9th U.S. National Congress on Computational Mechanics

Non-Invasive Sensing of Subsurface Properties

Numerical Simulation of 3D Borehole Resistivity Measurements Using a Fourier Series Expansion in a Non-Orthogonal System of Coordinates

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OVERVIEW

1. Motivation: Resistivity logging measurements.

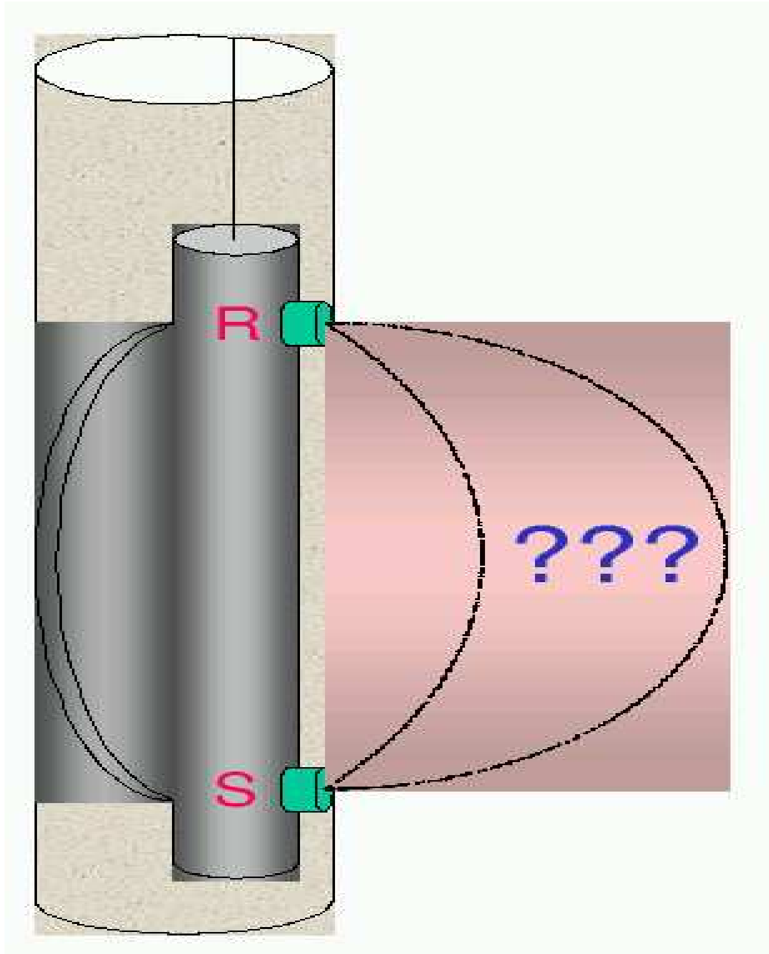
2. Methodology:
   - Fourier series expansion.
   - Non-orthogonal system of coordinates.
   - 2D goal-oriented self-adaptive hp-FEM.

3. Numerical results: 3D DC measurements in deviated wells.

4. Conclusions and future work.

MOTIVATION (APPLICATIONS)

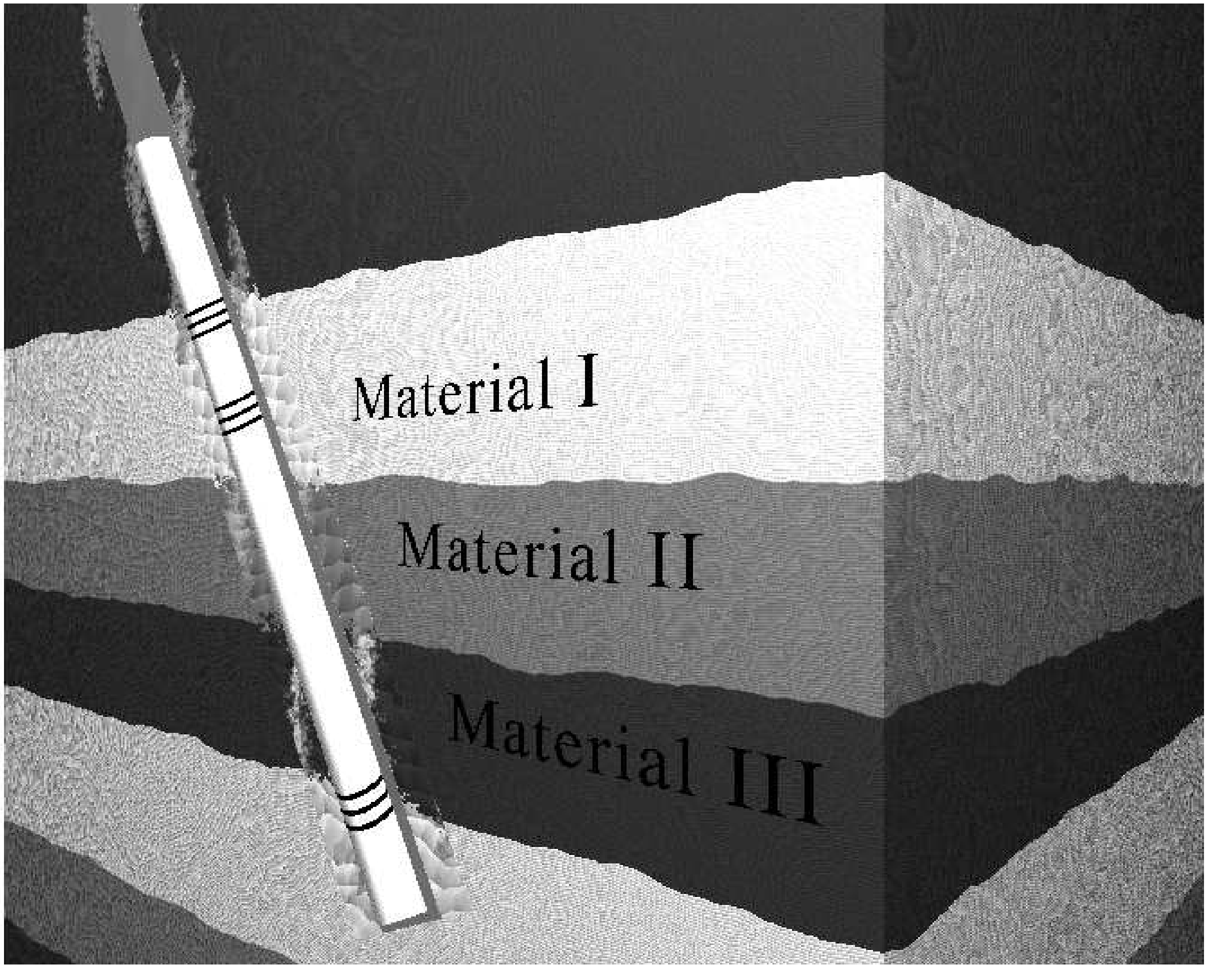
Main Objective: To Solve an Inverse Problem



A software for solving the DIRECT problem is essential in order to solve the INVERSE problem

MOTIVATION (APPLICATIONS)

