PIERS 2008, Cambridge, MA, USA

Design of Frequency Domain EM Finite Elements for Geophysical Applications

D. Pardo, M. J. Nam, C. Torres-Verdín, M. Paszynski

July 2, 2008

Department of Petroleum and Geosystems Engineering

THE UNIVERSITY OF TEXAS AT AUSTIN

GEOPHYSICAL APPLICATIONS

Type of Problems We Can Solve with our FE Software

Applications	Borehole Logging | Controlled Source EM
Spatial Dimensions	2D | 3D
Well Type	Vertical Well | Deviated Well | Eccentered Tool
Logging Instruments	LWD/MWD | Normal/Laterolog | Dual Laterolog Triaxial Induction | Dielectric Instruments | Cross-Well
Frequency	0-1 GHz
Materials	Isotropic | Anisotropic
Physical Devices	Magnetic Buffers | Insulators | Casing Casing Imperfections | Displacement Currents | Combination of All
Sources	Finite Size Antennas | Dipoles in Any Direction | Electrodes Solenoidal Antennas | Toroidal Antennas | Combination of All
Invasion	Water | Oil | etc.

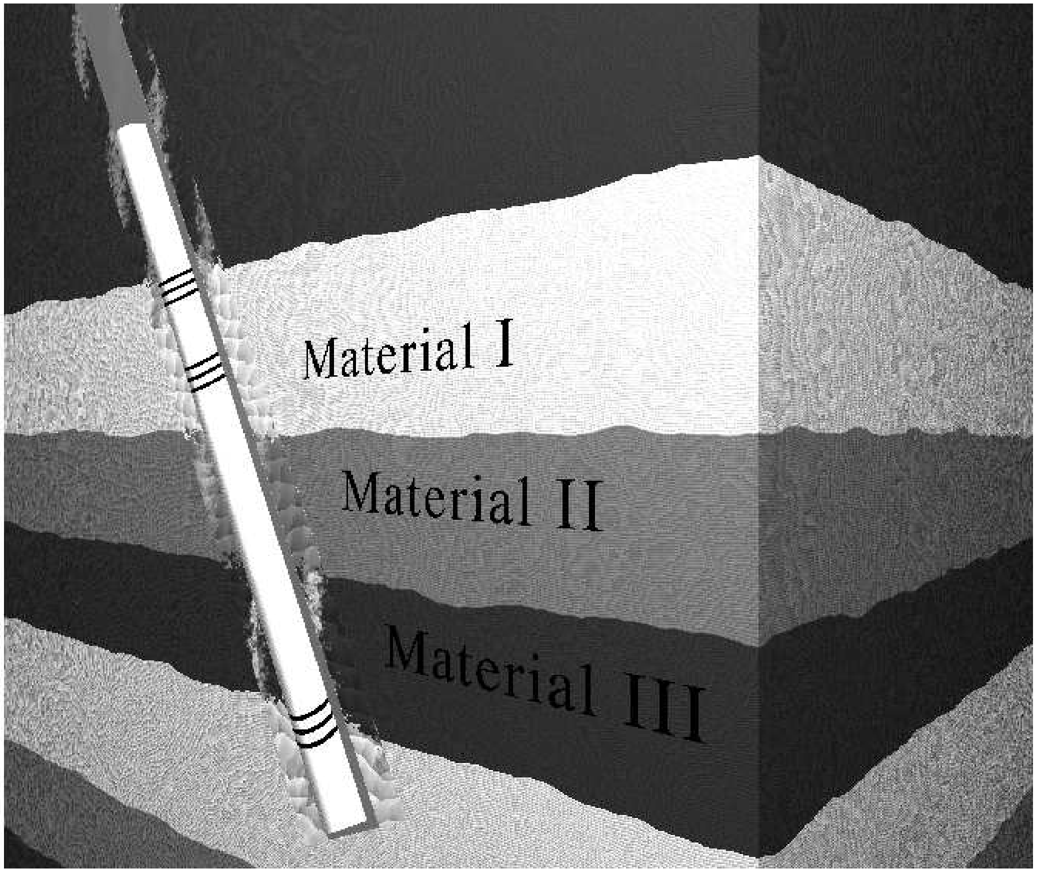
MOST (OIL-INDUSTRY) GEOPHYSICAL PROBLEMS

OVERVIEW

1. **Motivation:**
   - Simulation of borehole logging measurements.
   - Simulation of marine controlled source electromagnetics (CSEM).
2. **Mathematical Formulation Using a Fourier-Finite-Element Method.**
3. **Discretization:**
   - H^1 and $H(\mathrm{curl})$ finite elements.
   - Self-adaptive goal-oriented hp-refinements.
4. **Parallel Implementation.**
5. **Conclusions and Future Work.**

MOTIVATION (BOREHOLE LOGGING)

Deviated Wells (Forward Problem)

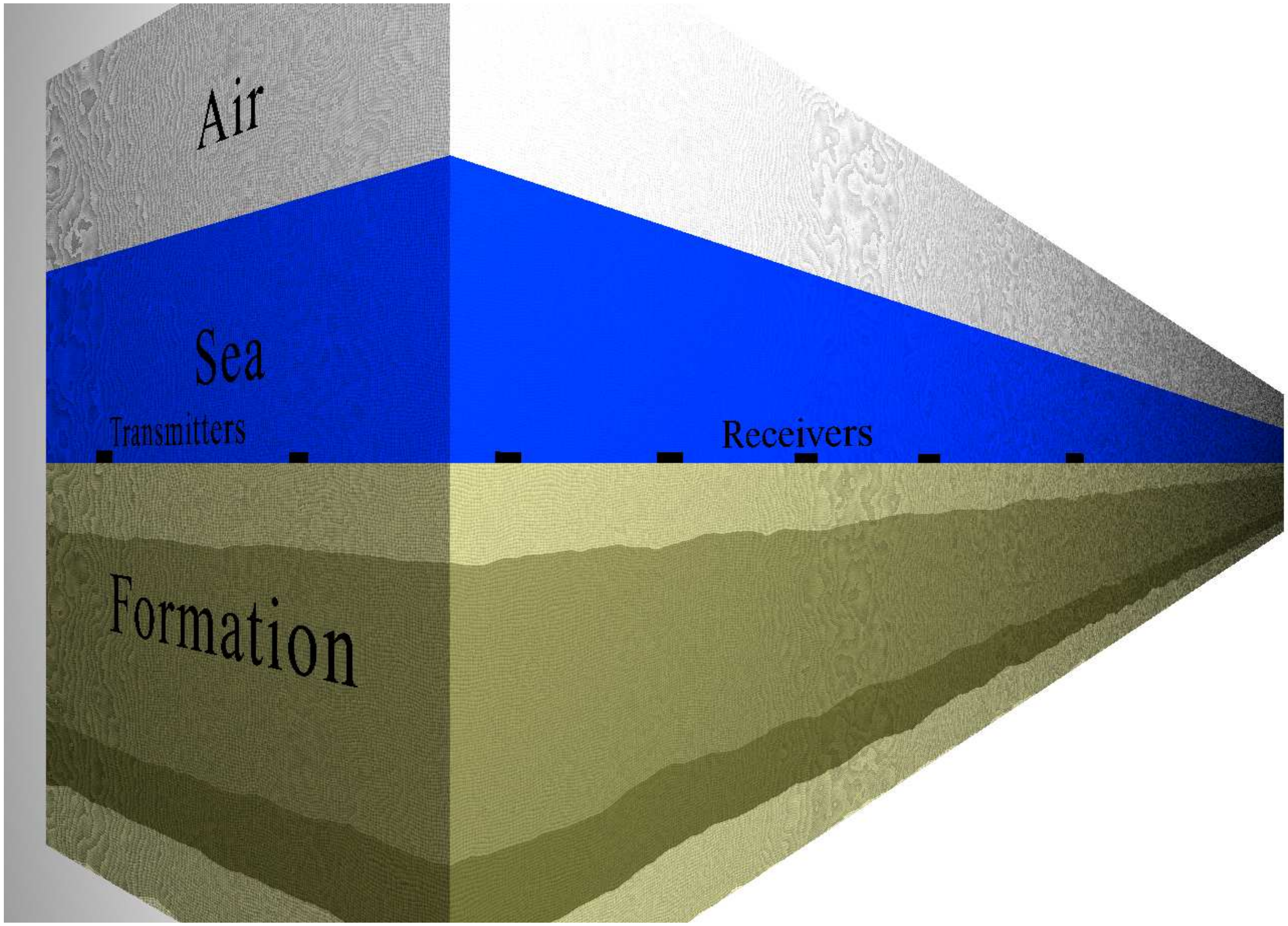


- Dip Angle
- Invasion
- Anisotropy
- Triaxial Induction
- Eccentricity
- Laterolog
- Through-Casing
- Induction-LWD
- Induction-Wireline
- Inverse Problems
- Multi-Physics

Objective: Find solution at the receiver antennas.

MOTIVATION (CSEM)

Marine Controlled Source EM



Objective: Find solution at the receiver antennas.

