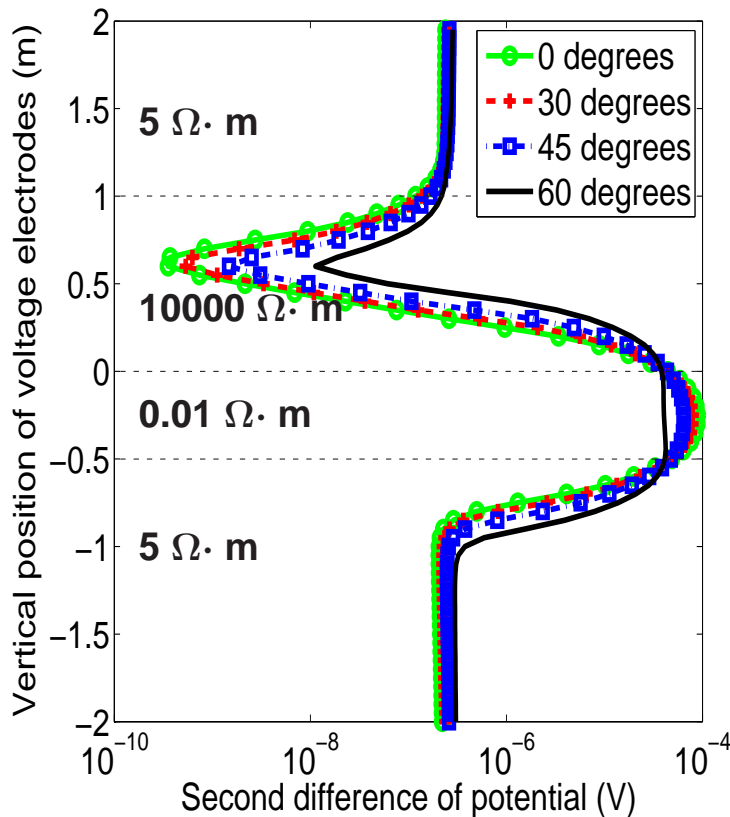
Fourier series expansion + change of coordinates + 2D goal-oriented hp-FEM

NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Casing Conductivity)

Casing Resistivity = $10^{-5} \Omega \cdot m$

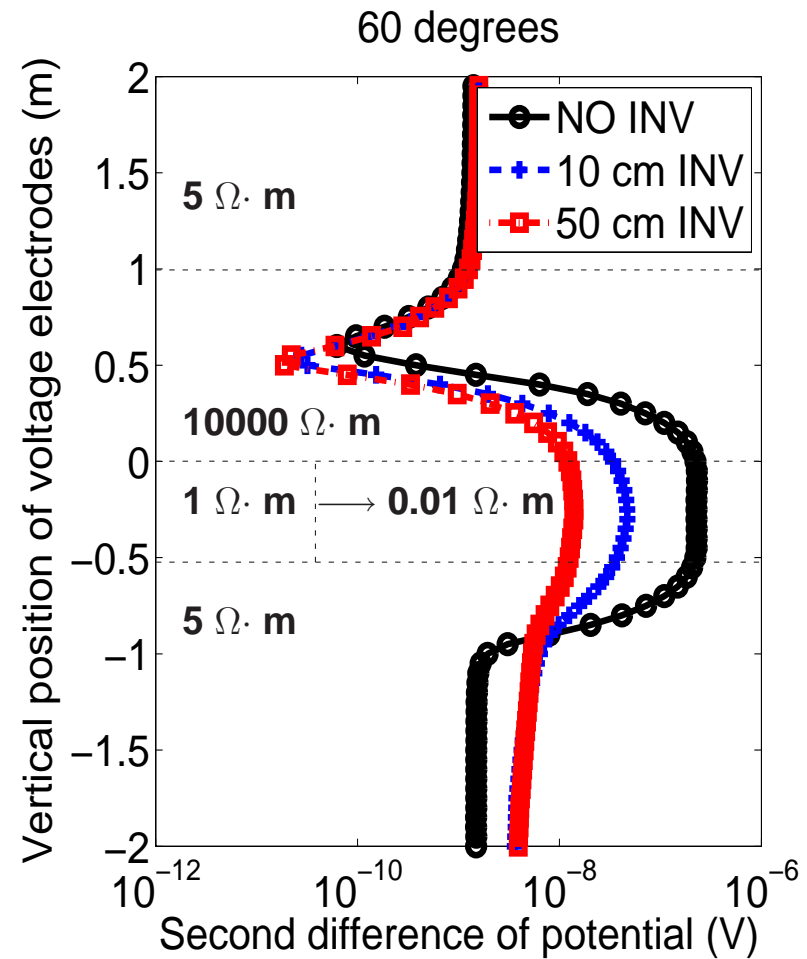
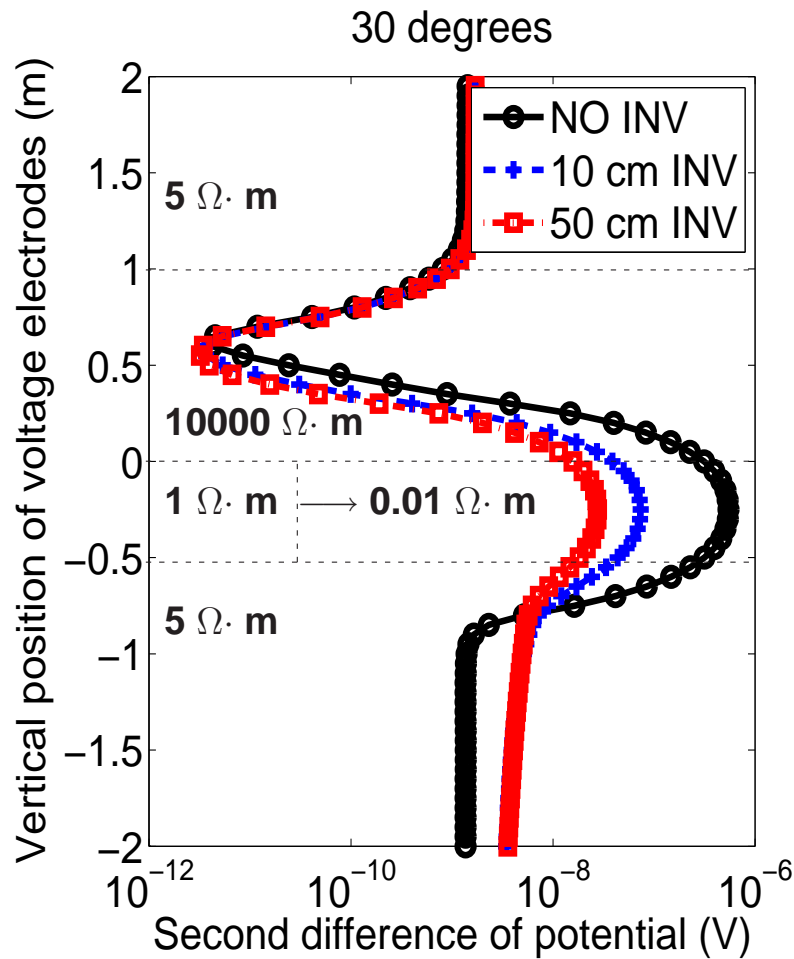
Casing Resistivity = $2.3 \times 10^{-7} \Omega \cdot m$



Qualitatively, results for various casing conductivities are similar even for deviated wells.

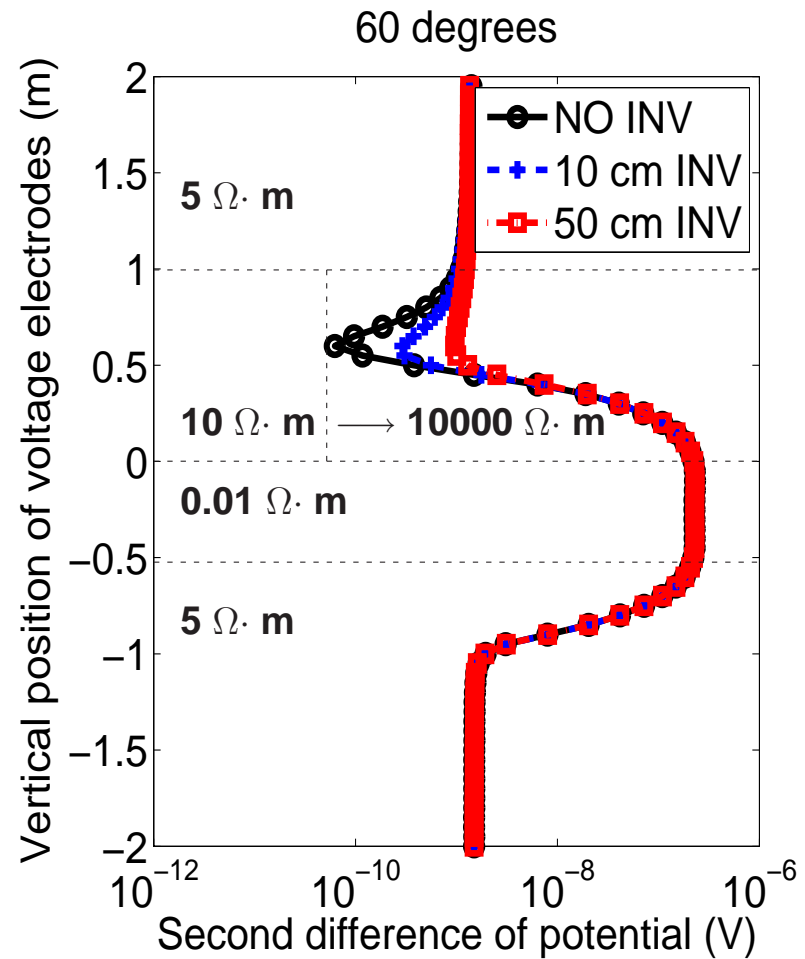
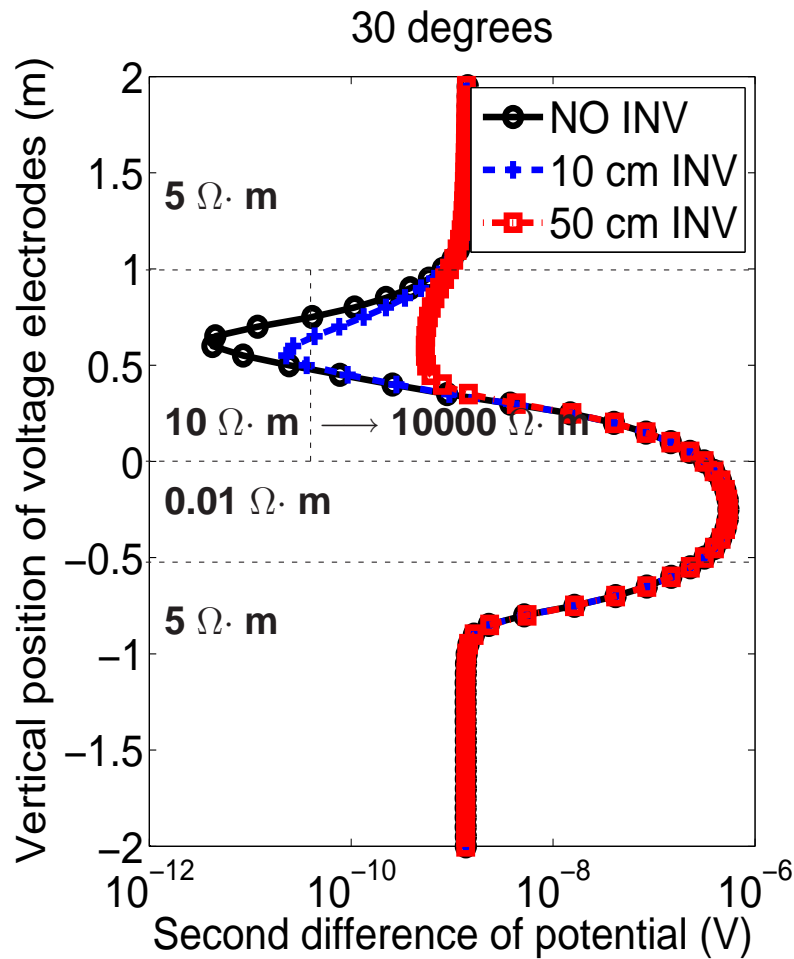
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)

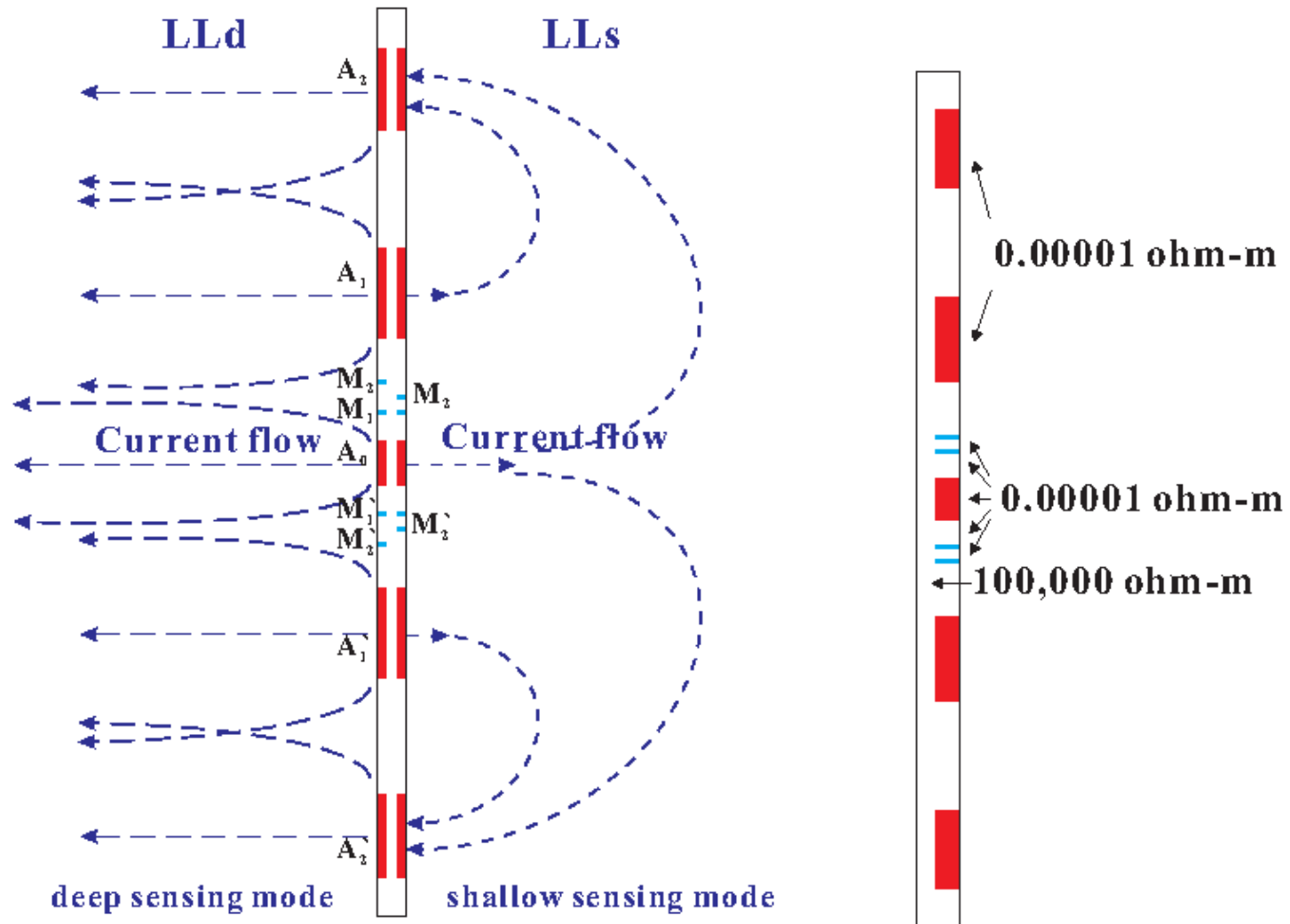


NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)