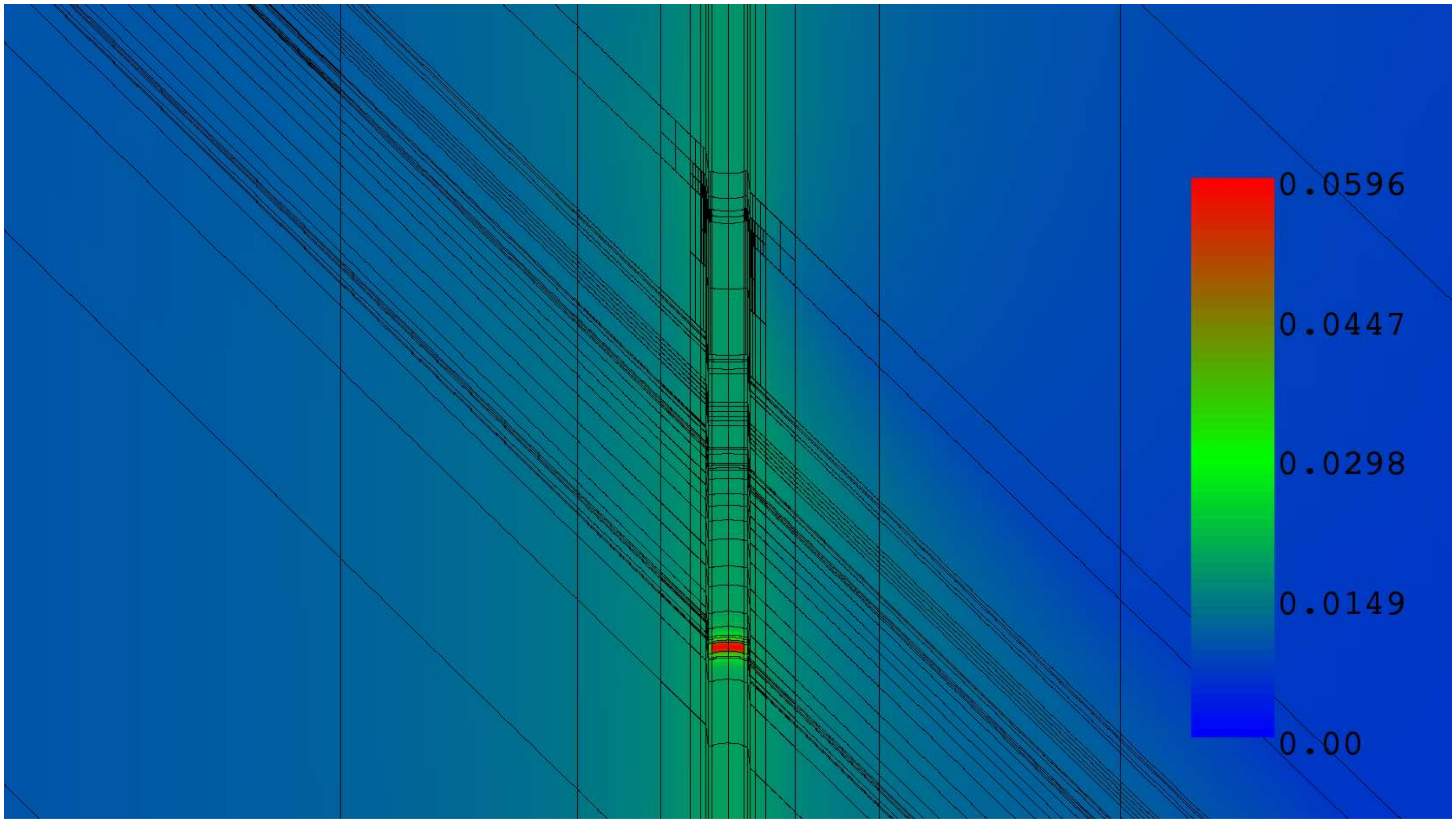
Deviated Wells (Forward Problem)



Objective: Find solution at the receiver antennas.

MOTIVATION (APPLICATIONS)

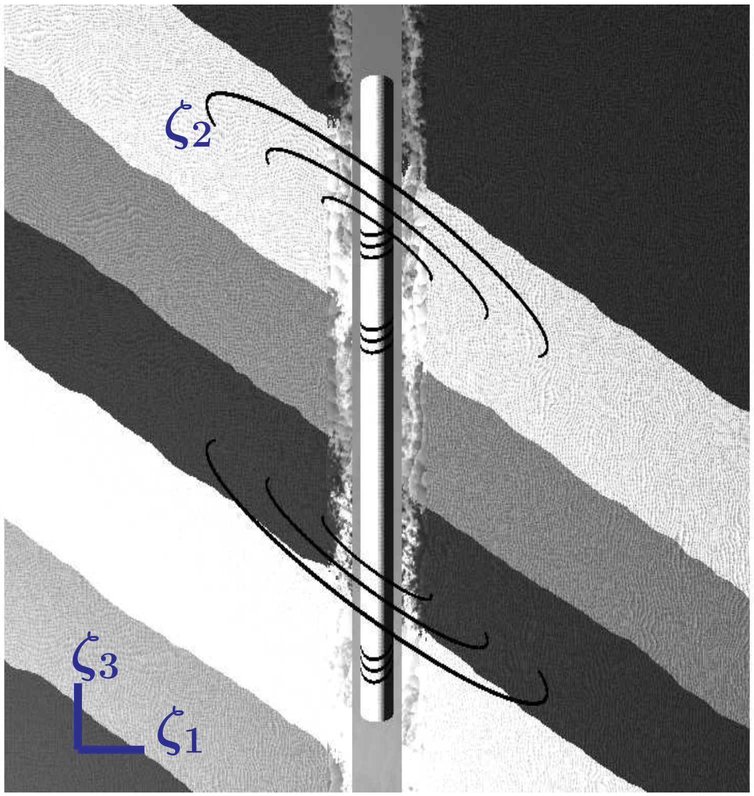
Example: Solution in a 60 degrees deviated well ($-\nabla \sigma \nabla u = f$)



Several hours to obtain one solution (3D forward simulation).
Several months needed to solve the inverse problem.

METHODOLOGY: MAIN IDEA

Non-Orthogonal System of Coordinates



Material coefficients are constant with respect to the quasi-azimuthal direction ζ_2

Fourier Series Expansion in ζ_2

DC Problems: $-\nabla \sigma \nabla u = f$

$$u(\zeta_1, \zeta_2, \zeta_3) = \sum_{l=-\infty}^{l=\infty} u_l(\zeta_1, \zeta_3) e^{jl\zeta_2}$$

$$\sigma(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-\infty}^{m=\infty} \sigma_m(\zeta_1, \zeta_3) e^{jm\zeta_2}$$

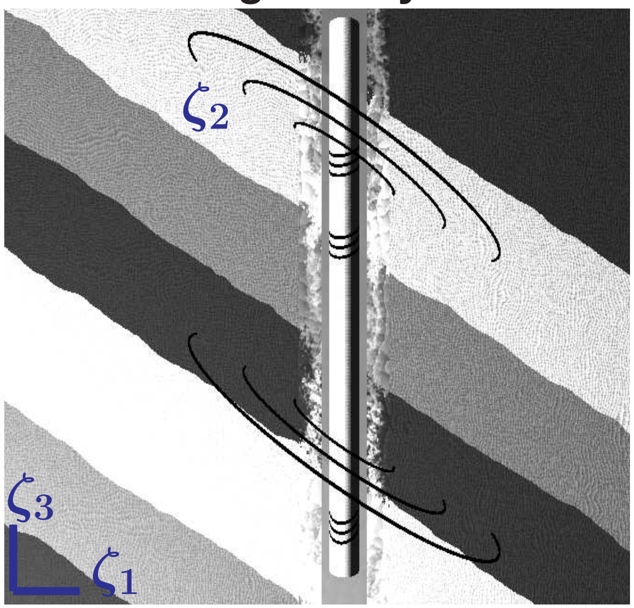
$$f(\zeta_1, \zeta_2, \zeta_3) = \sum_{n=-\infty}^{n=\infty} f_n(\zeta_1, \zeta_3) e^{jn\zeta_2}$$

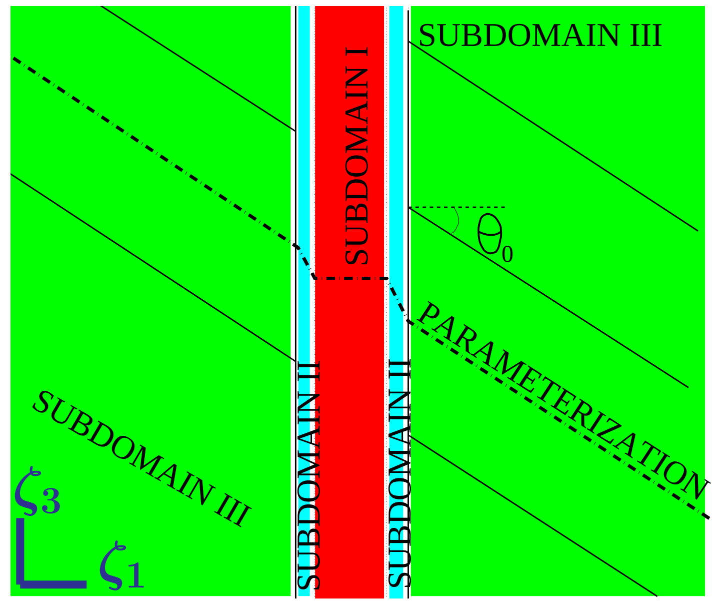
Fourier modes $e^{jl\zeta_2}$ are orthogonal high-order basis functions that are (almost) invariant with respect to the gradient operator.

METHODOLOGY: NEW SYSTEM OF COORDINATES

Cartesian system of coordinates: $\mathrm{x} = (x_1, x_2, x_3)$.

New non-orthogonal system of coordinates: $\zeta = (\zeta_1, \zeta_2, \zeta_3)$.





