Joint Industry Research Consortium on Formation Evaluation

Self-Adaptive Goal-Oriented hp-Finite Element Simulations of Induction and Laterolog Measurements in the Presence of Steel Casing”

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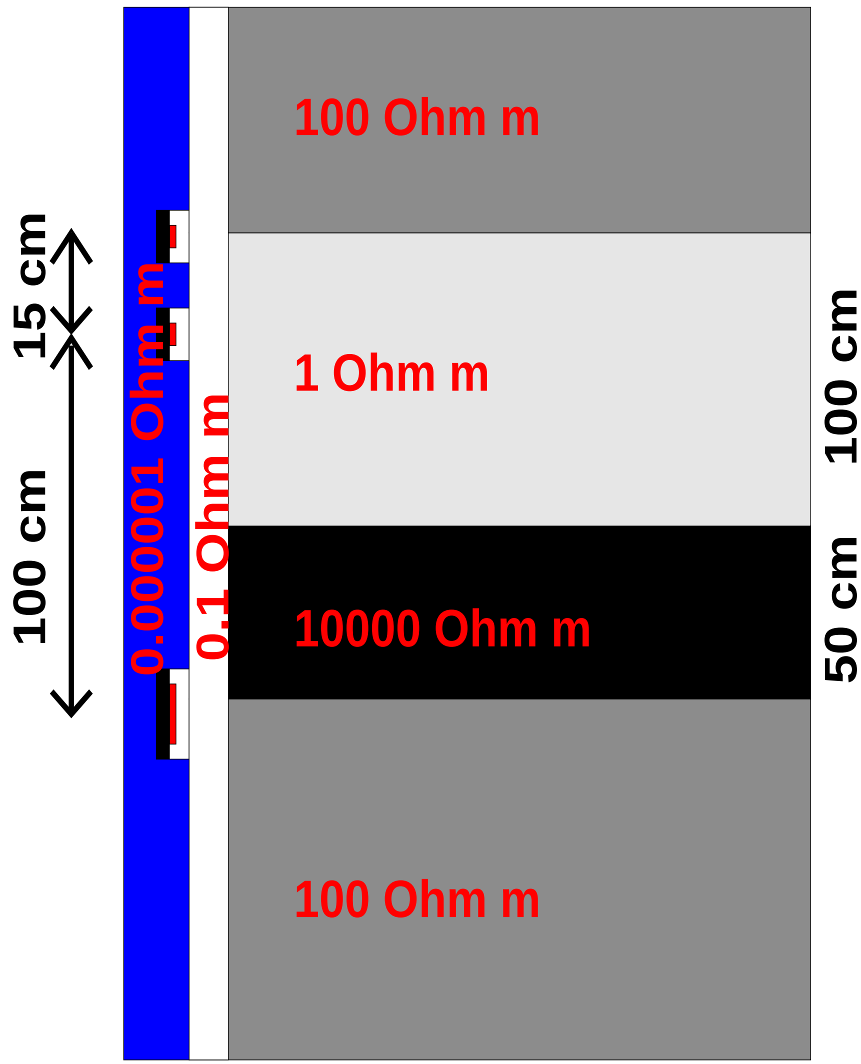
The University of Texas at Austin

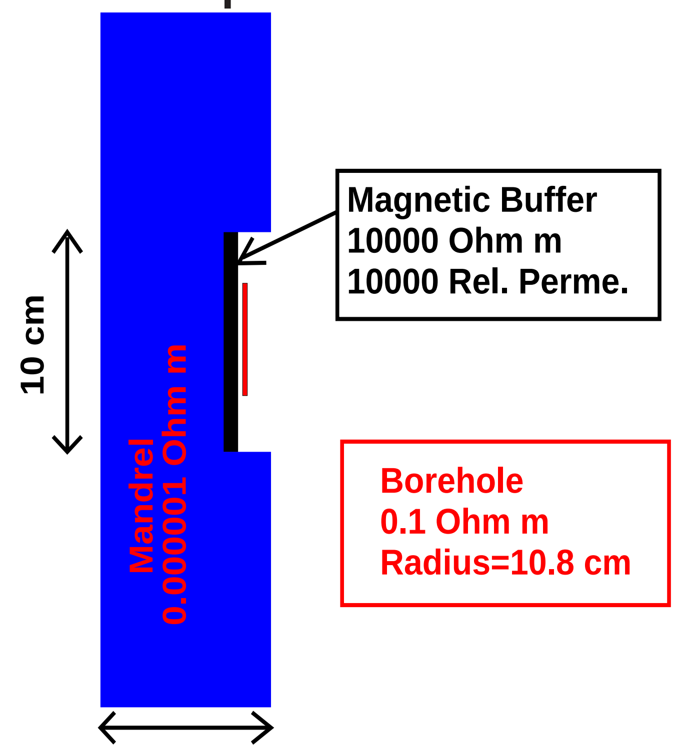
OVERVIEW

1. **Motivation**
2. **Numerical Methodology**
   - hp-Finite Elements (Exponential convergence)
   - Automatic Goal-Oriented Refinements (in the quantity of interest)
3. **Current Stage of the 2D High Performance FE Software**
   - Flexibility
   - Reliability
   - Accuracy
   - Performance
4. **Simulations in Presence of Metal Casing of:**
   - Induction Instruments
   - Laterolog Measurements
5. **Conclusions and Future Work**

MOTIVATION



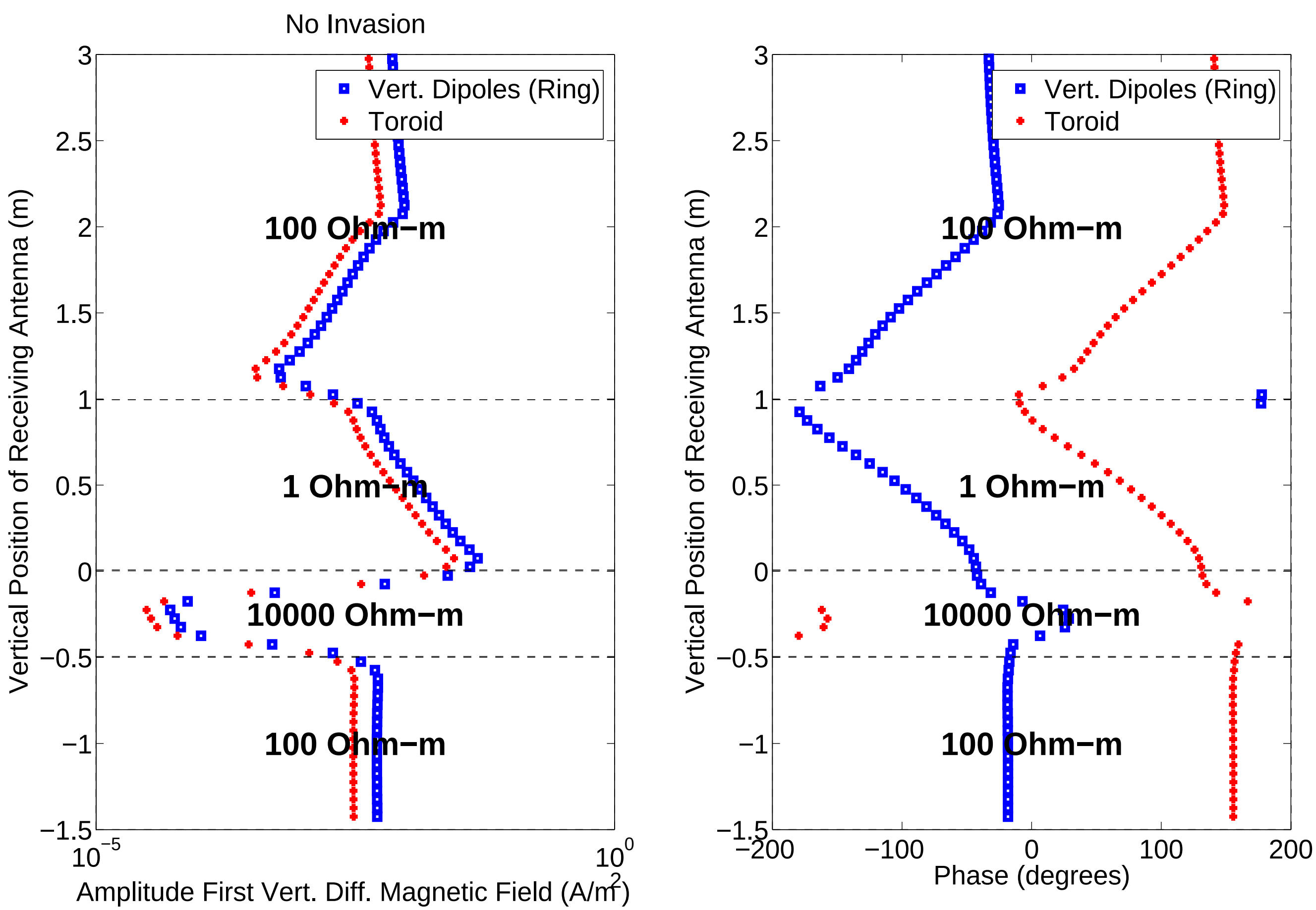
Description of Antennas



Goal: To Study the Effect of Invasion, Anistotropy, and Magnetic Permeability.

MOTIVATION

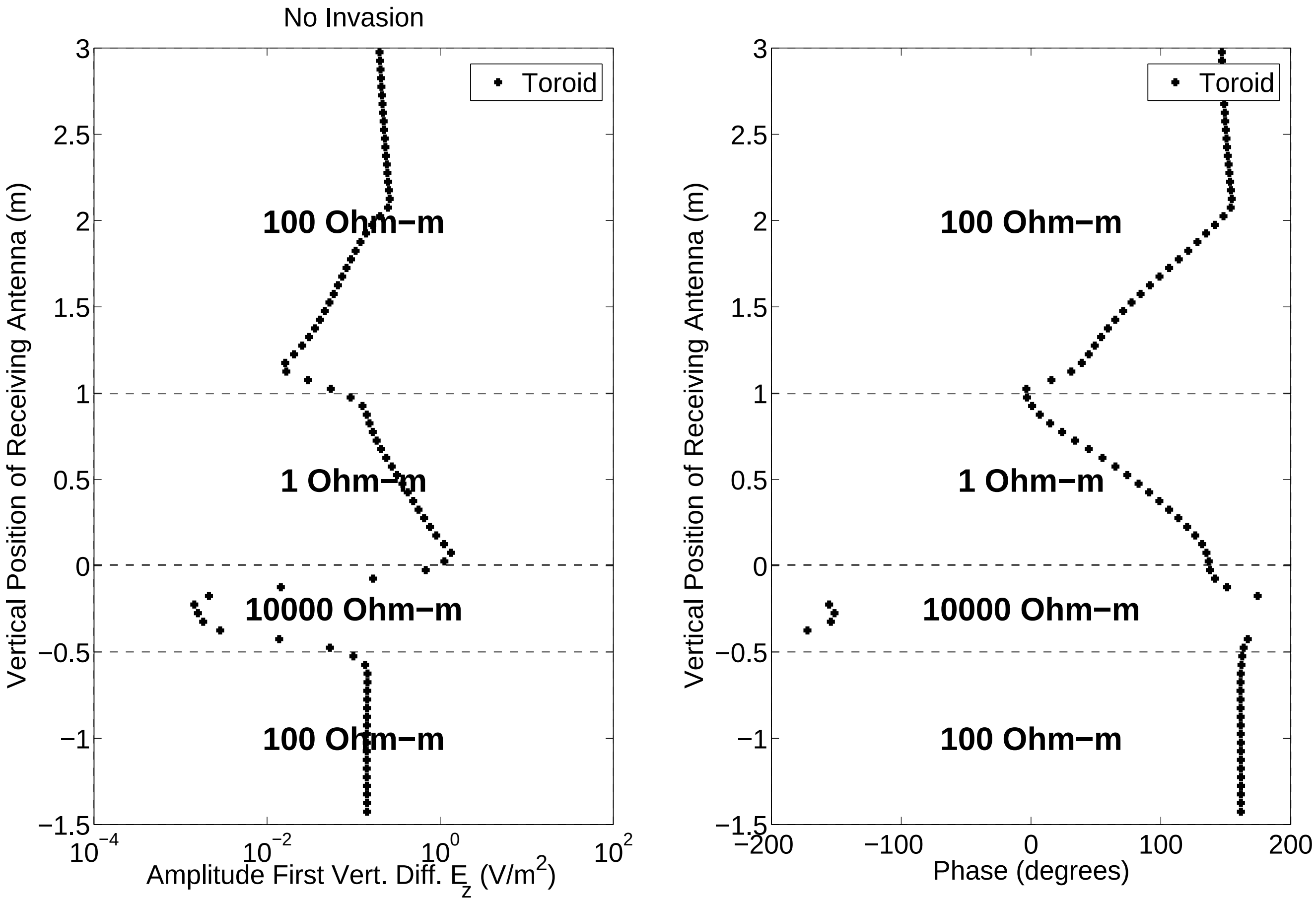
First Vert. Diff. H_ϕ for different antennas



In LWD instruments, we obtain similar results using toroids or a ring of vert. dipoles

MOTIVATION

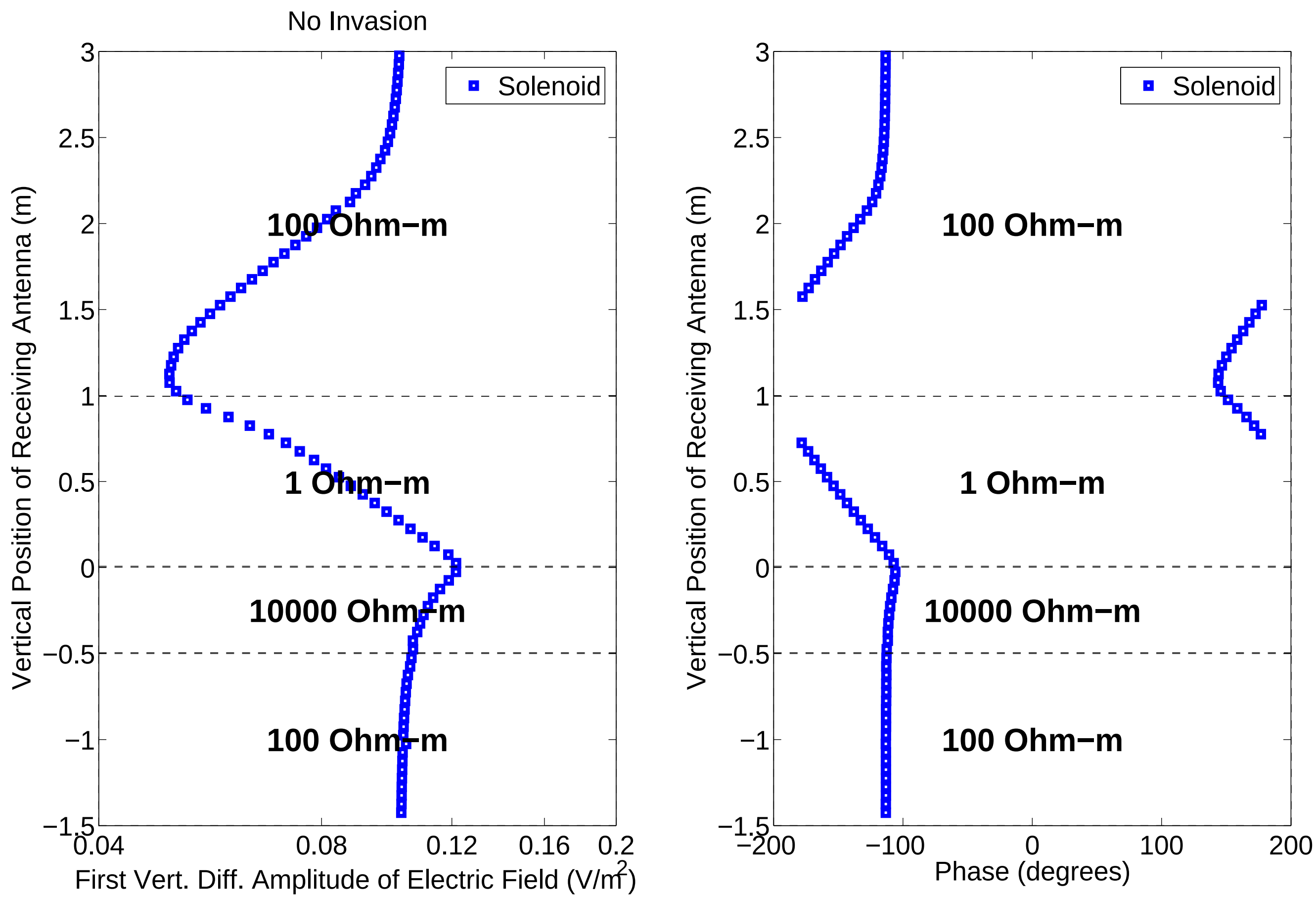
First Vert. Diff. E_z for a toroid antenna