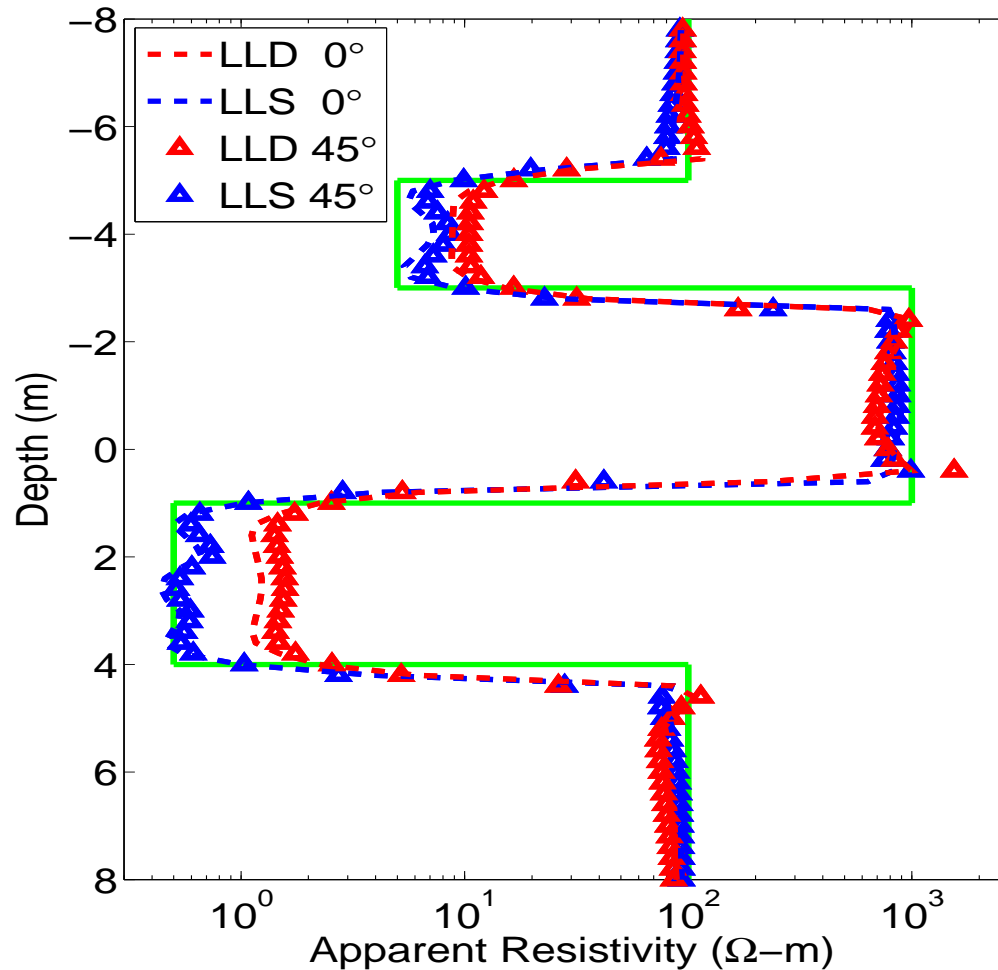


NUMERICAL RESULTS: DC LATEROLOG



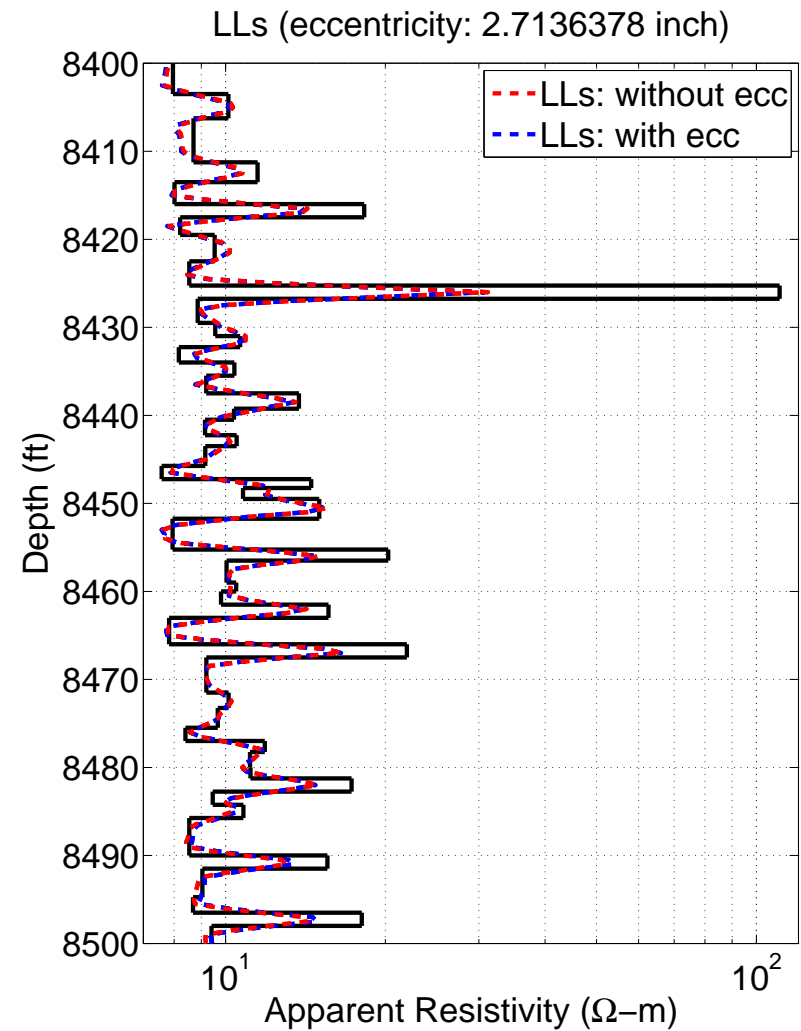
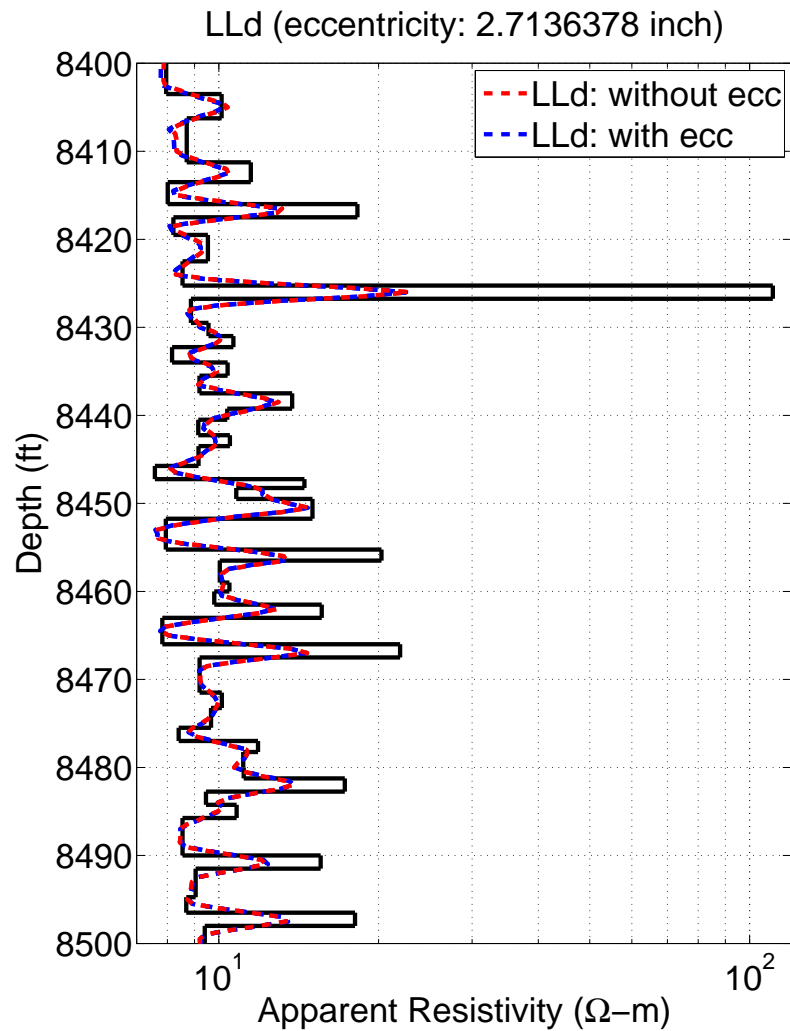
NUMERICAL RESULTS: DC LATEROLOG