Subdomain I ; **Subdomain II** ; **Subdomain III**

$$\begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 \end{cases} ; \quad \begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 + \tan \theta_0 \dfrac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \end{cases} ; \quad \begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 + \tan \theta_0 \zeta_1 \end{cases}$$

METHODOLOGY: NEW SYSTEM OF COORDINATES

Final Variational Formulation

We define the Jacobian matrix $\mathcal{J} = \dfrac{\partial(x_1, x_2, x_3)}{\partial(\zeta_1, \zeta_2, \zeta_3)}$ **and its determinant** $|\mathcal{J}| = \det(\mathcal{J})$**.**

Variational formulation in the new system of coordinates:

$$\begin{cases} \text{Find } u \in u_D + H_D^1(\Omega) \text{ such that:} \\ \left\langle \dfrac{\partial v}{\partial \zeta} \,,\, \tilde{\sigma} \dfrac{\partial u}{\partial \zeta} \right\rangle_{L^2(\Omega)} = \left\langle v \,,\, \tilde{f} \right\rangle_{L^2(\Omega)} \quad \forall v \in H_D^1(\Omega)\,, \end{cases}$$

where:

$$\tilde{\sigma} := \mathcal{J}^{-1} \sigma \mathcal{J}^{-1^T} |\mathcal{J}| \quad ; \quad \tilde{f} := f |\mathcal{J}|\,.$$

Same variational formulation with new materials and load data

METHODOLOGY: FOURIER SERIES EXPANSION

For a mono-modal test function $v = v_k e^{jk\zeta_2}$, we have:

$$\begin{cases} \text{Find } u \in u_D + H^1_D(\Omega) \text{ such that:} \\ \displaystyle\sum_{m,n} \left\langle \left(\frac{\partial v}{\partial \zeta}\right)_k e^{jk\zeta_2} \, , \, \tilde{\sigma}_m \left(\frac{\partial u}{\partial \zeta}\right)_n e^{j(m+n)\zeta_2} \right\rangle_{L^2(\Omega)} = \\ \displaystyle = \sum_l \left\langle v_k e^{jk\zeta_2} \, , \, \tilde{f}_l e^{jl\zeta_2} \right\rangle_{L^2(\Omega)} \quad \forall v_k e^{jk\zeta_2} \in H^1_D(\Omega) \end{cases}$$

Using the L^2-orthogonality of Fourier modes:

$$\begin{cases} \text{Find } u \in u_D + H^1_D(\Omega) \text{ such that:} \\ \displaystyle\sum_n \left\langle \left(\frac{\partial v}{\partial \zeta}\right)_k \, , \, \tilde{\sigma}_{k-n} \left(\frac{\partial u}{\partial \zeta}\right)_n \right\rangle_{L^2(\Omega_{2D})} = \left\langle v_k \, , \, \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})} \quad \forall v_k \end{cases}$$

METHODOLOGY: FOURIER SERIES EXPANSION

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

$$\tilde{\sigma}(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-2}^{m=2} \tilde{\sigma}_m(\zeta_1, \zeta_3) e^{jm\zeta_2}$$

METHODOLOGY: FOURIER SERIES EXPANSION

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

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Final Variational Formulation

$$\begin{cases} \text{Find } u \in u_D + H_D^1(\Omega) \text{ such that:} \\ \displaystyle\sum_{n=k-2}^{n=k+2} \left\langle \left(\frac{\partial v}{\partial \zeta}\right)_k , \tilde{\sigma}_{k-n} \left(\frac{\partial u}{\partial \zeta}\right)_n \right\rangle_{L^2(\Omega_{2D})} = \left\langle v_k , \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})} \quad \forall v_k \end{cases}$$

METHODOLOGY: IMPLEMENTATION

Example (7 Fourier Modes)

$$\sum_{n=k-2}^{n=k+2} \underbrace{\left\langle \left(\frac{\partial v}{\partial \zeta}\right)_k , \tilde{\sigma}_{k-n} \left(\frac{\partial u}{\partial \zeta}\right)_n \right\rangle_{L^2(\Omega_{2D})}}_{(k,\, k-n,\, n)} = \left\langle v_k , \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})}$$

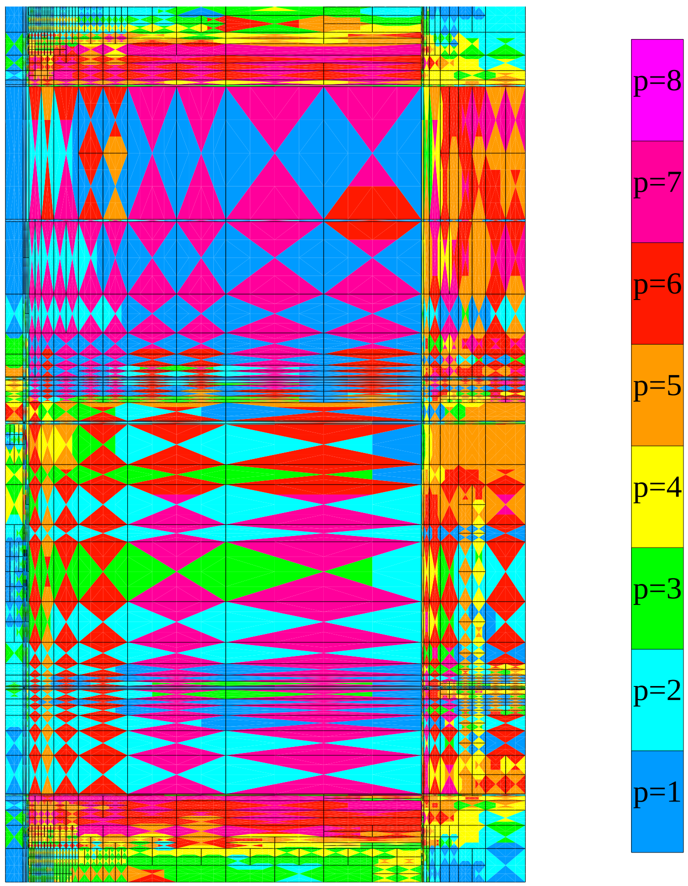
Stiffness Matrix:

$$\begin{pmatrix}
(-3,0,-3) & (-3,-1,-2) & (-3,-2,-1) & 0 & 0 & 0 & 0 \\
(-2,1,-3) & (-2,0,-2) & (-2,-1,-1) & (-2,-2,0) & 0 & 0 & 0 \\
(-1,2,-3) & (-1,1,-2) & (-1,0,-1) & (-1,-1,0) & (-1,-2,1) & 0 & 0 \\
0 & (0,2,-2) & (0,1,-1) & (0,0,0) & (0,-1,1) & (0,-2,2) & 0 \\
0 & 0 & (1,2,-1) & (1,1,0) & (1,0,1) & (1,-1,2) & (1,-2,3) \\
0 & 0 & 0 & (2,2,0) & (2,1,1) & (2,0,2) & (2,-1,3) \\
0 & 0 & 0 & 0 & (3,2,1) & (3,1,2) & (3,0,3)
\end{pmatrix}$$

METHODOLOGY: 2D hp-FEM

A Self-Adaptive Goal-Oriented hp-FEM

Optimal 2D Grid
(Through Casing Resistivity Problem)



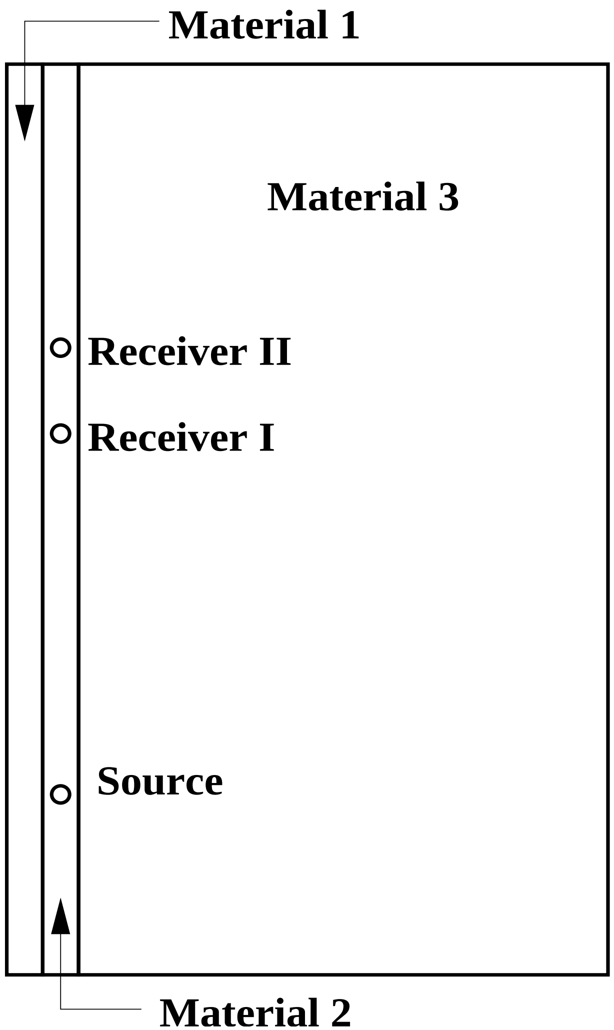
We vary locally the element size h and the polynomial order of approximation p throughout the grid.