MATHEMATICAL FORMULATION (3D)

3D Variational Formulation

Time-Harmonic Maxwell's Equations

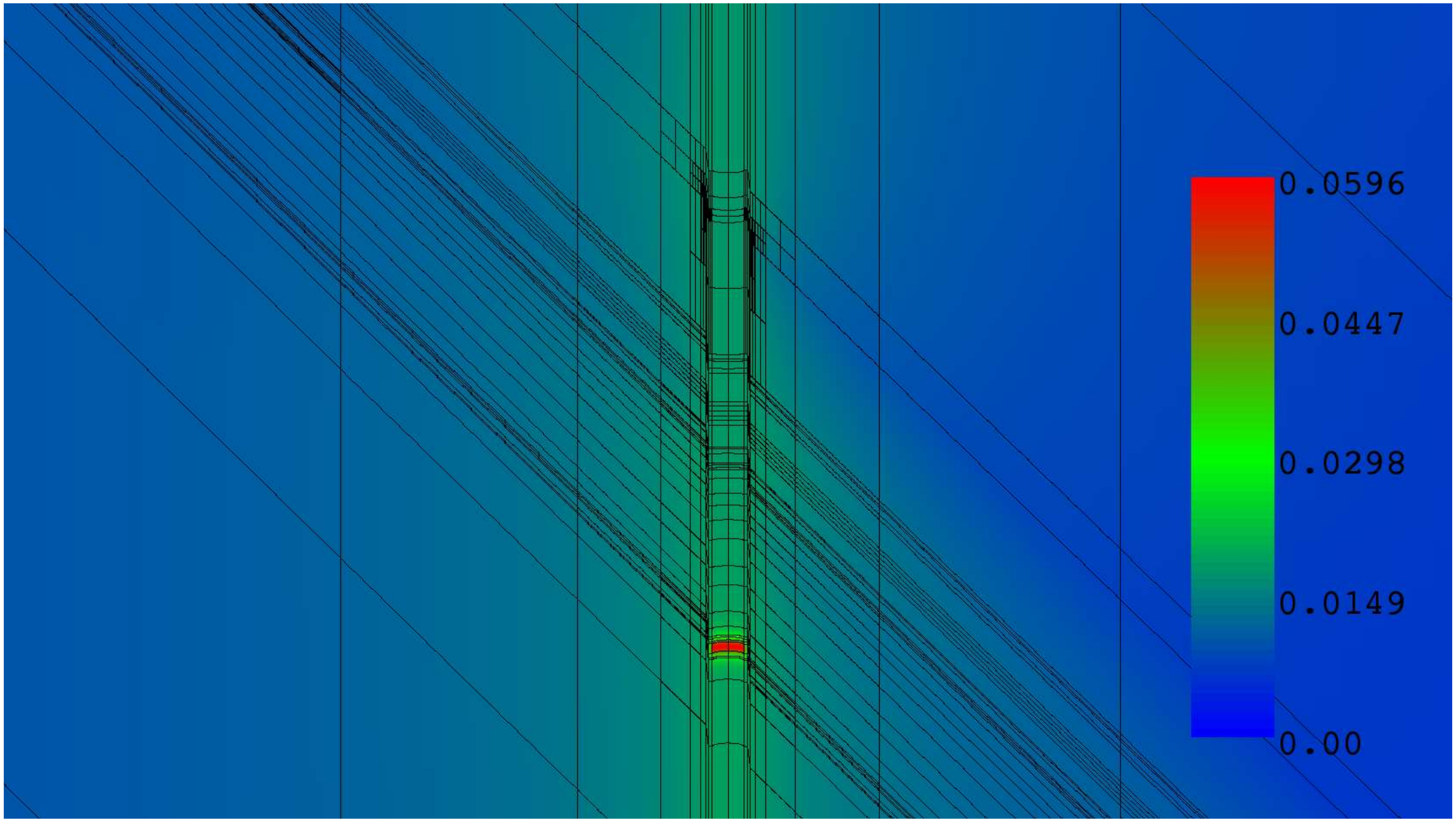
$\nabla \times \mathrm{H} = \mathring{\sigma}\mathrm{E} + \mathrm{J}^{imp}$	Ampere's law ($\mathring{\sigma} = \sigma + j\omega\epsilon$)
$\nabla \times \mathrm{E} = \mathring{\mu}\mathrm{H} + \mathrm{M}^{imp}$	Faraday's law ($\mathring{\mu} = -j\omega\mu$)
$\nabla \cdot (\epsilon \mathrm{E}) = \rho$	Gauss' law of Electricity
$\nabla \cdot (\mu \mathrm{H}) = 0$	Gauss' law of Magnetism

E-VARIATIONAL FORMULATION:

$$\left\{ \begin{array}{l} \text{Find } \mathrm{E} \in \mathrm{E}_{\Gamma_E} + H_{\Gamma_E}(\mathbf{curl};\Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \nabla \times \mathrm{E} \right\rangle_{L^2(\Omega)} - \left\langle \mathrm{F}, \mathring{\sigma}\mathrm{E} \right\rangle_{L^2(\Omega)} = \left\langle \mathrm{F}, \mathrm{J}^{imp} \right\rangle_{L^2(\Omega)} \\ \quad - \left\langle \mathrm{F}_t, \mathrm{J}^{imp}_{\Gamma_H} \right\rangle_{L^2(\Gamma_H)} + \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \mathrm{M}^{imp} \right\rangle_{L^2(\Omega)} \quad \forall\, \mathrm{F} \in H_{\Gamma_E}(\mathbf{curl};\Omega) \end{array} \right.$$

MATHEMATICAL FORMULATION (3D)

Example: Solution in a 60-degree deviated well ($-\nabla\sigma\nabla u = f$)



Several hours to obtain one solution (3D forward simulation).
Several months needed to solve the inverse problem.

FOURIER ANALYSIS

Dimensionality Reduction for Maxwell's Equations

Solving a 3D problem is CPU time and memory intensive. In some cases, we may reduce the complexity of the problem by using Fourier analysis.

Borehole Problems

Cylindrical Coordinates

Fourier Series Expansion

$$\mathrm{E}(\phi) := \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E}) e^{jn\phi}$$

X-Well, CSEM Problems

Cartesian Coordinates

Fourier Transform

$$\mathrm{E}(x_1) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathcal{F}_r(\mathrm{E}) e^{jrx_1} dx_1$$

FOURIER ANALYSIS

Fourier Series Expansion

Fourier series expansion **Inverse Fourier series expansion**

$$\mathcal{F}_n(\mathrm{E}) := \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \mathrm{E}(\phi) e^{-jn\phi} d\phi \quad ; \quad \mathrm{E}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E}) e^{jn\phi}.$$

Main properties

- **Compatibility with differentiation** $\mathcal{F}_n(\frac{\partial \mathrm{E}}{\partial \phi}) = jn\mathcal{F}_n(\mathrm{E})$:

$$\mathcal{F}_n(\nabla \times \mathrm{E}) = \nabla^n \times (\mathcal{F}_n(\mathrm{E})),$$

where

$$\nabla^n \times \mathrm{E} := \left(\frac{jnE_z}{\rho} - \frac{\partial E_\phi}{\partial z}, \frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}, \frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{jnE_\rho}{\rho} \right),$$

- L_2**-Orthogonality:**

$$\frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{jn\phi} e^{-jm\phi} d\phi = \sqrt{2\pi}\delta_{nm} \; .$$

FOURIER ANALYSIS

Fourier Transform

Fourier transform **Inverse Fourier transform**

$$\mathcal{F}_r(\mathrm{E}) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathrm{E}(x) e^{-jrx} dx \quad ; \quad \mathrm{E}(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathcal{F}_r(\mathrm{E}) e^{jrx} dr.$$

Main properties

- **Compatibility with differentiation** $\mathcal{F}_r(\frac{\partial \mathrm{E}}{\partial x}) = jr\mathcal{F}_r(\mathrm{E})$**:**

$$\mathcal{F}_r(\nabla \times \mathrm{E}) = \nabla^r \times (\mathcal{F}_r(\mathrm{E})),$$

where

$$\nabla^r \times \mathrm{E} := \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - jrE_z, jrE_y - \frac{\partial E_x}{\partial y} \right),$$

- L_2**-Orthogonality:**

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{jrx} e^{-jsx} = \sqrt{2\pi}\delta_{sr} \ .$$

FOURIER ANALYSIS

E-Variational Formulations (Cylindrical Coordinates)

FINITE ELEMENT —3D—:

$$\begin{cases} \text{Find } \mathrm{E} \in \mathrm{E}_{\Gamma_E} + H_{\Gamma_E}(\mathrm{curl};\Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \nabla \times \mathrm{E} \right\rangle_{L^2(\Omega)} - \left\langle \mathrm{F}, \mathring{\sigma}\mathrm{E} \right\rangle_{L^2(\Omega)} = \left\langle \mathrm{F}, \mathrm{J}^{imp} \right\rangle_{L^2(\Omega)} \\ \quad - \left\langle \mathrm{F}_t, \mathrm{J}^{imp}_{\Gamma_H} \right\rangle_{L^2(\Gamma_H)} + \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \mathrm{M}^{imp} \right\rangle_{L^2(\Omega)} \quad \forall\, \mathrm{F} \in H_{\Gamma_E}(\mathrm{curl};\Omega) \end{cases}$$

FOURIER FINITE ELEMENT —3D = Sequence of Coupled 2D Problems—:

$$\begin{cases} \text{Find } \mathrm{E} = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E})\, e^{jn\phi}, \text{ where for each } n: \\ \mathcal{F}_n(\mathrm{E}) \in \mathcal{F}_n(\mathrm{E}_{\Gamma_{E,1D}}) + H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}), \text{ and} \\ \sum_{m=-\infty}^{\infty} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F})\, ,\, \mathcal{F}_{n-m}(\mathring{\mu}^{-1}) \nabla^m \times \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F})\, ,\, \mathcal{F}_{n-m}(\mathring{\sigma}) \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} \\ \quad = \left\langle \mathcal{F}_n(\mathrm{F})\, ,\, \mathcal{F}_n(\mathrm{J}^{imp}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}_t)\, ,\, \mathcal{F}_n(\mathrm{J}_S^{imp}) \right\rangle_{L^2(\Gamma_{H,1D})} \\ \quad + \sum_{m=-\infty}^{\infty} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F})\, ,\, \mathcal{F}_{n-m}(\mathring{\mu}^{-1}) \mathcal{F}_m(\mathrm{M}^{imp}) \right\rangle_{L^2(\Omega_{2D})} \quad \forall\, \mathcal{F}_n(\mathrm{F}) \in H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}) \end{cases}$$