Laterolog Measurements



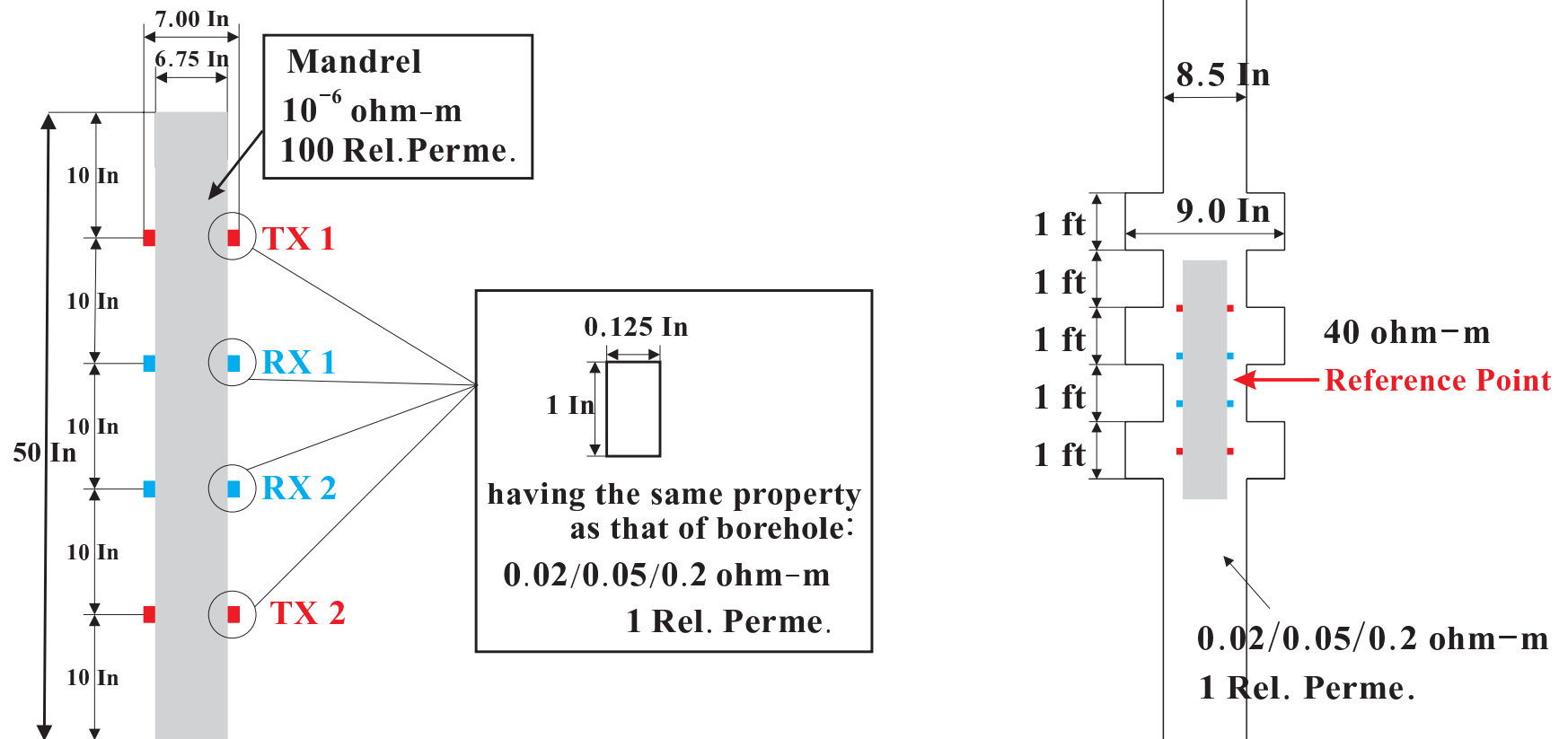
NUMERICAL RESULTS: DC LATEROLOG

ECCENTRICITY STUDY



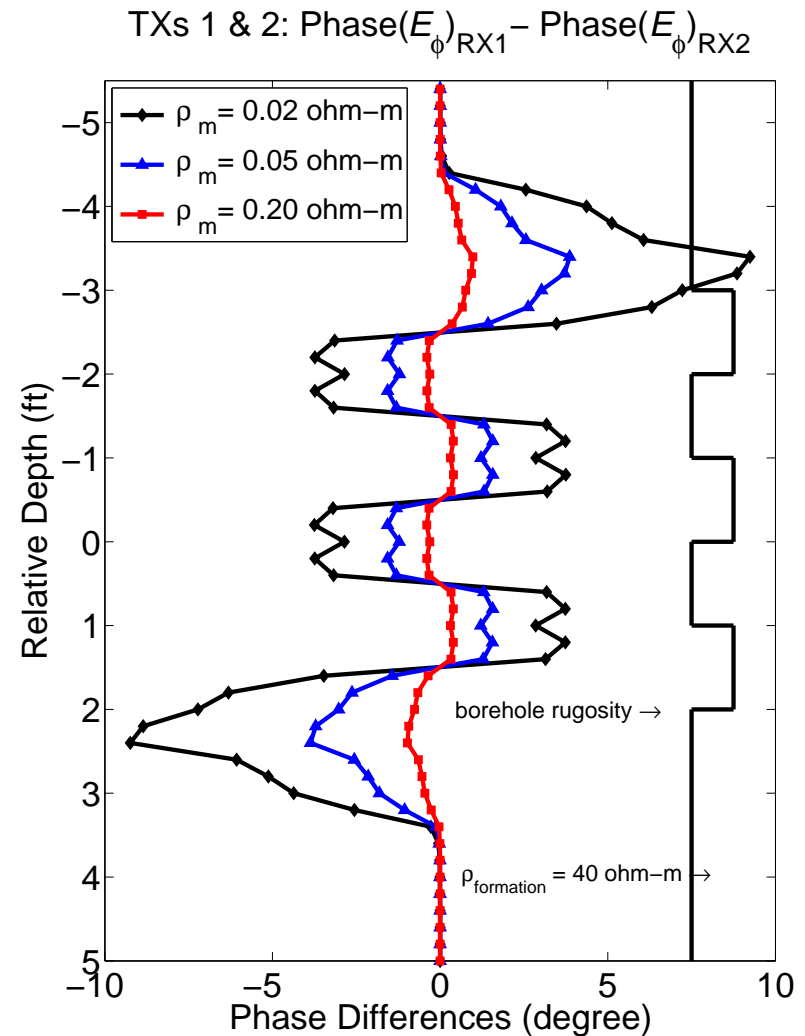
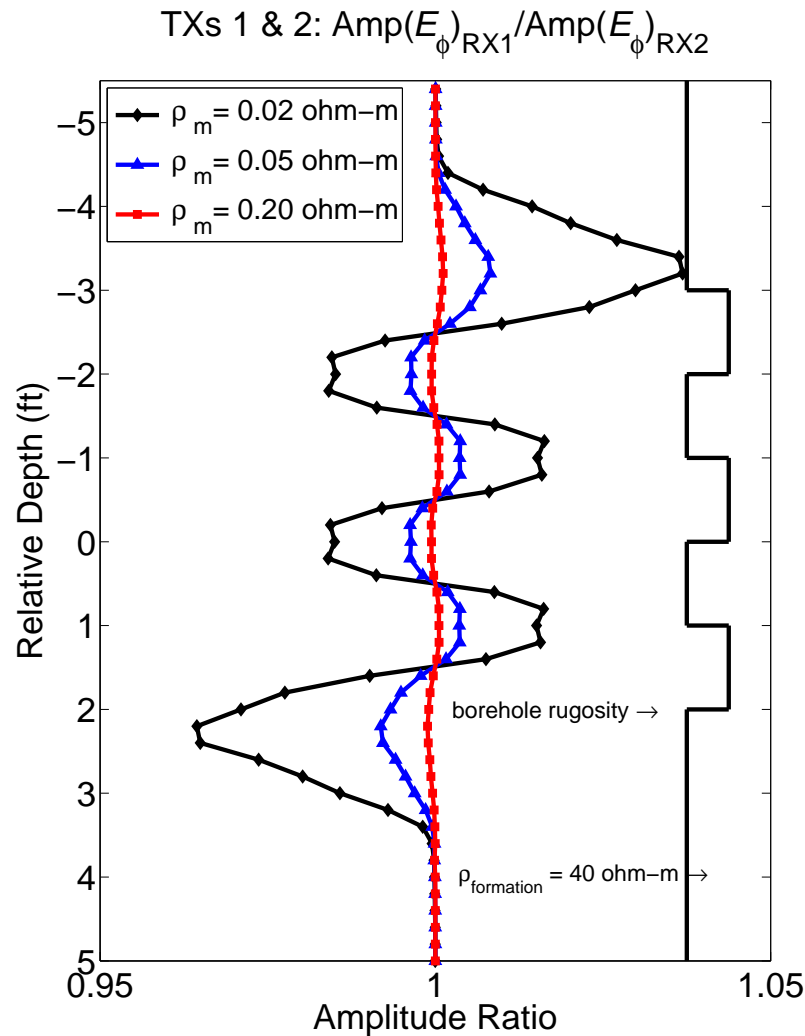
NUMERICAL RESULTS: INDUCTION

RUGOSITY STUDY: MODEL PROBLEM AT 2 MHz



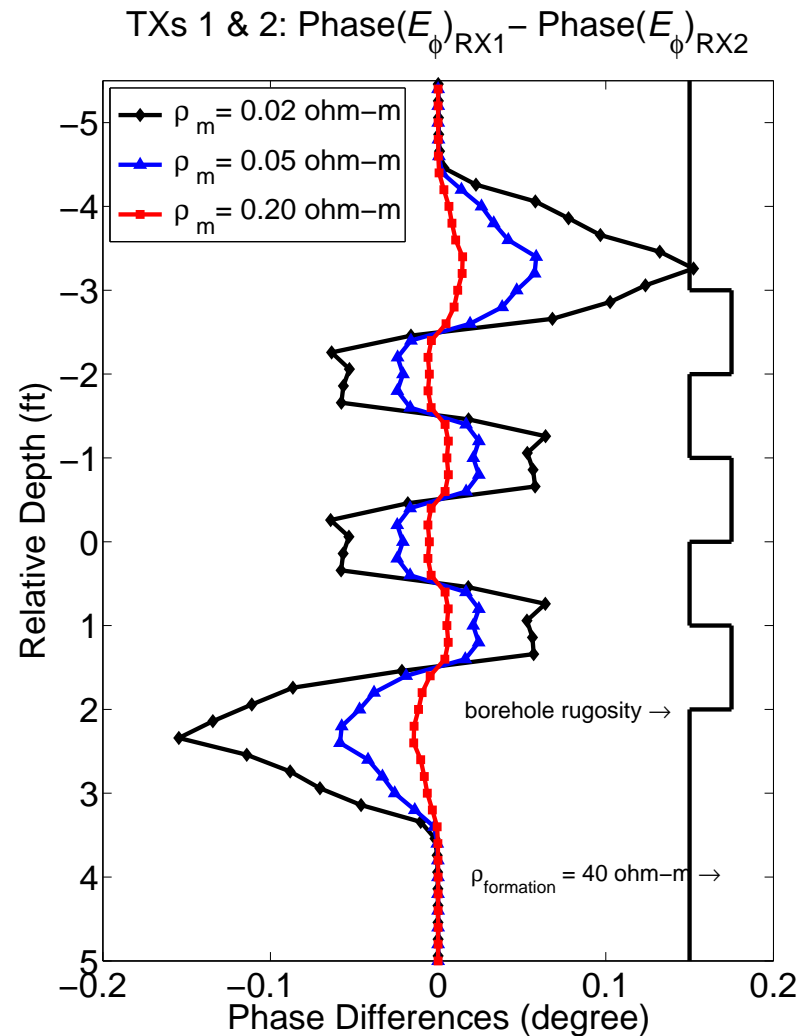
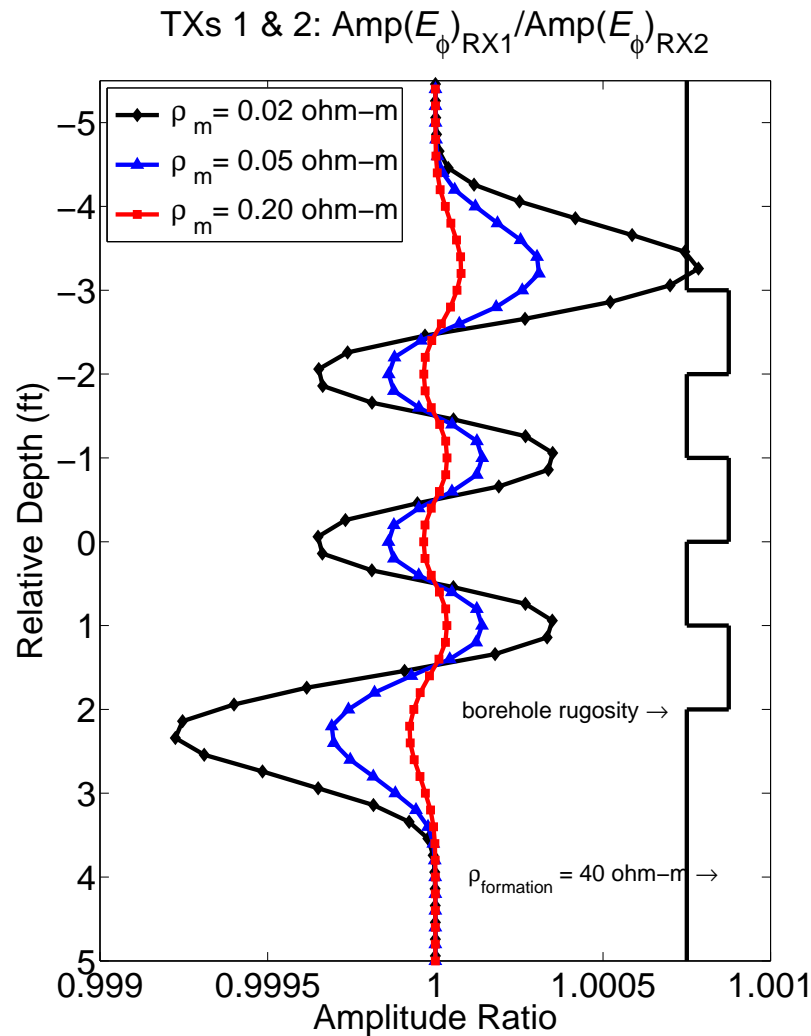
NUMERICAL RESULTS: INDUCTION

RUGOSITY STUDY (2 MHz)



NUMERICAL RESULTS: INDUCTION

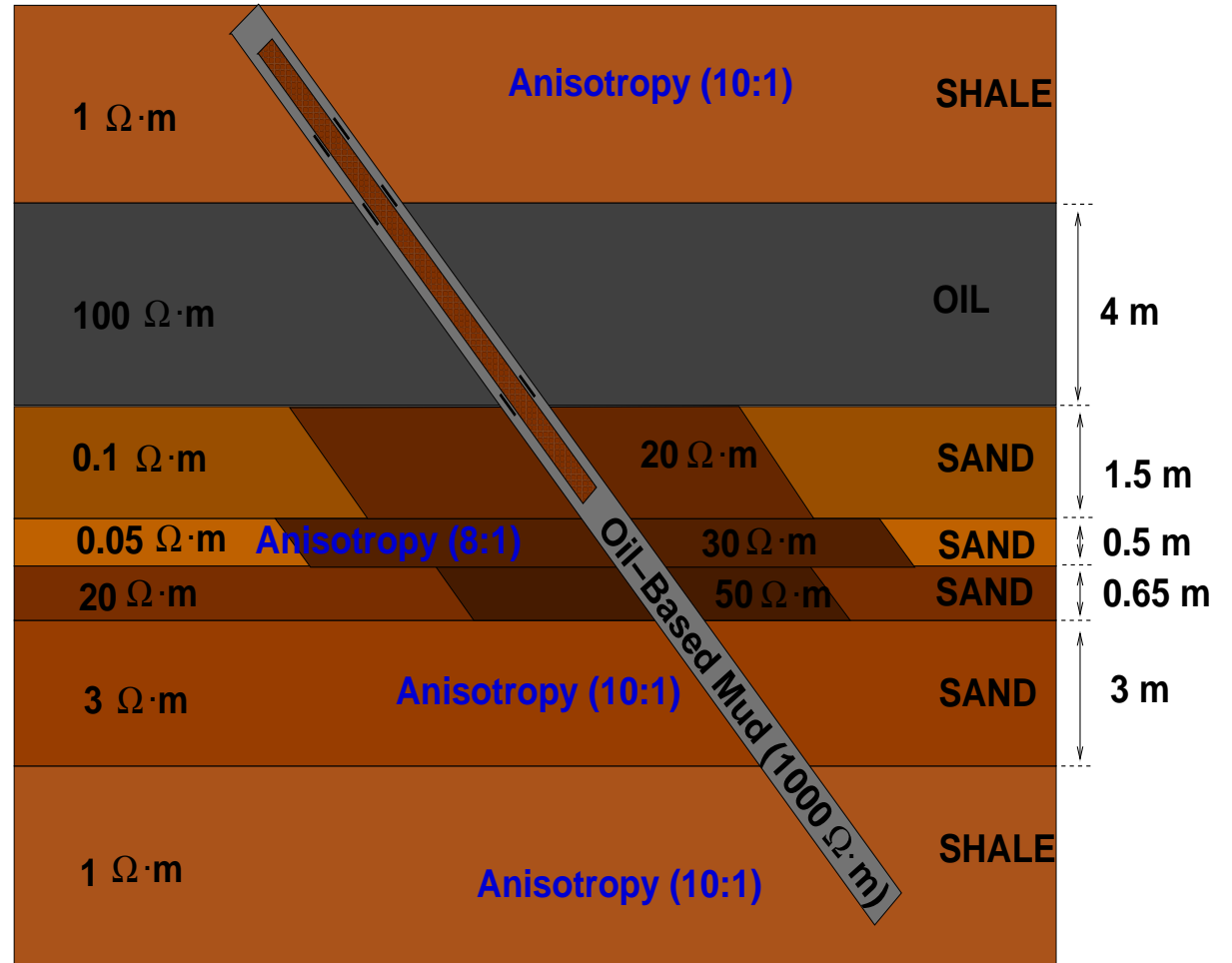
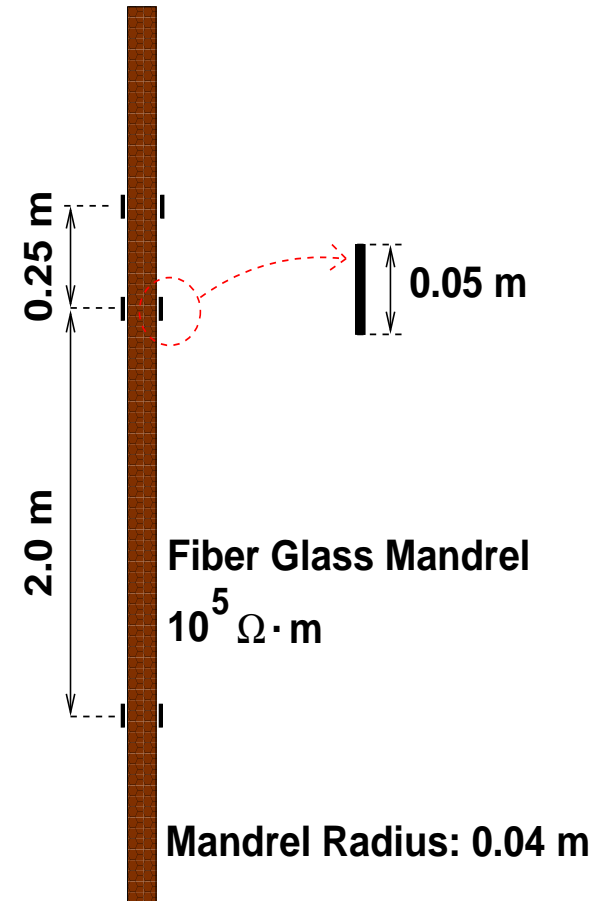
RUGOSITY STUDY (20 kHz)