Optimal grids are automatically generated by the computer.

The self-adaptive goal-oriented hp-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.

NUMERICAL RESULTS: VALIDATION

Three Model Problems



Problem I **(Uniform Materials)**

Material 1:1 Ω−m
Material 2:1 Ω−m
Material 3:1 Ω−m

Problem II

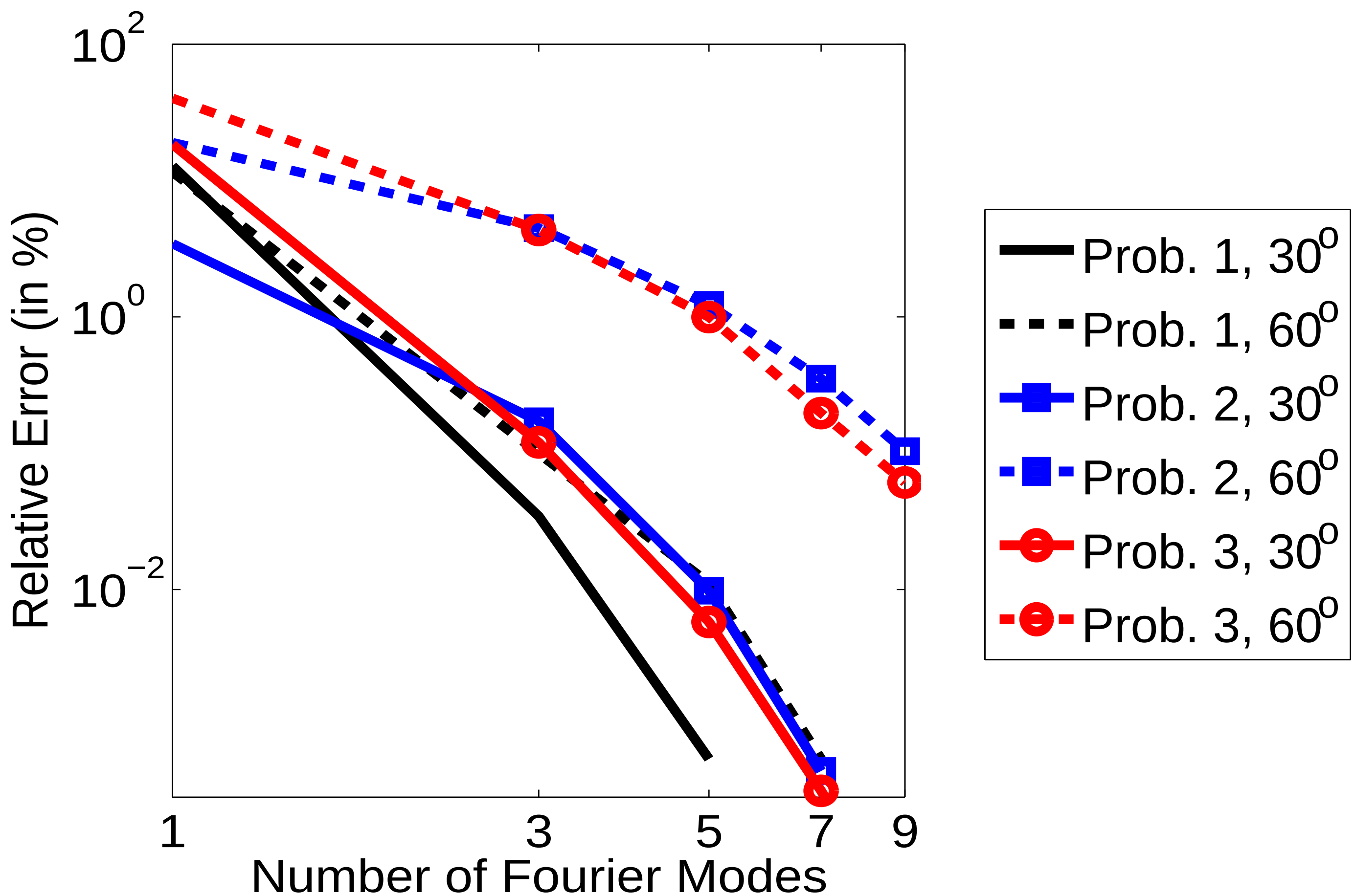
Material 1:0.00001 Ω−m
Material 2:10 Ω−m
Material 3:1 Ω−m

Problem III

Material 1:10000 Ω−m
Material 2: 0.2 Ω−m
Material 3:1 Ω−m

NUMERICAL RESULTS: VALIDATION

Three Model Problems

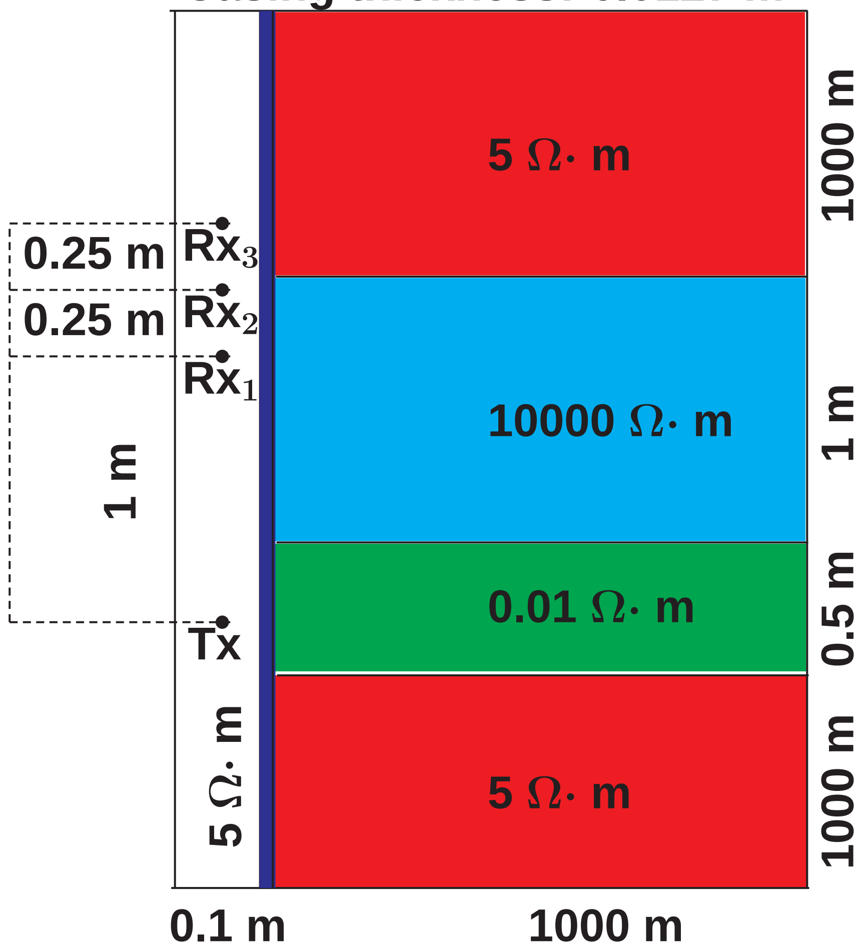


Exponential Convergence in terms of the Number of Fourier Modes

NUMERICAL RESULTS: DC RESULTS

Simulation of Through Casing Resistivity Measurements

Casing resistivity: $10^{-5} - 10^{-7} \Omega\cdot$ m
Casing thickness: 0.012 7 m



Left Figure:
- Axial-symmetric model
- One current electrode (emitter)
- Three voltage electrodes (collectors)

Objective:
- Compute second diff. of potential for various depth angles and possibly with water invasion

Method of solution:
- Fourier series expansion + change of coordinates + 2D goal-oriented hp-FEM