Toroids are adequate for identifying highly resistive layers

MOTIVATION

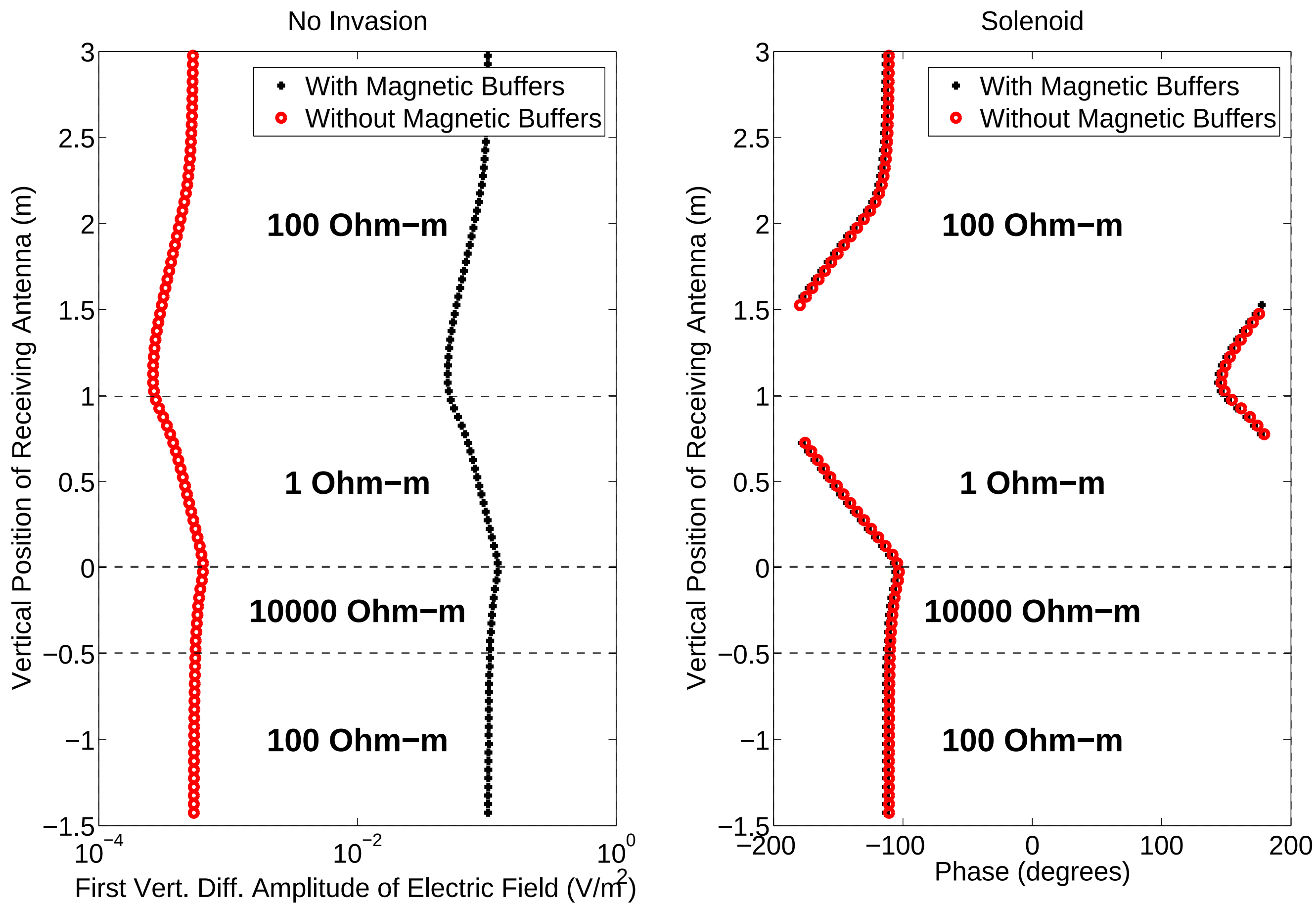
First Vert. Diff. E_ϕ for a solenoid antenna



Solenoids are adequate for identifying low resistive layers

MOTIVATION

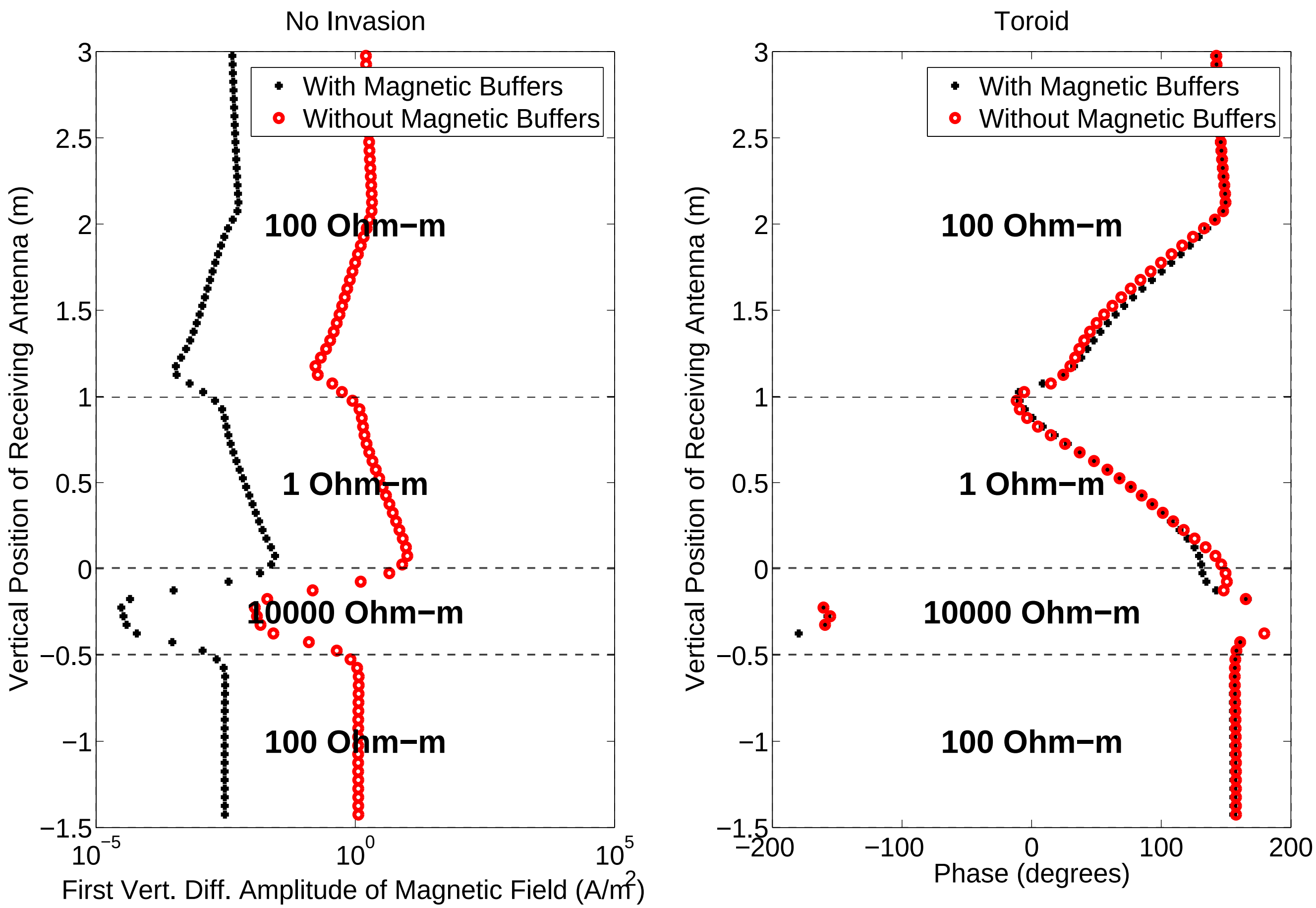
Use of Magnetic Buffers (E_ϕ for a solenoid)



Use of magnetic buffers strengthen the signal in combination with solenoids

MOTIVATION

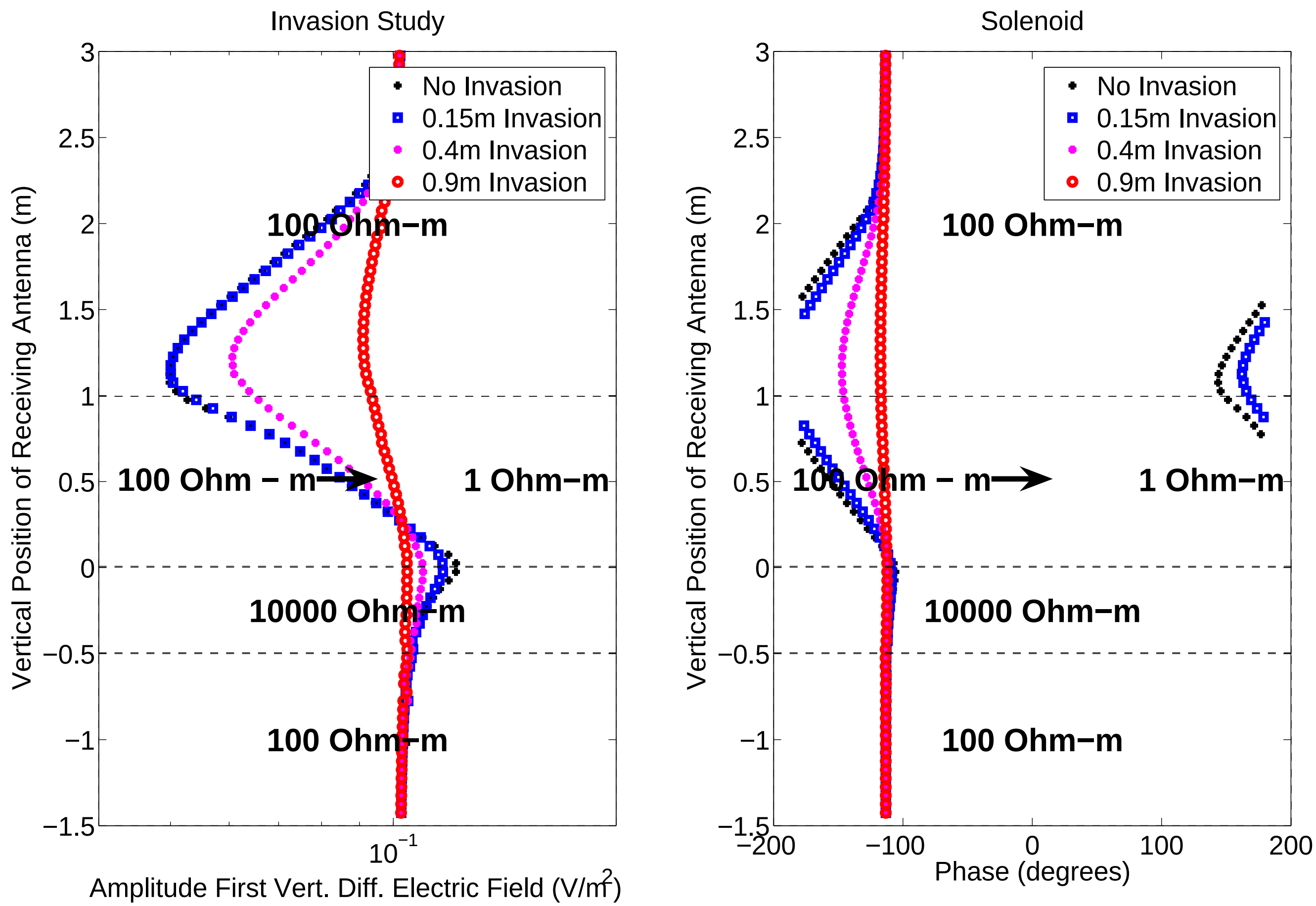
Use of Magnetic Buffers (H_ϕ for a toroid)



However, magnetic buffers weaken the signal in combination with toroids

MOTIVATION

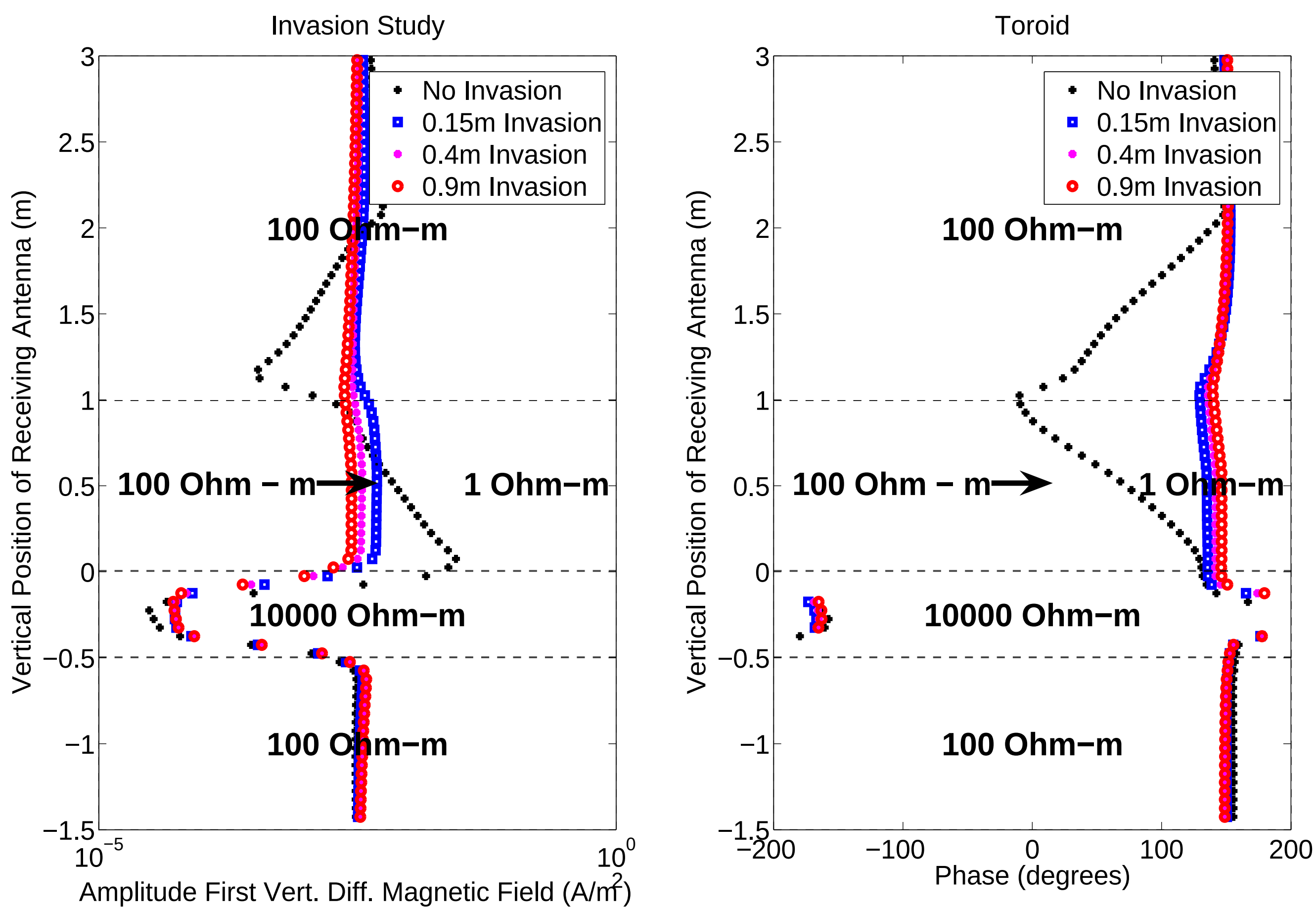
Invasion study (E_ϕ for a solenoid)