NUMERICAL RESULTS: INDUCTION

Model Problem

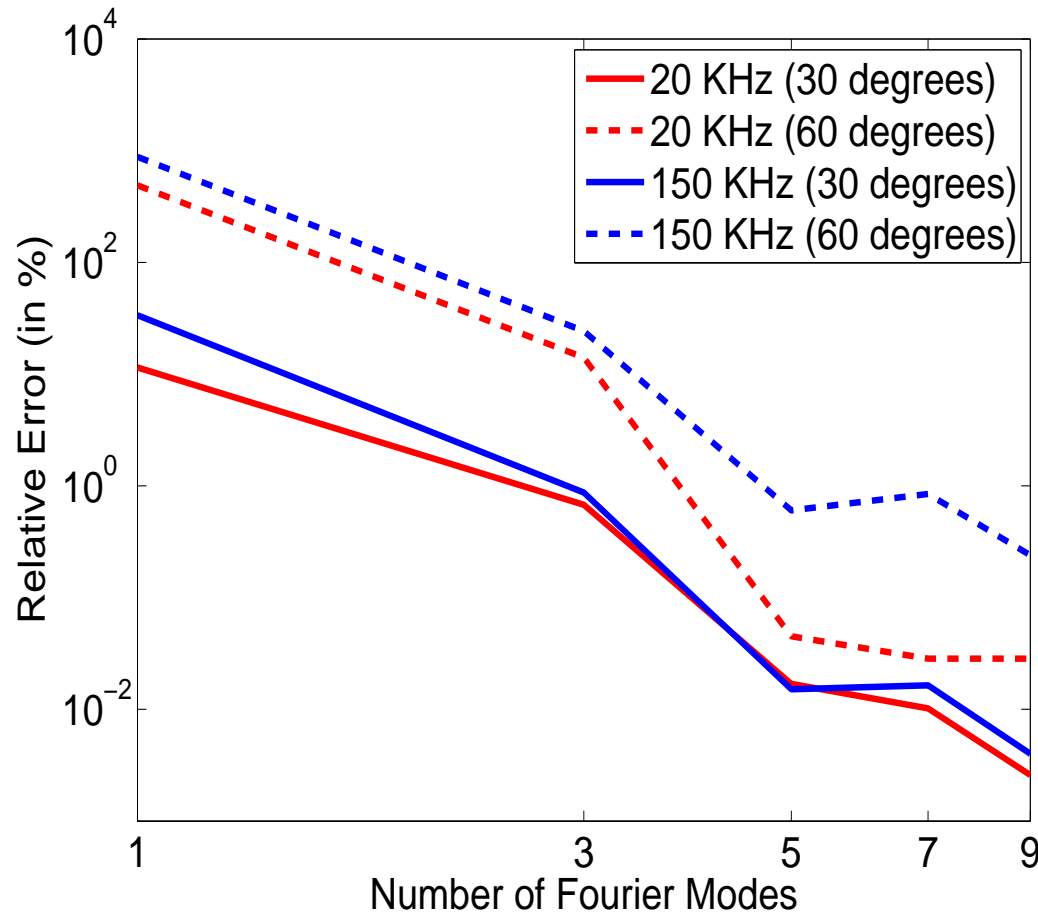
20 kHz (Wireline)



NUMERICAL RESULTS: INDUCTION

Verification

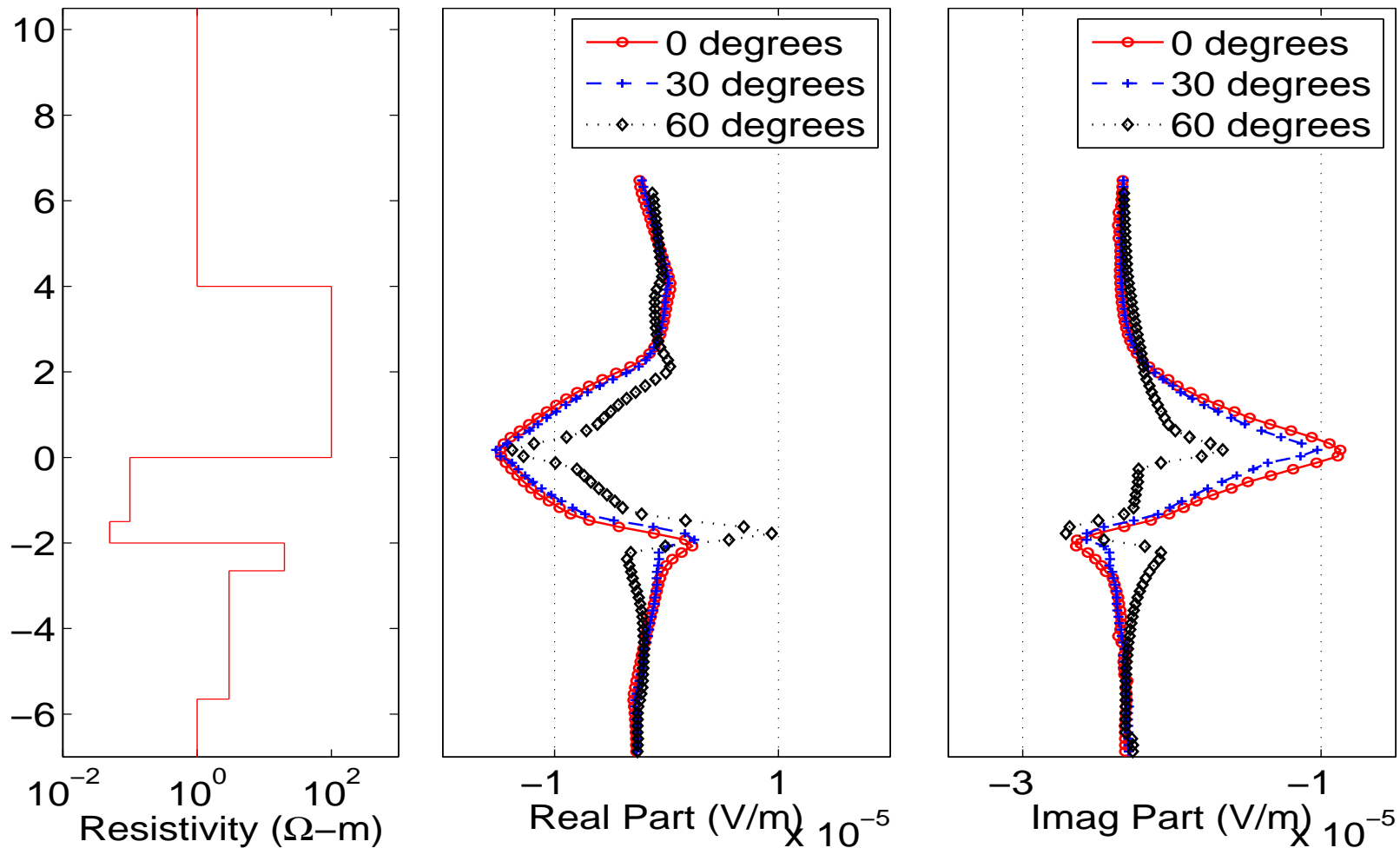
Logging Instrument in a Homogeneous Formation



NUMERICAL RESULTS: INDUCTION

Dip Angle

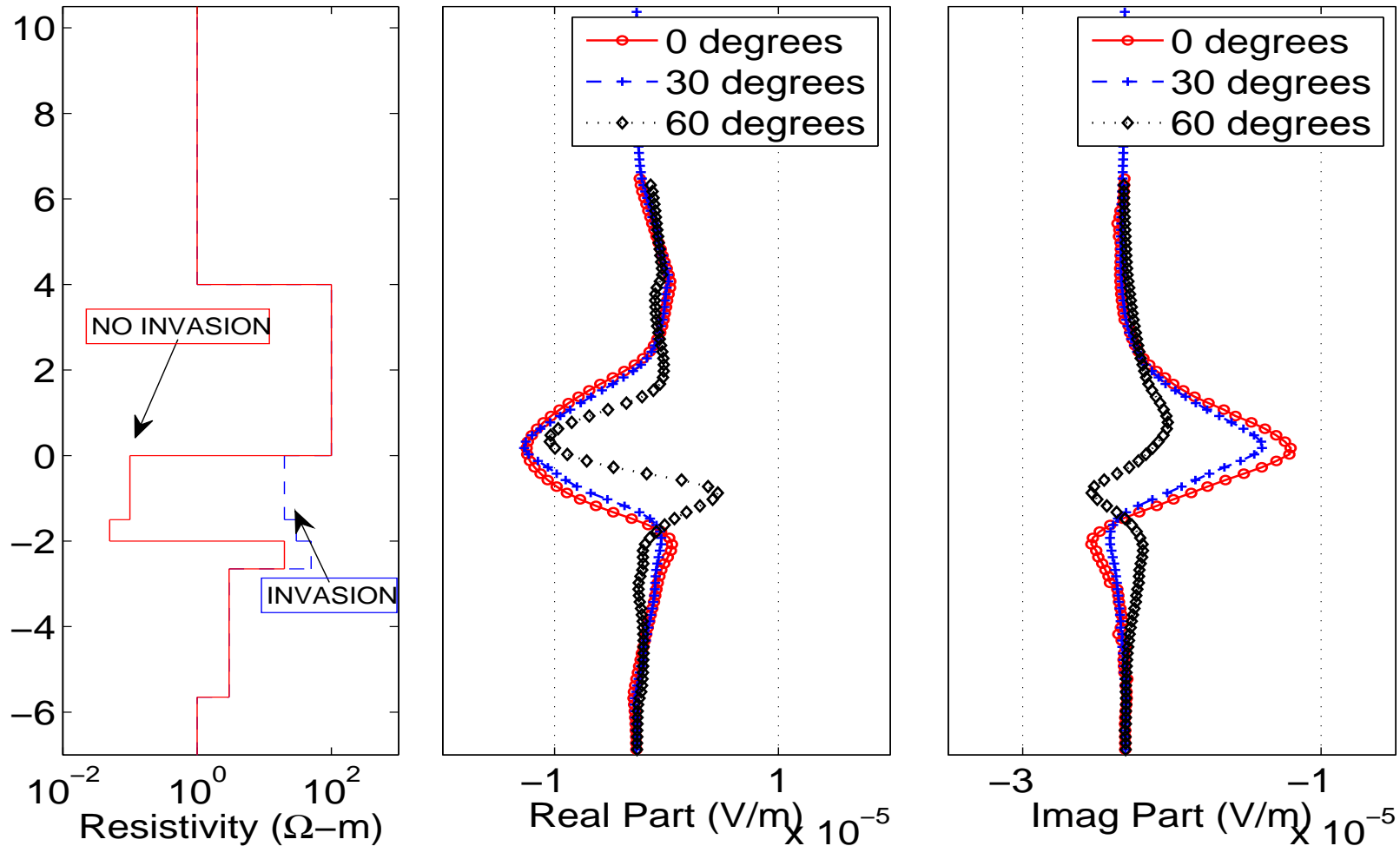
Wireline, 20 Khz.



NUMERICAL RESULTS: INDUCTION

Dip Angle + Invasion

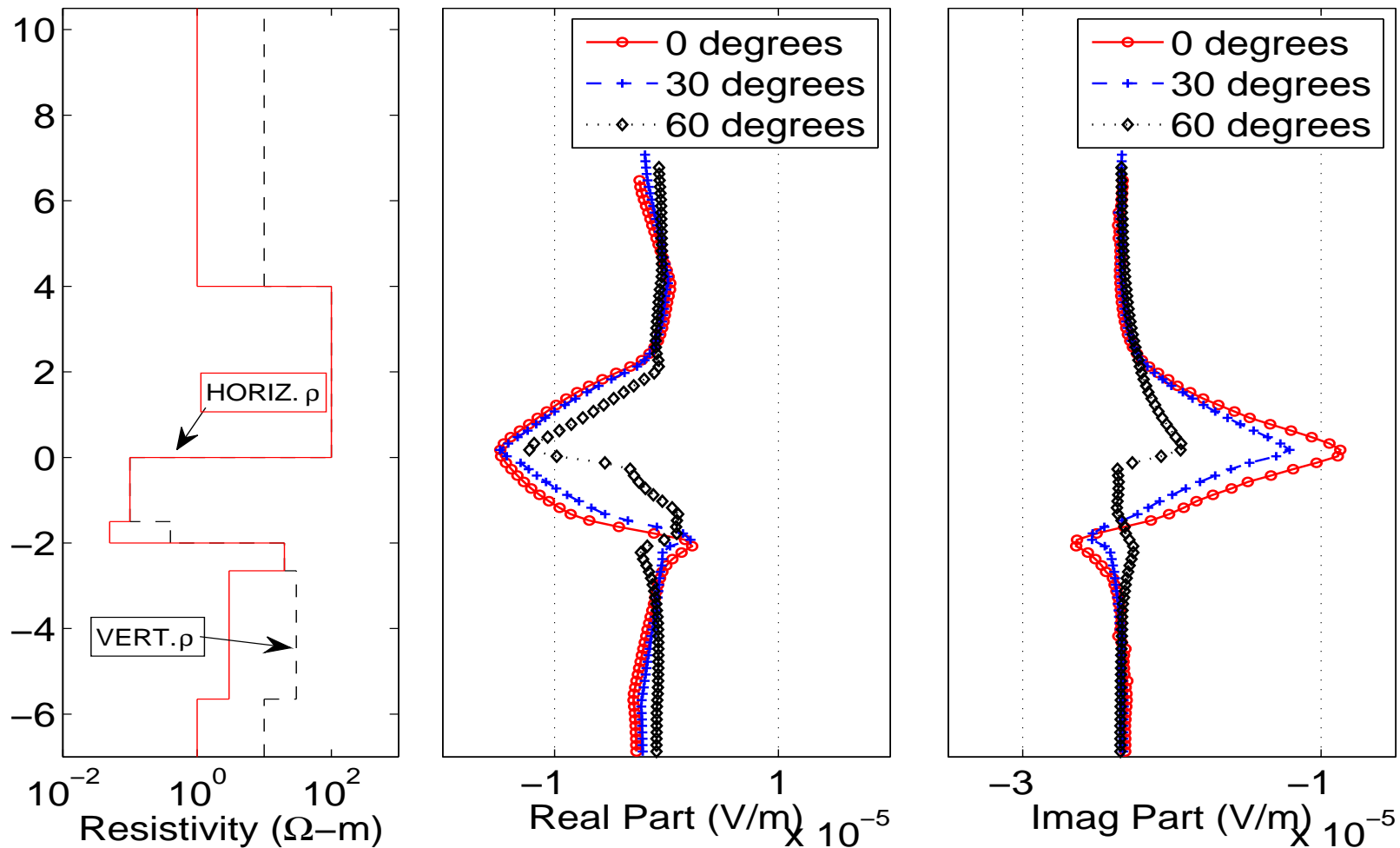
Wireline, 20 Khz.



