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A Parallel, Fourier Finite-Element Formulation with an Iterative Solver for the Simulation of 3D LWD Measurements Acquired in Deviated Wells

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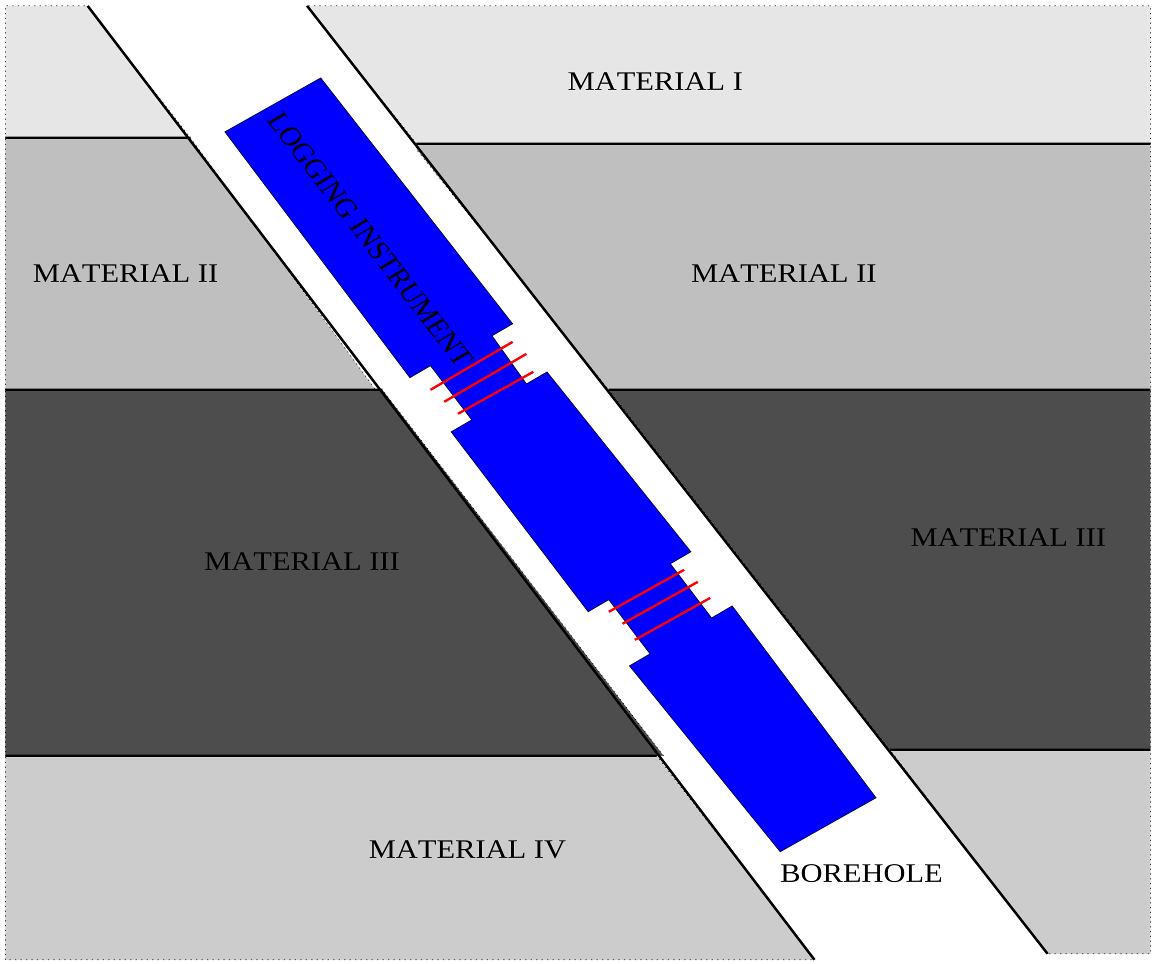
OVERVIEW

1. Motivation: Simulation of logging-while-drilling (LWD) measurements acquired in deviated wells.
2. Method:
   - Fourier-finite-element formulation in a non-orthogonal system of coordinates.
   - Goal-oriented self-adaptive hp-FE method.
3. Iterative solver.
4. Numerical simulation of LWD measurements.
5. Conclusions and future work.

MOTIVATION (LOGGING-WHILE-DRILLING)

Induction Instrument in a Deviated Well



Goal: Determine EM field at the receiver antennas.

FOURIER-FINITE-ELEMENT FORMULATION

3D Variational Formulation

Time-Harmonic Maxwell's Equations

$\nabla \times \mathrm{H} = \mathring{\sigma}\mathrm{E} + \mathrm{J}^{imp}$	**Ampere's law** ($\mathring{\sigma} = \sigma + j\omega\epsilon$)
$\nabla \times \mathrm{E} = \mathring{\mu}\mathrm{H} + \mathrm{M}^{imp}$	**Faraday's law** ($\mathring{\mu} = -j\omega\mu$)
$\nabla \cdot (\epsilon \mathrm{E}) = \rho$	**Gauss' law of Electricity**
$\nabla \cdot (\mu \mathrm{H}) = 0$	**Gauss' law of Magnetism**

E-VARIATIONAL FORMULATION:

$$\begin{cases} \text{Find } \mathrm{E} \in \mathrm{E}_{\Gamma_E} + H_{\Gamma_E}(\mathbf{curl};\Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \nabla \times \mathrm{E} \right\rangle_{L^2(\Omega)} - \left\langle \mathrm{F}, \mathring{\sigma}\mathrm{E} \right\rangle_{L^2(\Omega)} = \left\langle \mathrm{F}, \mathrm{J}^{imp} \right\rangle_{L^2(\Omega)} \\ \quad - \left\langle \mathrm{F}_t, \mathrm{J}^{imp}_{\Gamma_H} \right\rangle_{L^2(\Gamma_H)} + \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \mathrm{M}^{imp} \right\rangle_{L^2(\Omega)} \quad \forall\, \mathrm{F} \in H_{\Gamma_E}(\mathbf{curl};\Omega) \end{cases}$$

FOURIER-FINITE-ELEMENT FORMULATION

E-Variational Formulations (Cylindrical Coordinates)

FINITE ELEMENT —3D—:

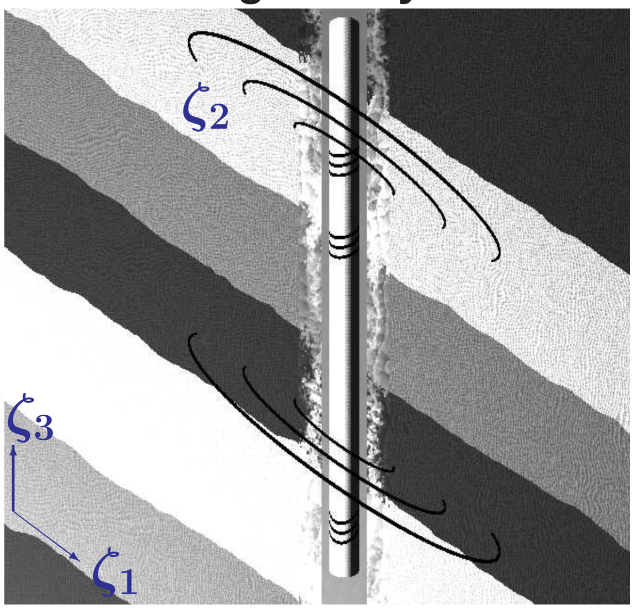
$$\begin{cases} \text{Find } \mathrm{E} \in \mathrm{E}_{\Gamma_E} + H_{\Gamma_E}(\mathrm{curl};\Omega) \text{ such that:} \\ \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \nabla \times \mathrm{E} \right\rangle_{L^2(\Omega)} - \left\langle \mathrm{F}, \mathring{\sigma} \mathrm{E} \right\rangle_{L^2(\Omega)} = \left\langle \mathrm{F}, \mathrm{J}^{imp} \right\rangle_{L^2(\Omega)} \\ \quad - \left\langle \mathrm{F}_t, \mathrm{J}^{imp}_{\Gamma_H} \right\rangle_{L^2(\Gamma_H)} + \left\langle \nabla \times \mathrm{F}, \mathring{\mu}^{-1} \mathrm{M}^{imp} \right\rangle_{L^2(\Omega)} \quad \forall\, \mathrm{F} \in H_{\Gamma_E}(\mathrm{curl};\Omega) \end{cases}$$