Large invasion effects can be sensed using solenoids

MOTIVATION

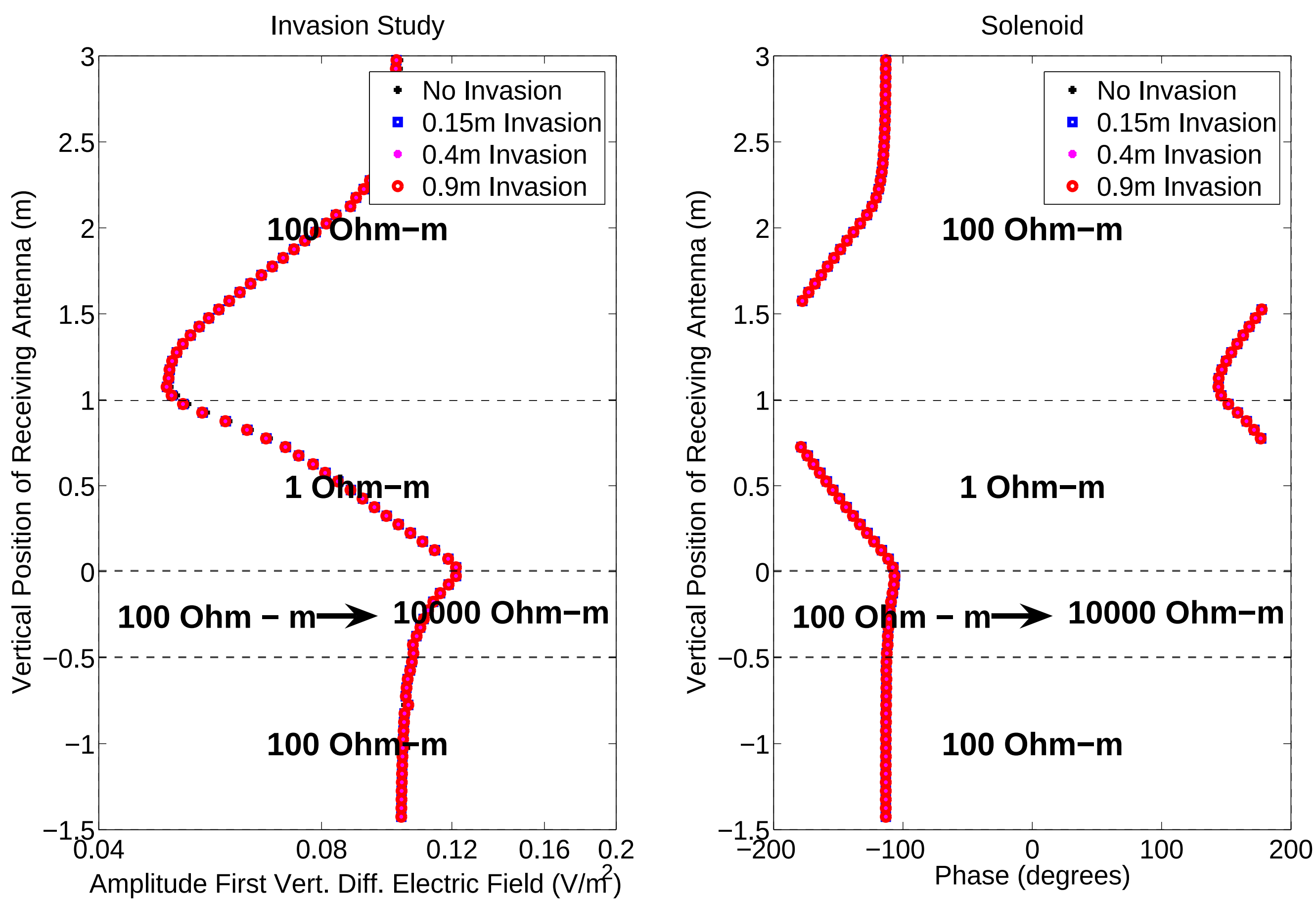
Invasion study (H_ϕ for a toroid)



Small invasion effects can be sensed using toroids

MOTIVATION

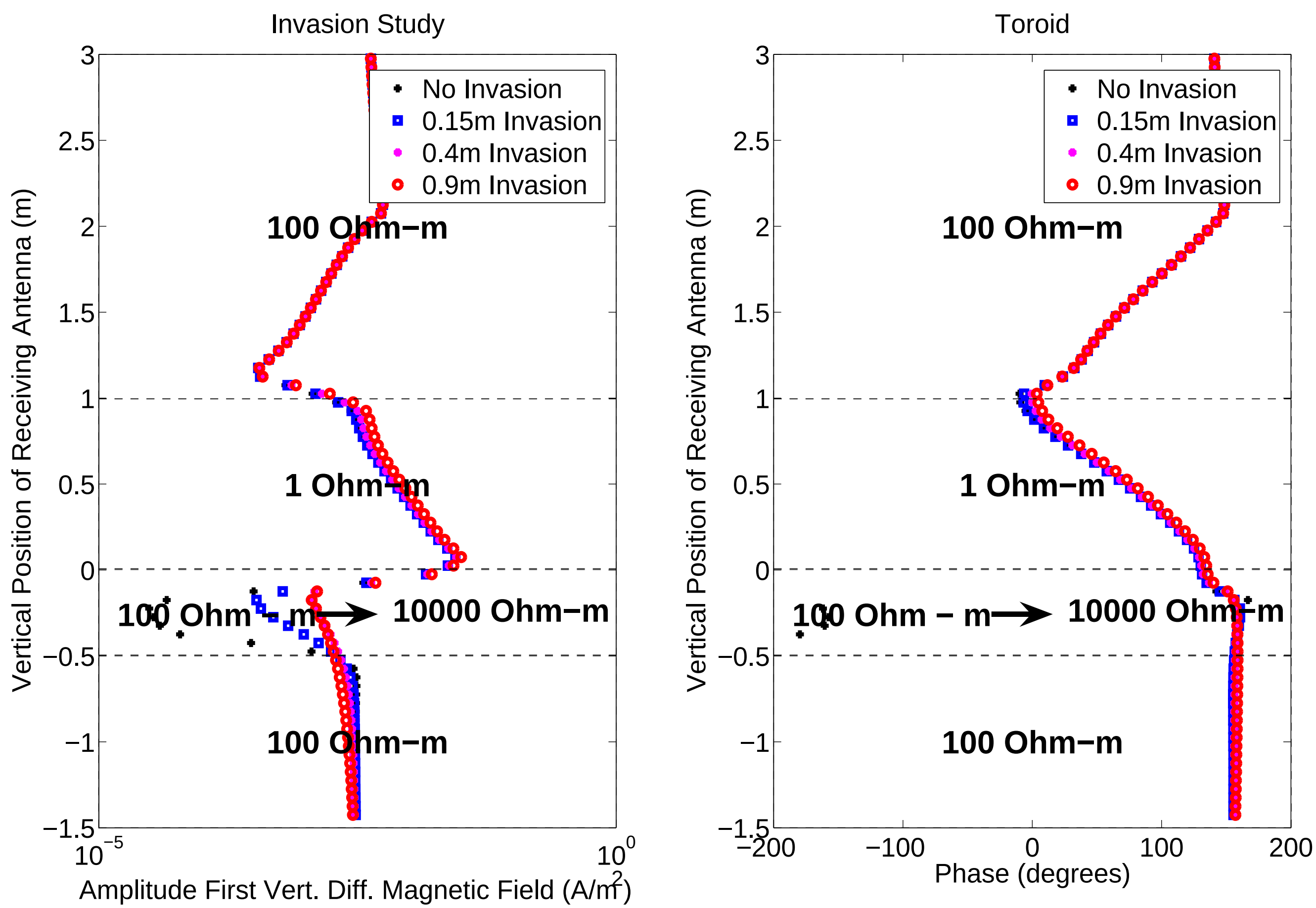
Invasion study (E_ϕ for a solenoid)



Invasion in resistive layers cannot be sensed using solenoids

MOTIVATION

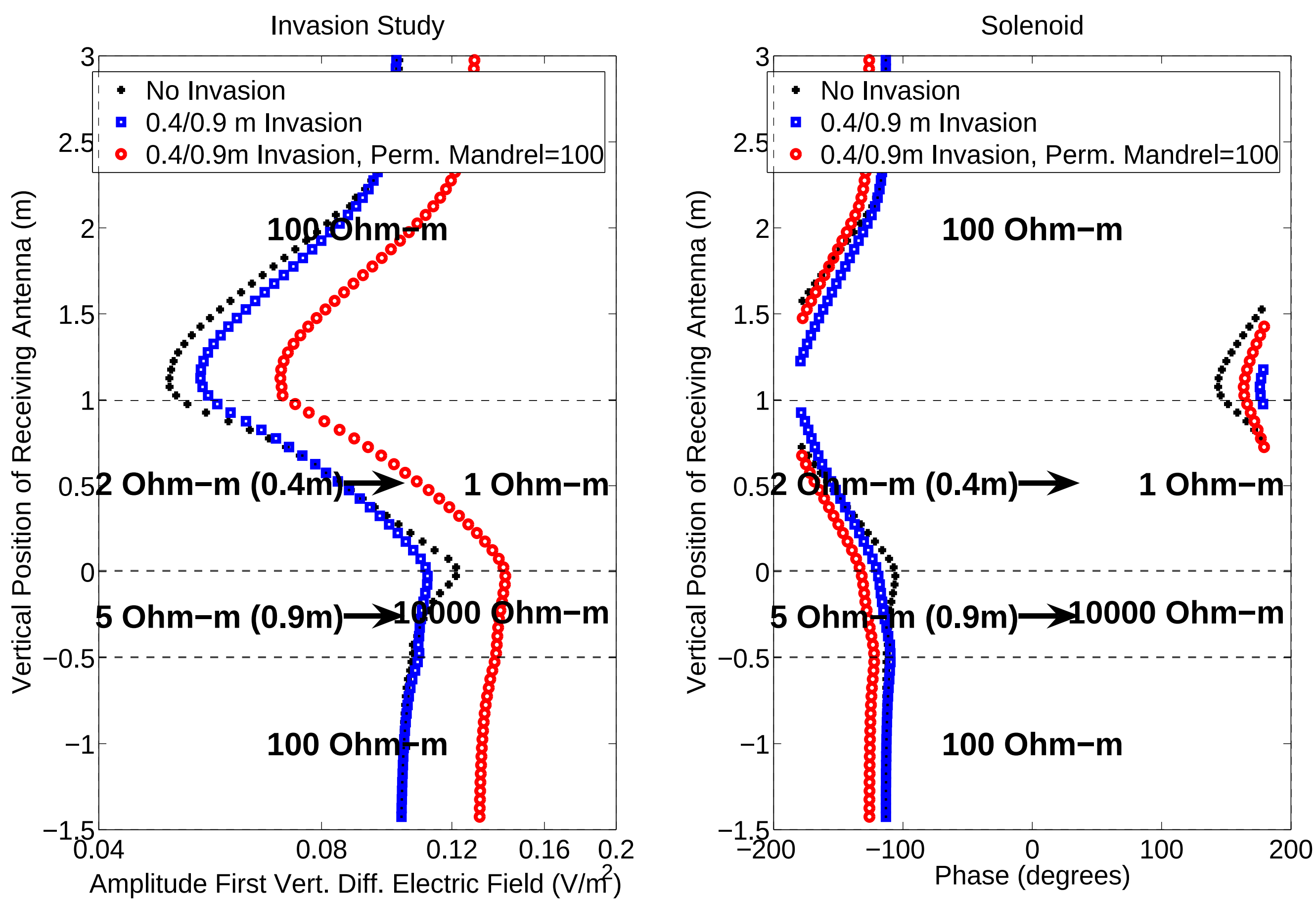
Invasion study (H_ϕ for a toroid)



Invasion in resistive layers should be studied using toroids

MOTIVATION

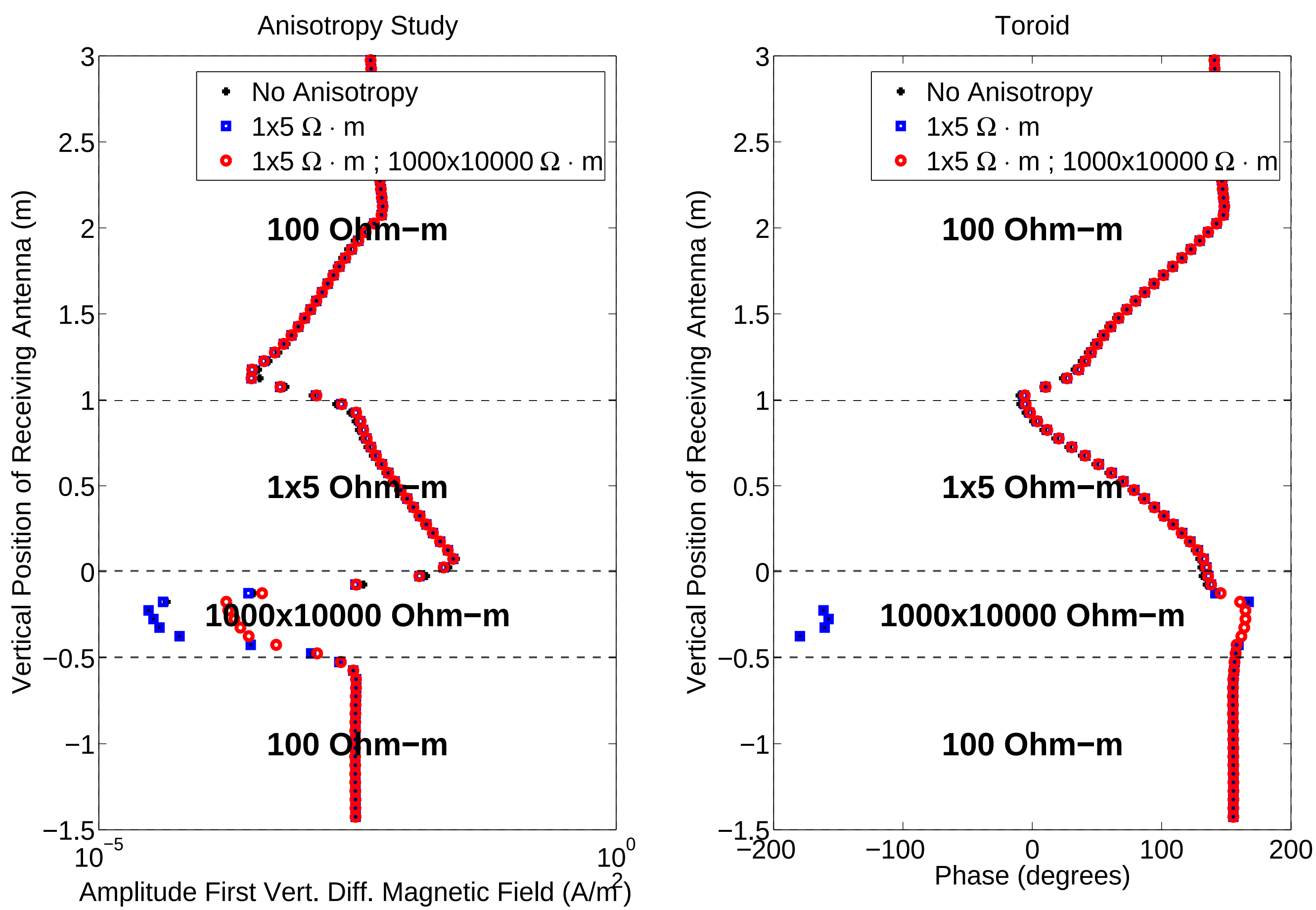
Invasion and mandrel magnetic permeab. (E_ϕ)



The effect of magnetic permeability on the mandrel is similar to the effect of magnetic buffers