NUMERICAL RESULTS: INDUCTION

Dip Angle + Anisotropy

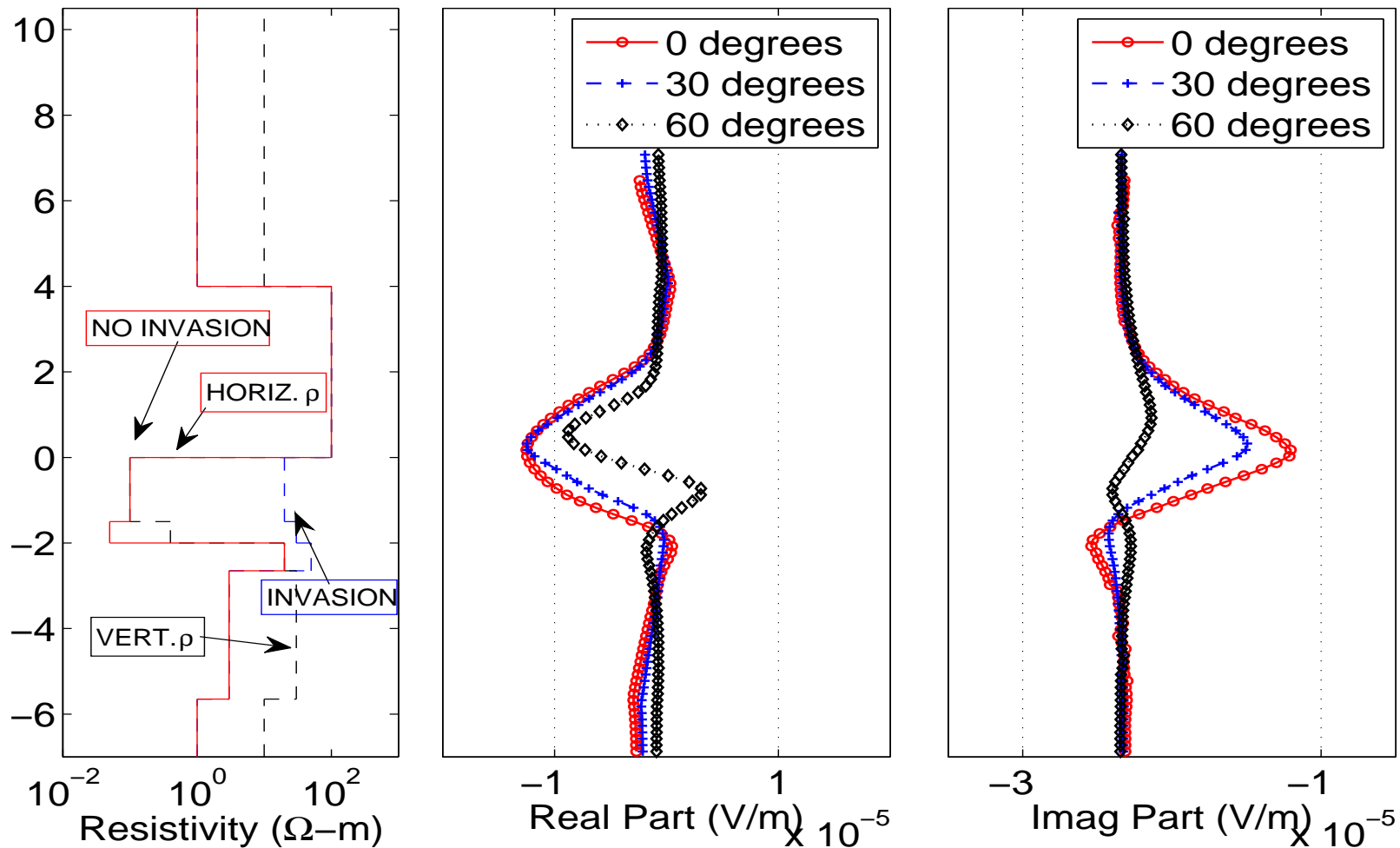
Wireline, 20 Khz.



NUMERICAL RESULTS: INDUCTION

Dip Angle + Invasion + Anisotropy

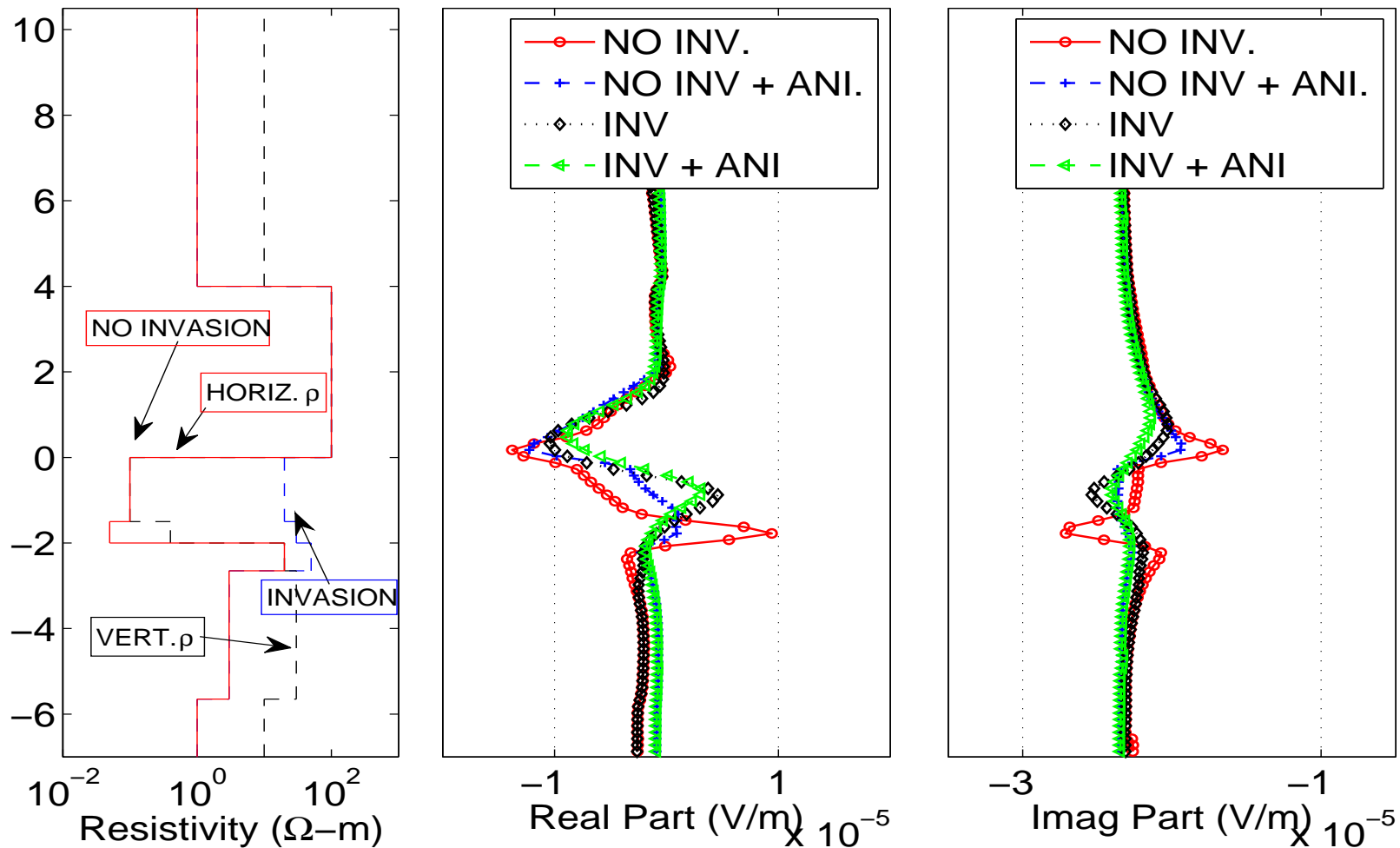
Wireline, 20 Khz.



NUMERICAL RESULTS: INDUCTION

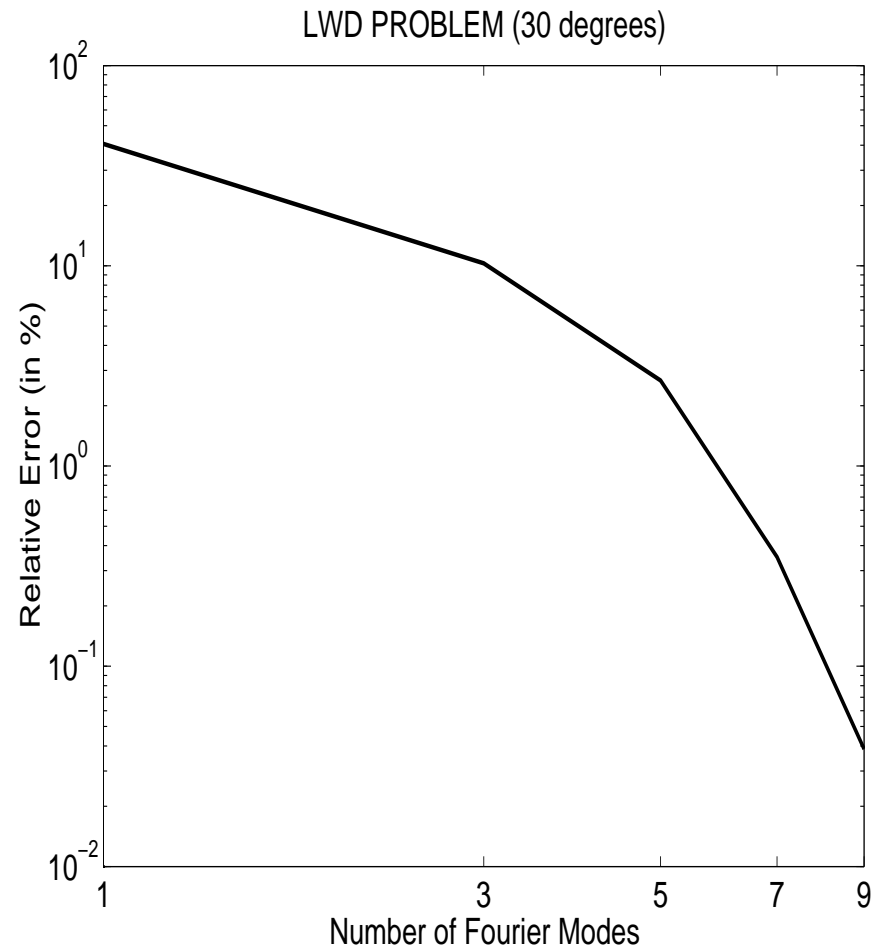
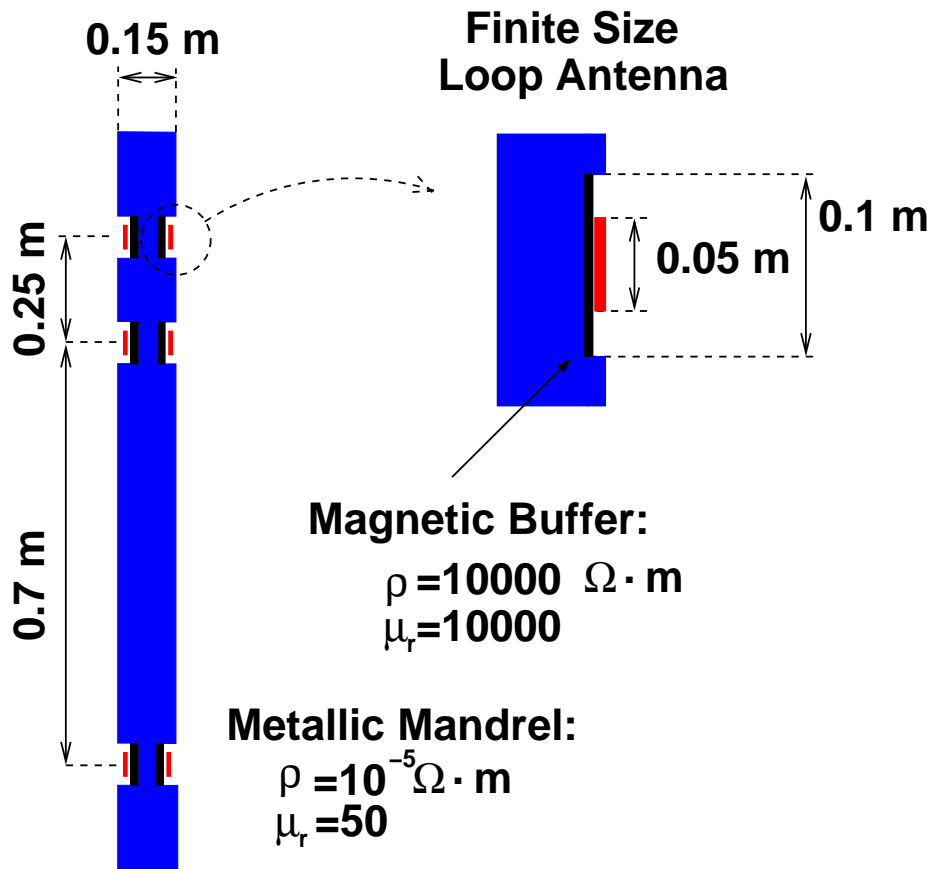
60-Degree Deviated Well

Wireline, 20 Khz.