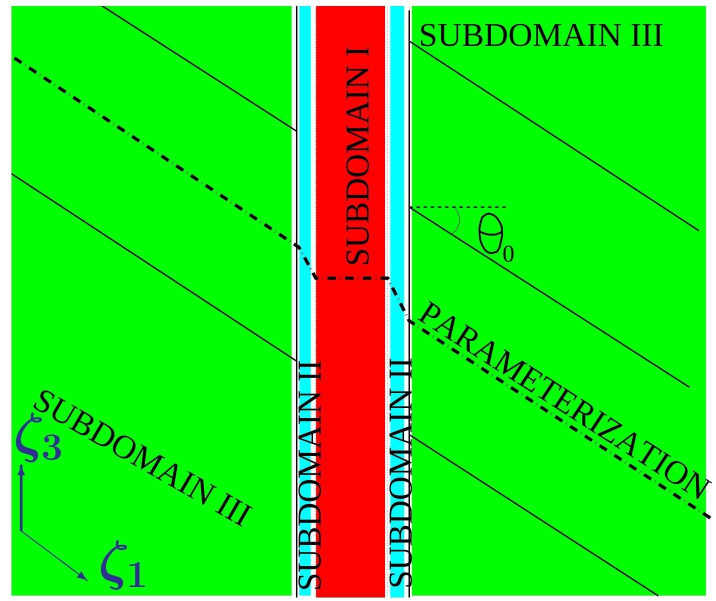
FOURIER FINITE ELEMENT —3D = Sequence of Coupled 2D Problems—:

$$\begin{cases} \text{Find } \mathrm{E} = \dfrac{1}{\sqrt{2\pi}} \displaystyle\sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E})\, e^{jn\phi}, \text{ where for each } n: \\ \mathcal{F}_n(\mathrm{E}) \in \mathcal{F}_n(\mathrm{E}_{\Gamma_{E,1D}}) + H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}), \text{ and} \\ \displaystyle\sum_{m=-\infty}^{\infty} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\mu}^{-1}) \nabla^m \times \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\sigma}) \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} \\ \quad = \left\langle \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_n(\mathrm{J}^{imp}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}_t) \,,\, \mathcal{F}_n(\mathrm{J}^{imp}_S) \right\rangle_{L^2(\Gamma_{H,1D})} \\ \quad + \displaystyle\sum_{m=-\infty}^{\infty} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\mu}^{-1}) \mathcal{F}_m(\mathrm{M}^{imp}) \right\rangle_{L^2(\Omega_{2D})} \quad \forall\, \mathcal{F}_n(\mathrm{F}) \in H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}) \end{cases}$$

FOURIER-FINITE-ELEMENT FORMULATION

Cartesian system of coordinates: $\mathrm{x} = (x, y, z)$.
New non-orthogonal system of coordinates: $\zeta = (\zeta_1, \zeta_2, \zeta_3)$.





Subdomain I ; **Subdomain II** ; **Subdomain III**

$$\begin{cases} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 \end{cases} ; \quad \begin{cases} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 + \tan \theta_0 \dfrac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \end{cases} ; \quad \begin{cases} x = \zeta_1 \cos \zeta_2 \\ y = \zeta_1 \sin \zeta_2 \\ z = \zeta_3 + \tan \theta_0 \zeta_1 \end{cases}$$

FOURIER-FINITE-ELEMENT FORMULATION

E-Variational Formulation in the New System of Coordinates ζ

In the new system of coordinates, we obtain:

3D FOURIER FINITE ELEMENT FORMULATION
— Sequence of "Weakly" Coupled 2D Problems —

$$\left\{ \begin{array}{l} \text{Find } \mathrm{E} = \dfrac{1}{\sqrt{2\pi}} \displaystyle\sum_{n=-\infty}^{\infty} \mathcal{F}_n(\mathrm{E})\, e^{jn\zeta_2}, \text{ where for each } n\text{:} \\ \mathcal{F}_n(\mathrm{E}) \in \mathcal{F}_n(\mathrm{E}_{\Gamma_{E,1D}}) + H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}), \text{ and} \\ \displaystyle\sum_{m=-2}^{2} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\mu}_{mod}^{-1}) \nabla^m \times \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\sigma}_{mod}) \mathcal{F}_m(\mathrm{E}) \right\rangle_{L^2(\Omega_{2D})} \\ \quad = \left\langle \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_n(\mathrm{J}^{imp}) \right\rangle_{L^2(\Omega_{2D})} - \left\langle \mathcal{F}_n(\mathrm{F}_t) \,,\, \mathcal{F}_n(\mathrm{J}_S^{imp}) \right\rangle_{L^2(\Gamma_{H,1D})} \\ \quad + \displaystyle\sum_{m=-2}^{2} \left\langle \nabla^n \times \mathcal{F}_n(\mathrm{F}) \,,\, \mathcal{F}_{n-m}(\mathring{\mu}_{mod}^{-1}) \mathcal{F}_m(\mathrm{M}^{imp}) \right\rangle_{L^2(\Omega_{2D})} \quad \forall\, \mathcal{F}_n(\mathrm{F}) \in H_{\Gamma_{E,1D}}(\mathrm{curl}^n;\Omega_{2D}) \end{array} \right.$$

Five Fourier modes are sufficient to represent EXACTLY the new material coefficients resulting from incorporating the change of coordinates.

FOURIER-FINITE-ELEMENT FORMULATION

Main Advantages

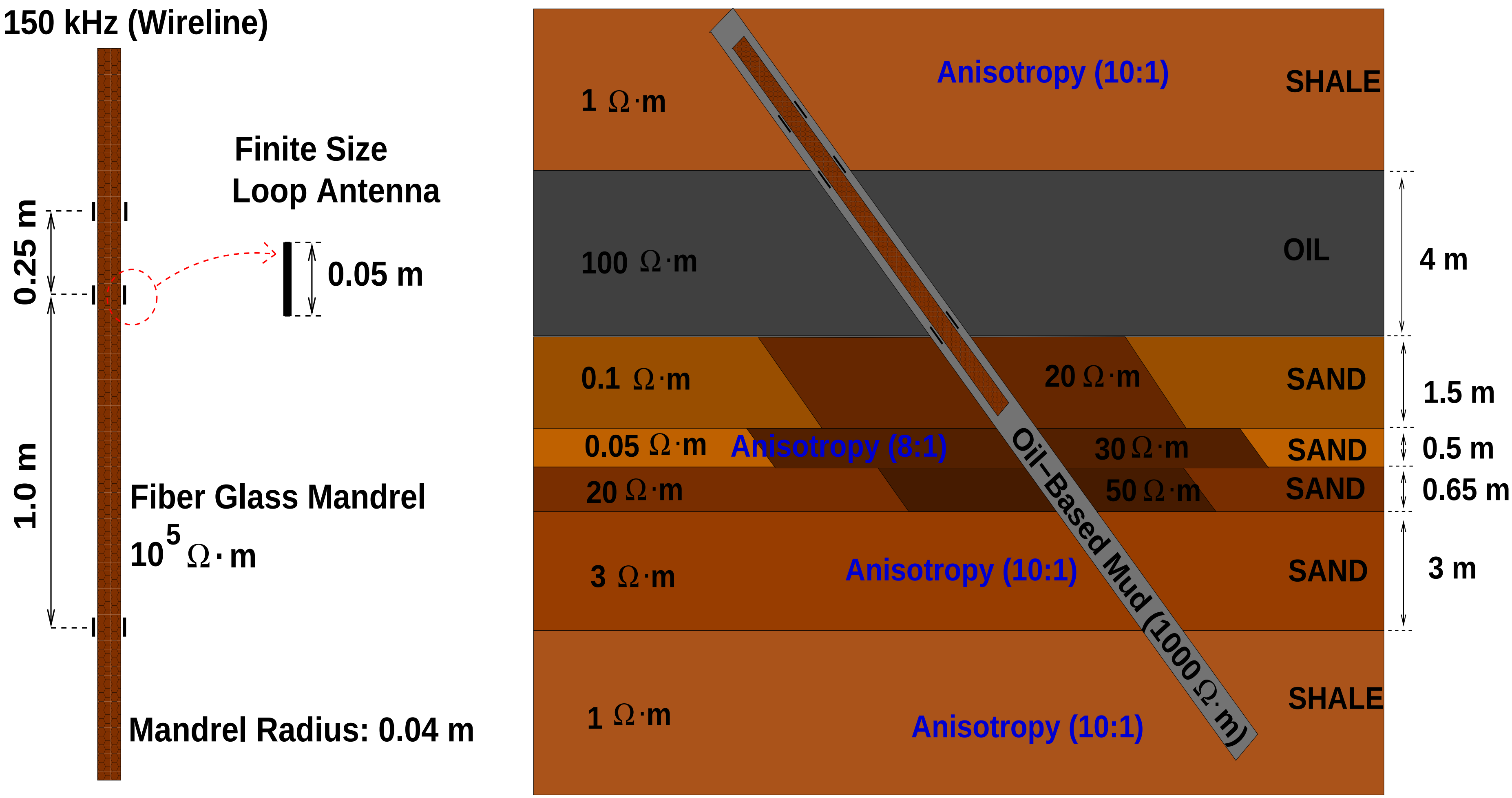
Using this non-orthogonal system of coordinates, we obtain:

- **Subdomain I :** $\mathcal{F}_{n-m}(\mathring{\mu}^{-1}) = \mathcal{F}_{n-m}(\mathring{\sigma}^{-1}) = 0 \quad \forall \ |n-m| > 0$ **(uncoupled 2D problems).**
- **Subdomain II :** $\mathcal{F}_{n-m}(\mathring{\mu}^{-1}) = \mathcal{F}_{n-m}(\mathring{\sigma}^{-1}) = 0 \quad \forall \ |n-m| > 2$ **(penta-diagonal coupling).**
- **Subdomain III :** $\mathcal{F}_{n-m}(\mathring{\mu}^{-1}) = \mathcal{F}_{n-m}(\mathring{\sigma}^{-1}) = 0 \quad \forall \ |n-m| > 1$ **(tri-diagonal coupling).**

SEVERAL SYSTEMS OF COORDINATES THAT LEAD TO A SIGNIFICANT COMPLEXITY REDUCTION CAN BE CONSTRUCTED FOR MANY GEOMETRIES ARISING IN CHALLENGING GEOPHYSICAL APPLICATIONS.