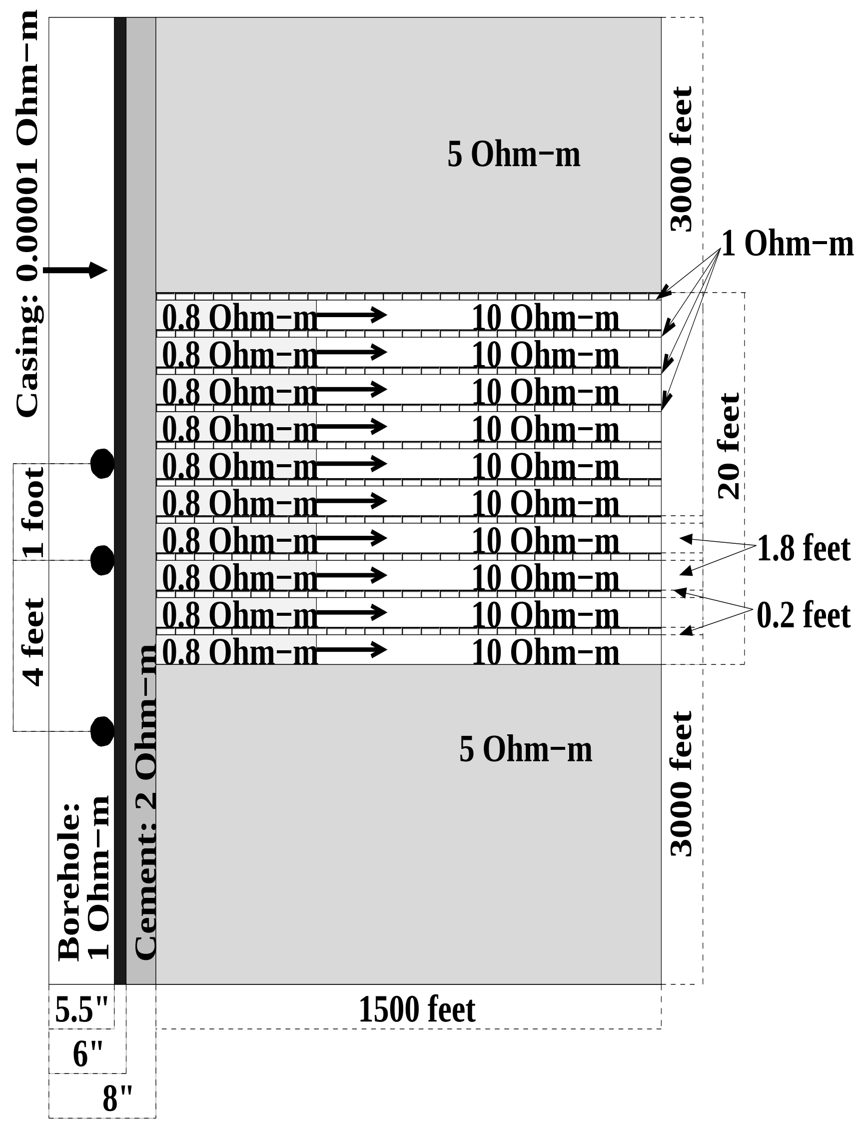
MOTIVATION

Anisotropy (H_ϕ)



Anisotropy effects may be important when studying resistive layers

MOTIVATION



Axisymmetric 3D problem.

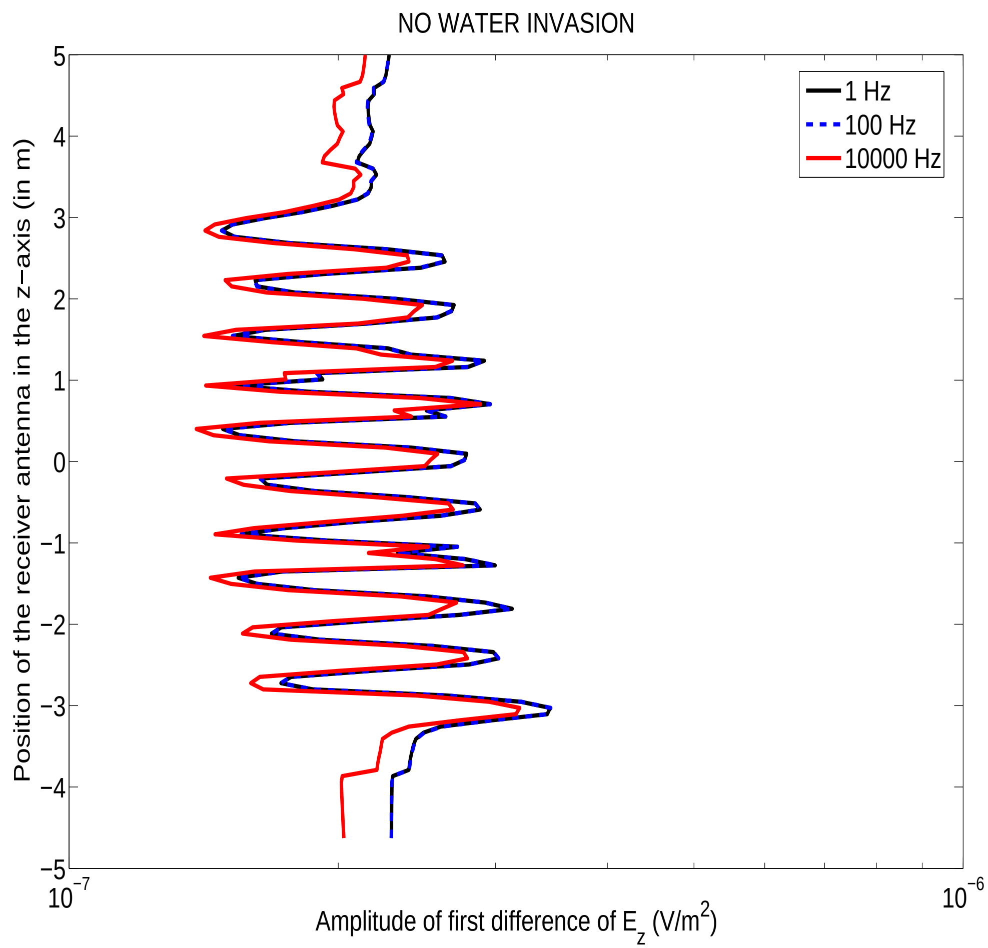
Seven different materials.

Through casing resistivity instrument.

Large variations on resistivity.

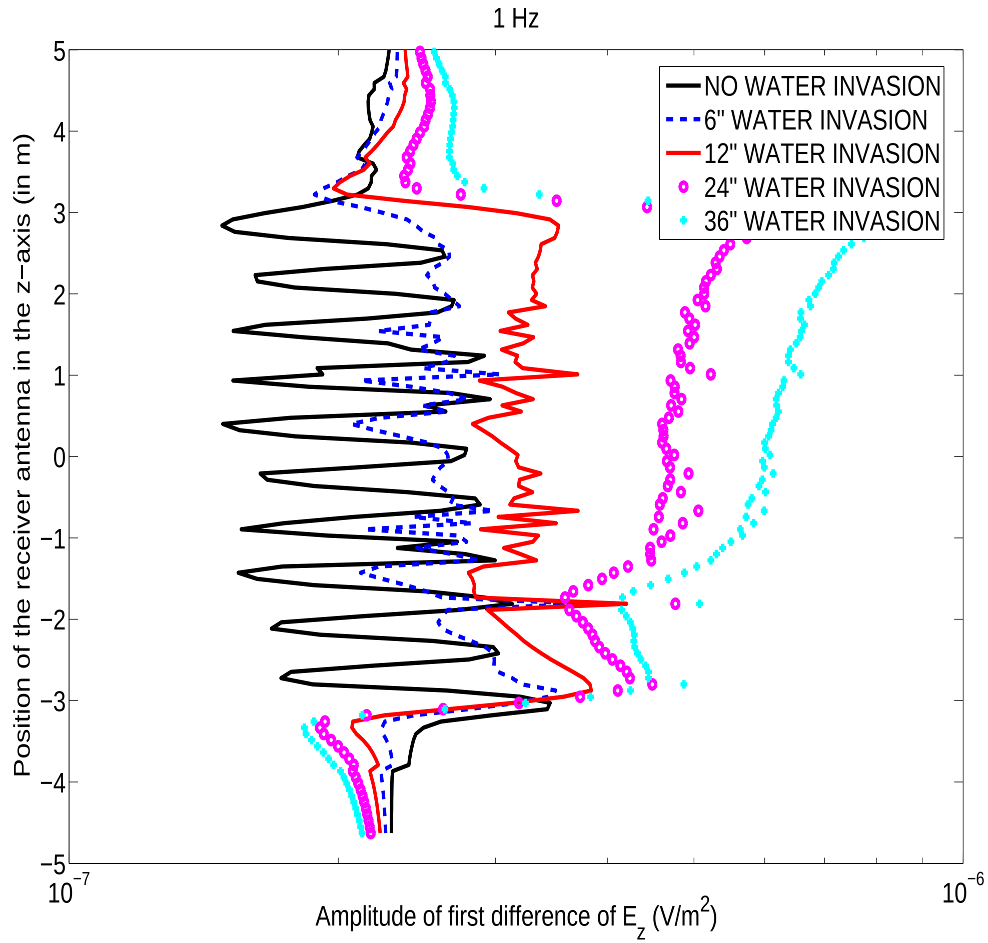
Objective: Study the effect of invasion THROUGH CASING on laminated sands.

MOTIVATION



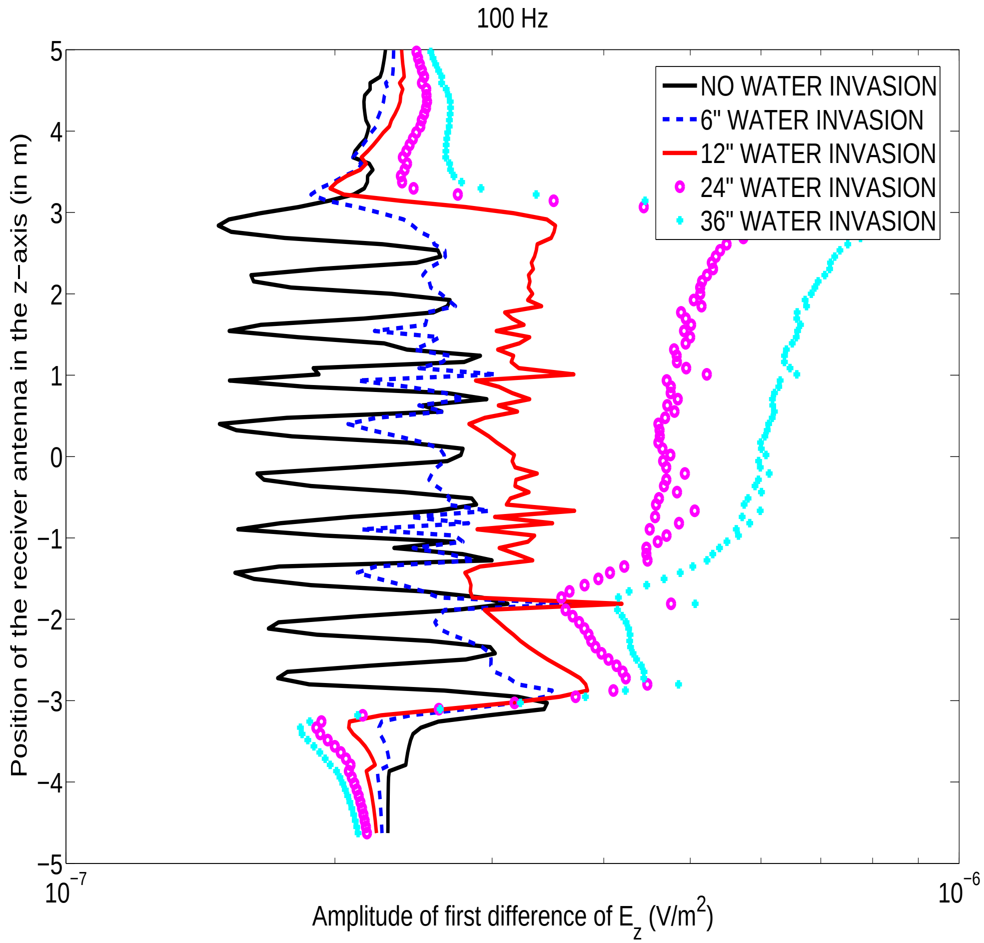
Variations due to frequency are small (below 5%)

MOTIVATION



Variations due to water ivasion are large

MOTIVATION



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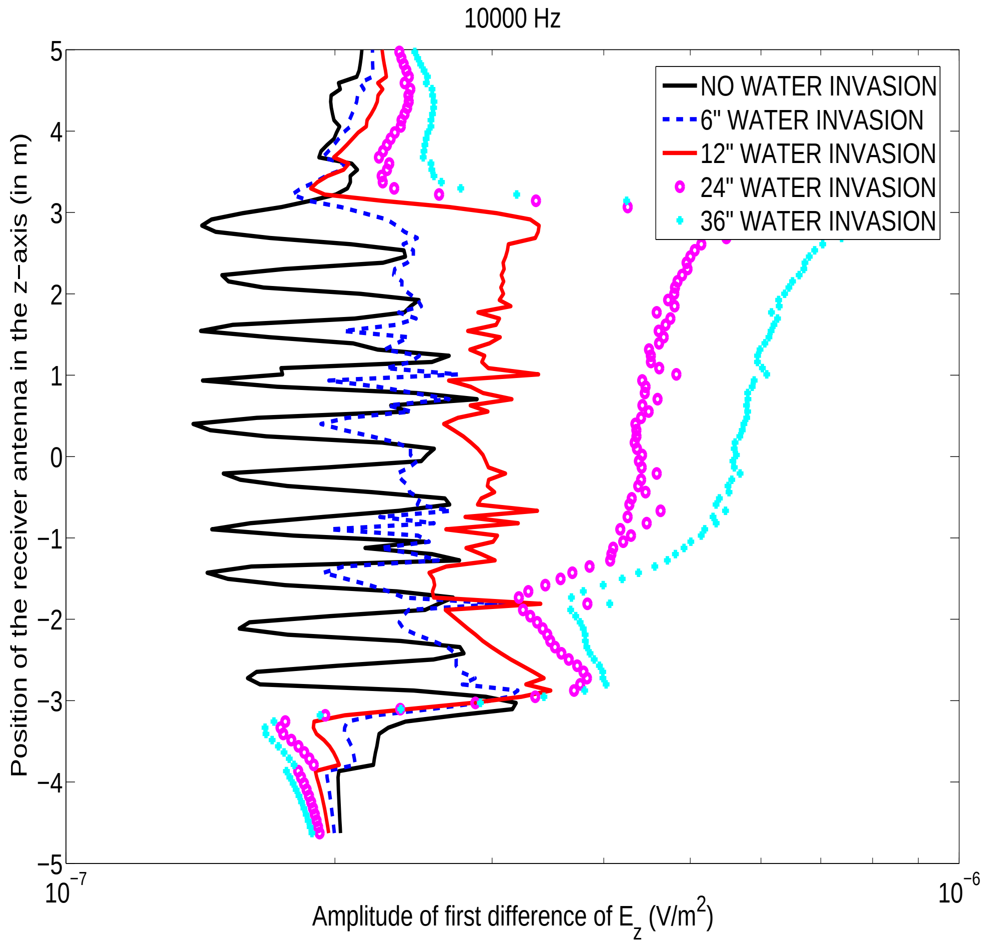

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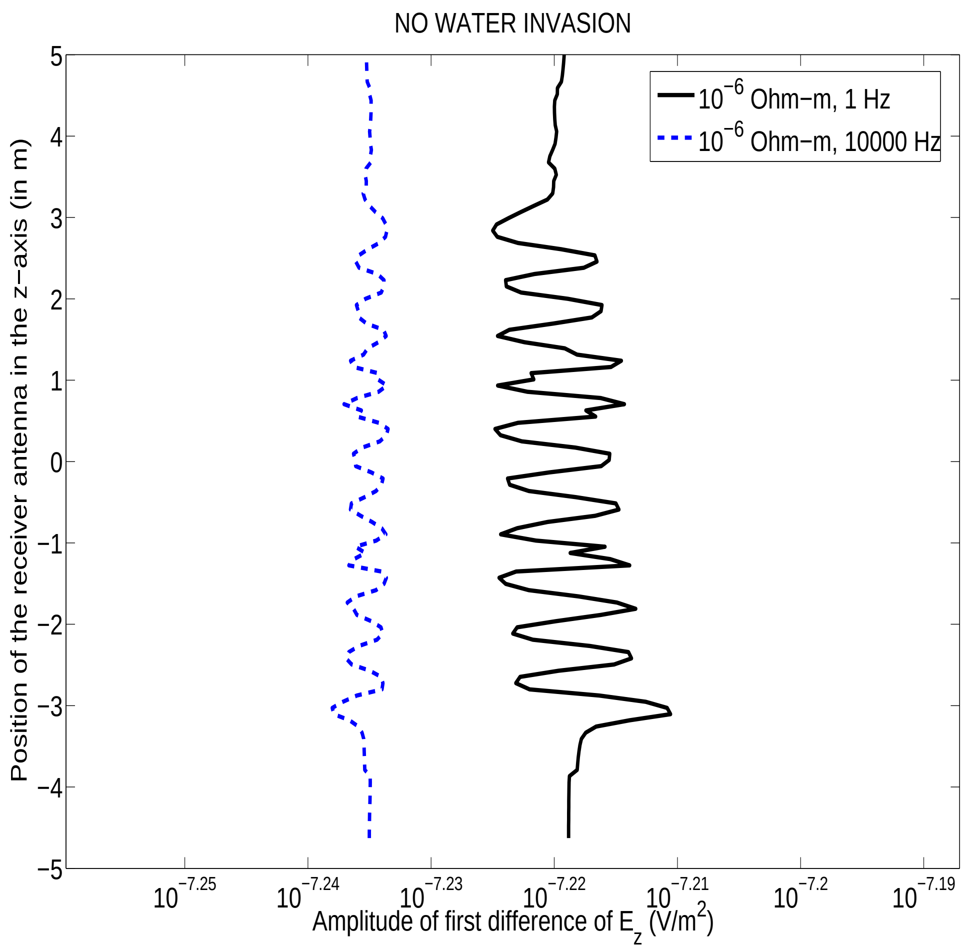