NUMERICAL RESULTS: INDUCTION

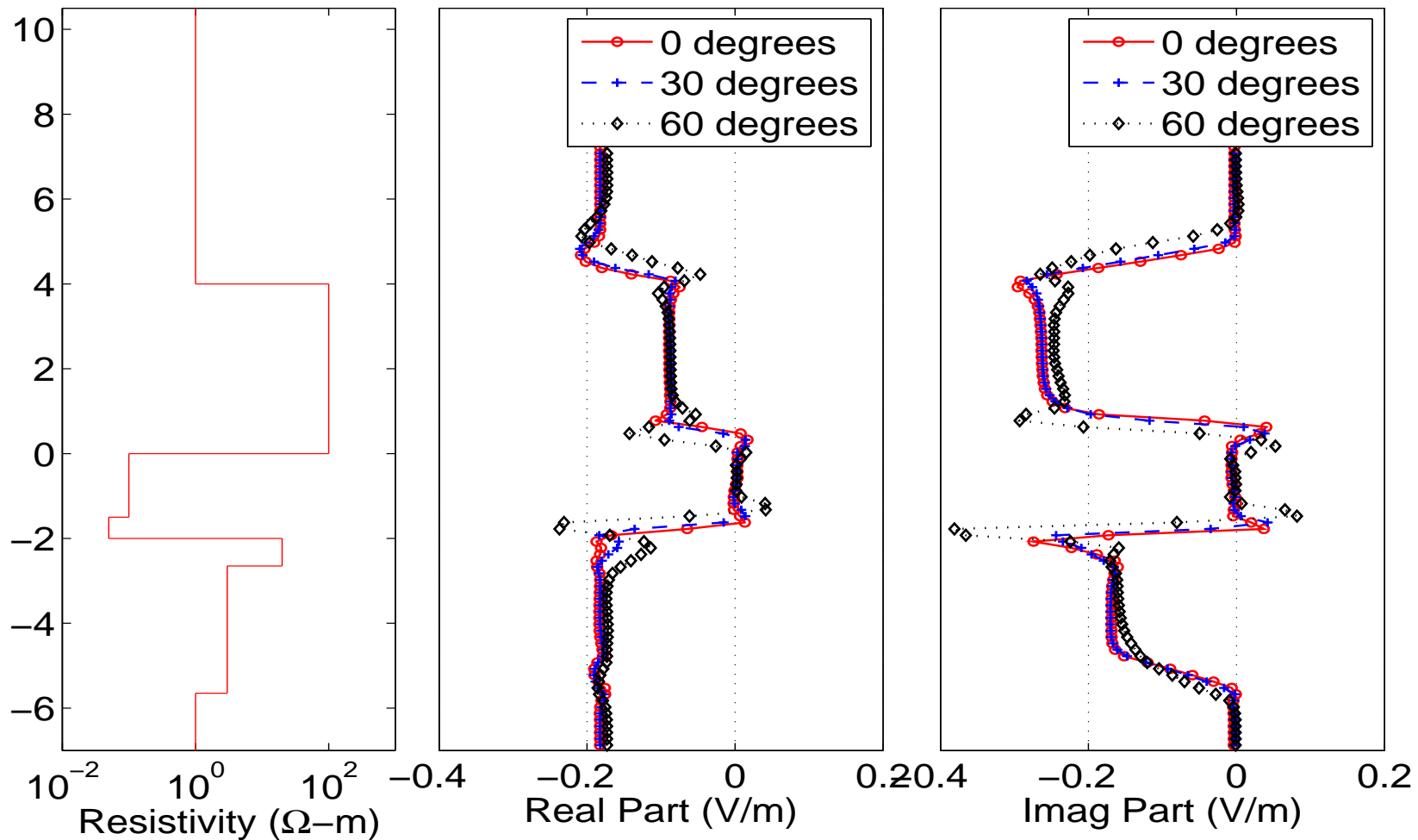
Model Problem and Verification



NUMERICAL RESULTS: INDUCTION

Dip Angle

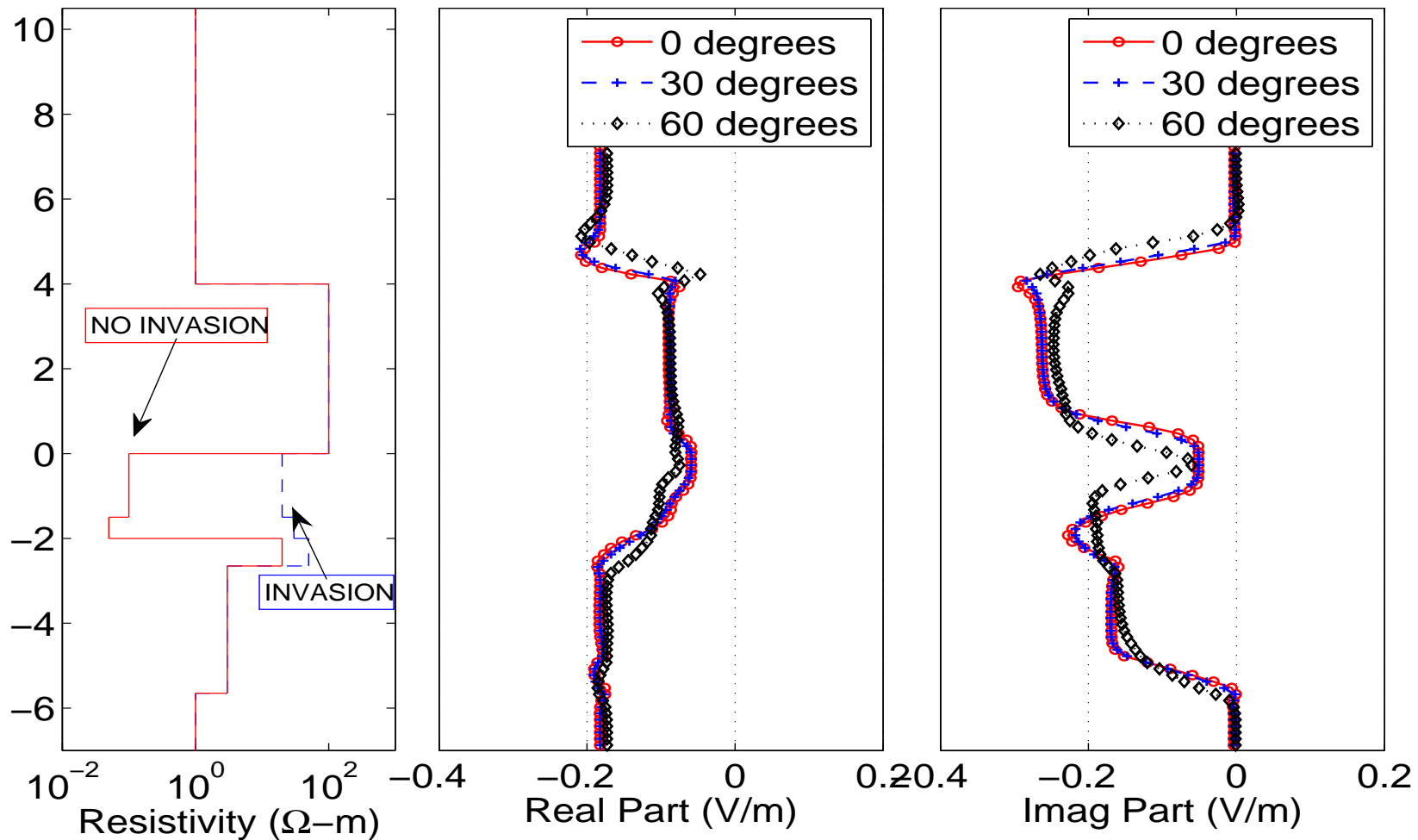
LWD, 2 Mhz



NUMERICAL RESULTS: INDUCTION

Dip Angle + Invasion

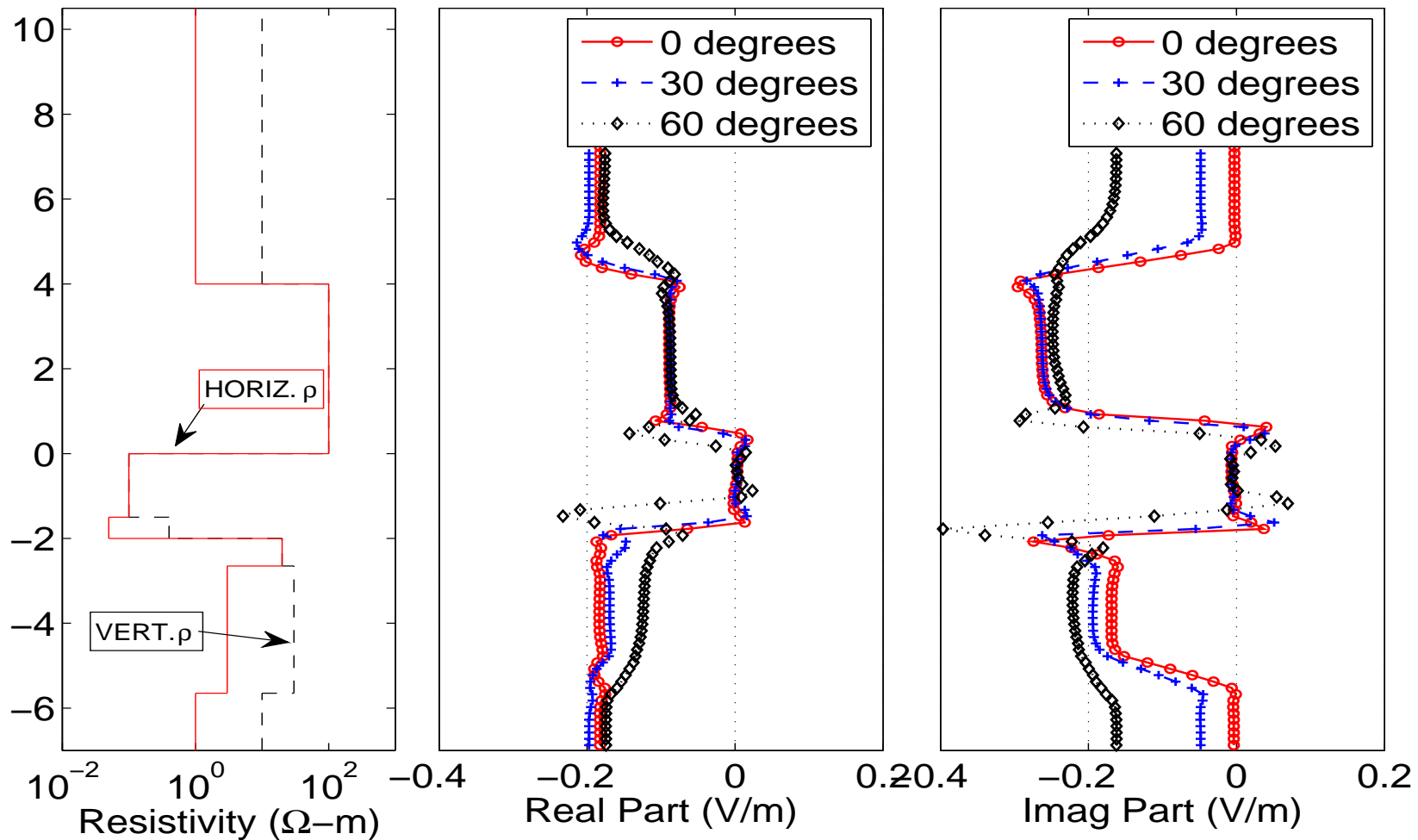
LWD, 2 Mhz



NUMERICAL RESULTS: INDUCTION

Dip Angle + Anisotropy

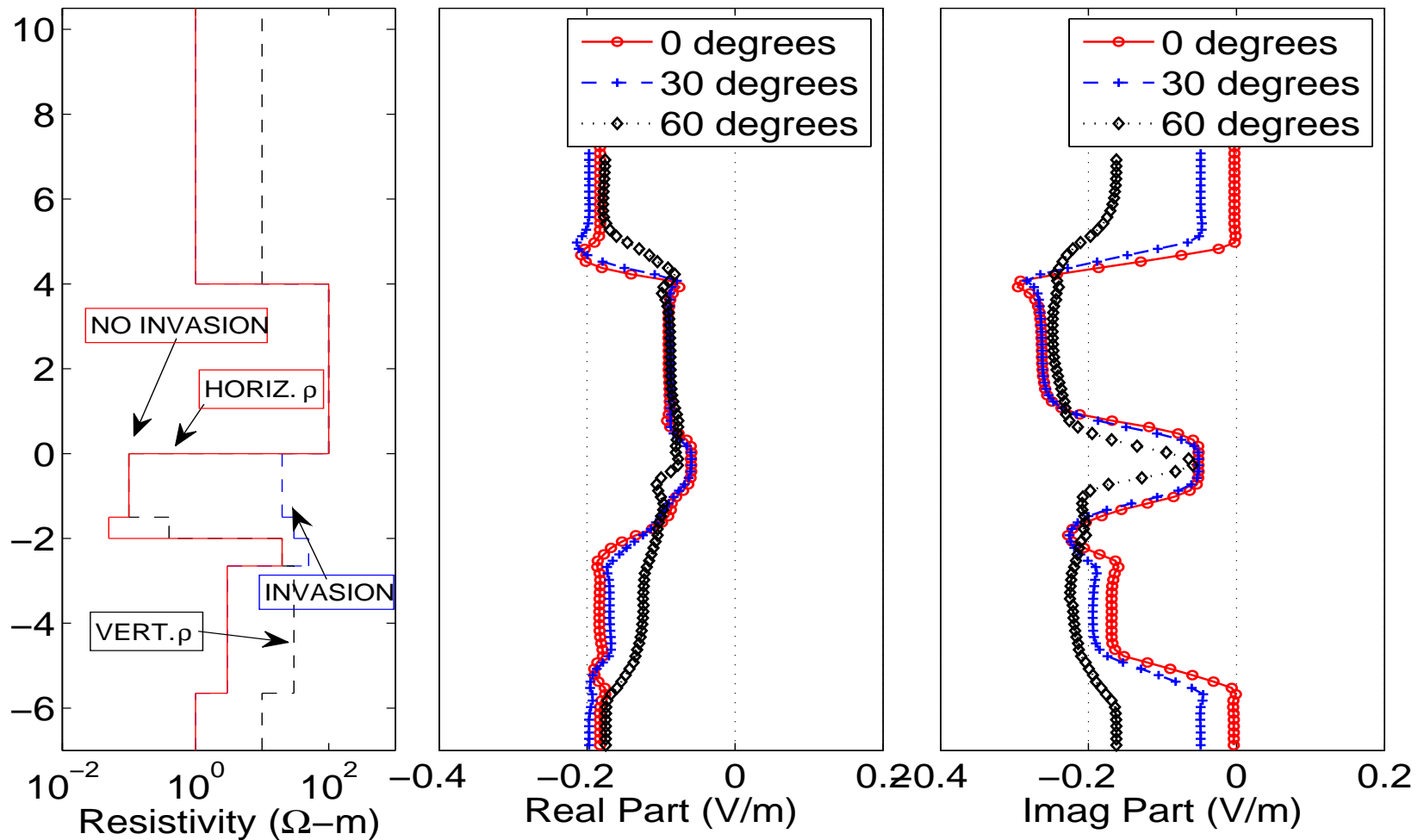
LWD, 2 Mhz



NUMERICAL RESULTS: INDUCTION

Dip Angle + Invasion + Anisotropy

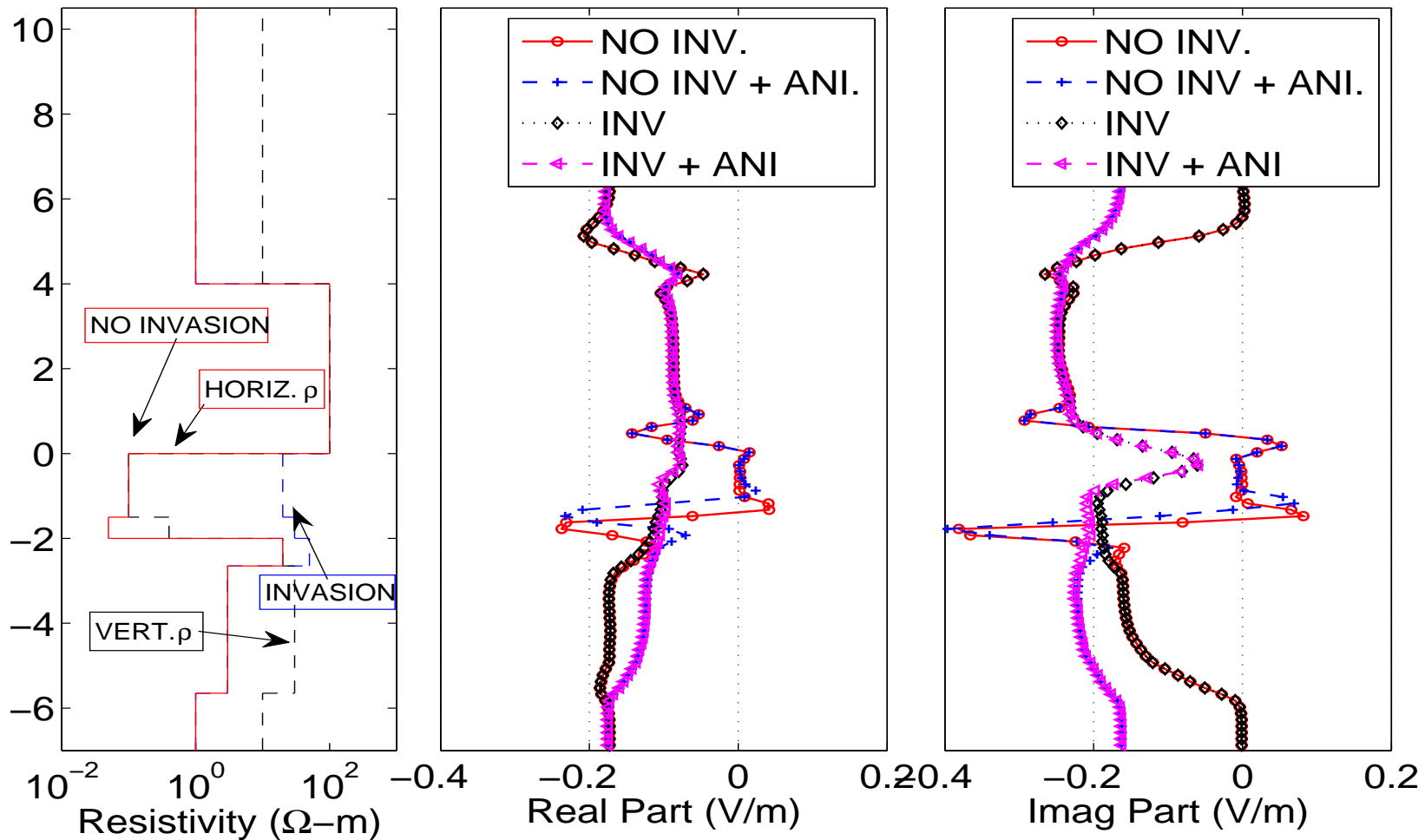
LWD, 2 Mhz



NUMERICAL RESULTS: INDUCTION

60-Degree Deviated Well

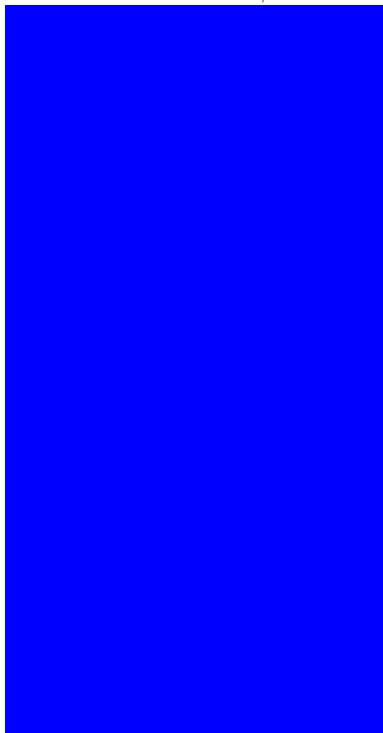
LWD, 2 Mhz



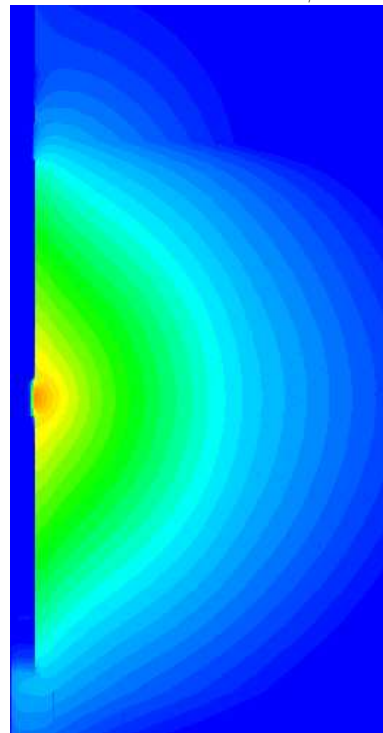