FOURIER-FINITE-ELEMENT: ILLUSTRATION

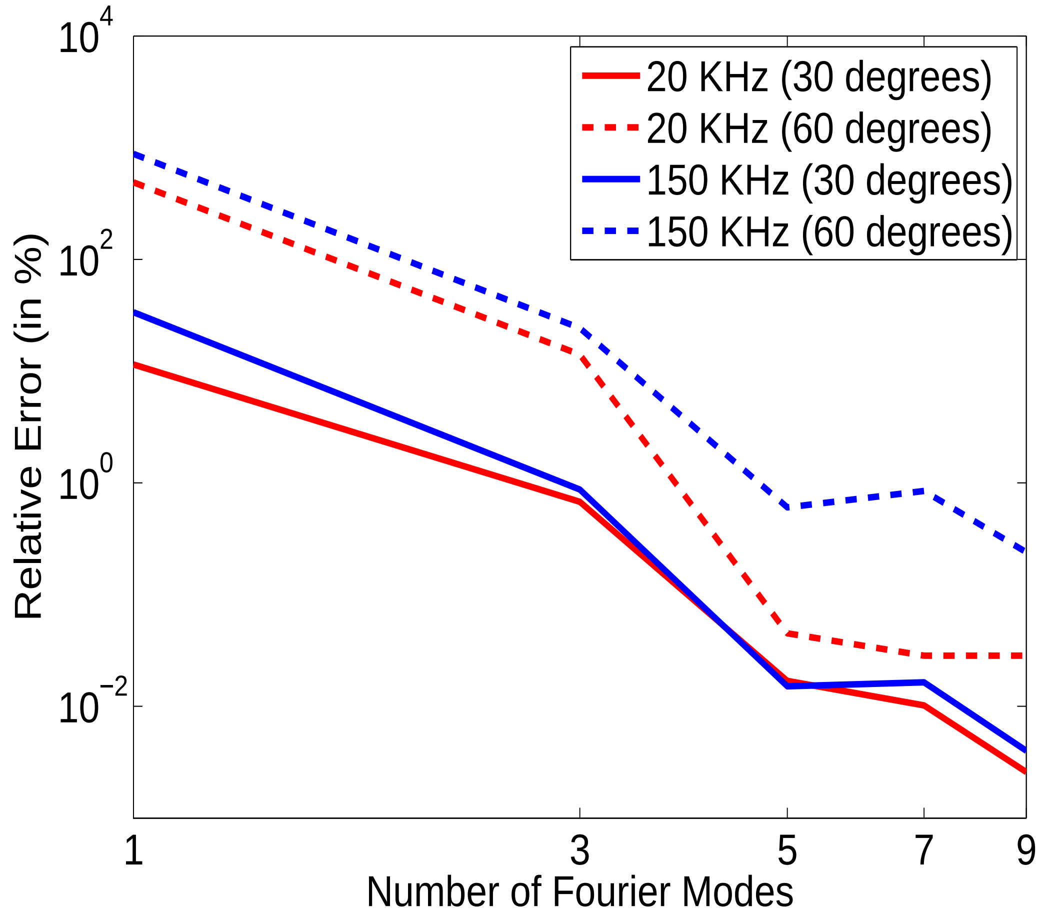
Model Problem



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

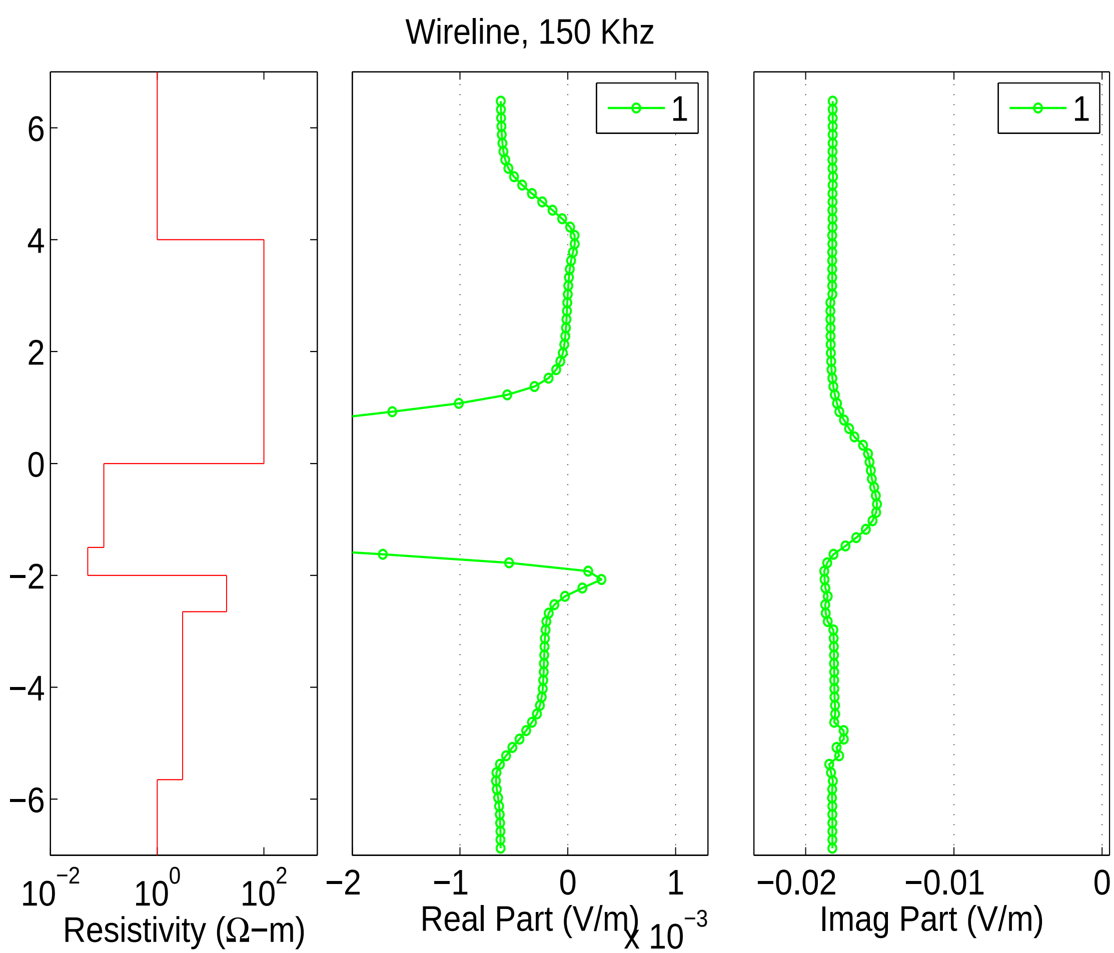
Logging Instrument in a Homogeneous Formation