Casing resistivity can be analyzed from different frequency measurements

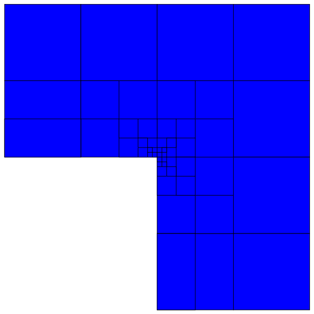
RESISTIVITY LOGGING PROBLEMS

Type of Problems We Can Solve with 2Dhp90 (v.7.2)

Physical Devices	Magnetic Buffers	Insulators	Displacement Currents
	Casing	Casing Imperfections	Combination of all
Materials	Isotropic	Anisotropic*	
Sources	Toroidal Antennas	Solenoidal Antennas	Dipoles in Any Direction
	Electrodes	Finite Size Antennas	Combination of All
Logging Instruments	LWD/MWD	Laterolog	Normal
	Induction	Dielectric Instruments	Cross-well
Frequency	0-10 Ghz.		
Invasion	Water	Oil	etc.

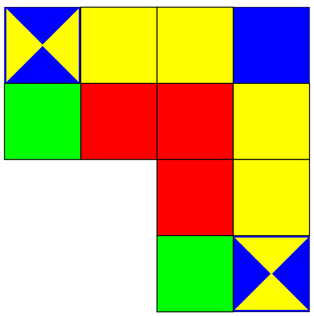
ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

THE hp-FINITE ELEMENT METHOD



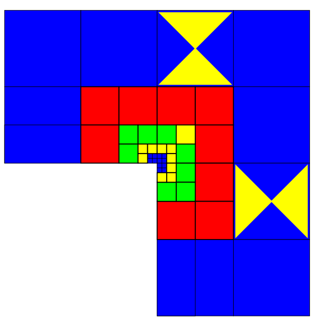
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal *h*-grids do NOT converge exponentially in real applications.
3. They may "lock" (100% error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.

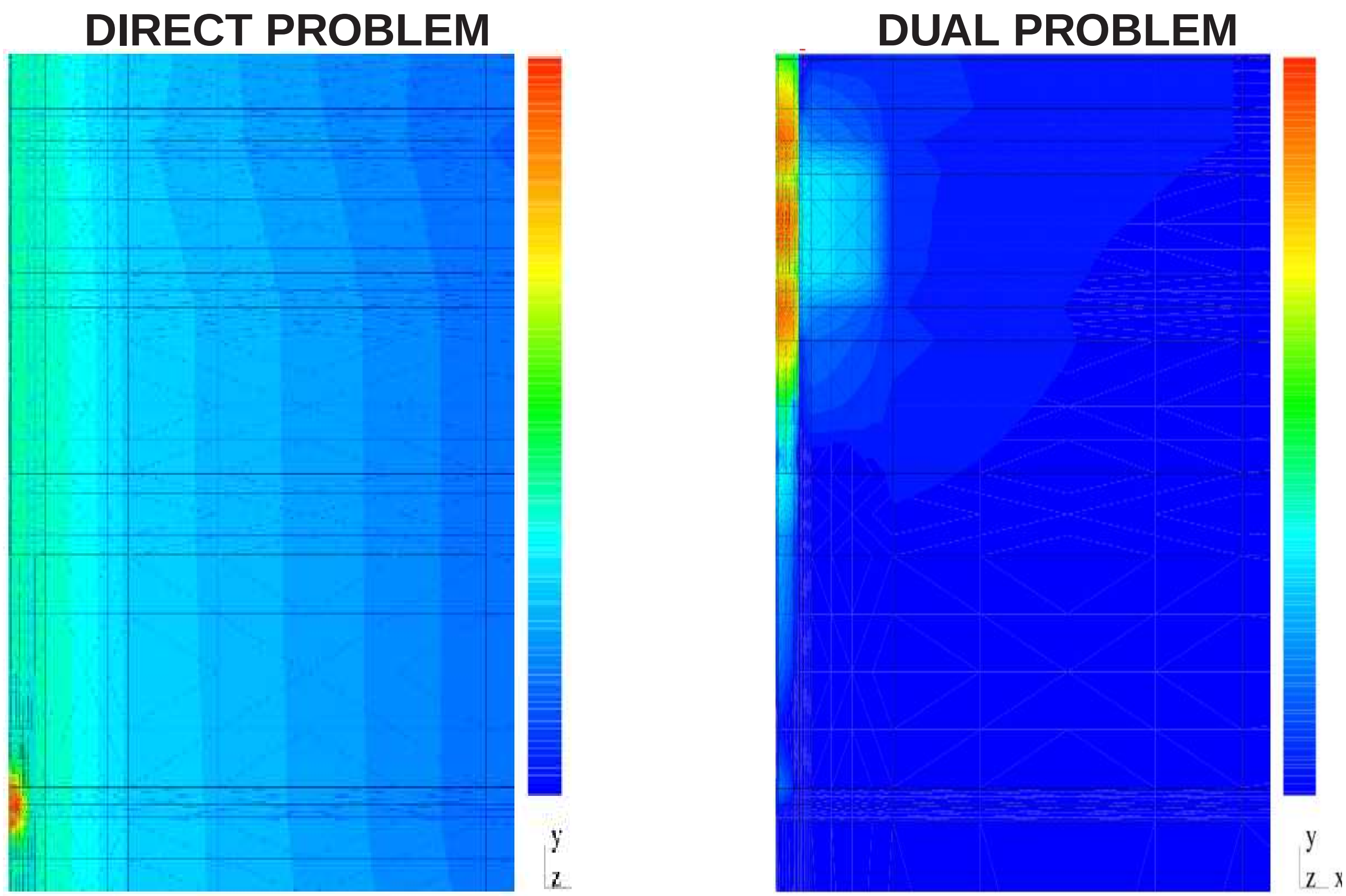


The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

GOAL-ORIENTED ADAPTIVITY

Mathematical Formulation (Goal-Oriented Adaptivity)

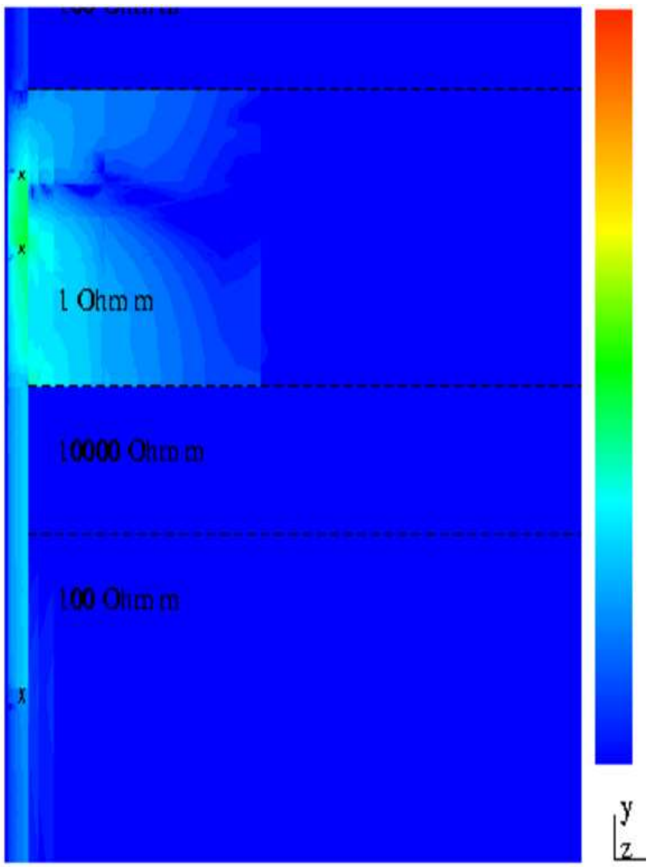


$$L(\Psi)=b(\Psi,G)$$

GOAL-ORIENTED ADAPTIVITY

Movie Presentation (Sensitivity Functions)

We want to study resolution and depth of investigation of a logging instrument.



We have: $|L(\Psi)| = |\int S\, dV| \leq \int |S|\, dV$.

In the next movies, we display: $\log_{10} |S|$.

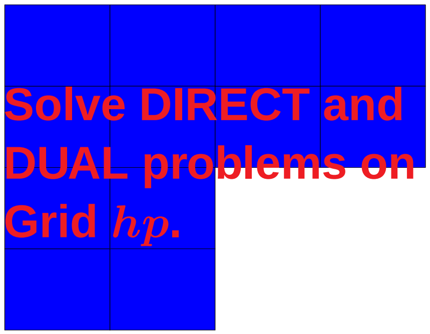
Scales:

- **Red** $\longrightarrow |S| = |L(\Psi)| * 10^4$.
- **Blue** $\longrightarrow |S| = |L(\Psi)| * 10^{-2}$.

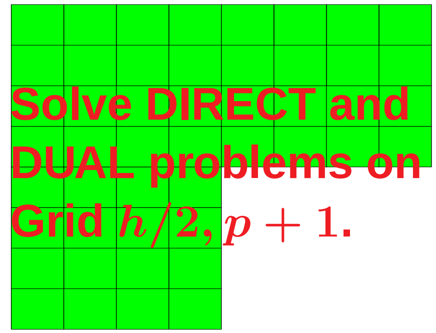
Direct Current

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Algorithm for Goal-Oriented Adaptivity



$\longrightarrow$

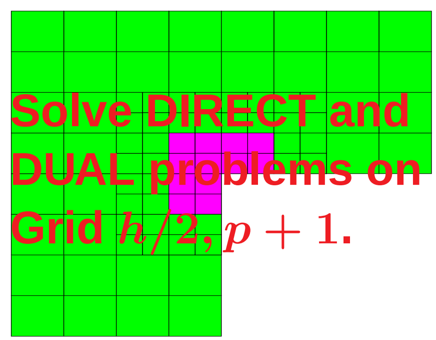


Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, **and** $\tilde{e} = \Psi_{h/2,p+1} - \Pi_{hp}\Psi_{h/2,p+1}$.
Compute $\epsilon = G_{h/2,p+1} - G_{hp}$, **and** $\tilde{\epsilon} = G_{h/2,p+1} - \Pi_{hp}G_{h/2,p+1}$.

$$|L(e)| = |b(e,\epsilon)| \sim |b(\tilde{e},\tilde{\epsilon})| \leq \sum_K |b_K(\tilde{e},\tilde{\epsilon})| \leq \sum_K \| \tilde{e} \|_{E,K} \| \tilde{\epsilon} \|_{E,K}.$$

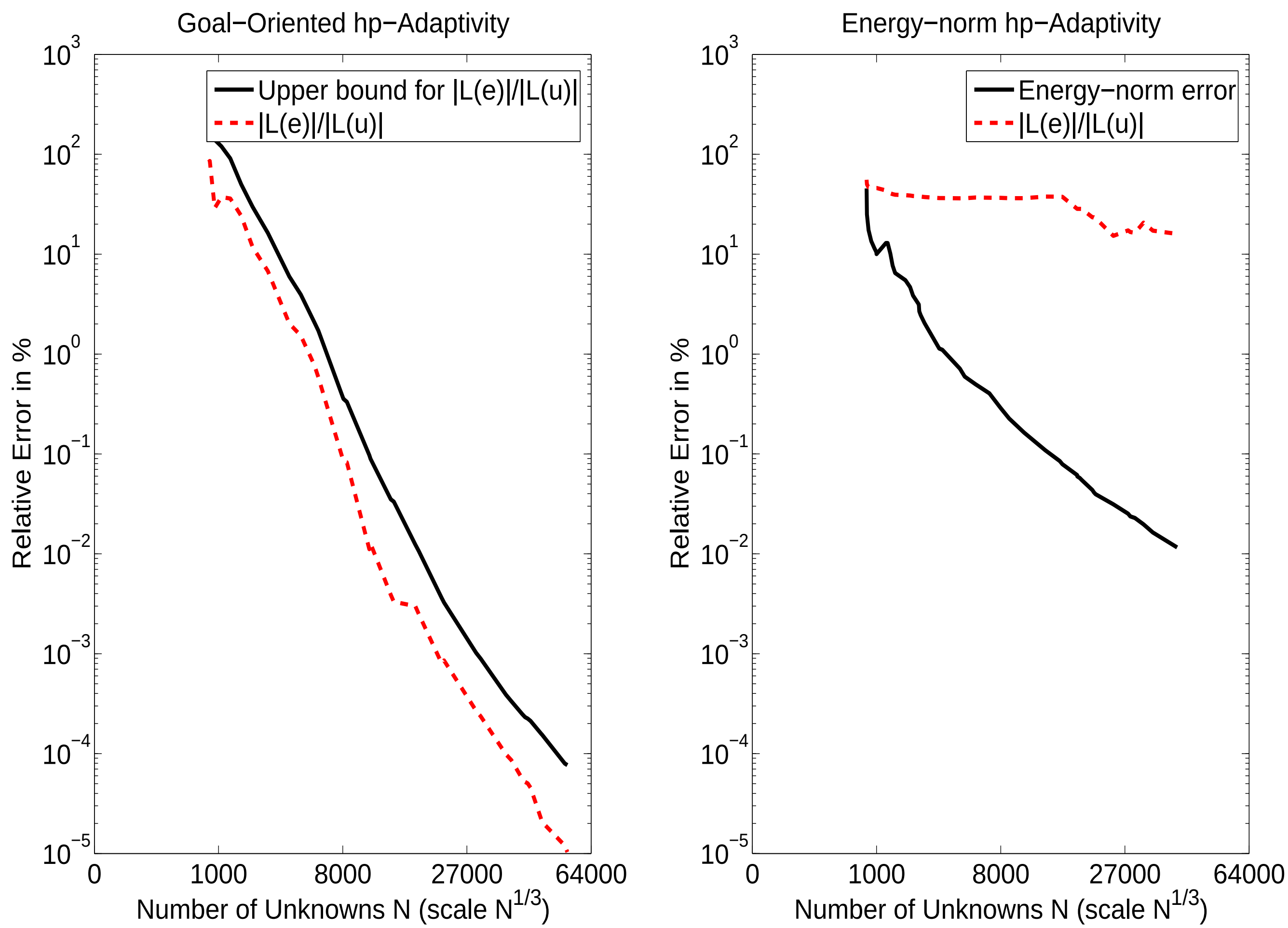
Apply the fully automatic hp-adaptive algorithm.

$\longrightarrow$



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Goal-Oriented vs. Energy-norm hp-Adaptivity

Problem with Mandrel at 2 Mhz.