TRIAxIAL INDUCTION

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole

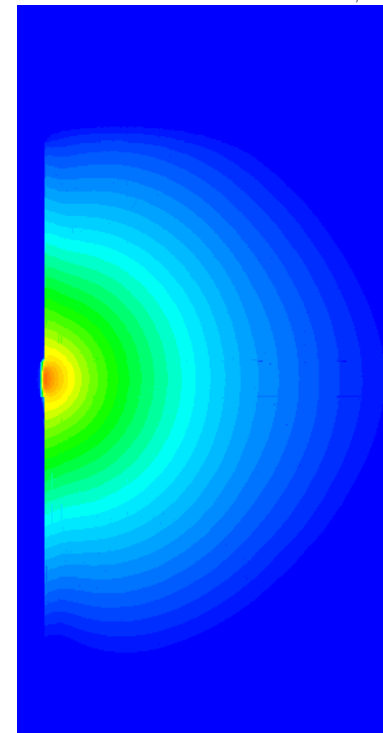
Mode 0 (H_ϕ)



Mode 1 (H_ϕ)



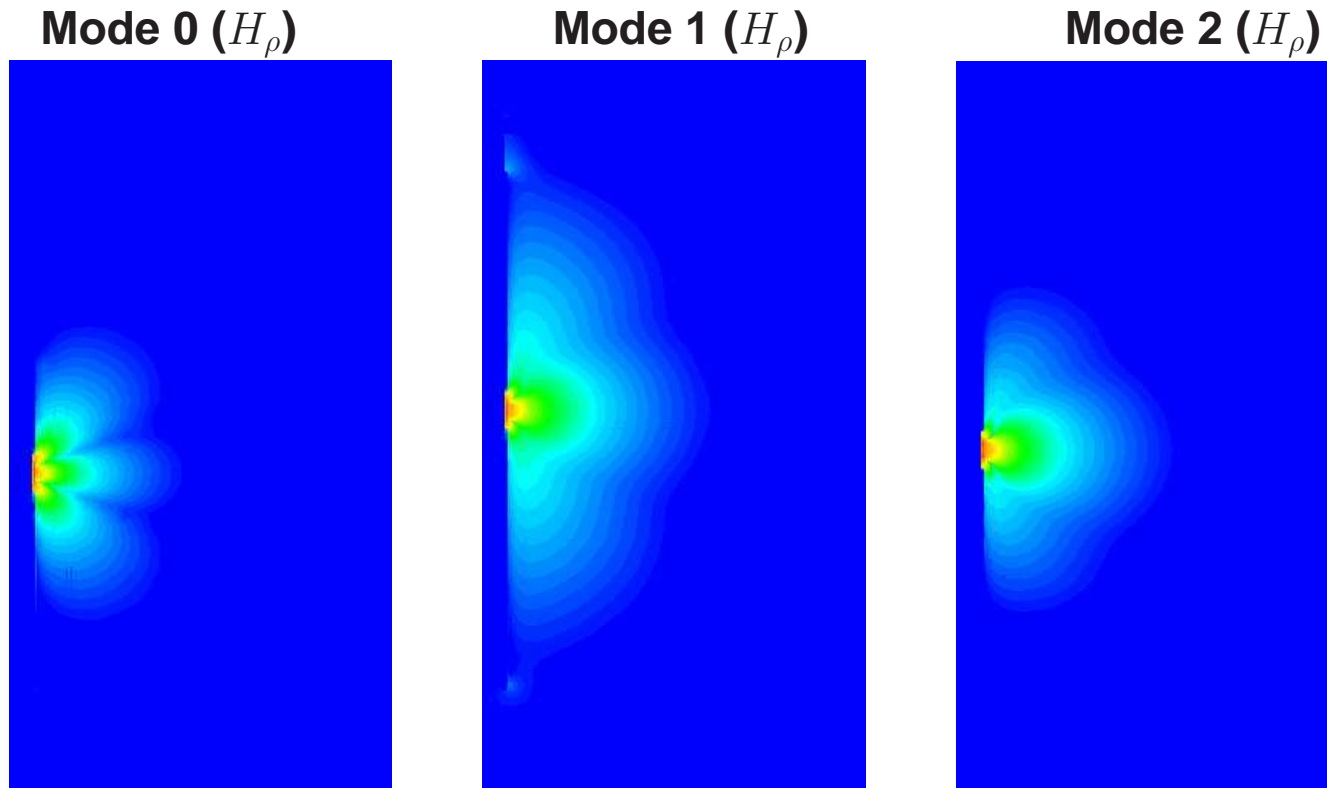
Mode 2 (H_ϕ)



EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

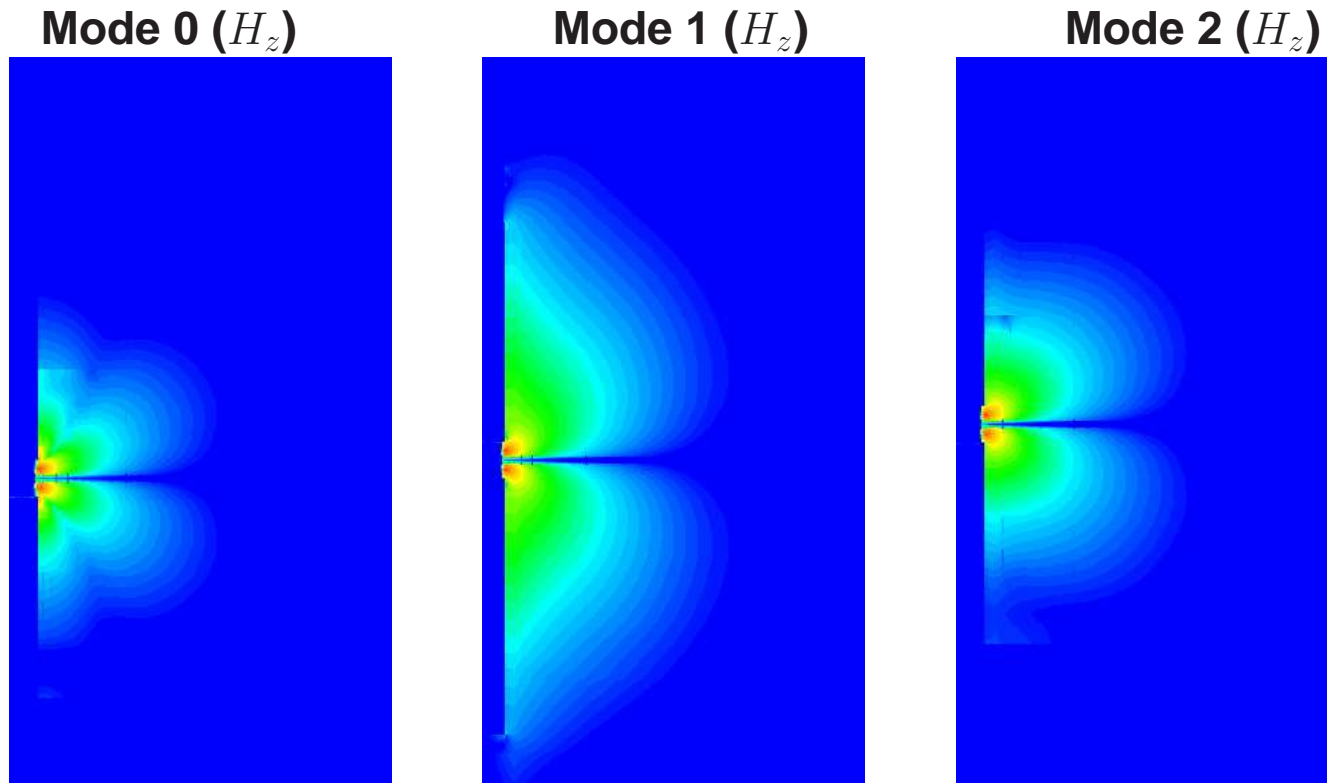
2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole

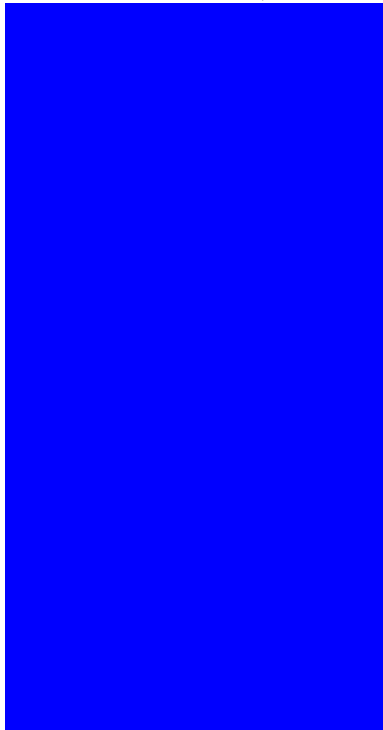


EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

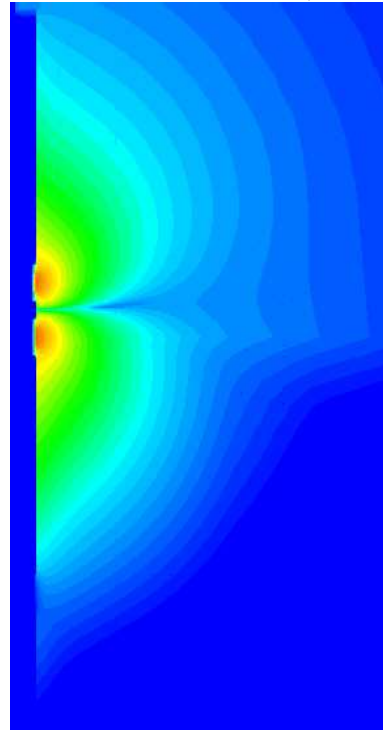
TRIAxIAL INDUCTION

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole

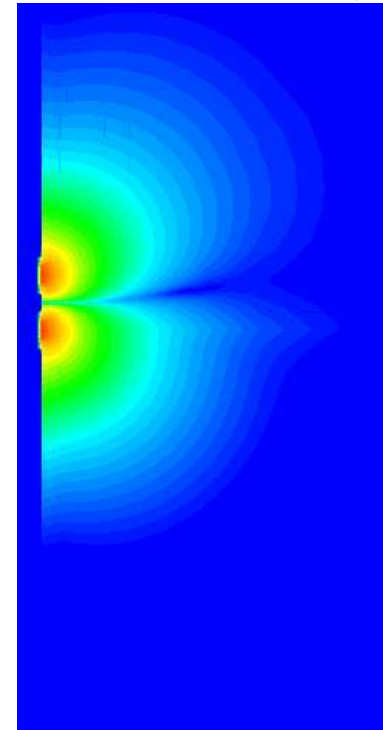
Mode 0 (H_ϕ)



Mode 1 (H_ϕ)



Mode 2 (H_ϕ)