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

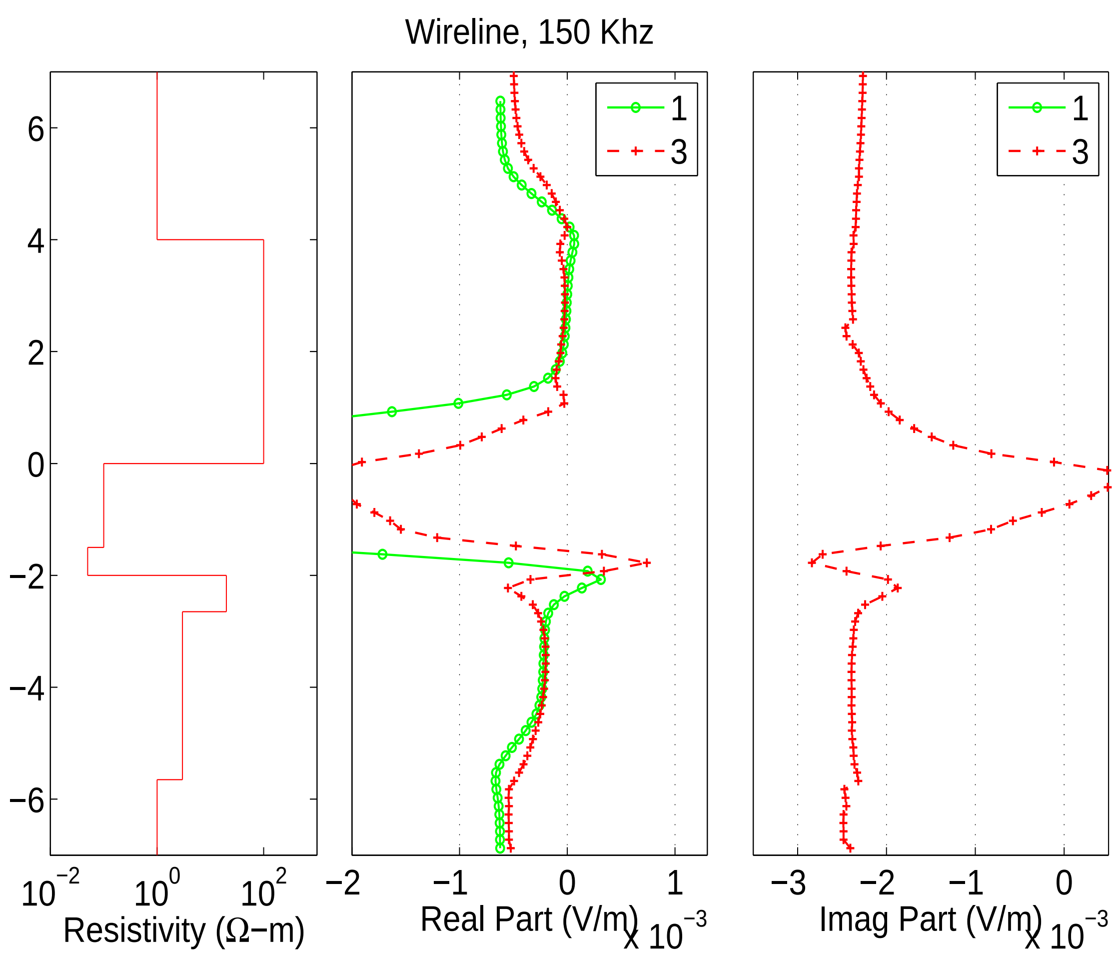
Logging Instrument in a Homogeneous Formation



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

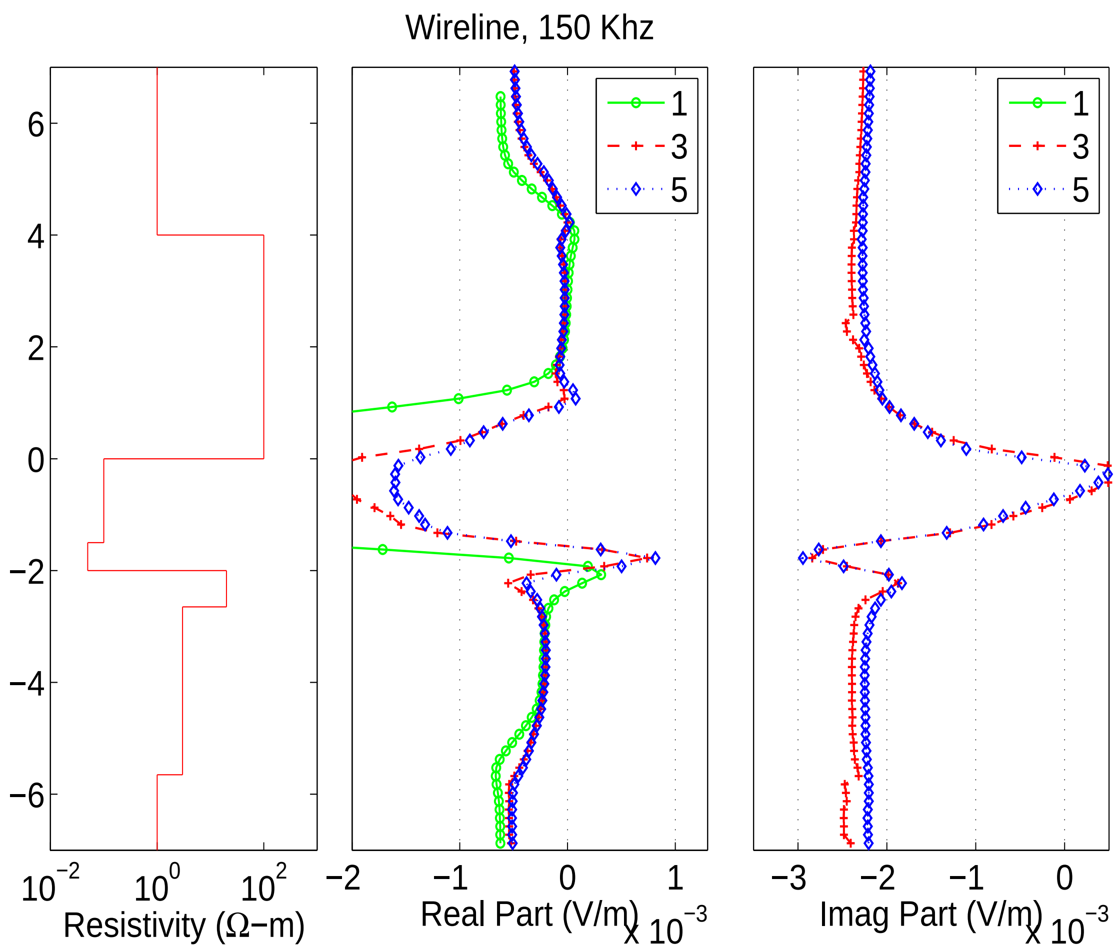
Logging Instrument in a Homogeneous Formation



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

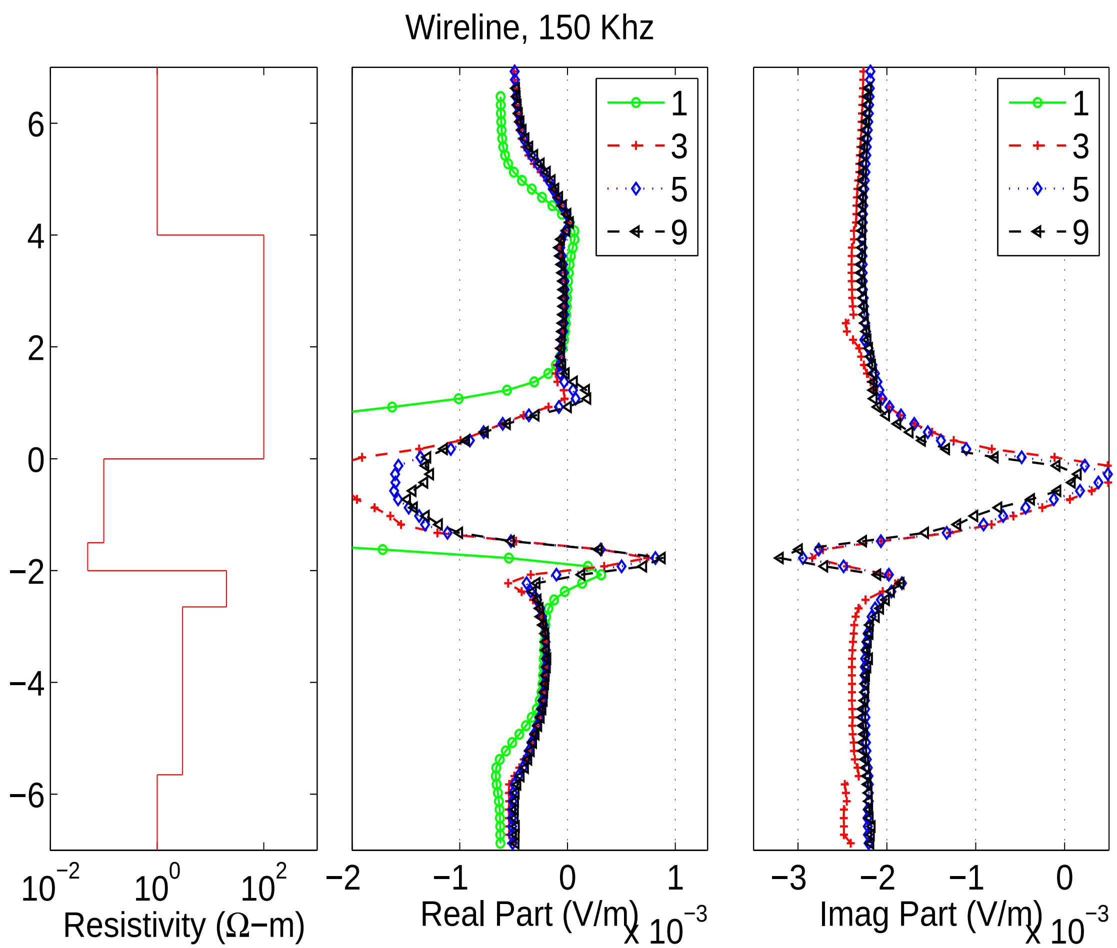
Logging Instrument in a Homogeneous Formation