FOURIER ANALYSIS

E-Variational Formulations (Cylindrical Coordinates)

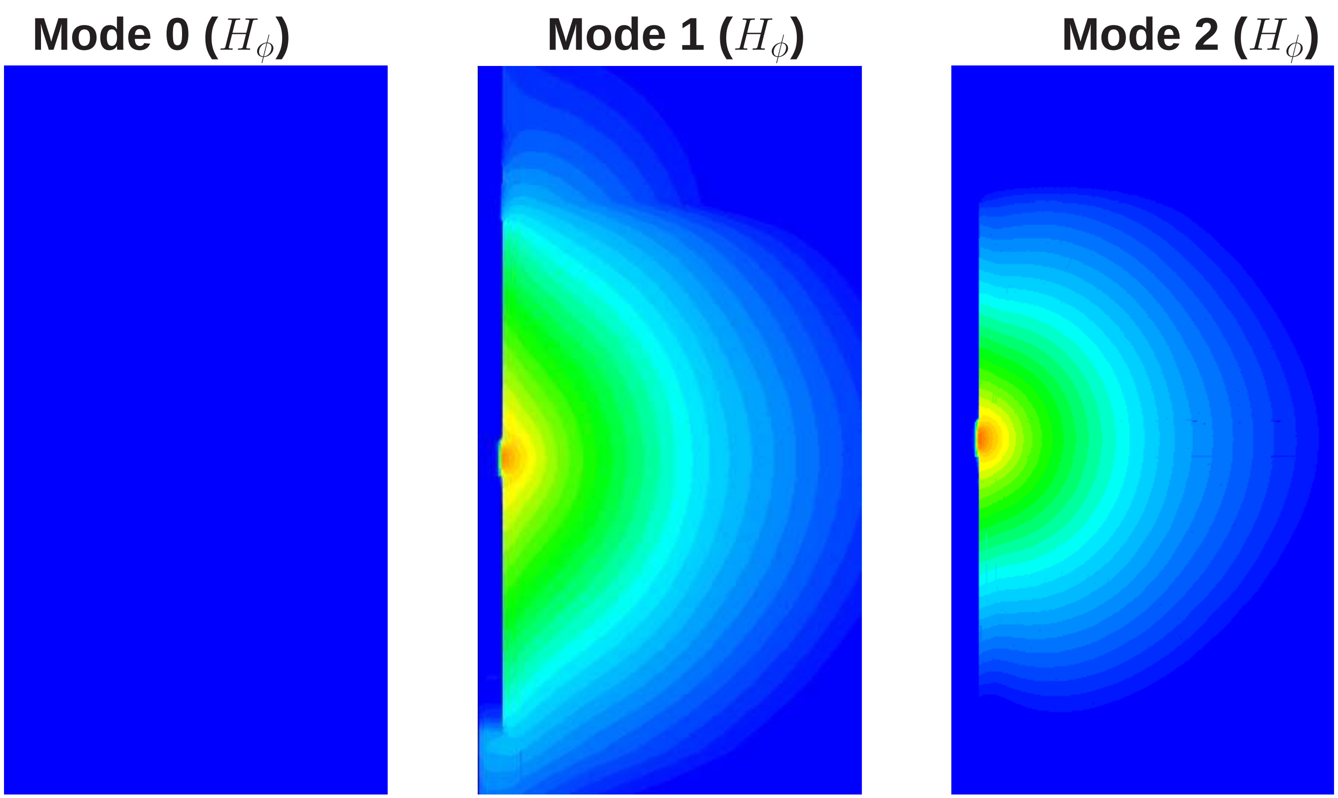
Assumption: For $n \neq m$ **we assume** $\mathcal{F}_{n-m}(\mathring{\mu}^{-1}) = \mathcal{F}_{n-m}(\mathring{\sigma}^{-1}) = 0$.

FOURIER FINITE ELEMENT —2.5D = Sequence of Uncoupled 2D Problems—:

$$\begin{cases} \text{Find } \mathrm{E} = \dfrac{1}{\sqrt{2\pi}} \displaystyle\sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E})\, e^{jn\phi}, \text{ where for each } n: \\ \mathcal{F}_n(\mathrm{E}) \in \mathcal{F}_n(\mathrm{E}_{\Gamma_{E,1D}}) + H_{\Gamma_{E,1D}}(\mathrm{curl}^n; \Omega_{2D}), \text{ and} \\ \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \, , \, \mathcal{F}_n(\mathring{\mu}^{-1}) \nabla^n \times \mathcal{F}_n(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}) \, , \, \mathcal{F}_n(\mathring{\sigma}) \mathcal{F}_n(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} \\ \quad = \left\langle \mathcal{F}_n(\mathrm{F}) \, , \, \mathcal{F}_n(\mathrm{J}^{imp}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}_t) \, , \, \mathcal{F}_n(\mathrm{J}_S^{imp}) \right\rangle_{L^2(\Gamma_{H,1D})} \\ \quad + \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \, , \, \mathcal{F}_n(\mathring{\mu}^{-1}) \mathcal{F}_n(\mathrm{M}^{imp}) \right\rangle_{L^2(\Omega_{2D})} \quad \forall\, \mathcal{F}_n(\mathrm{F}) \in H_{\Gamma_{E,1D}}(\mathrm{curl}^n; \Omega_{2D}) \end{cases}$$