NUMERICAL RESULTS: DC RESULTS

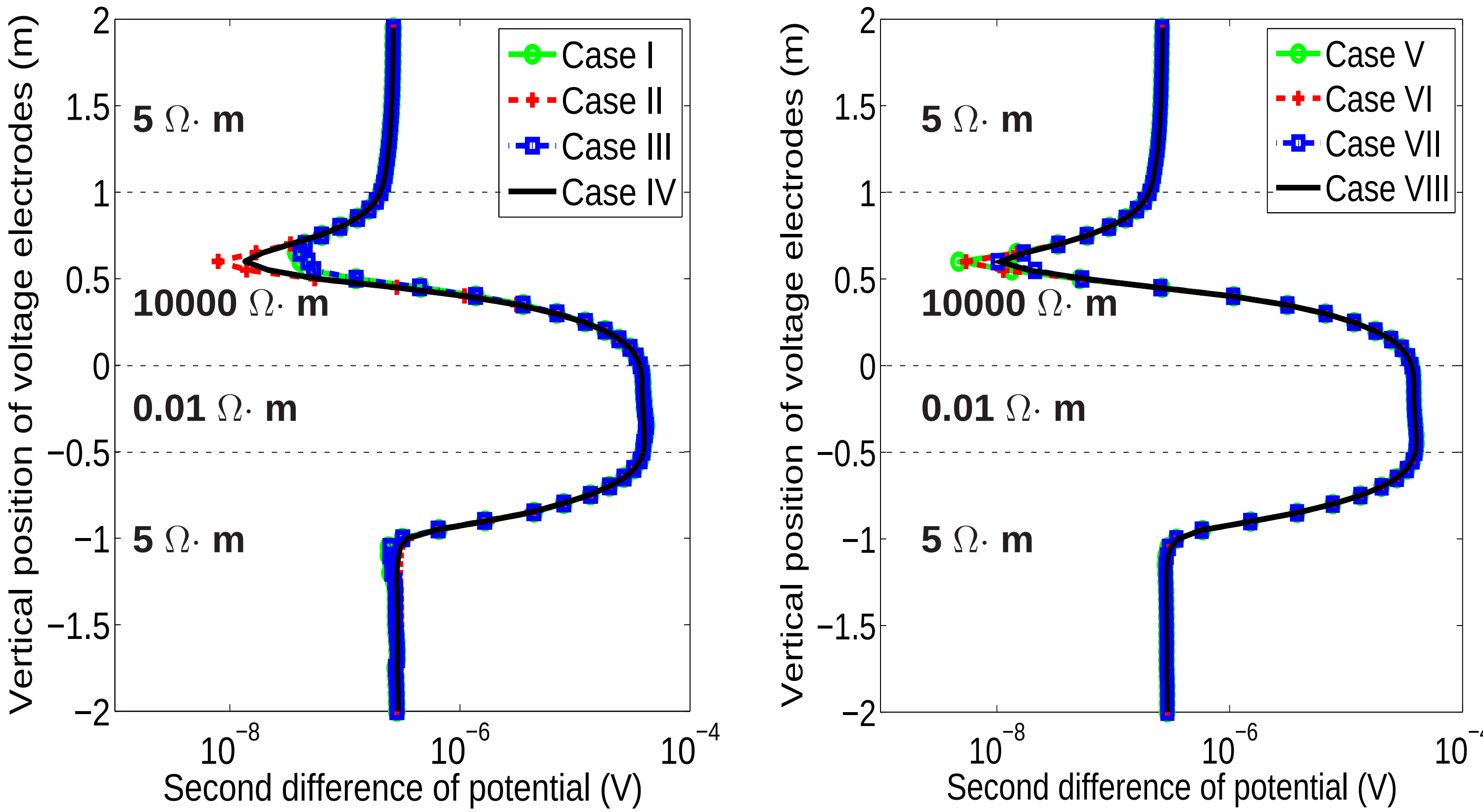
Simulation of Through Casing Resistivity Measurements

Algorithm (Case) Number	I	II	III	IV	V	VI	VII	VIII
1 Fourier mode used for adaptivity	X	X	X	X				
5 Fourier modes used for adaptivity					X	X	X	X
Final hp-grid NOT p-enriched	X		X		X		X	
Final hp-grid globally p-enriched		X		X		X		X
9 Fourier modes used for the final solution	X	X			X	X		
15 Fourier modes used for the final solution			X	X			X	X

Different algorithms provide different levels of accuracy

NUMERICAL RESULTS: DC RESULTS

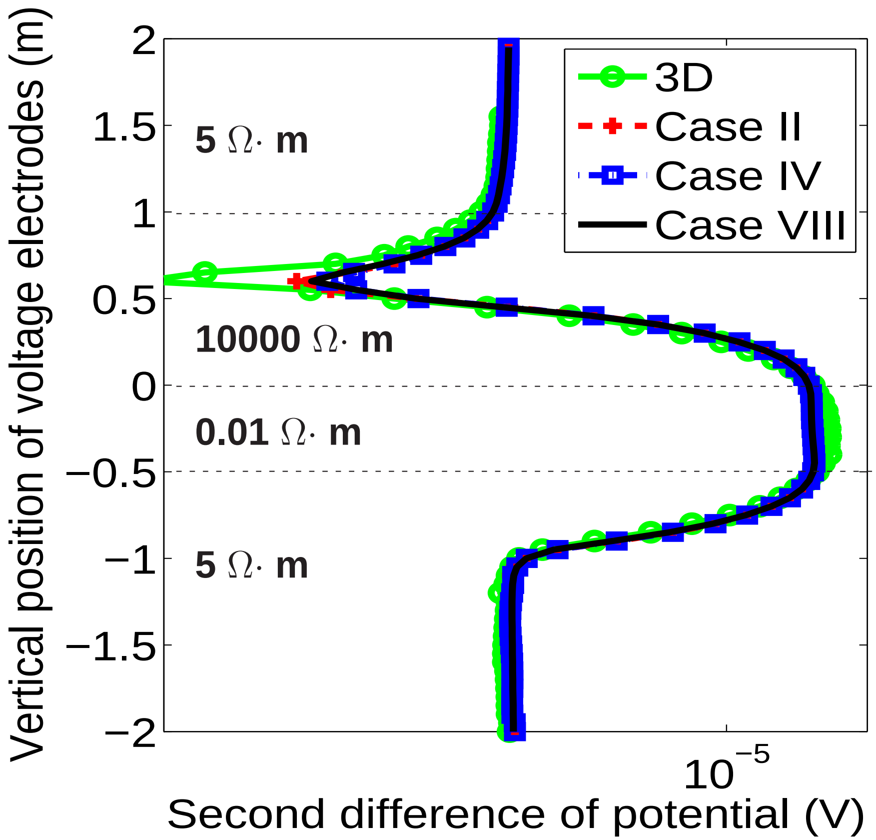
Through Casing Resistivity Measurements (60-Degree Deviated Well)



Case Number	I	II	III	IV	V	VI	VII	VIII
Total Time (minutes)	21'	40'	39'	109'	244'	290'	286'	432'

NUMERICAL RESULTS: DC RESULTS

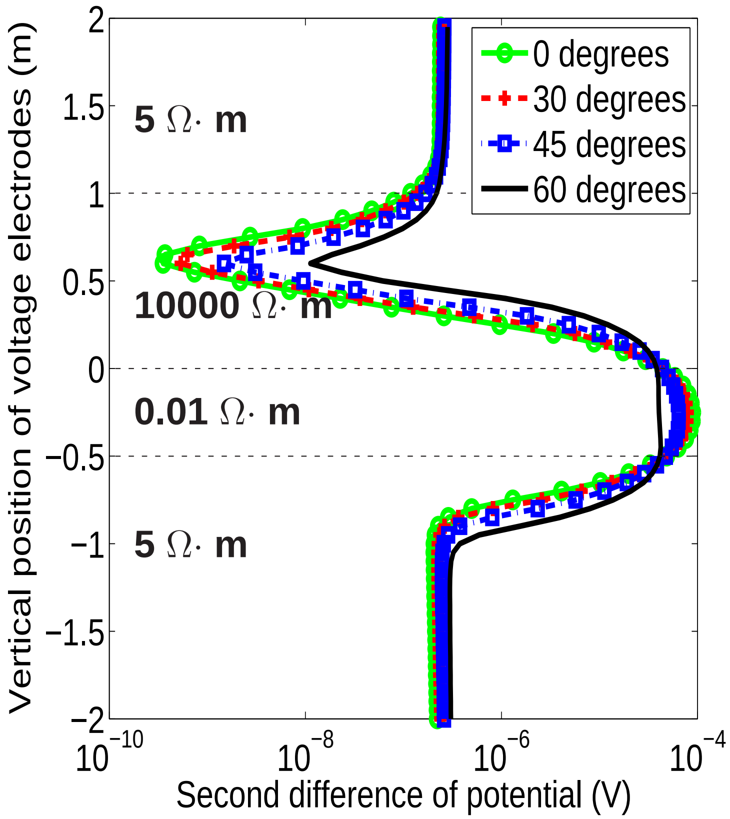
Through Casing Resistivity Measurements (60-Degree Deviated Well)