Continuous Elements (Goal-Oriented Adaptivity)

Quantity of Interest	Real Part	Imag Part
COARSE GRID	-0.1629862203E-01	-0.4016944732E-02
FINE GRID	-0.1629862347E-01	-0.4016944223E-02

Continuous Elements (Energy-norm Adaptivity)

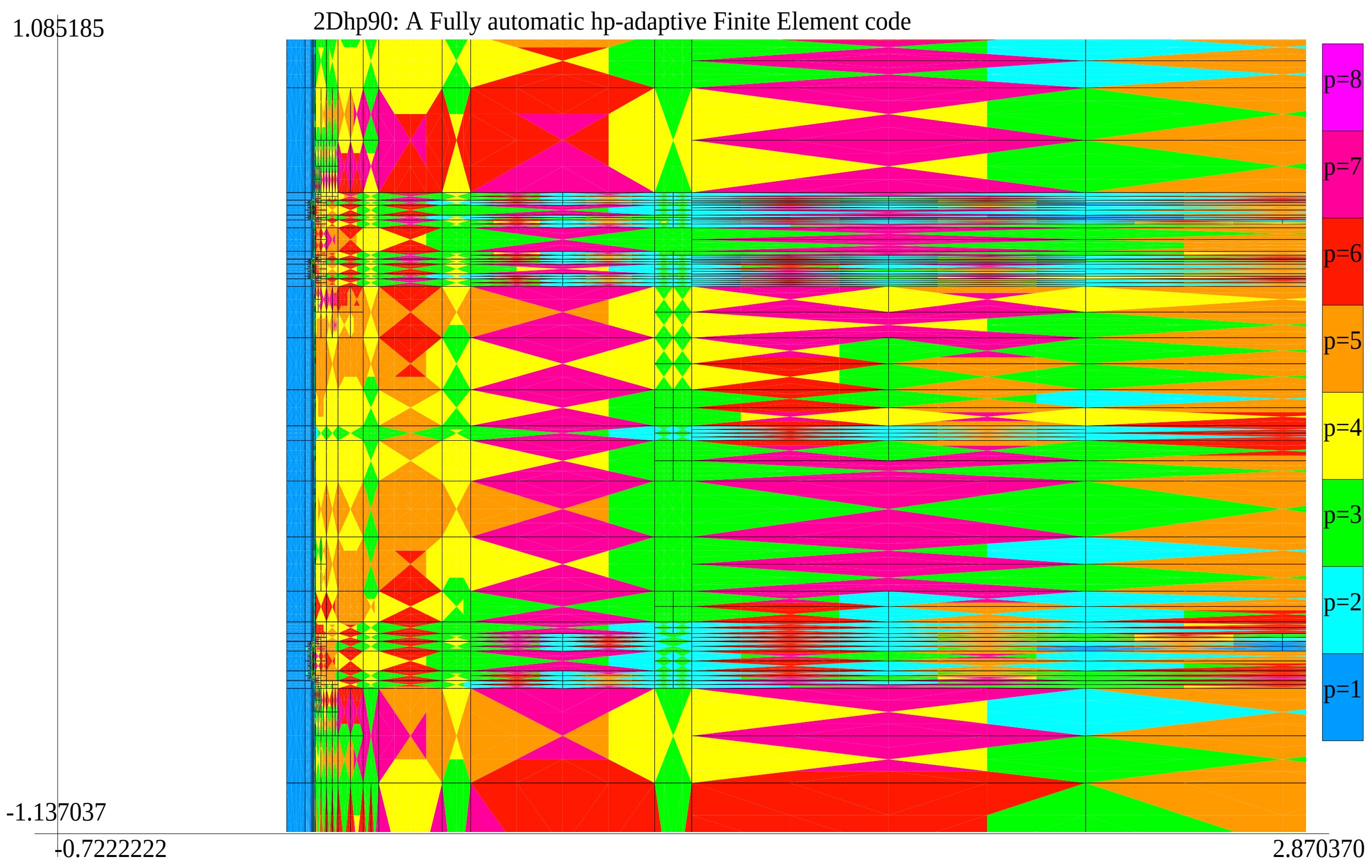
Quantity of Interest	Real Part	Imag Part
0.01% ENERGY ERROR	-0.1382759158E-01	-0.2989492851E-02

It is critical to use GOAL-ORIENTED adaptivity.

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

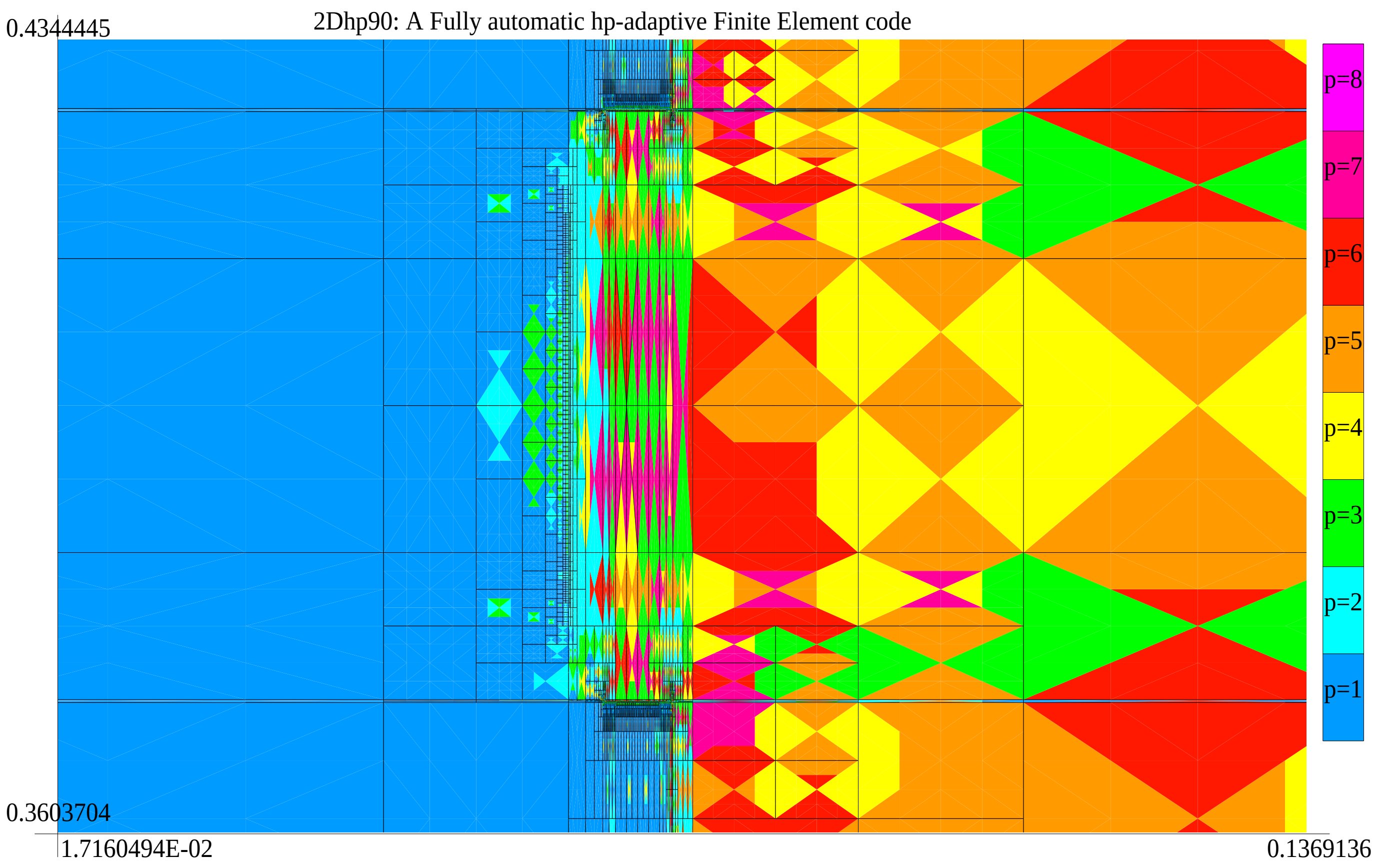
GOAL-ORIENTED HP-ADAPTIVITY



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

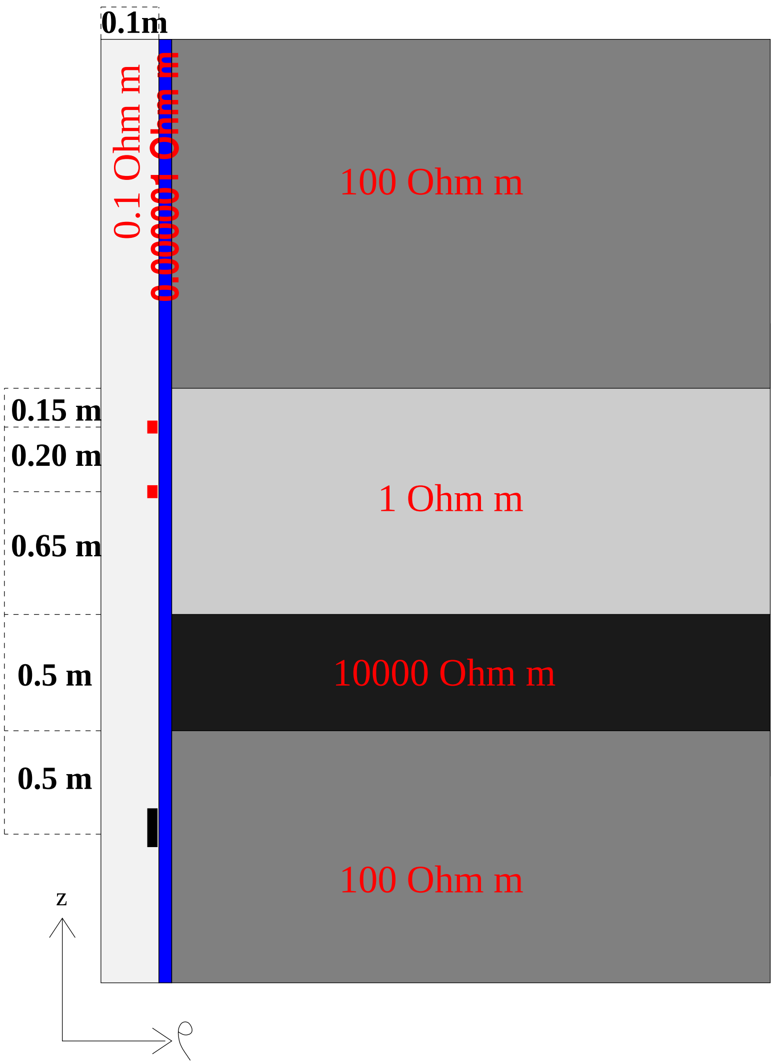
First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

GOAL-ORIENTED HP-ADAPTIVITY (ZOOM TOWARDS FIRST RECEIVER ANTENNA)



CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Model Problem with Steel Casing



Frequency: 10 Hz - 10 kHz.

Casing resistivity: 10^{-6} Ohm · m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 $\%$

Toroidal antennas

Size (domain): 500m x 4000m

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

Time-Harmonic Maxwell's Equations

$\nabla \times \mathrm{H} = (\bar{\bar{\sigma}} + j\omega\bar{\bar{\epsilon}})\mathrm{E} + \mathrm{J}^{imp}$	Ampere's law
$\nabla \times \mathrm{E} = -j\omega\bar{\bar{\mu}}\mathrm{H} - \mathrm{M}^{imp}$	Faraday's law
$\nabla \cdot (\bar{\bar{\epsilon}}\mathrm{E}) = \rho$	Gauss' law of Electricity
$\nabla \cdot (\bar{\bar{\mu}}\mathrm{H}) = 0$	Gauss' law of Magnetism

E-VARIATIONAL FORMULATION:

$$\left\{ \begin{array}{l} \text{Find } \mathrm{E} \in \mathrm{E}_D + H_D(\mathrm{curl};\Omega) \text{ such that:} \\ \displaystyle\int_\Omega (\bar{\bar{\mu}}^{-1}\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\bar{\bar{k}}^2 \mathrm{E}) \cdot \bar{\mathrm{F}}\, dV = -j\omega \int_\Omega \mathrm{J}^{imp} \cdot \bar{\mathrm{F}}\, dV \\ \displaystyle +j\omega \int_{\Gamma_N} \mathrm{J}^{imp}_{\Gamma_N} \cdot \bar{\mathrm{F}}_t\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}\mathrm{M}^{imp}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV \quad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega) \end{array} \right.$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

AXISYMMETRIC PROBLEMS

E_ϕ **-Variational Formulation (Azimuthal)**

$$\left\{\begin{array}{l}\text{Find } E_\phi \in E_{\phi,D} + \tilde{H}^1_D(\Omega) \text{ such that:}\\ \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \nabla \times \mathrm{E}_\phi) \cdot (\nabla \times \bar{\mathrm{F}}_\phi)\, dV - \int_\Omega (\bar{\bar{k}}^2_\phi \mathrm{E}_\phi) \cdot \bar{\mathrm{F}}_\phi\, dV = -j\omega \int_\Omega J^{imp}_\phi\, \bar{F}_\phi\, dV \\ \qquad + j\omega \int_{\Gamma_N} J^{imp}_{\phi,\Gamma_N}\, \bar{F}_\phi\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \mathrm{M}^{imp}_{\rho,z}) \cdot \bar{\mathrm{F}}_\phi\, dV \qquad \forall\, F_\phi \in \tilde{H}^1_D(\Omega)\end{array}\right.$$

$\mathrm{E}_{\rho,z}$ **-Variational Formulation (Meridian)**

$$\left\{\begin{array}{l}\text{Find } (E_\rho, E_z) \in \mathrm{E}_D + \tilde{H}_D(\mathrm{curl};\Omega) \text{ such that:}\\ \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \nabla \times \mathrm{E}_{\rho,z}) \cdot (\nabla \times \bar{\mathrm{F}}_{\rho,z})\, dV - \int_\Omega (\bar{\bar{k}}^2_{\rho,z} \mathrm{E}_{\rho,z}) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV = \\ \qquad -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z\, dV + j\omega \int_{\Gamma_N} J^{imp}_{\rho,\Gamma_N} \bar{F}_\rho + J^{imp}_{z,\Gamma_N} \bar{F}_z\, dS \\ \qquad - \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \mathrm{M}^{imp}_\phi) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV \qquad \forall\, (F_\rho, F_z) \in \tilde{H}_D(\mathrm{curl};\Omega)\end{array}\right.$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

- **Physical Devices: Casing, Casing Imperfections, Mandrel, Magnetic Buffers, Insulators, Displacement Currents, Combination of All, etc.**
- **Materials: Isotropic, Anisotropic*.**
- **Sources: Toroidal Antennas, Solenoidal Antennas, Dipoles in Any Direction, Electrodes, Finite Size Antennas, Combination of All, etc.**
- **Logging Instruments: Logging While Drilling (LWD), Laterolog, Normal, Induction, Dielectric Instruments, Cross-well, etc.**
- **Any Frequency (0-10 Ghz).**

ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
  2. Exact solution in a homogeneous media with a mandrel.
  3. Exact solution in a homogeneous media with casing.
- **Verification of Physical Properties.**
  1. Reciprocity principle (Gregory Itskovich).
  2. Discrete divergence free approximation for edge elements.
- **Numerical Verifications.**
  1. Different size of domain and antennas.
  2. Comparison against other numerical software (Yang Wei).
  3. Error control provided by the fine grid solution.
  4. Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

Continuous Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516098429E-08	-0.1456374493E-08
FINE GRID	0.1516094029E-08	-0.1456390824E-08

Edge Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516060872E-08	-0.1456337248E-08
FINE GRID	0.1516093804E-08	-0.1456390864E-08

Error control provided by the fine grid solution.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

Continuous Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516098429E-08	-0.1456374493E-08
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FINE GRID	0.1516093804E-08	-0.1456390864E-08

Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
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HIGHLY RELIABLE SOFTWARE

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Performance (How Fast Can We Solve the Problems?)