FOURIER ANALYSIS

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole

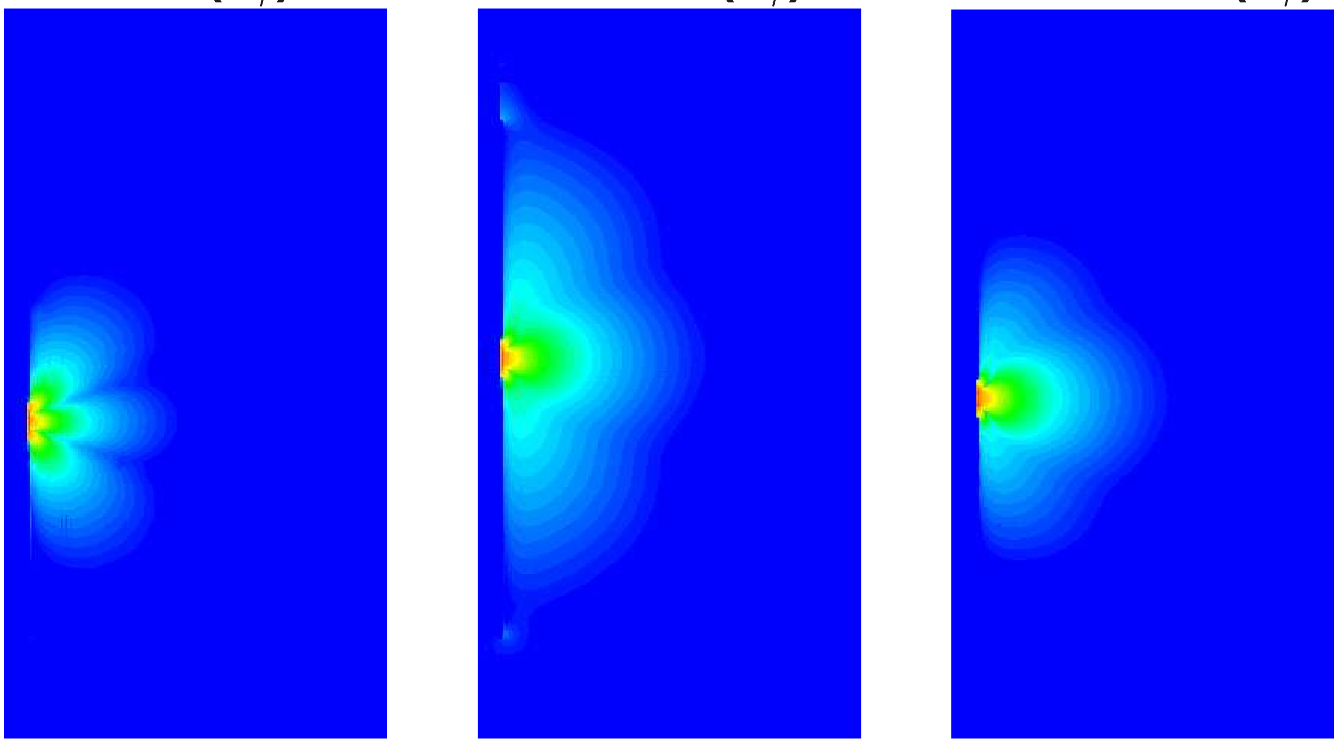


EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

FOURIER ANALYSIS

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole

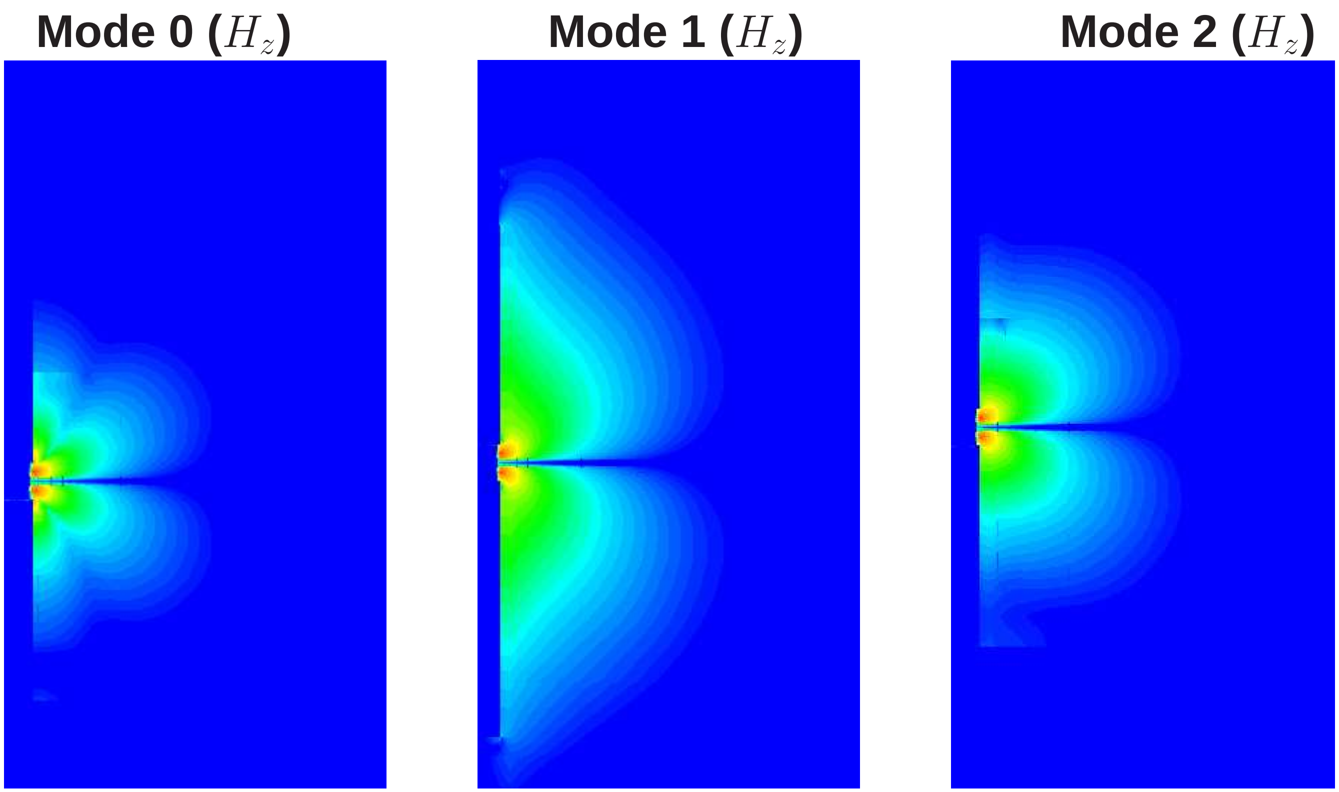
Mode 0 (H_ρ) **Mode 1 (H_ρ)** **Mode 2 (H_ρ)**



EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

FOURIER ANALYSIS

2.5D Problem (Triaxial Induction). Source: Horizontal Magnetic Dipole



EXAMPLE OF 2.5D TRIAXIAL INDUCTION LWD PROBLEM

FOURIER ANALYSIS

2D Variational Formulation (Axi-symmetric Problems)

If we further assume that $\mathcal{F}_n(\mathrm{J}^{imp}) = \mathcal{F}_n(\mathrm{J}_S^{imp}) = \mathcal{F}_n(\mathrm{M}^{imp}) = 0 \quad \forall n \neq 0$**, then we obtain one uncoupled 2D problem. Now,** $\mathrm{E} = \mathcal{F}_0(\mathrm{E})$**.**

E_ϕ **-Variational Formulation (Azimuthal)**

$$\begin{cases} \text{Find } E_\phi \in E_{\phi,D} + \tilde{H}_D^1(\Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}_\phi, \mathring{\mu}_{\rho,z}^{-1} \nabla \times \mathrm{E}_\phi \right\rangle_{L^2(\Omega_{2D})} - \langle \mathrm{F}_\phi, \mathring{\sigma}_\phi \mathrm{E}_\phi \rangle_{L^2(\Omega_{2D})} = \left\langle F_\phi, J_\phi^{imp} \right\rangle_{L^2(\Omega_{2D})} \\ \quad - \left\langle F_\phi, J_{\phi,\tilde{\Gamma}_H}^{imp} \right\rangle_{L^2(\tilde{\Gamma}_H)} + \left\langle F_\phi, \mathring{\mu}_{\rho,z}^{-1} \mathrm{M}_{\rho,z}^{imp} \right\rangle_{L^2(\Omega_{2D})} \qquad \forall\, F_\phi \in \tilde{H}_D^1(\Omega) \end{cases}$$

$\mathrm{E}_{\rho,z}$ **-Variational Formulation (Meridian)**

$$\begin{cases} \text{Find } \mathrm{E}_{\rho,z} = (E_\rho, E_z) \in \mathrm{E}_D + \tilde{H}_D(\mathrm{curl}; \Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}_{\rho,z}, \mathring{\mu}_\phi^{-1} \nabla \times \mathrm{E}_{\rho,z}) \right\rangle_{L^2(\Omega_{2D})} - \langle \mathrm{F}_{\rho,z}, \mathring{\sigma}_{\rho,z} \mathrm{E}_{\rho,z} \rangle_{L^2(\Omega_{2D})} = \\ \quad \left\langle \mathrm{F}_{\rho,z}, \mathrm{J}_{\rho,z}^{imp} \right\rangle_{L^2(\Omega_{2D})} - \left\langle (\mathrm{F}_{\rho,z})_t, \mathrm{J}_{\rho,z,\tilde{\Gamma}_H}^{imp} \right\rangle_{L^2(\tilde{\Gamma}_H)} \\ \quad + \left\langle \mathrm{F}_{\rho,z}, \mathring{\mu}_\phi^{-1} \mathrm{M}_\phi^{imp} \right\rangle_{L^2(\Omega_{2D})} \qquad \forall\, (F_\rho, F_z) \in \tilde{H}_D(\mathrm{curl}; \Omega) \end{cases}$$

FOURIER ANALYSIS

2D Formulations

2D problem with casing at 10 Hz.

E_ϕ Formulation

Quantity of Interest	Real Part	Imag Part
GRID 1	0.1516098429E-08	-0.1456374493E-08
GRID 2	0.1516094029E-08	-0.1456390824E-08

$\mathrm{E}_{\rho,z}$ Formulation

Quantity of Interest	Real Part	Imag Part
GRID 3	0.1516060872E-08	-0.1456337248E-08
GRID 4	0.1516093804E-08	-0.1456390864E-08

The two formulations can be employed as a verification method (and perhaps an an error control method).

DEVIATED WELLS

E-Variational Formulations (Cylindrical Coordinates)

FINITE ELEMENT —3D—:

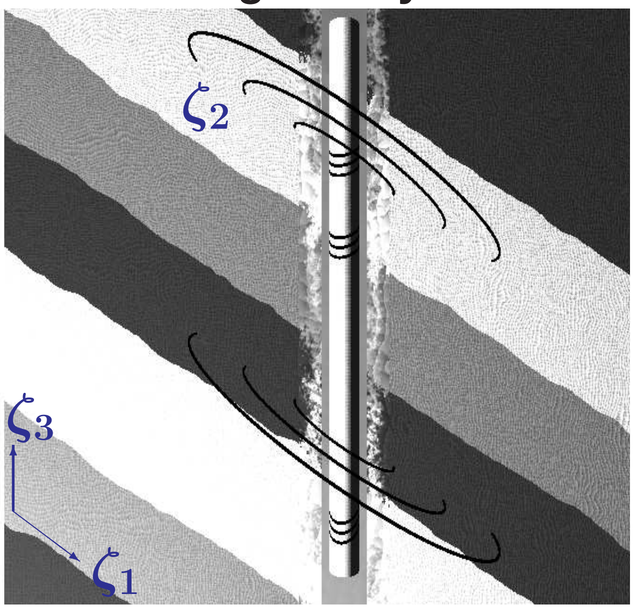
$$\begin{cases} \text{Find } \mathrm{E} \in \mathrm{E}_{\Gamma_E} + H_{\Gamma_E}(\mathrm{curl};\Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \nabla \times \mathrm{E} \right\rangle_{L^2(\Omega)} - \left\langle \mathrm{F}, \mathring{\sigma}\mathrm{E} \right\rangle_{L^2(\Omega)} = \left\langle \mathrm{F}, \mathrm{J}^{imp} \right\rangle_{L^2(\Omega)} \\ \quad - \left\langle \mathrm{F}_t, \mathrm{J}^{imp}_{\Gamma_H} \right\rangle_{L^2(\Gamma_H)} + \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1}\mathrm{M}^{imp} \right\rangle_{L^2(\Omega)} \quad \forall\, \mathrm{F} \in H_{\Gamma_E}(\mathrm{curl};\Omega) \end{cases}$$

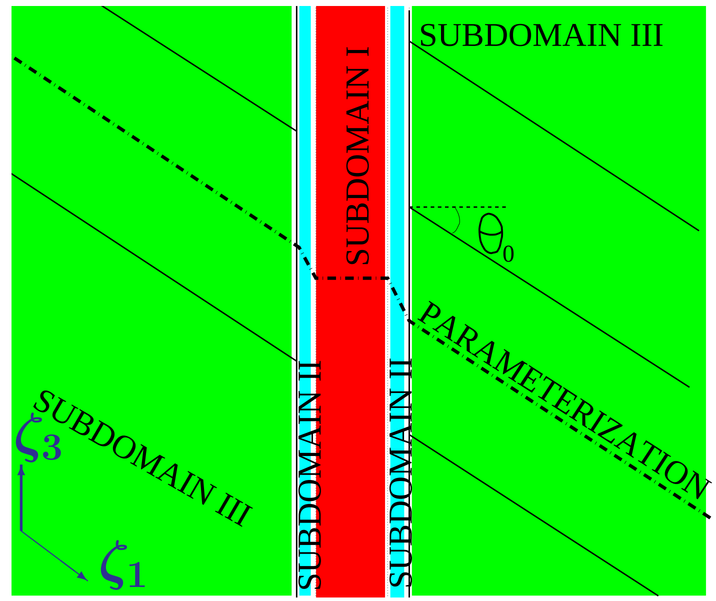
FOURIER FINITE ELEMENT —3D = Sequence of Coupled 2D Problems—:

$$\begin{cases} \text{Find } \mathrm{E} = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E})\, e^{jn\phi}, \text{ where for each } n\text{:} \\ \mathcal{F}_n(\mathrm{E}) \in \mathcal{F}_n(\mathrm{E}_{\Gamma_{E,1D}}) + H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}), \text{ and} \\ \sum_{m=-\infty}^{\infty} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\mu}^{-1}) \nabla^m \times \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\sigma}) \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} \\ \quad = \left\langle \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_n(\mathrm{J}^{imp}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}_t) \,,\, \mathcal{F}_n(\mathrm{J}^{imp}_S) \right\rangle_{L^2(\Gamma_{H,1D})} \\ \quad + \sum_{m=-\infty}^{\infty} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\mu}^{-1}) \mathcal{F}_m(\mathrm{M}^{imp}) \right\rangle_{L^2(\Omega_{2D})} \quad \forall\, \mathcal{F}_n(\mathrm{F}) \in H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}) \end{cases}$$

DEVIATED WELLS

Cartesian system of coordinates: $\mathrm{x} = (x, y, z)$.
New non-orthogonal system of coordinates: $\zeta = (\zeta_1, \zeta_2, \zeta_3)$.