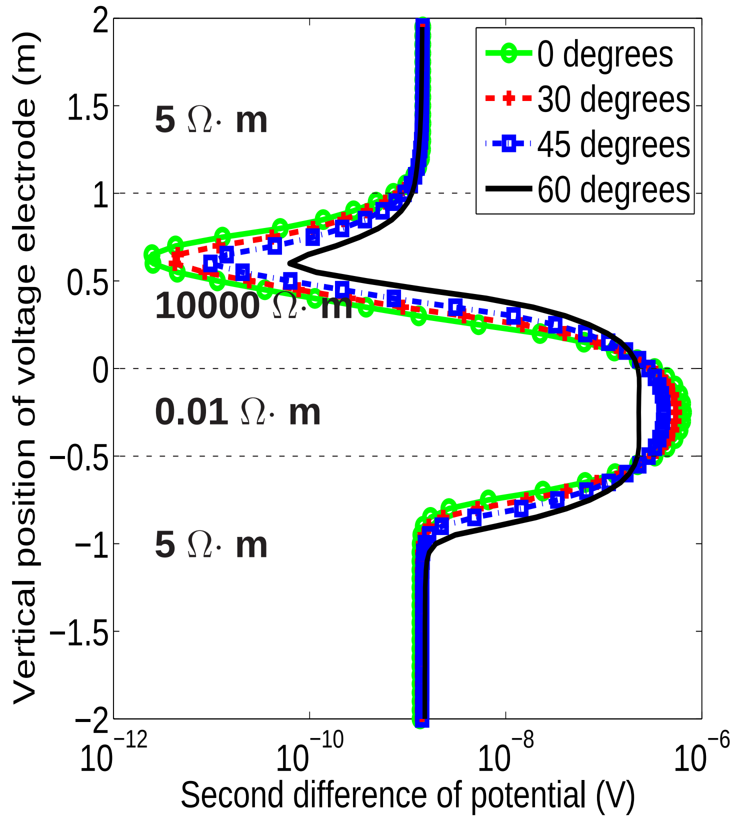


Results with the new methodology seem more accurate than those obtained with the 3D software. In addition, with the new methology we reduce the CPU time from several days to two hours.

NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Casing Conductivity)

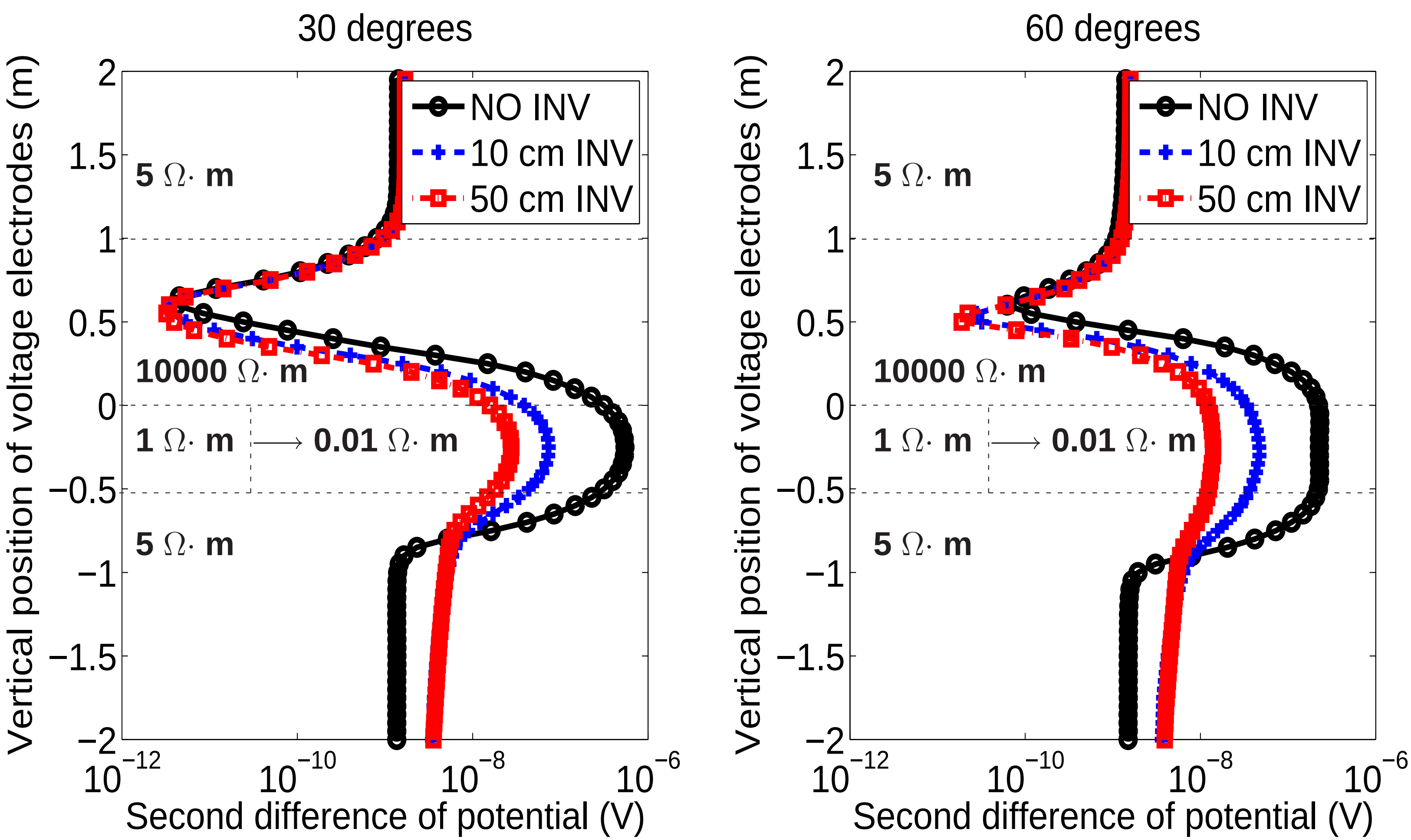




Qualitatively, results for various casing conductivities are similar even for deviated wells.