80 Vert. Pos.	$10^{-6}\Omega \cdot m$	$10^{-5}\Omega \cdot m$
Toroid (10 Khz)	19' 46"	16' 28"
Ring of Vert. Dipoles (10 Khz)	22' 47"	17' 02"
Ring of Horiz. Dipoles (10 Khz)	19' 25"	13' 25"
Electrodes (0 Hz)	10' 10"	8' 35"

IBM Power 4 compiler 1.3 Ghz (4 years old)

Possible improvements in performance:

- **To use a 3.4 Ghz processor.**
- **To execute the code in 8 processors (10 positions per processor).**
- **To improve implementation.**

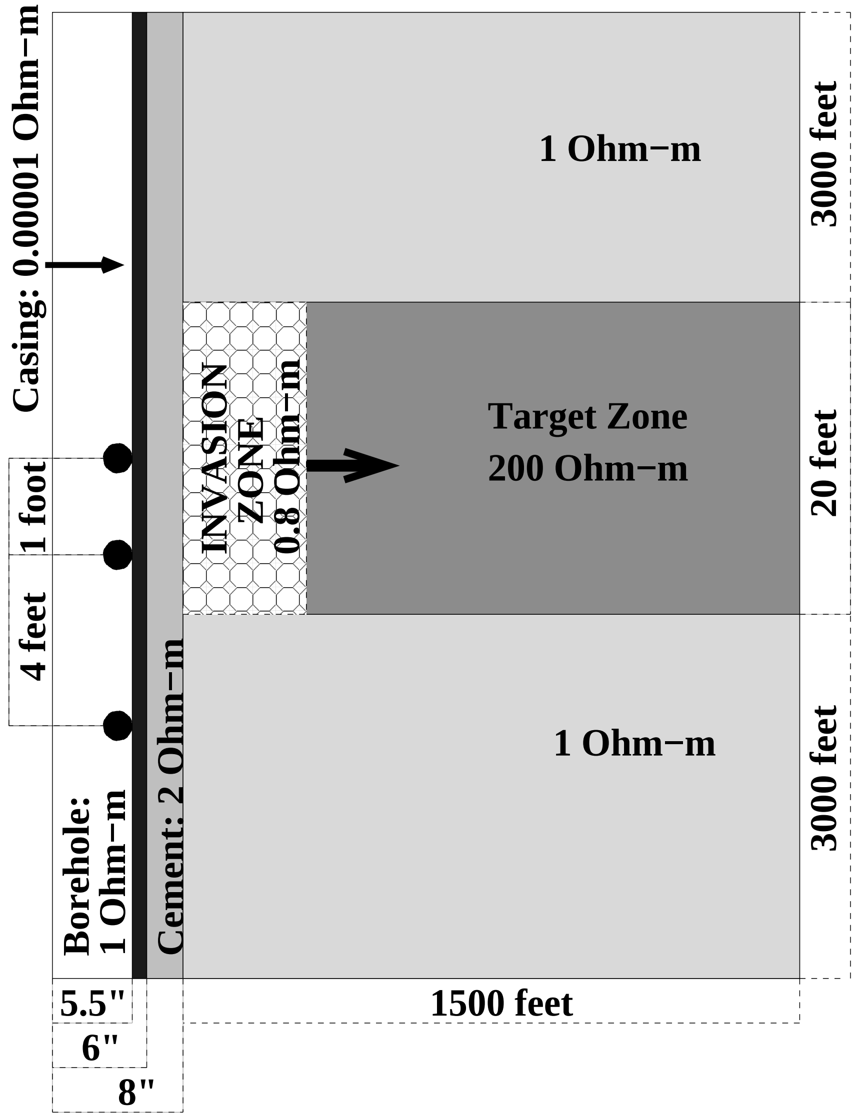
HIGH PERFORMANCE SOFTWARE

SIMULATION OF LOGGING INSTRUMENTS

List of Four Model Problems Solved with 2Dhp90 (v.7.2)

- **Problem I:** Through Casing Resistivity Tool (TCRT). Study of water invasion in an oil-based formation.
- **Problem II:** Study of oil invasion on an anisotropic formation with laminated sands.
- **Problem III:** Detection of oil-water contact below the position of an induction logging instrument.
- **Problem IV:** Through Casing Resistivity Tool (TCRT). Study of anisotropy and water invasion effects on a model formation typical from the Gulf of Mexico.

SIMULATION OF LOGGING INSTRUMENTS



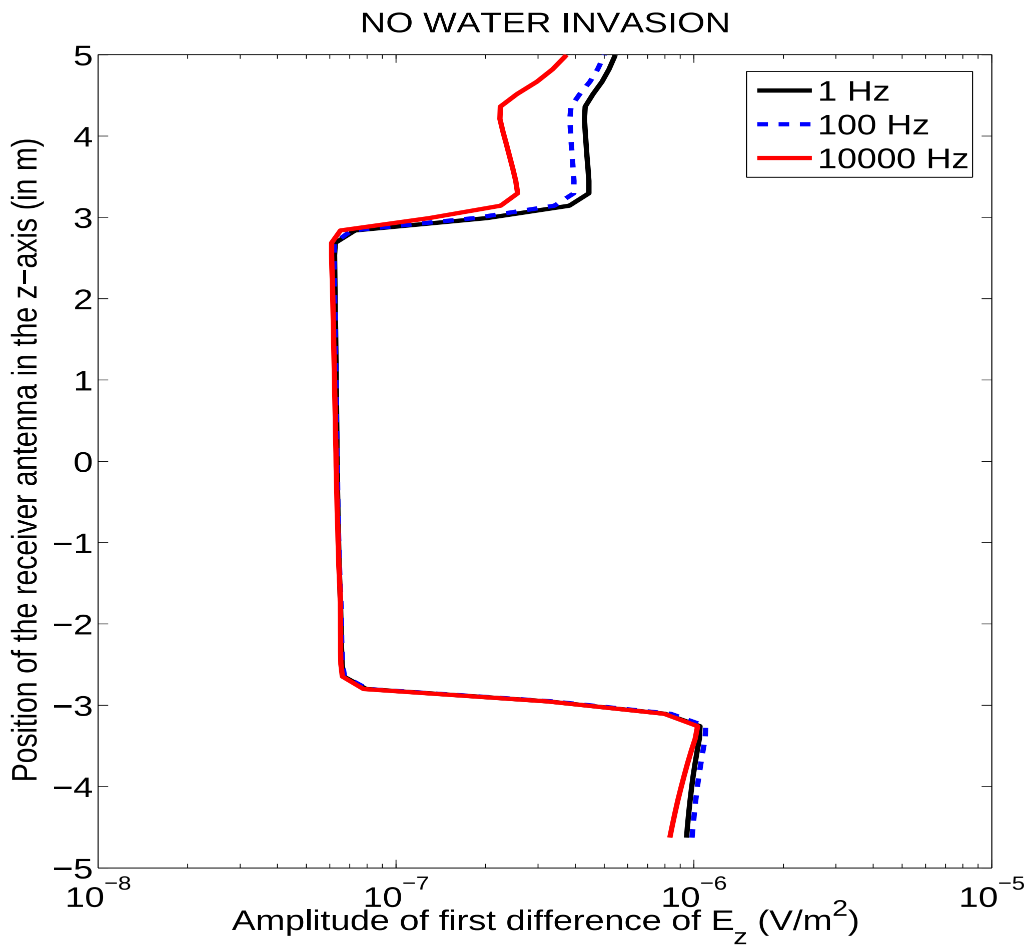
Axisymmetric 3D problem.

Seven different materials.

Through casing resistivity instrument.

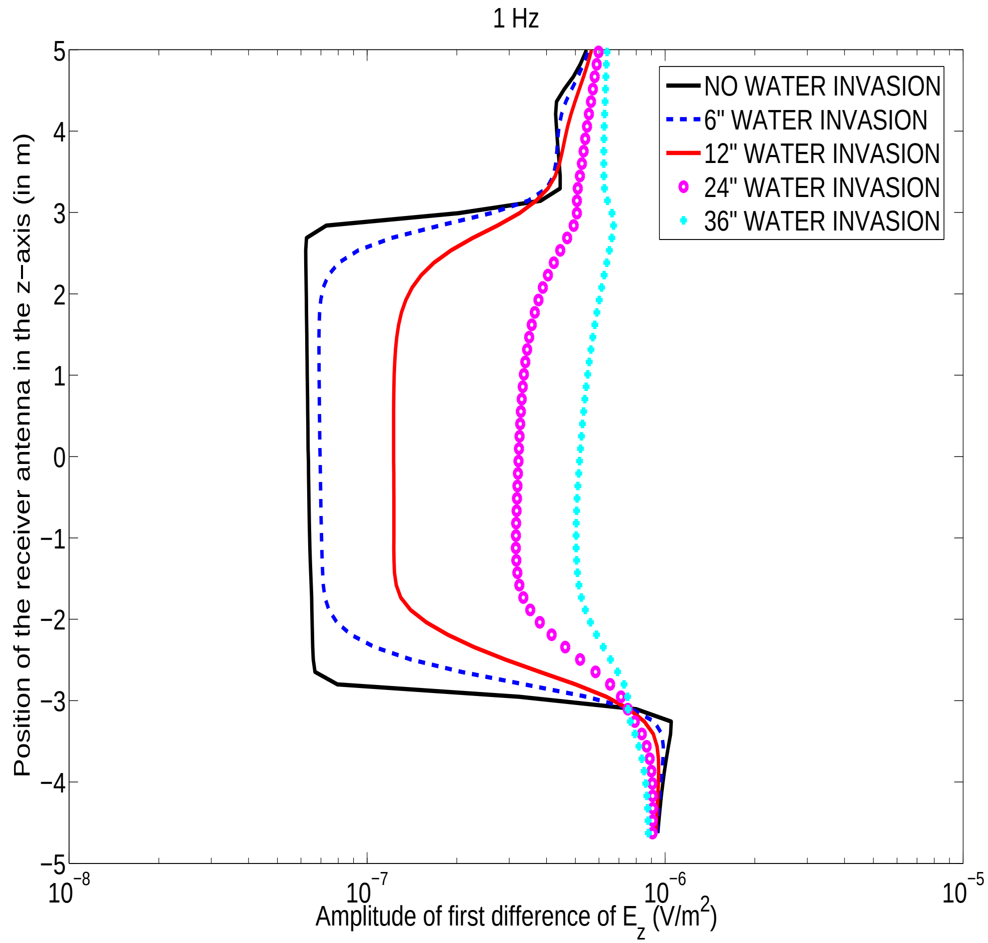
Objective: Study the effect of water invasion THROUGH CASING.

SIMULATION OF LOGGING INSTRUMENTS



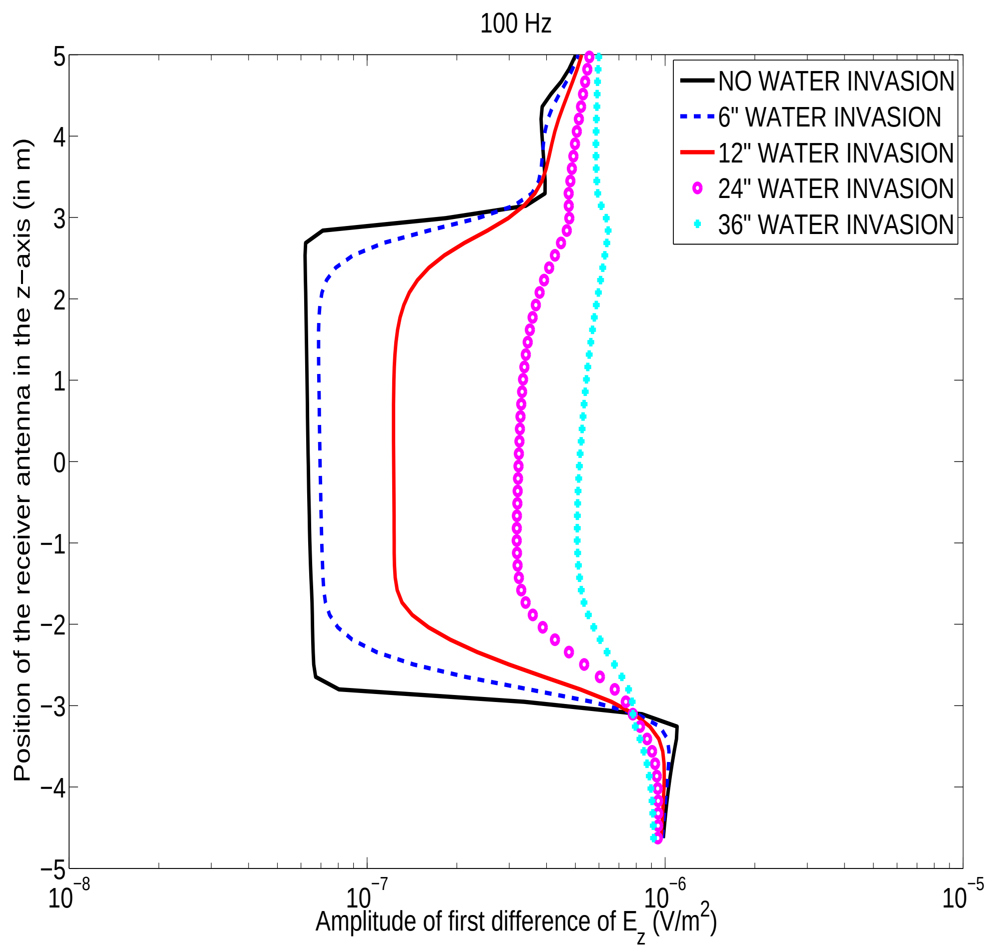
Variations due to frequency are small (below 5%)

SIMULATION OF LOGGING INSTRUMENTS



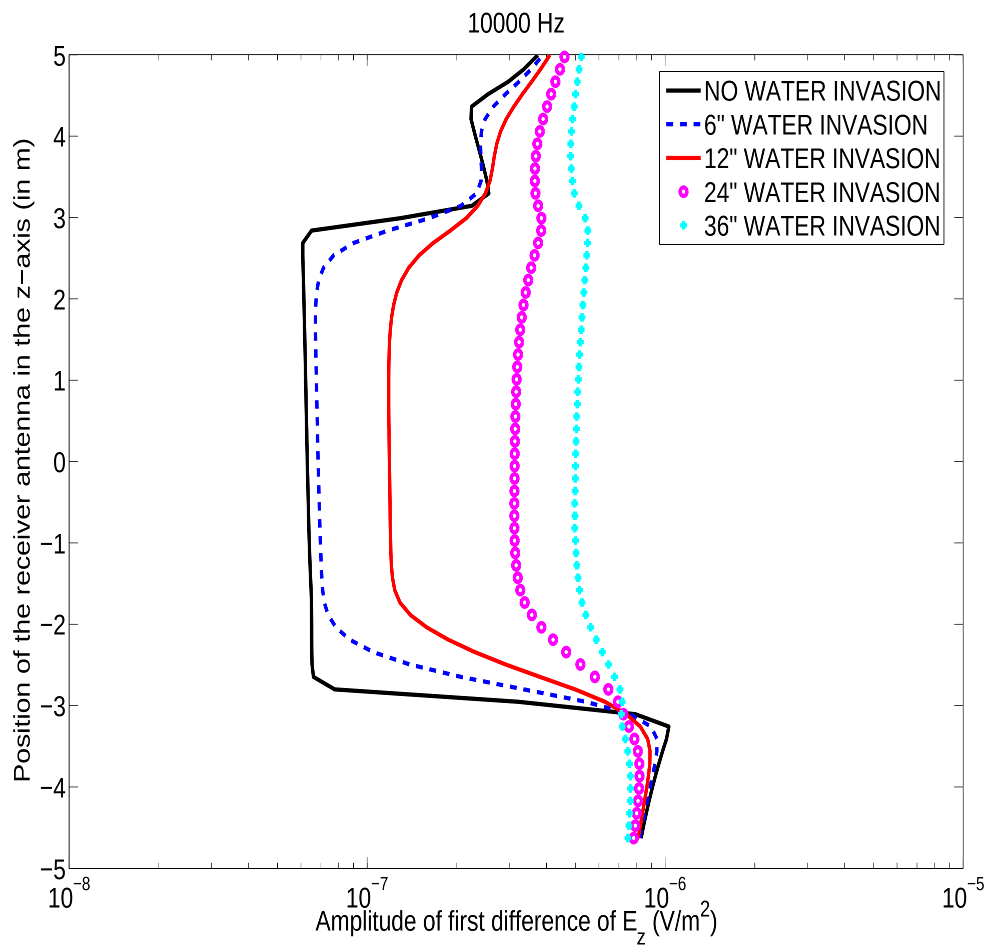
Water invasion through casing can be accurately assessed