Subdomain I ; **Subdomain II** ; **Subdomain III**

$$\begin{cases} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 \end{cases} ; \quad \begin{cases} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 + \tan \theta_0 \dfrac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \end{cases} ; \quad \begin{cases} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 + \tan \theta_0 \zeta_1 \end{cases}$$

DISCRETIZATION

2D Finite Elements + 1D Fourier

3D Problem (using a Fourier Finite Element Method):

- $H(\mathrm{curl})$ **(Nedelec elements) for the meridian components (**$\mathrm{E}_{\rho,z}$**), and**
- H^1 **(Lagrange elements) for the azimuthal component (**E_ϕ**).**

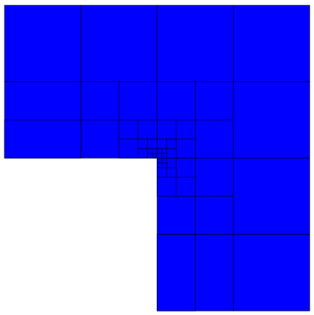
2.5D Problem (using a Fourier Finite Element Method):

- $H(\mathrm{curl})$ **(Nedelec elements) for the meridian components (**$\mathrm{E}_{\rho,z}$**), and**
- H^1 **(Lagrange elements) for the azimuthal component (**E_ϕ**).**

2D Problem:

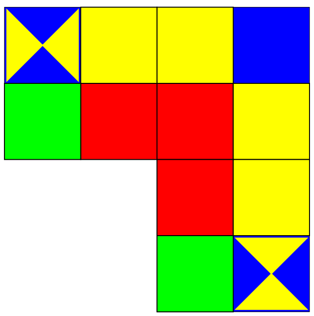
- $H(\mathrm{curl})$ **(Nedelec elements) in terms of the meridian components (**$\mathrm{E}_{\rho,z}$**), or**
- H^1 **(Lagrange elements) in terms of the azimuthal component (**E_ϕ**).**

DISCRETIZATION



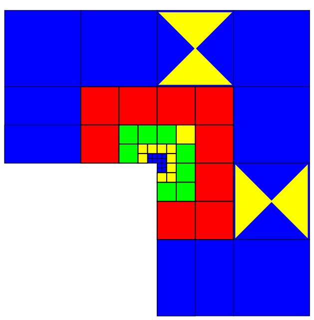
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal h-grids do NOT converge exponentially in real applications.
3. They may "lock" (100% error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.



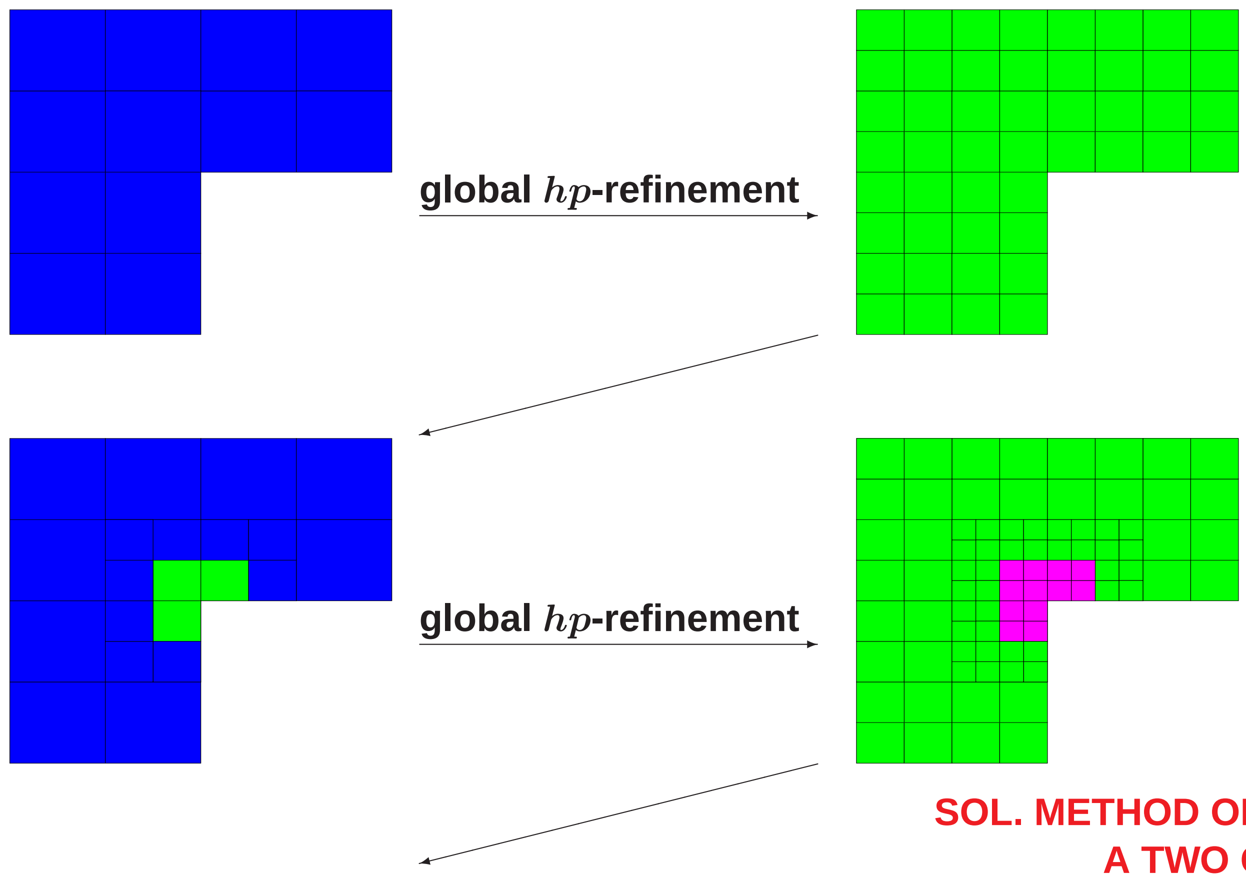
The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

DISCRETIZATION

Energy norm based fully automatic *hp*-adaptive strategy

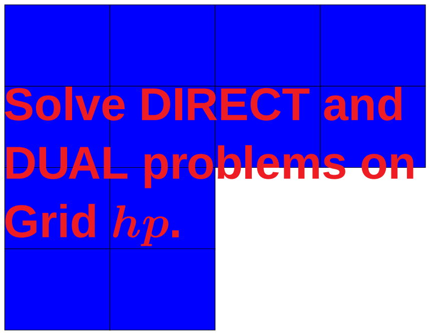
Coarse grids (hp) **Fine grids** ($h/2, p+1$)



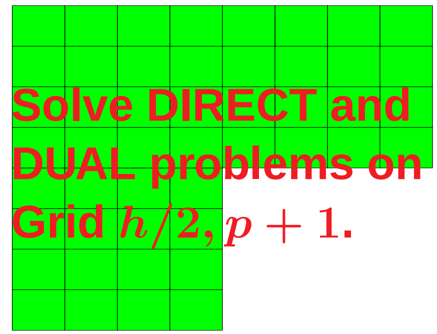
SOL. METHOD ON FINE GRIDS:
A TWO GRID SOLVER

DISCRETIZATION

Algorithm for Goal-Oriented Adaptivity



$\longrightarrow$



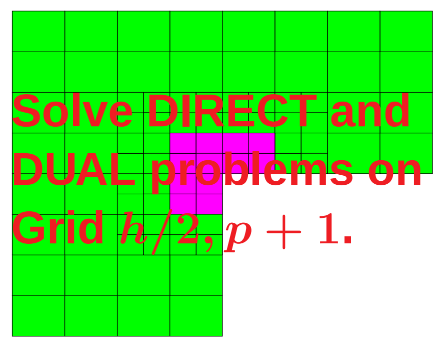
Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, **and** $\tilde{e} = \Psi_{h/2,p+1} - \Pi_{hp}\Psi_{h/2,p+1}$.

Compute $\epsilon = G_{h/2,p+1} - G_{hp}$, **and** $\tilde{\epsilon} = G_{h/2,p+1} - \Pi_{hp}G_{h/2,p+1}$.