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

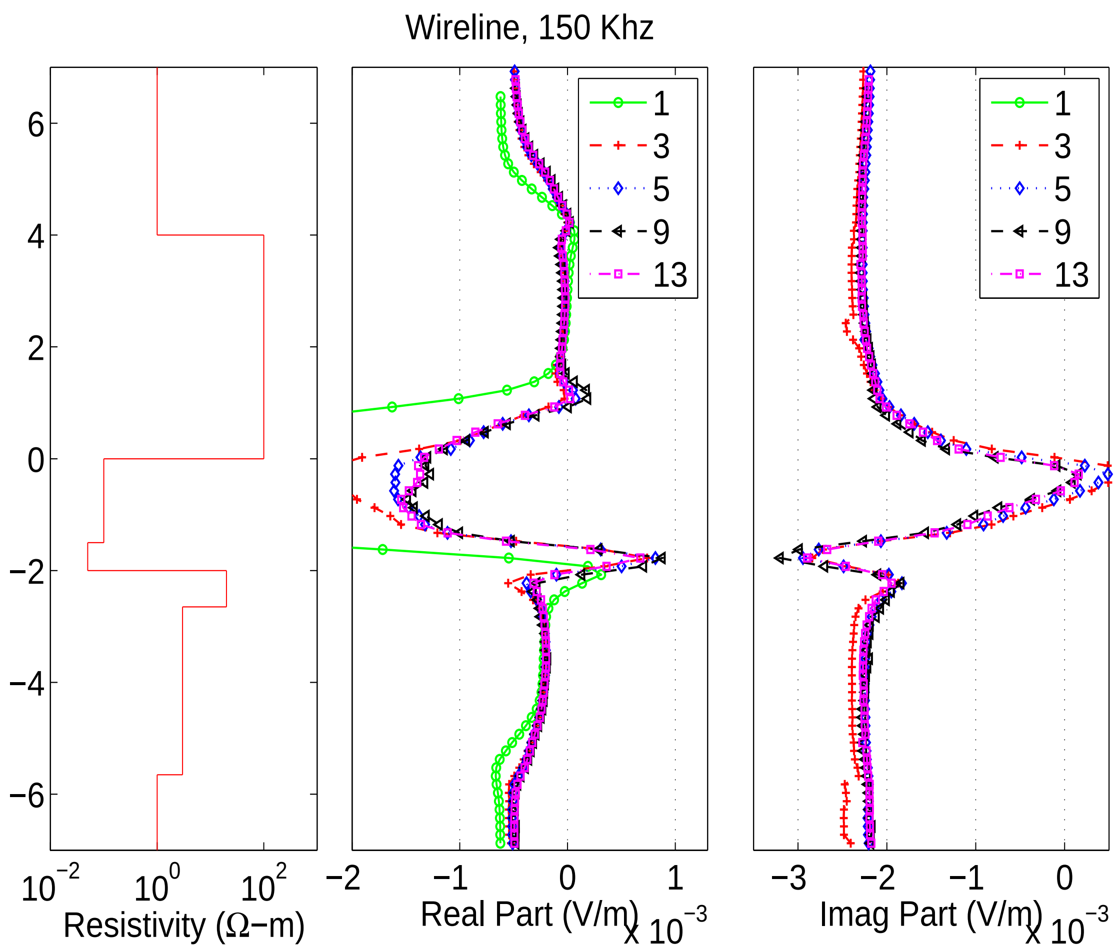
Logging Instrument in a Homogeneous Formation



FOURIER-FINITE-ELEMENT: ILLUSTRATION

Verification

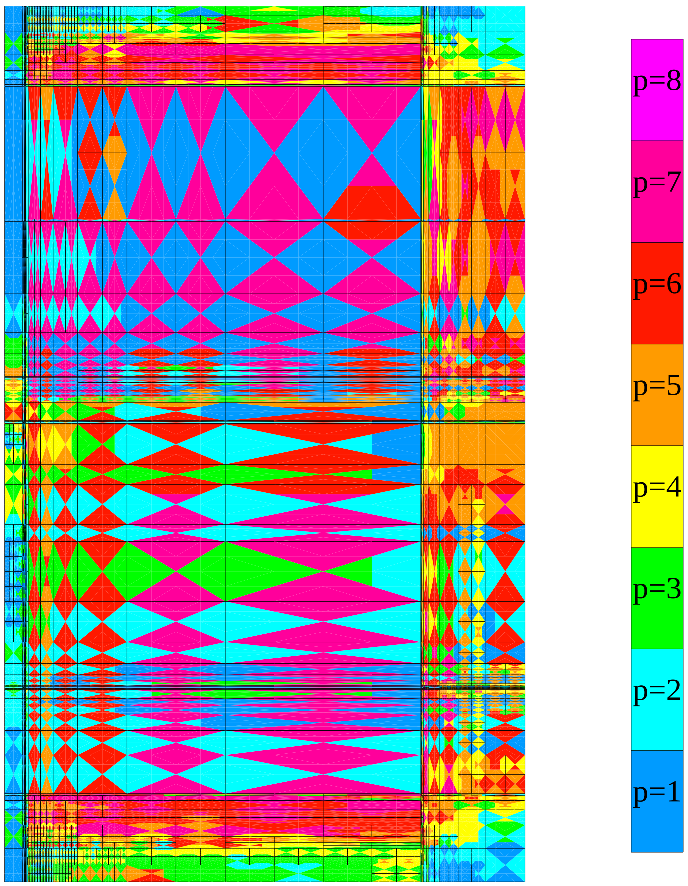
Logging Instrument in a Homogeneous Formation



SELF-ADAPTIVE GOAL-ORIENTED HP-FEM

A Self-Adaptive Goal-Oriented hp-FEM

Optimal 2D Grid
(Through Casing Resistivity Problem)



We vary locally the element size h and the polynomial order of approximation p throughout the grid.

Optimal grids are automatically generated by the computer.

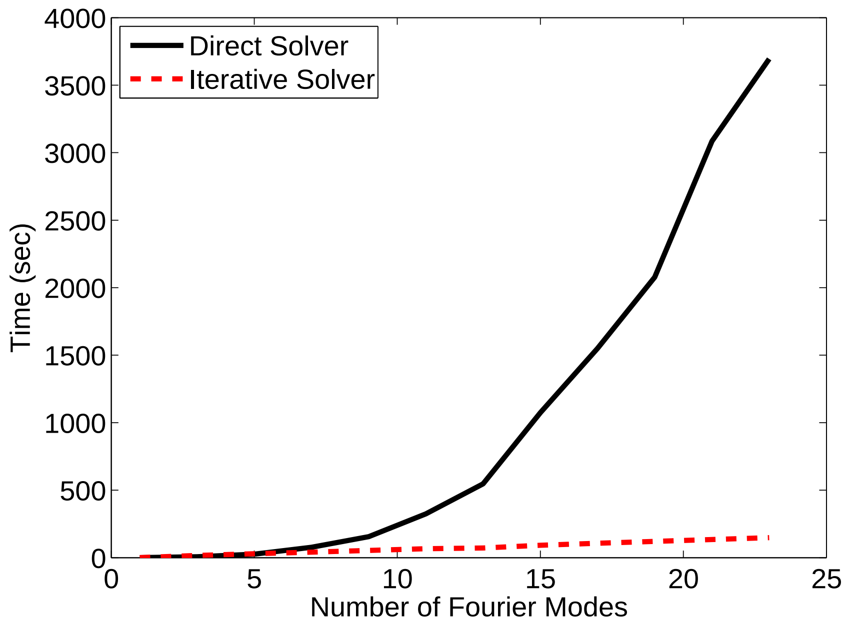
The self-adaptive goal-oriented hp-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.