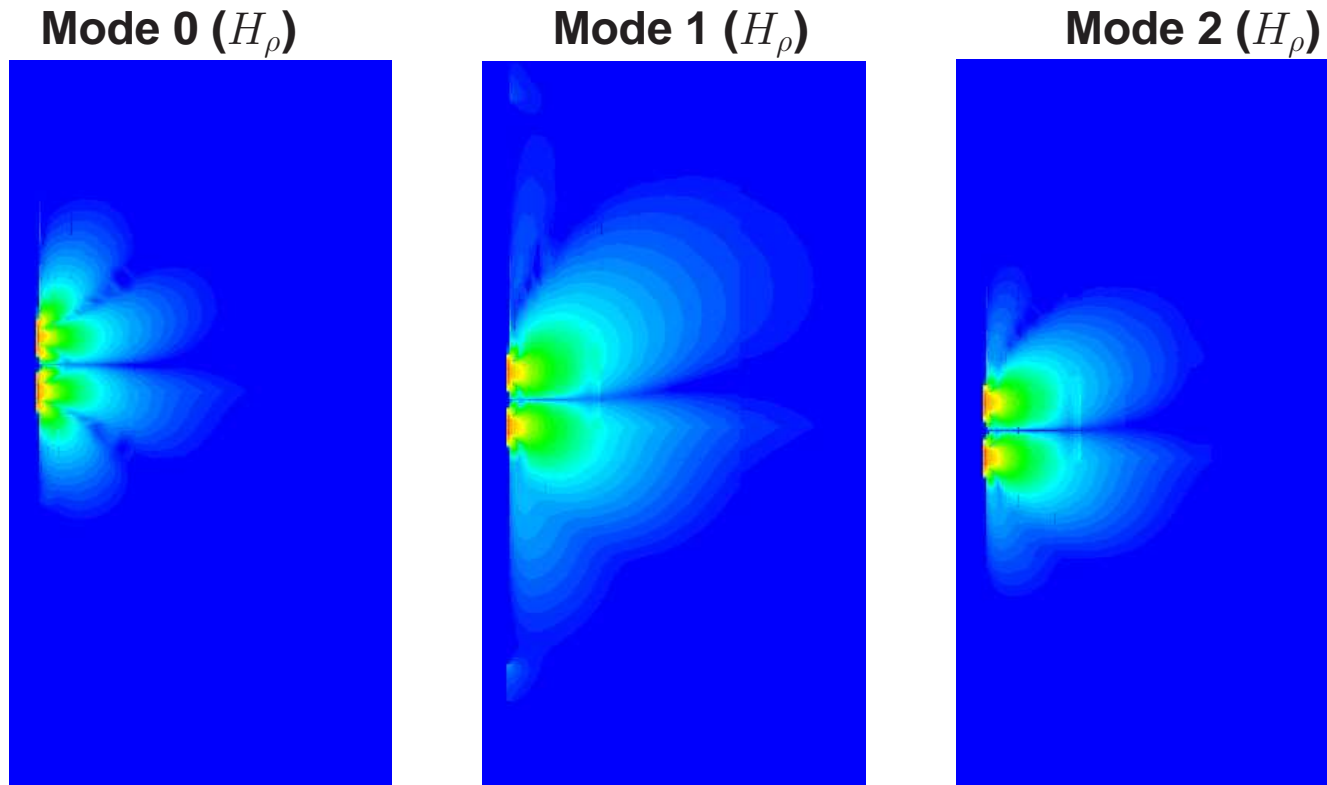


DUAL PROBLEM

EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

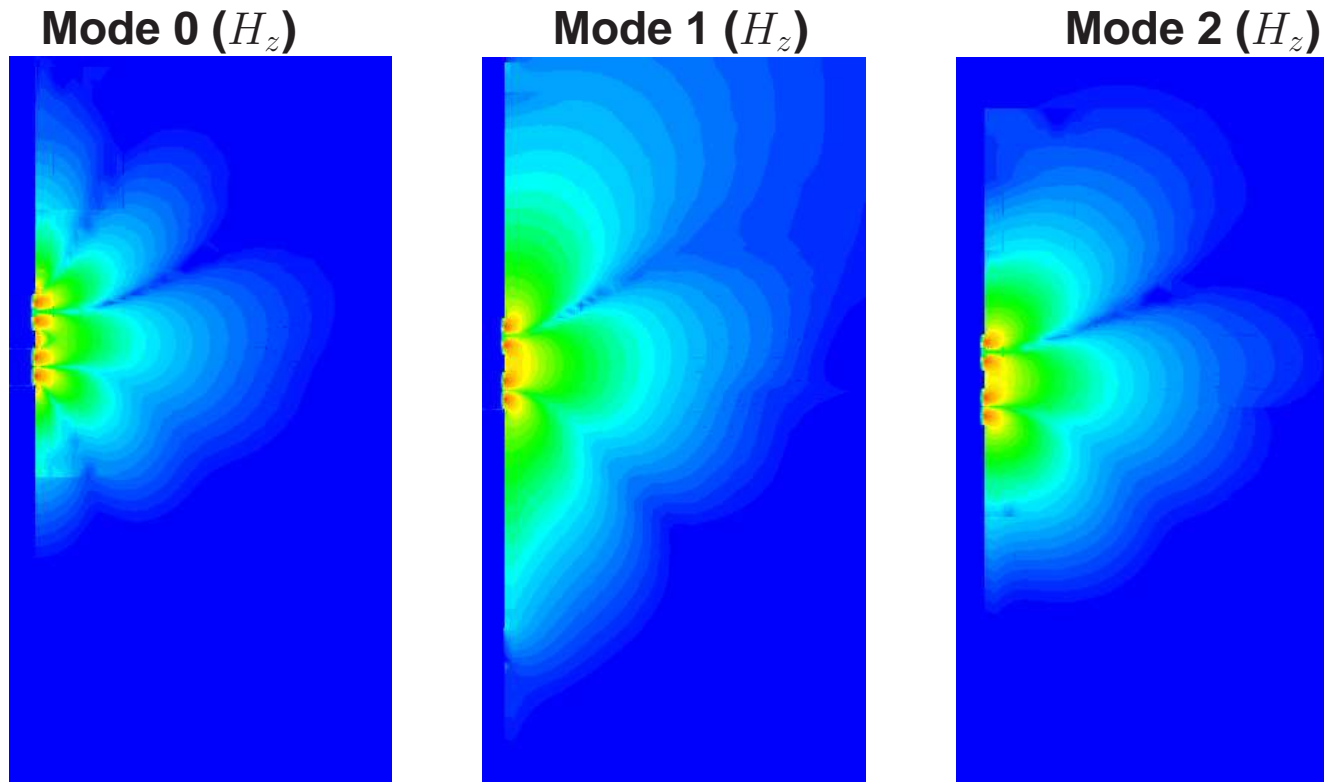
2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



DUAL PROBLEM
EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

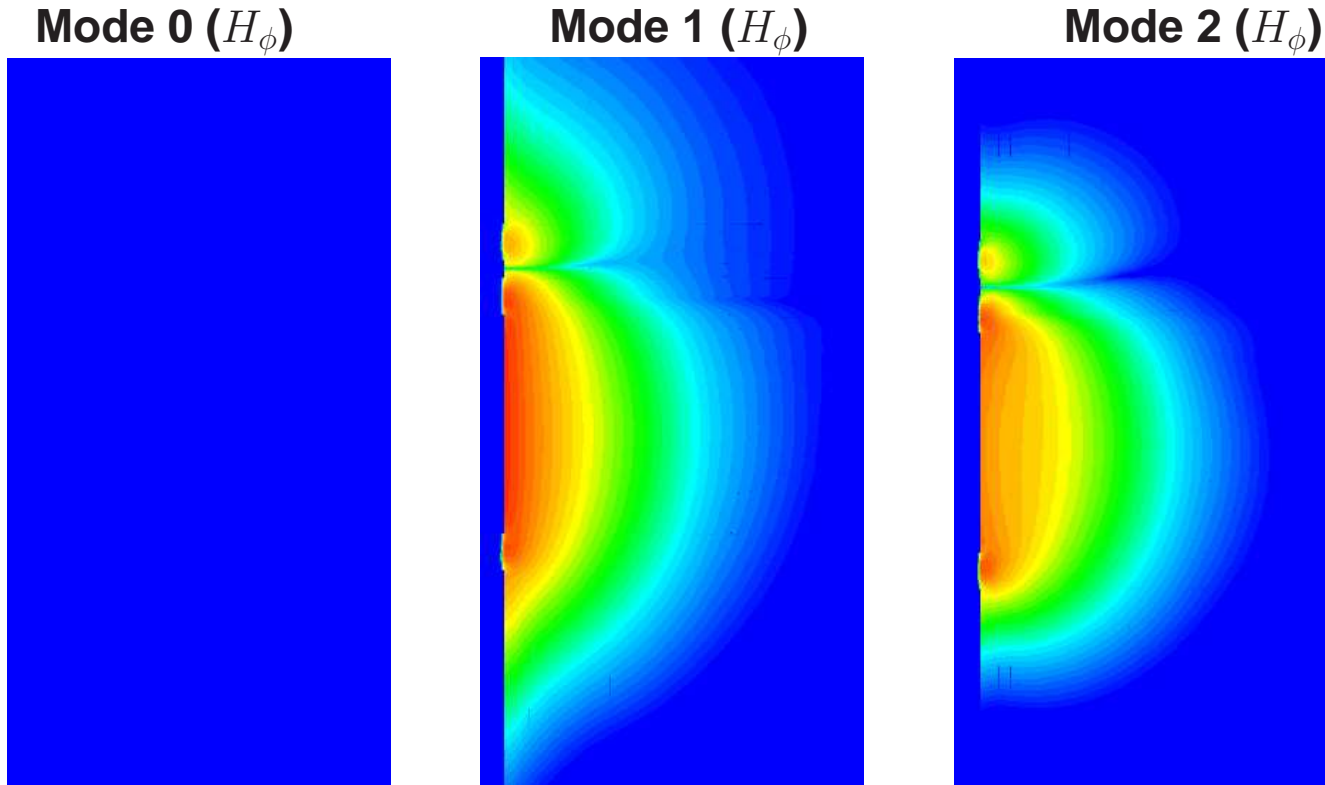
2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



DUAL PROBLEM
EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

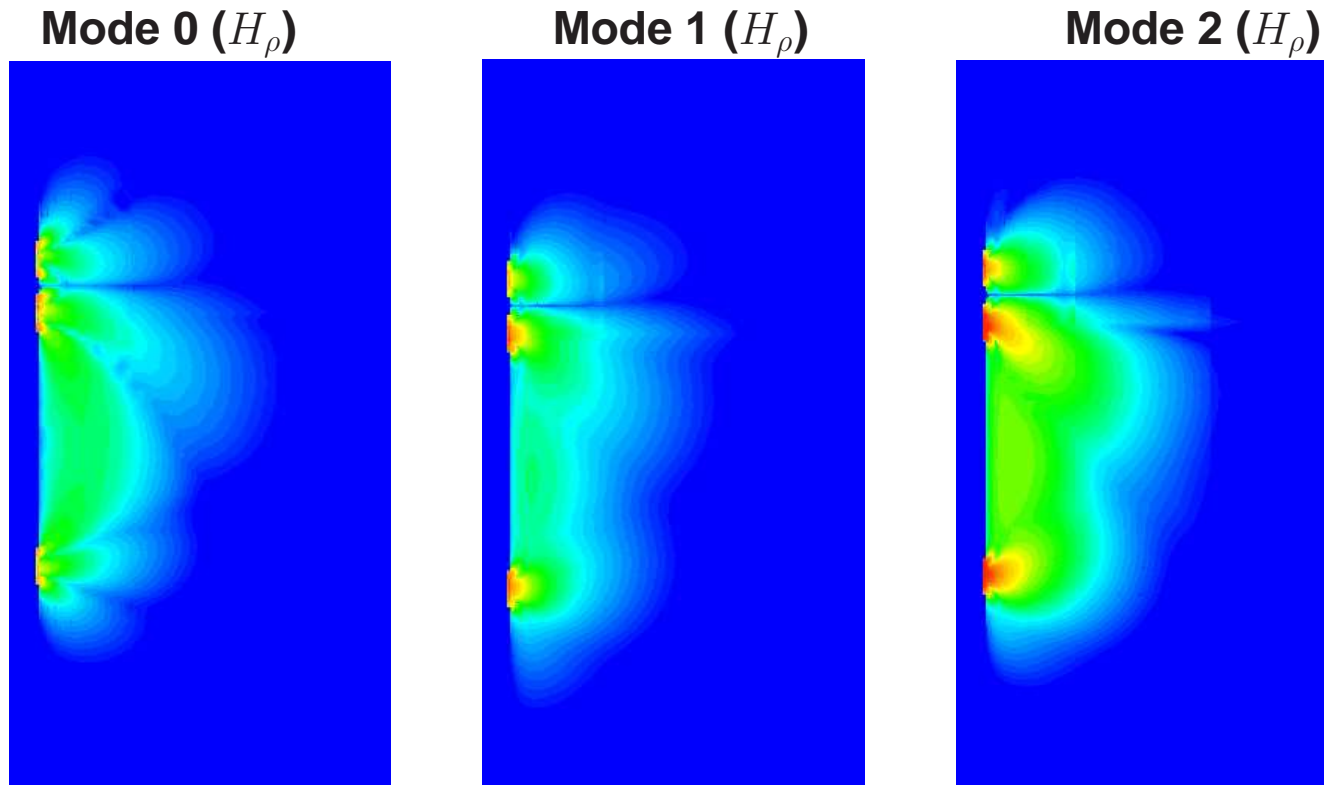
2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



DIRECT SOLUTION TIMES DUAL SOLUTION
EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

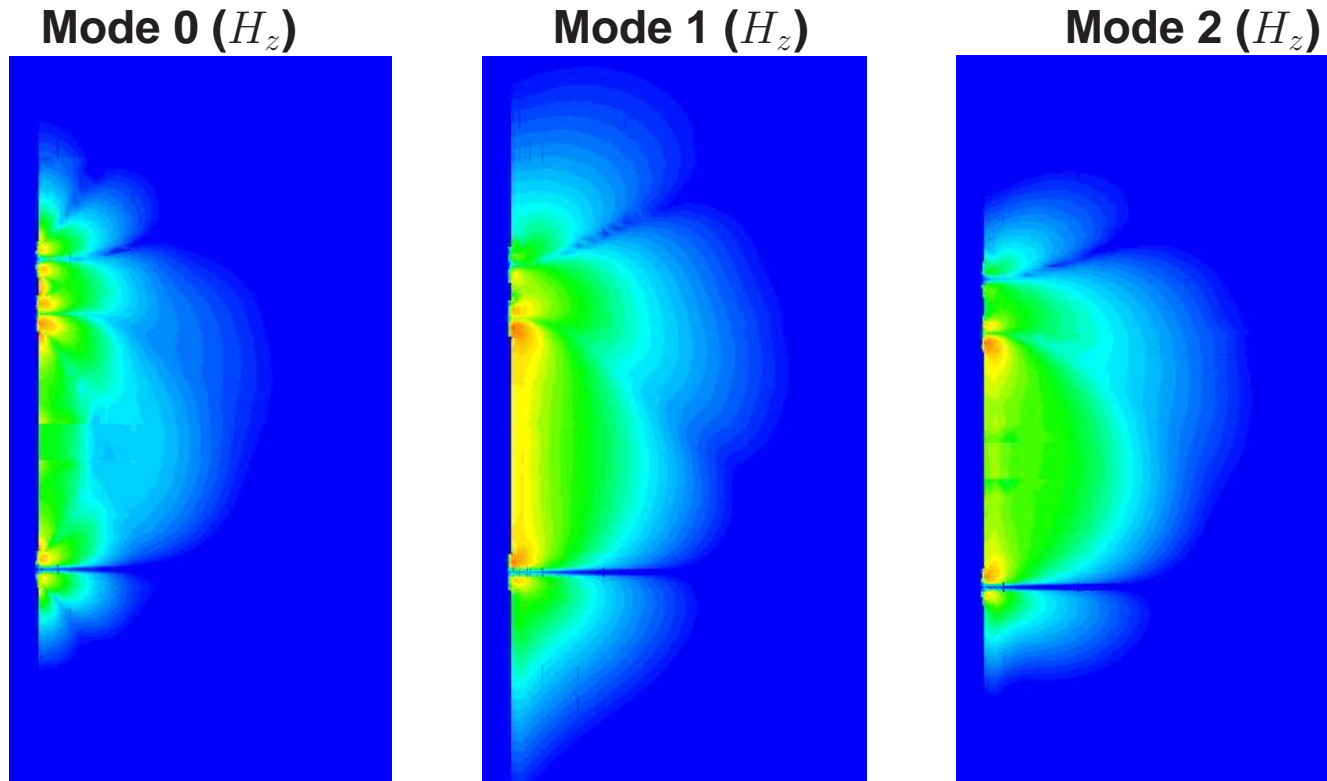
2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



DIRECT SOLUTION TIMES DUAL SOLUTION
EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

TRIAxIAL INDUCTION

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



DIRECT SOLUTION TIMES DUAL SOLUTION
EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

CONCLUSIONS AND FUTURE WORK

Conclusions

- A Fourier-Finite-Element method combined with a 2D parallel self-adaptive refinement strategy enables efficient and accurate simulations of a large class of geophysical resistivity problems.
- A unique software can be used to study the response of through-casing, dual laterolog, triaxial induction, and logging-while-drilling instruments in possibly deviated wells, in the presence of rugosity, anisotropy, invasion, etc.

Future Work

- Inversion
- Multi-physics
- Marine controlled source EM applications.

Department of Petroleum and Geosystems Engineering

ACKNOWLEDGMENTS

Sponsors of UT Austin's consortium on Formation Evaluation