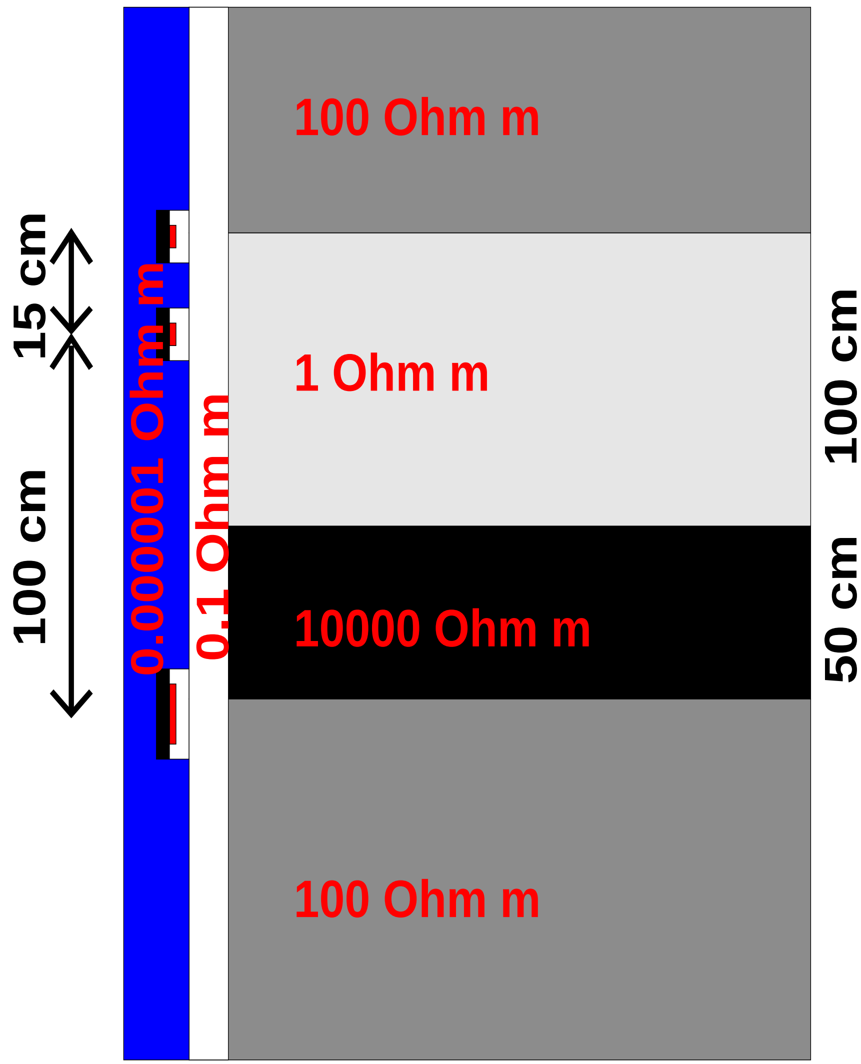
$$|L(e)| = |b(e,\epsilon)| \sim |b(\tilde{e},\tilde{\epsilon})| \leq \sum_K |b_K(\tilde{e},\tilde{\epsilon})| \leq \sum_K \parallel \tilde{e} \parallel_{E,K} \parallel \tilde{\epsilon} \parallel_{E,K}.$$

Apply the fully automatic hp-adaptive algorithm.

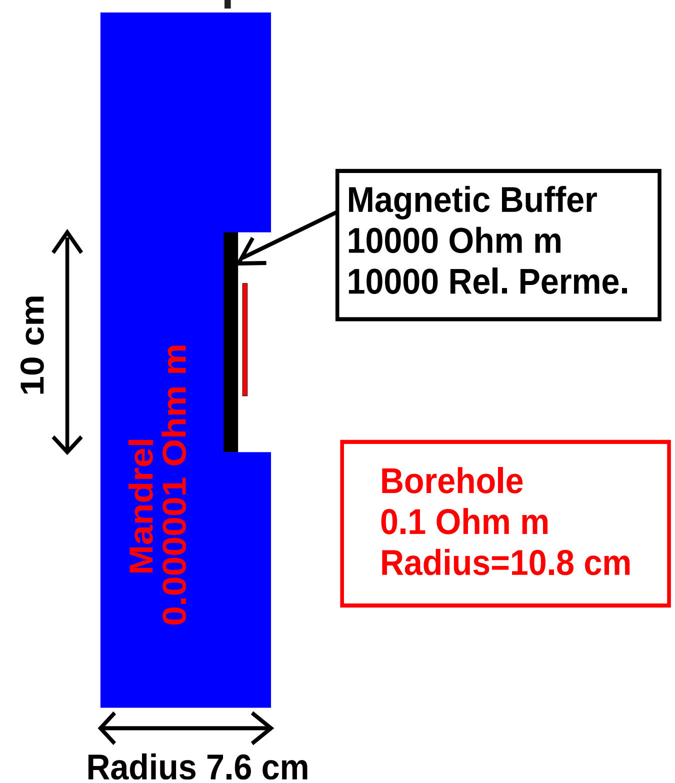
$\longrightarrow$



DISCRETIZATION



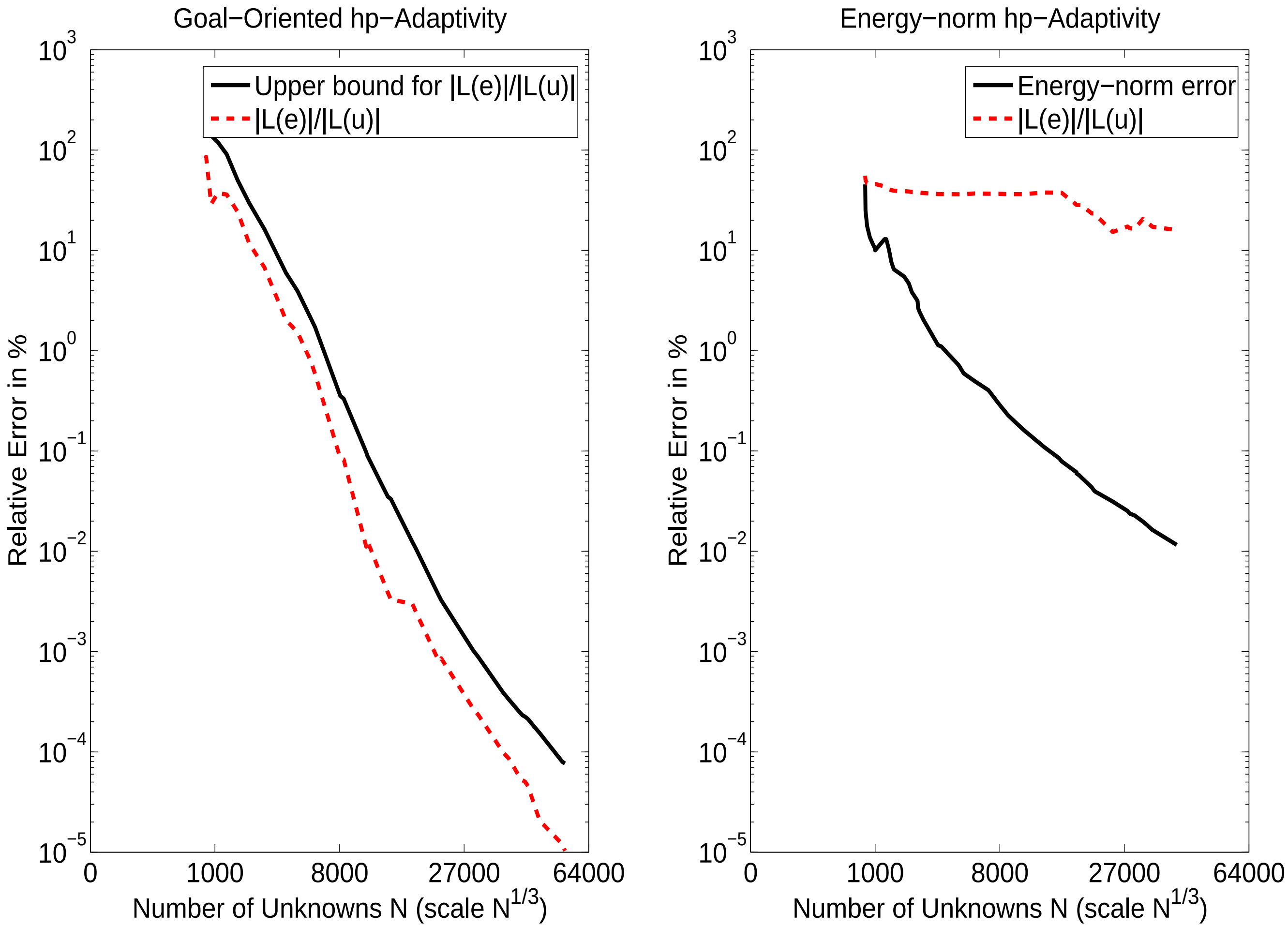
Description of Antennas



Goal: To Study the Effect of Invasion, Anistotropy, and Magnetic Permeability.

DISCRETIZATION

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m



DISCRETIZATION

Goal-Oriented vs. Energy-norm hp-Adaptivity

Problem with Mandrel at 2 Mhz.