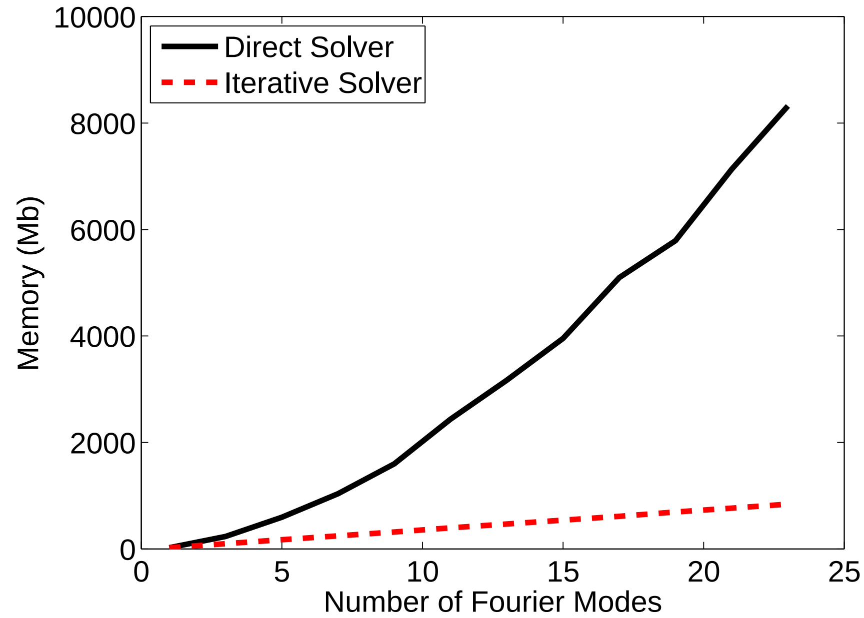
ITERATIVE SOLVER

Description and Performance of the Iterative Solver

Block-Jacobi Preconditioner + Krylov subspace optimization method (CG or GMRES).

The block Jacobi preconditioner consists of a 2.5D problem defined by ignoring the couplings ocurring between the different Fourier basis functions in the original problem.

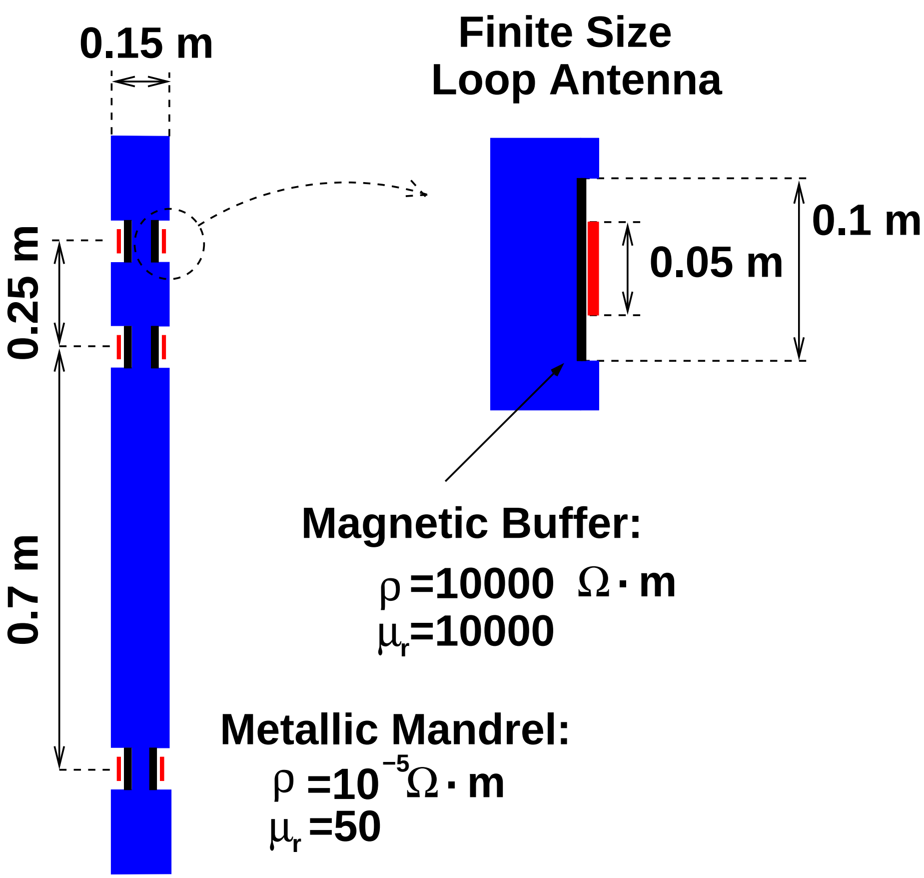


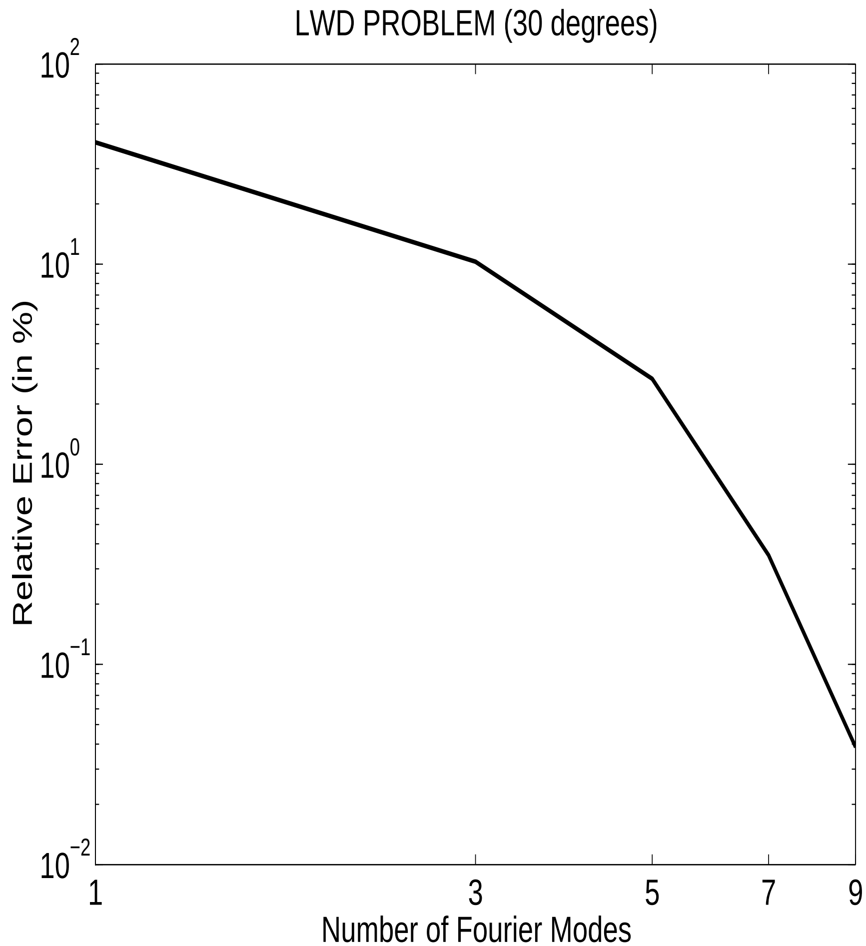


This simple iterative solver enables fast computations.