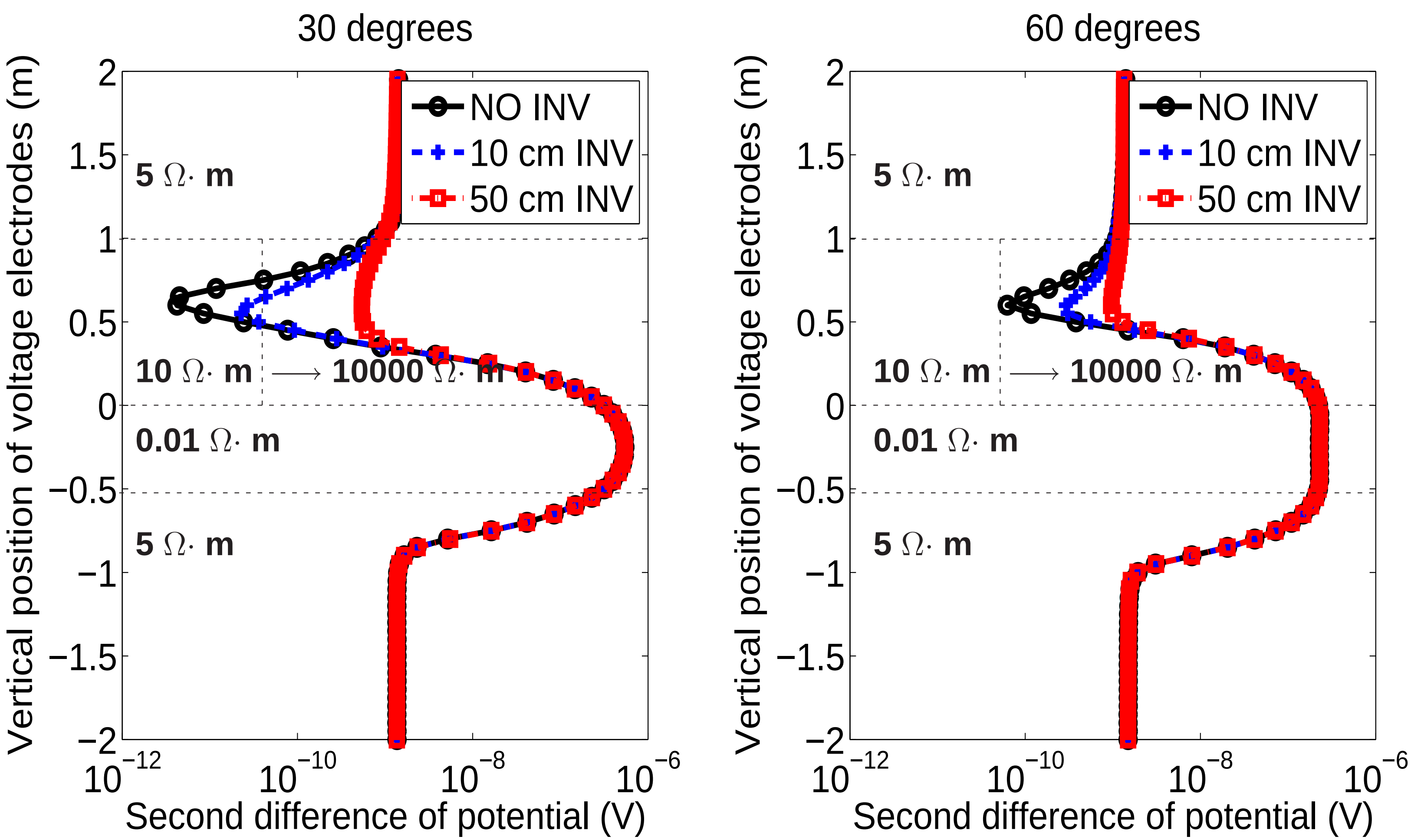
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)



NUMERICAL RESULTS: DC RESULTS

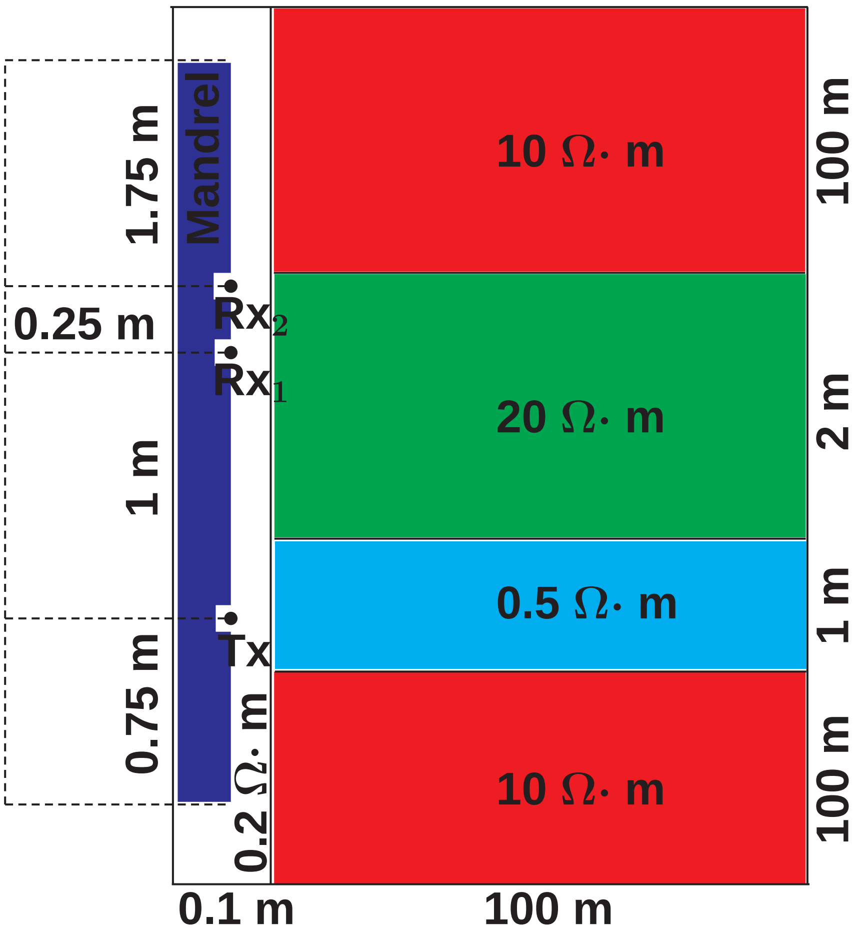
Through Casing Resistivity Measurements (Invasion)