Continuous Elements (Goal-Oriented Adaptivity)

Quantity of Interest	Real Part	Imag Part
COARSE GRID	-0.1629862203E-01	-0.4016944732E-02
FINE GRID	-0.1629862347E-01	-0.4016944223E-02

Continuous Elements (Energy-Norm Adaptivity)

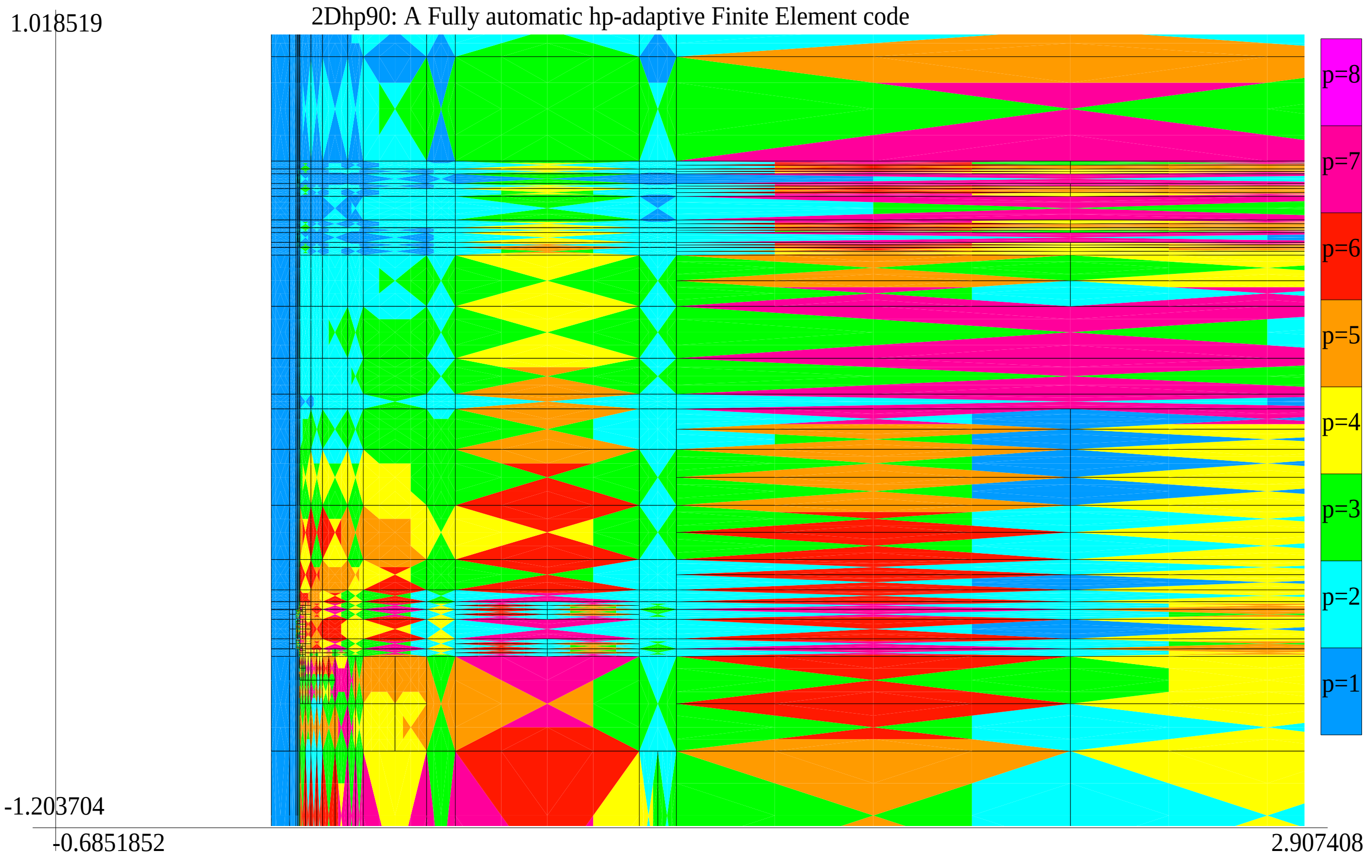
Quantity of Interest	Real Part	Imag Part
0.01% ENERGY ERROR	-0.1382759158E-01	-0.2989492851E-02

It is critical to use GOAL-ORIENTED adaptivity.