NUMERICAL RESULTS: LWD

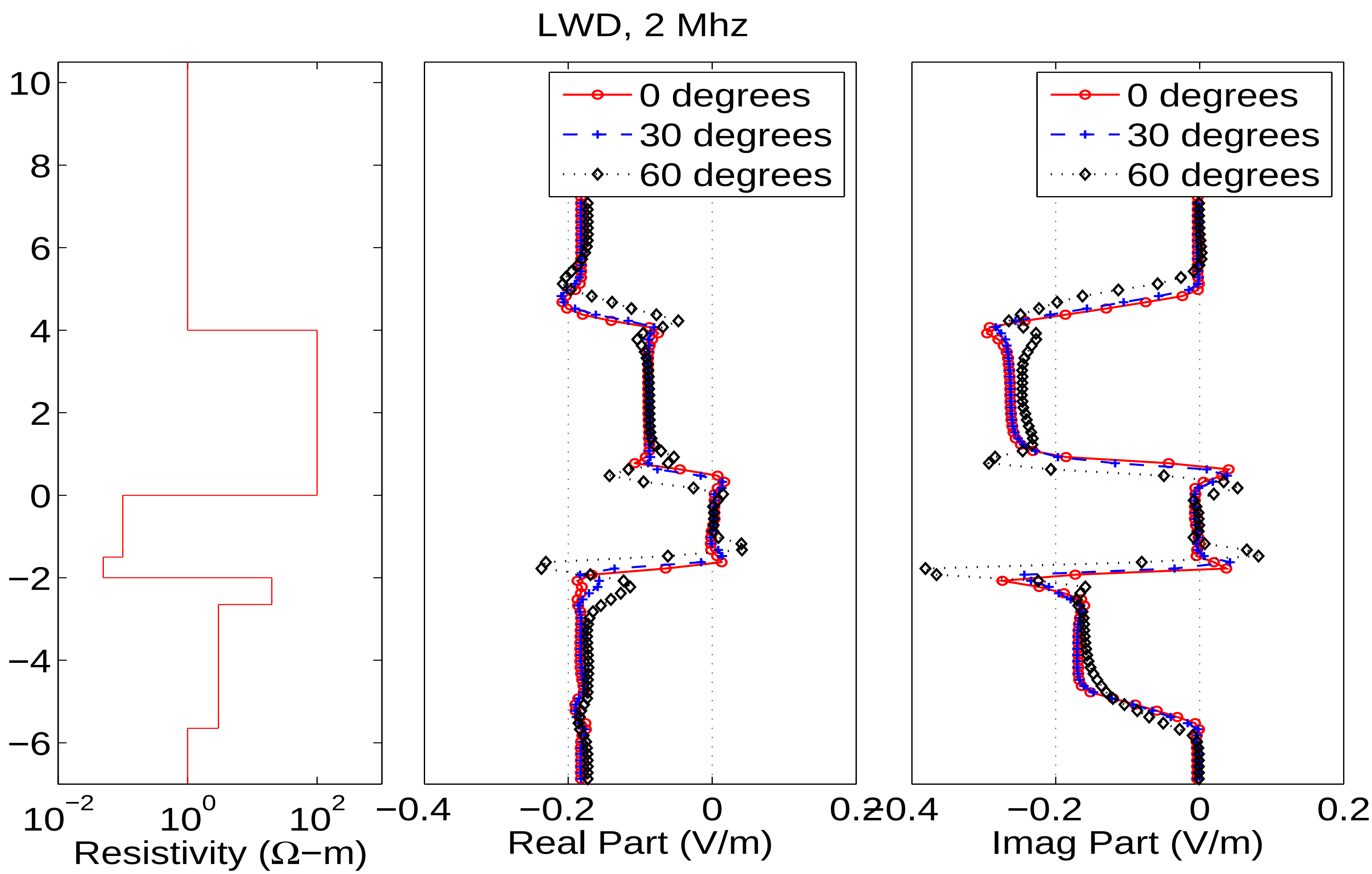
Model Problem and Verification





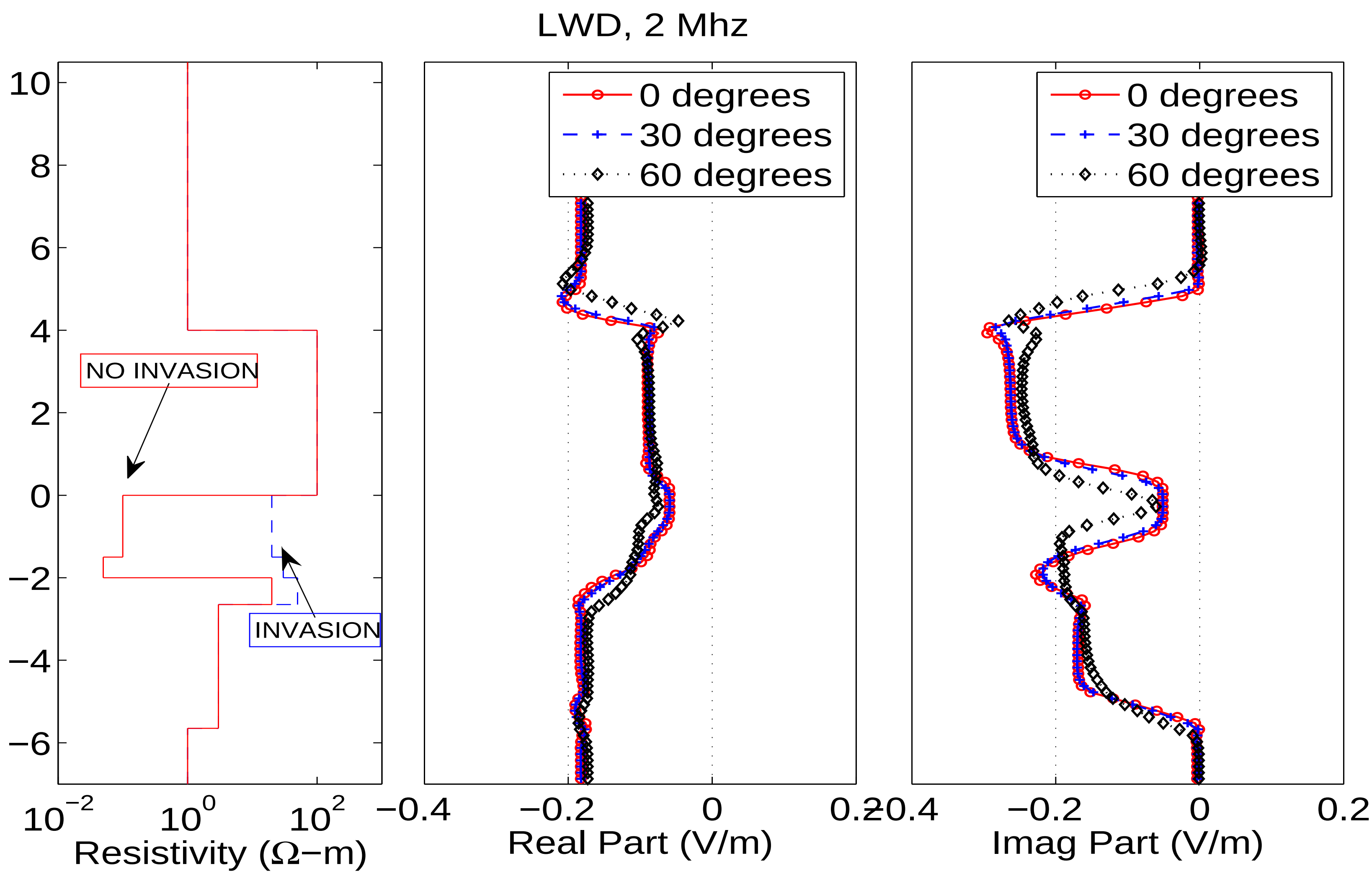
NUMERICAL RESULTS: LWD

Dip Angle



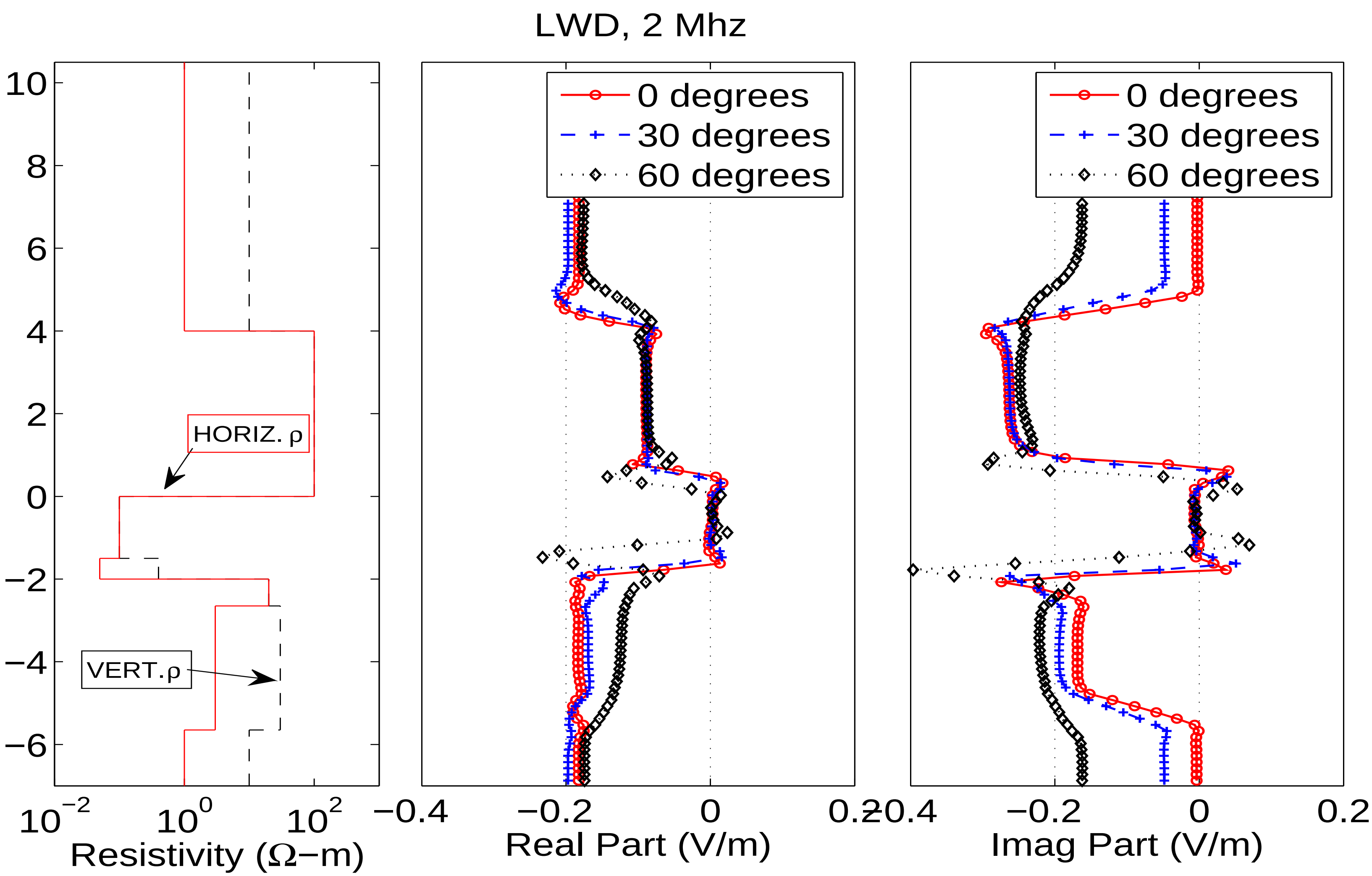
NUMERICAL RESULTS: LWD

Dip Angle + Invasion



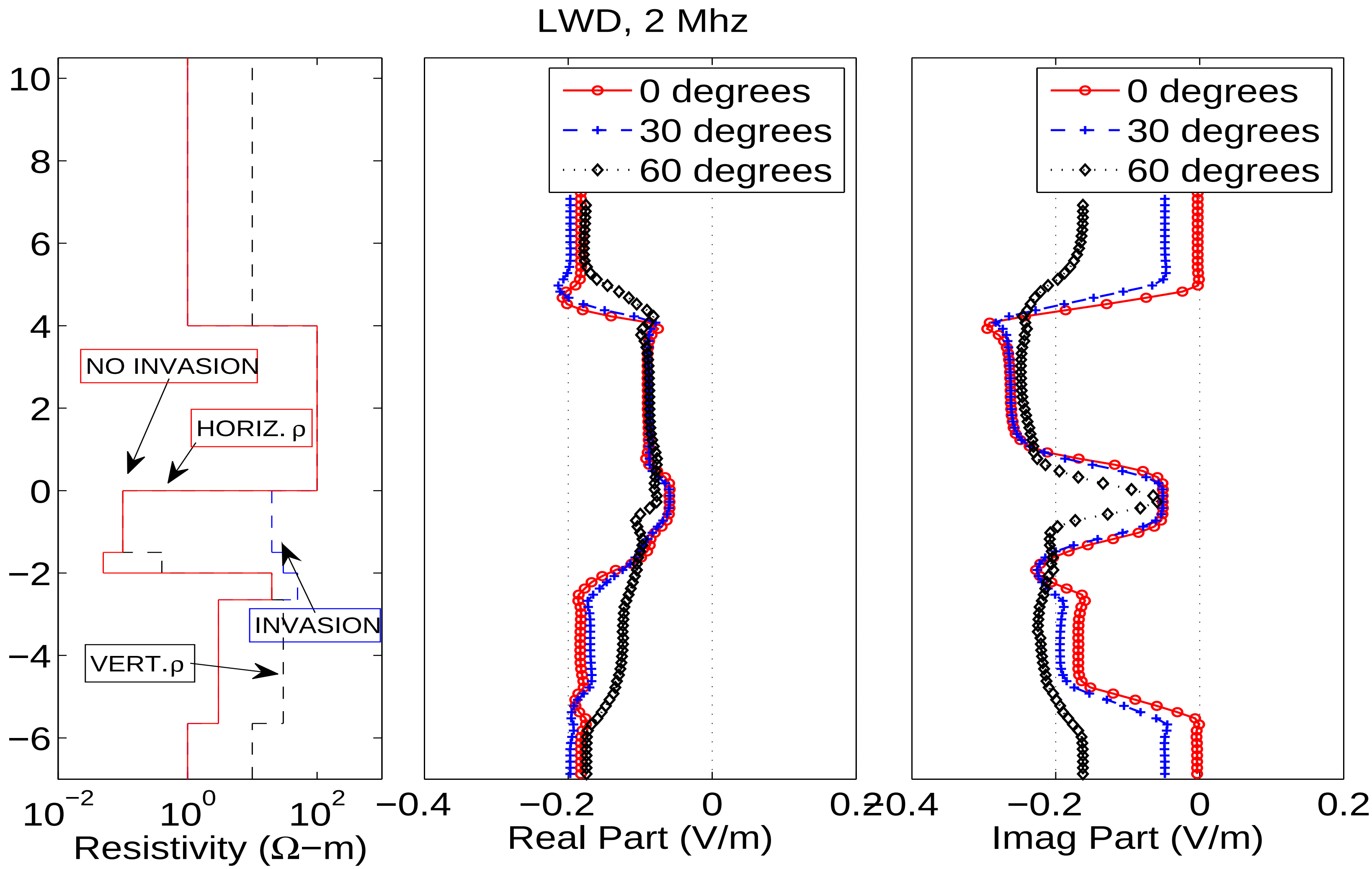
NUMERICAL RESULTS: LWD

Dip Angle + Anisotropy



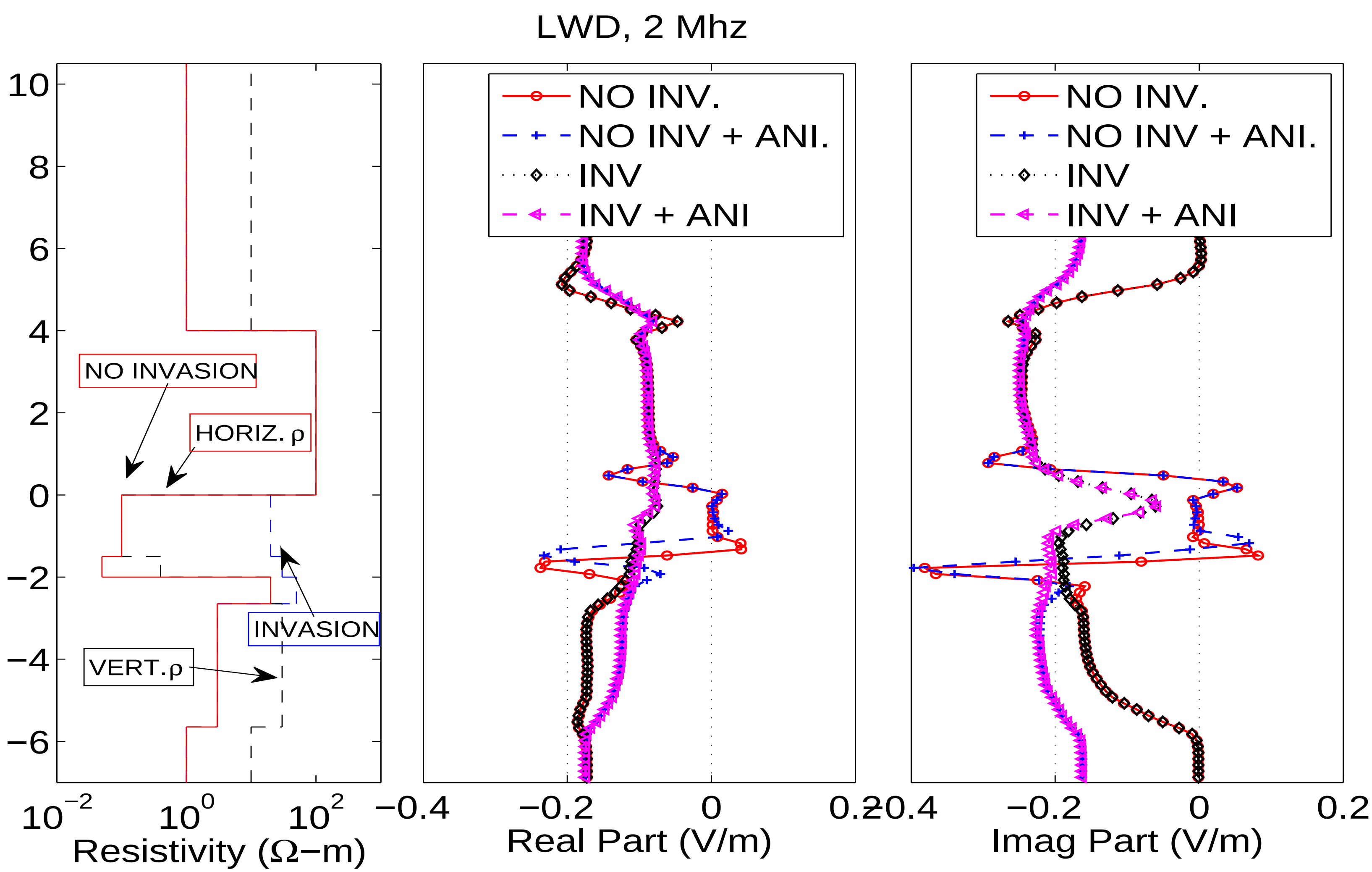
NUMERICAL RESULTS: LWD

Dip Angle + Invasion + Anisotropy



NUMERICAL RESULTS: LWD

60-Degree Deviated Well



CONCLUSIONS AND FUTURE WORK

Conclusions

- A Fourier-Finite-Element method in a non-orthogonal system of coordinates provides enhanced performance for simulating logging-while-drilling measurements in deviated wells.
- A simple iterative solver based on solving exactly a 2.5D problem provides excellent performance, even in presence of elongated elements and/or highly varying material properties.

Future Work

- Inversion
- Multi-physics
- Marine controlled source EM applications.

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