NUMERICAL RESULTS: DC RESULTS

Simulation of Mandrel Effects

Mandrel resistivity: 10000 $\Omega\cdot$ m
Mandrel thickness: 0.07 m



Left Figure:
- Axial-symmetric model
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Objective:
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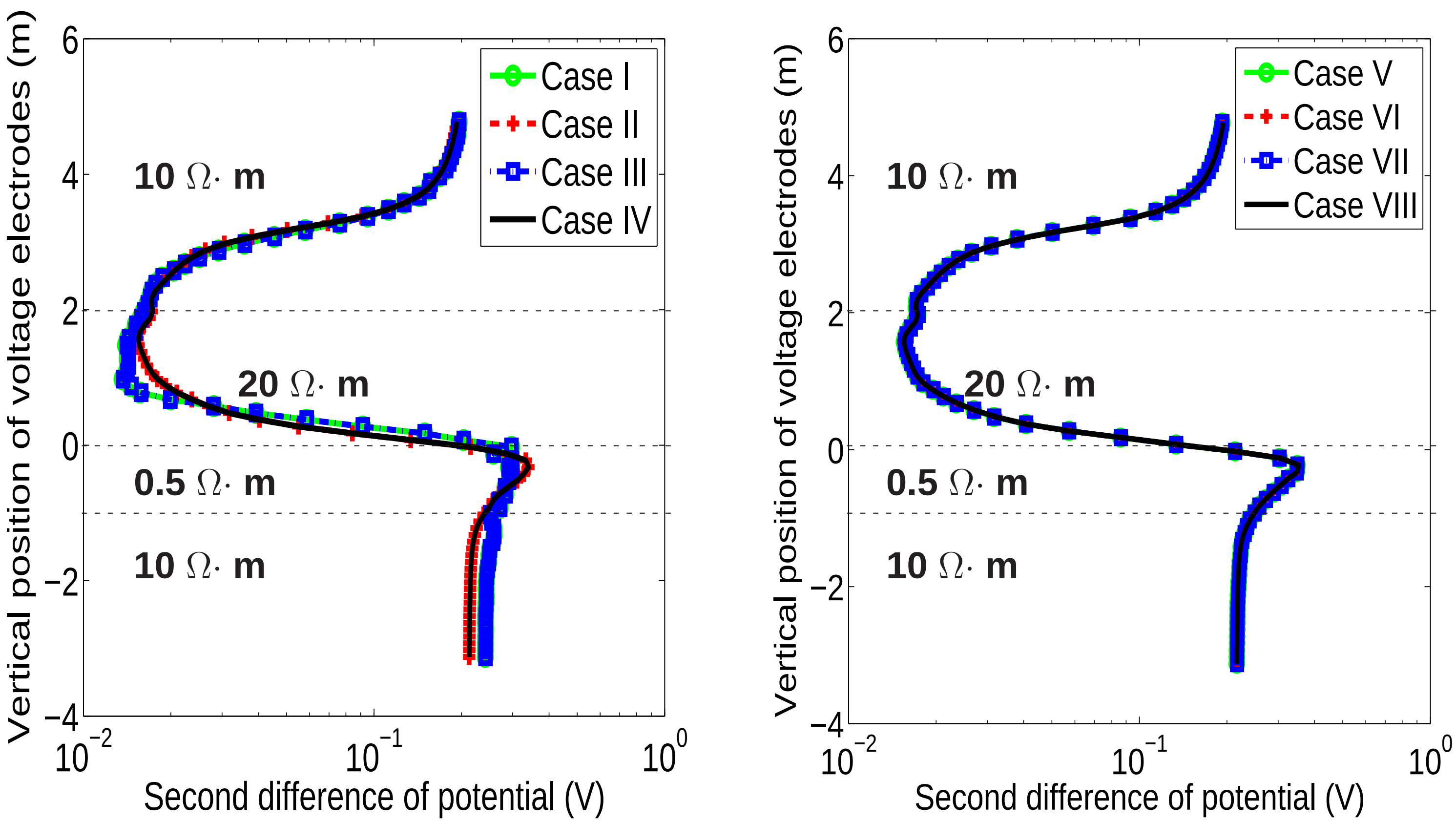
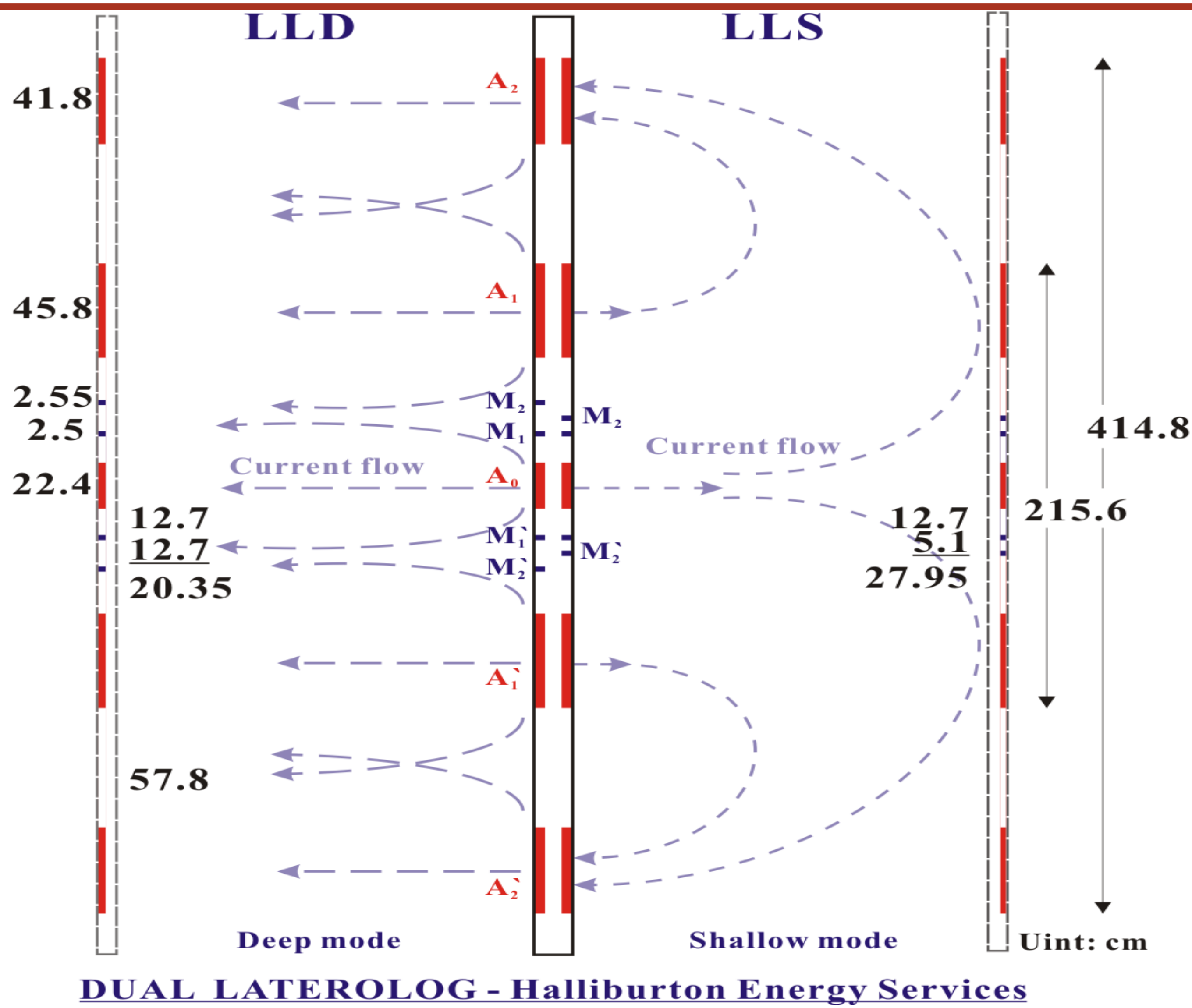
NUMERICAL RESULTS: DC RESULTS

Simulation of Mandrel Effects (60-Degree Deviated Well)



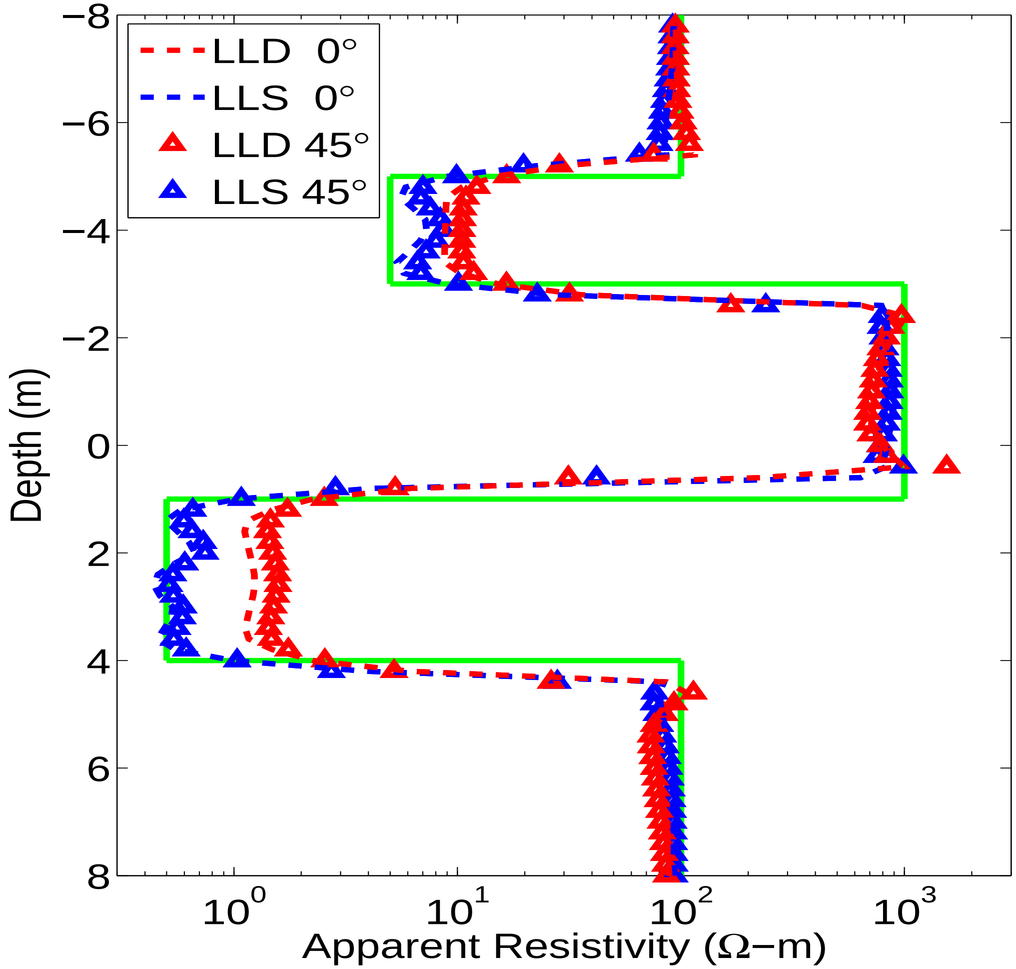
Case Number	I	II	III	IV	V	VI	VII	VIII
Total Time (minutes)	11'	25'	18'	83'	126'	153'	158'	279'

NUMERICAL RESULTS: DC RESULTS



NUMERICAL RESULTS: DC RESULTS

Laterolog Measurements



CONCLUSIONS AND FUTURE WORK

We have developed a new method based on a Fourier series expansion in a non-orthogonal system of coordinates.

- **LIMITATION: Geometry of the problem.**
- **ADVANTAGE: It combines exponential convergence with sparse (penta-diagonal) matrices.**
- **FURTHER APPLICABILITY OF THE METHOD:**
  - **Eccentric measurements and tilted antennas.**
  - **Multi-Physics: Resistivity logging instruments, sonic logging instruments (acoustics + elasticity), fluid-flow, geomechanics, etc.**
  - **Inverse problems.**

The new method enables simulations of challenging resistivity logging measurements that cannot be simulated otherwise.

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