DISCRETIZATION

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

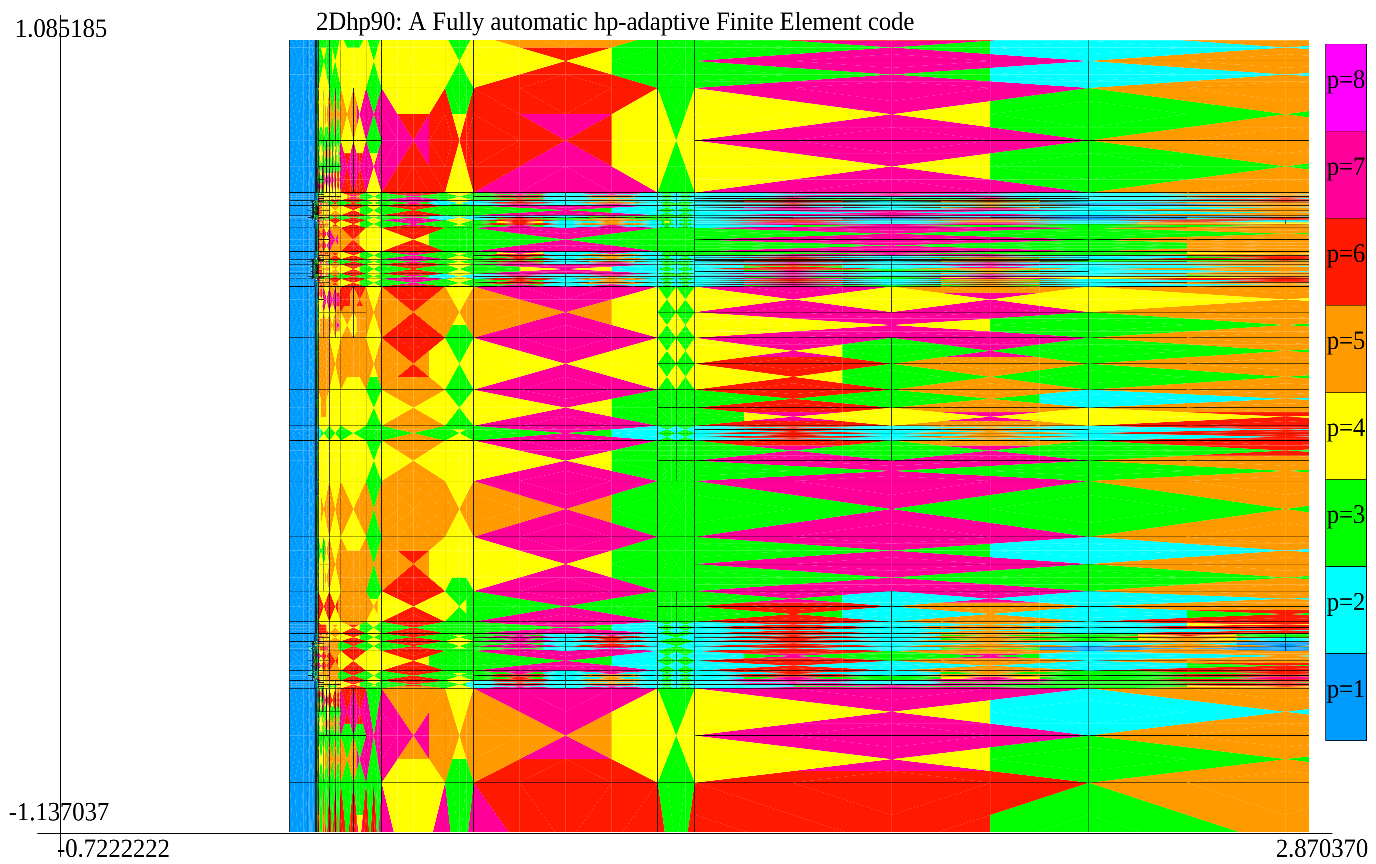
ENERGY-NORM HP-ADAPTIVITY



DISCRETIZATION

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

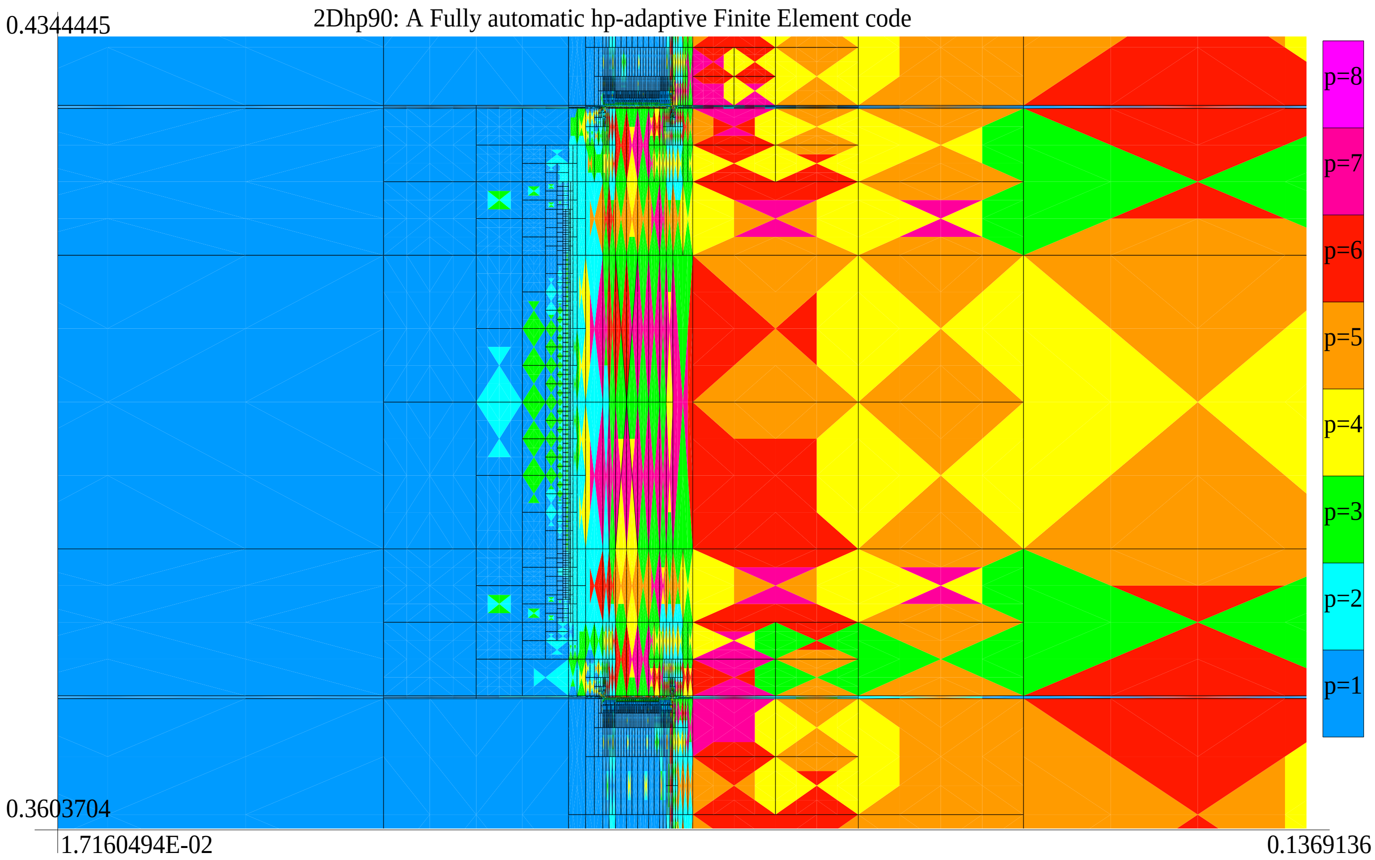
GOAL-ORIENTED HP-ADAPTIVITY



DISCRETIZATION

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

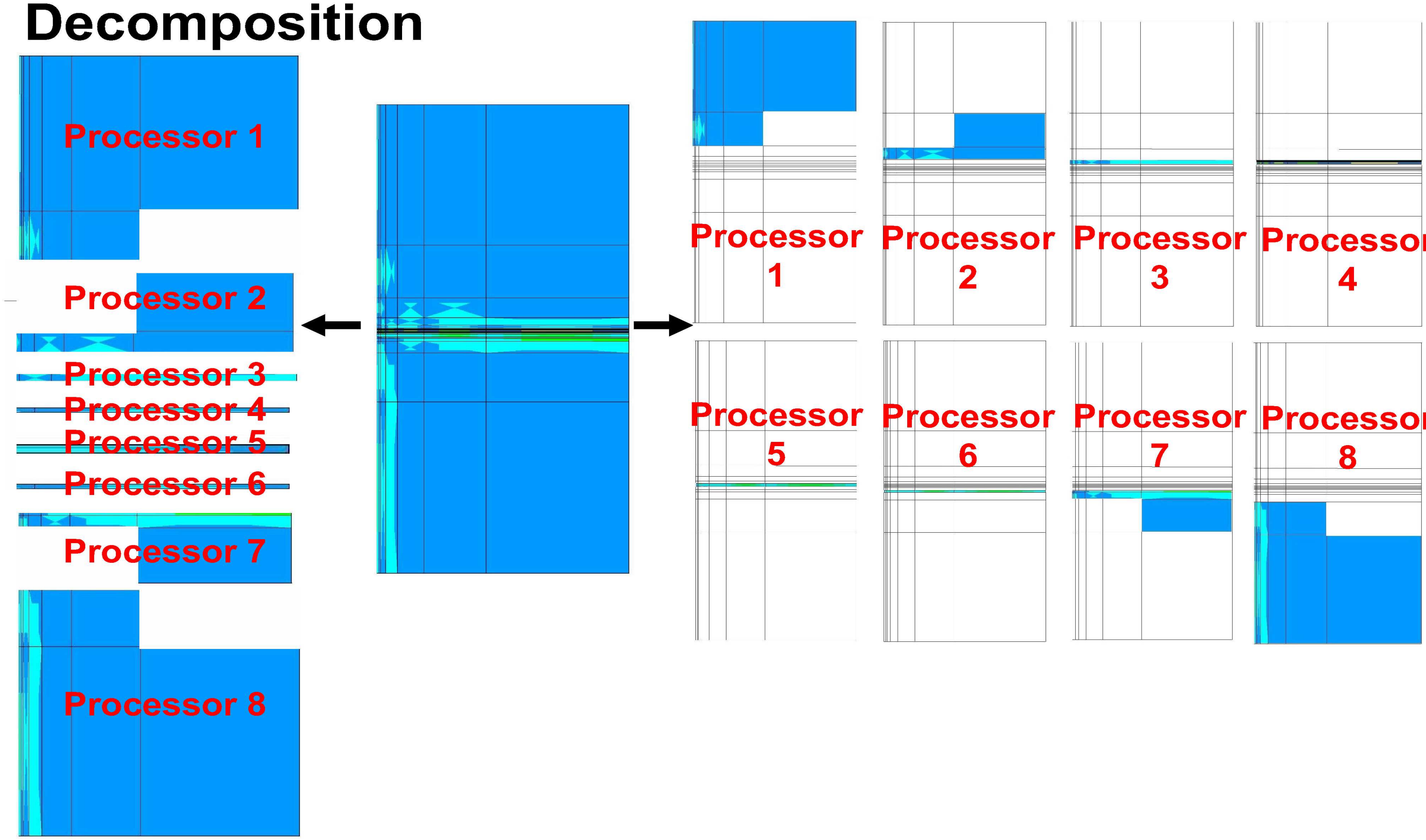
GOAL-ORIENTED HP-ADAPTIVITY (ZOOM TOWARDS FIRST RECEIVER ANTENNA)



PARALLEL IMPLEMENTATION

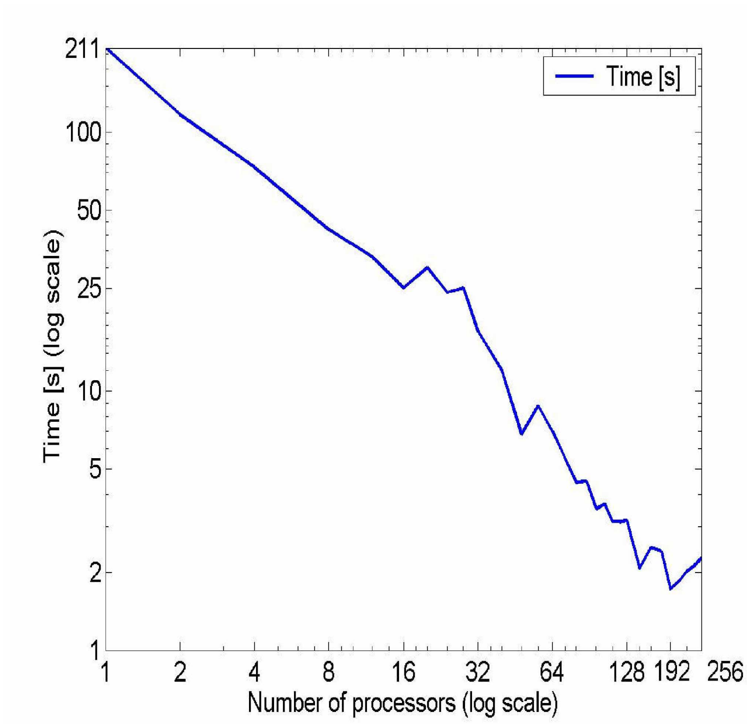
We Use Shared Domain Decomposition

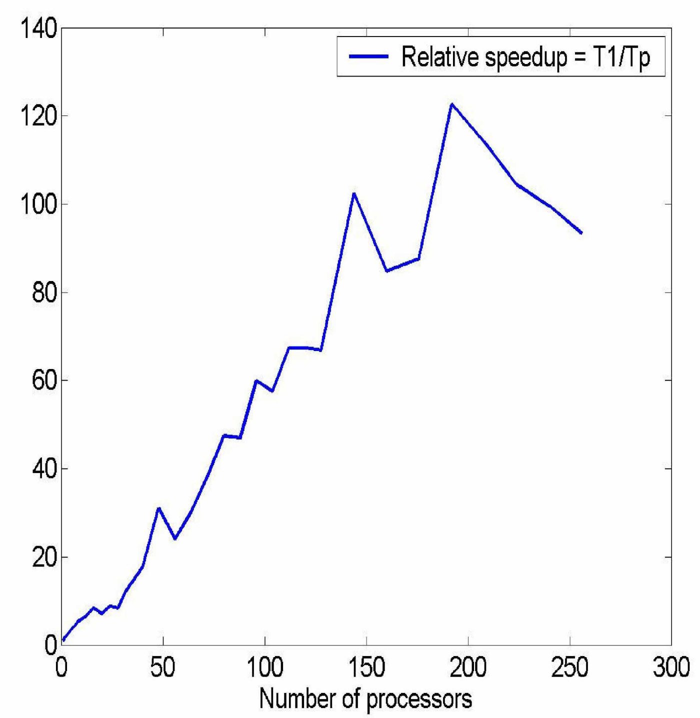
Distributed Domain Decomposition **Shared Domain Decomposition**



PARALLEL IMPLEMENTATION

Scalability of the Parallel Multi-Frontal Solver





Parallel computations performed on Texas Advanced Computing Center (TACC) 60 % relative efficiency up to 200 processors.

Parallel solver is 125 times faster on 200 processors.

CONCLUSIONS

- **A Fourier-Finite-Element method provides a suitable formulation for most frequency domain EM geophysical applications.**
- **The Fourier-Finite-Element method leads to discretizations based on mixed ($H(\mathrm{curl})$ and H^1) spaces.**
- **Spectral methods combined with adaptive refinements enable exponential rates of convergence.**
- **Goal-oriented refinements are essential in EM geophysical applications due to the dissipative nature of the earth.**
- **A parallel implementation based on a shared domain-decomposition is simple and provides acceptable scalability (over 50%) for a moderate number of processors.**

ACKNOWLEDGMENTS

Sponsors of UT Austin's consortium on Formation Evaluation

Anadarko Petroleum Corporation
ارامكو السعودية Saudi Aramco
Baker Hughes Baker Atlas
bhpbilliton
bp
Chevron
ConocoPhillips
Agip Eni
ExxonMobil
HALLIBURTON
IMP Innovar INSTITUTO MEXICANO DEL PETRÓLEO
Marathon Marathon Oil Corporation
BR PETROBRAS
Schlumberger
Shell
StatoilHydro
TOTAL
Weatherford