SIMULATION OF LOGGING INSTRUMENTS



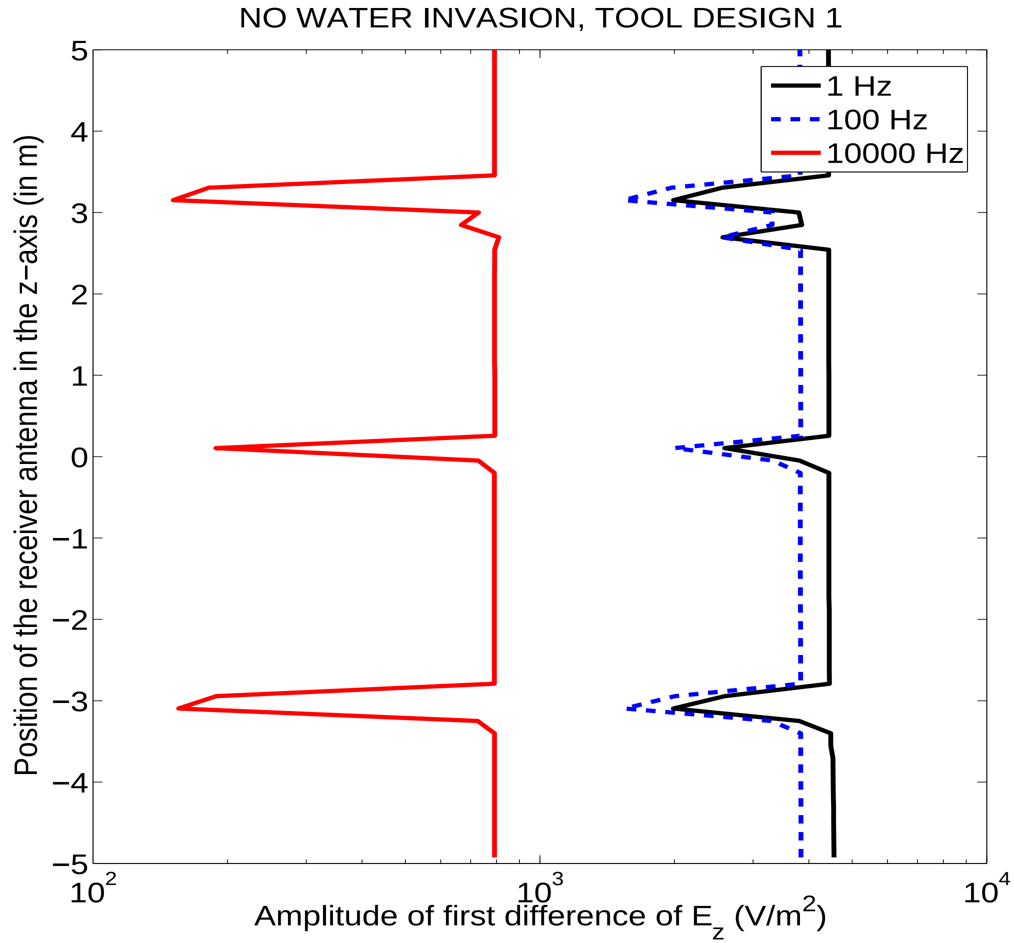
Water invasion through casing can be accurately assessed

SIMULATION OF LOGGING INSTRUMENTS



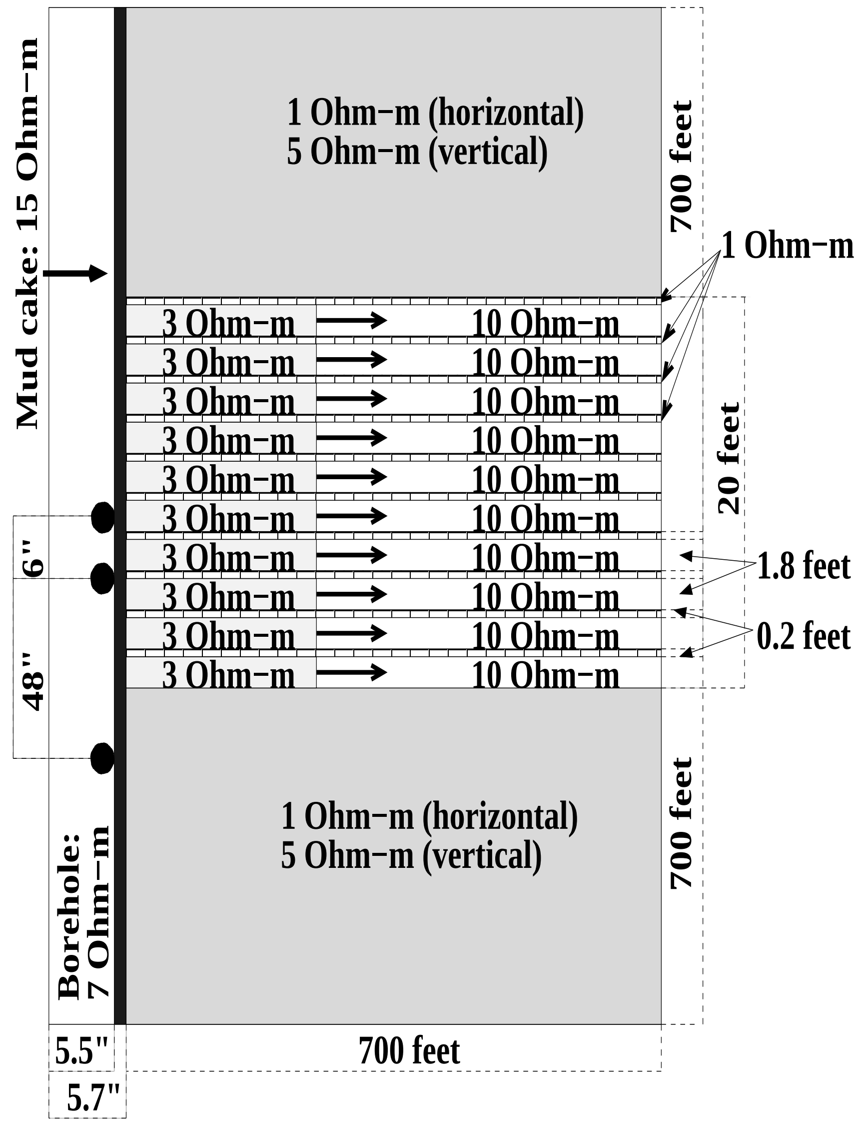
Water invasion through casing can be accurately assessed

SIMULATION OF LOGGING INSTRUMENTS



Mandrel Through Casing provides meaningless results

SIMULATION OF LOGGING INSTRUMENTS



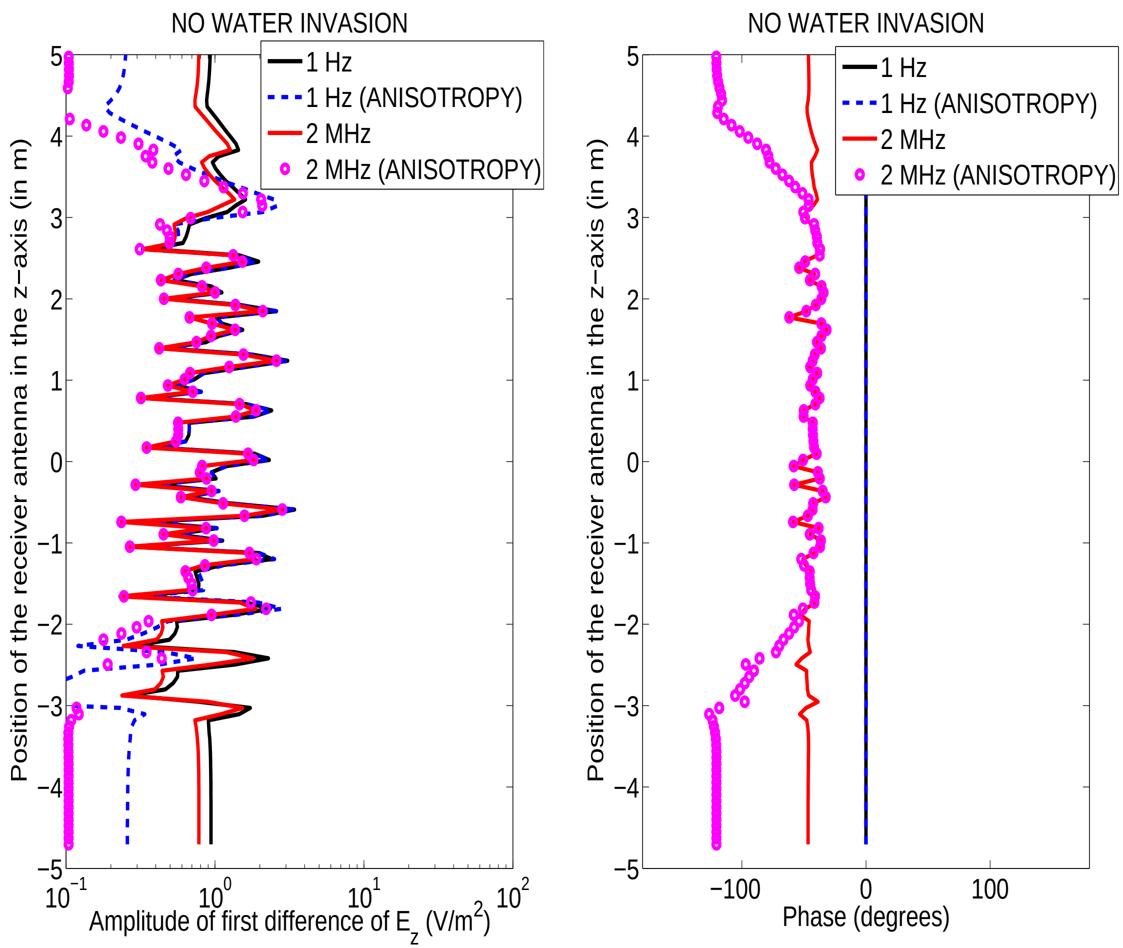
Axisymmetric 3D problem.

Seven different materials.

Laminated sands.

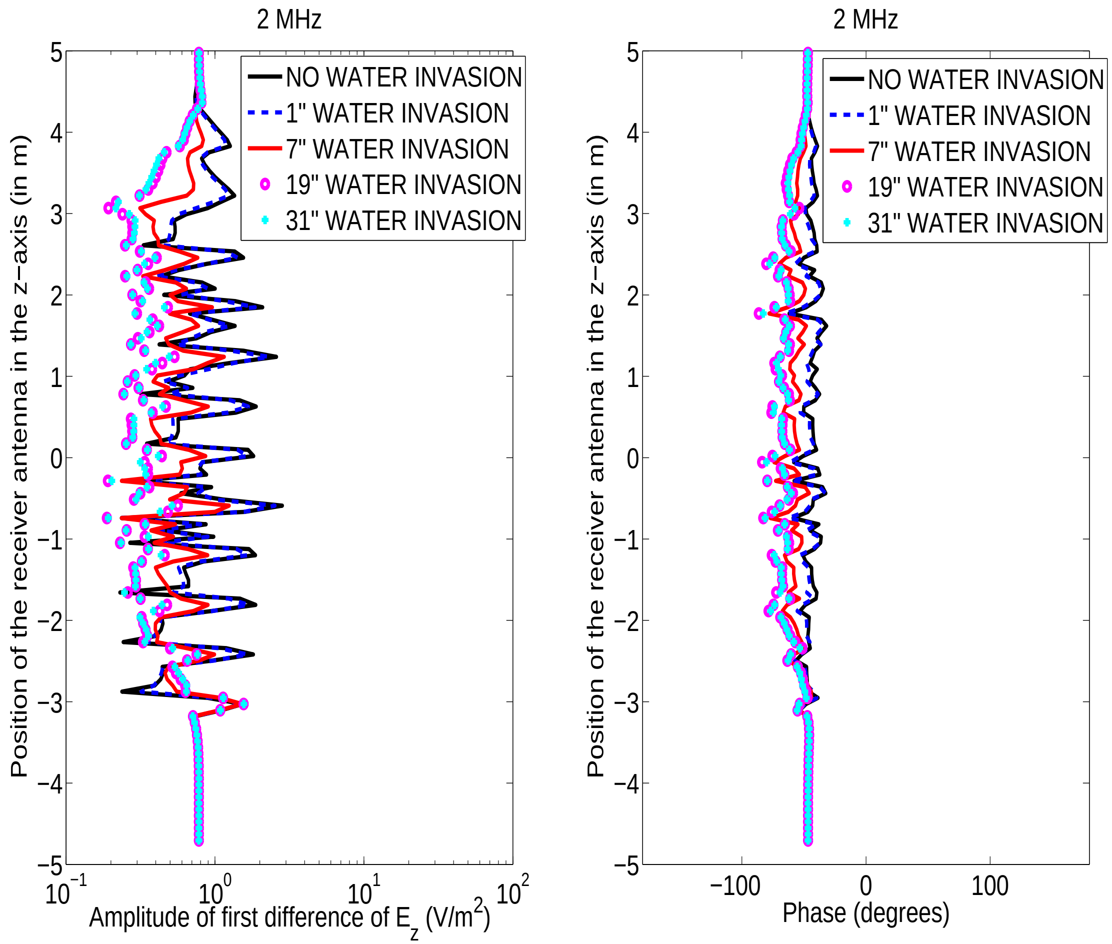
Objective: Study the effects of invasion and anisotropy.

SIMULATION OF LOGGING INSTRUMENTS



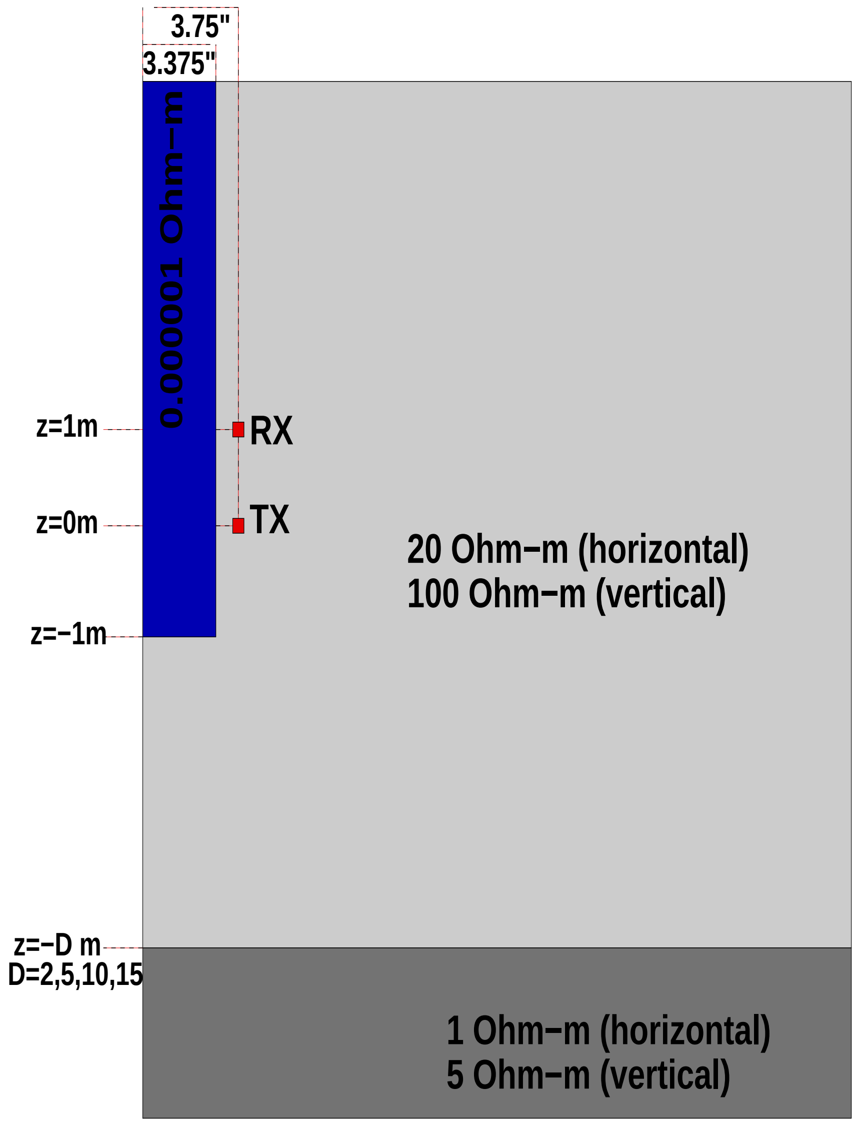
Anisotropy effects are significant. Frequency variations are below 10%

SIMULATION OF LOGGING INSTRUMENTS



Accurate software is needed for water invasion assessment

SIMULATION OF LOGGING INSTRUMENTS



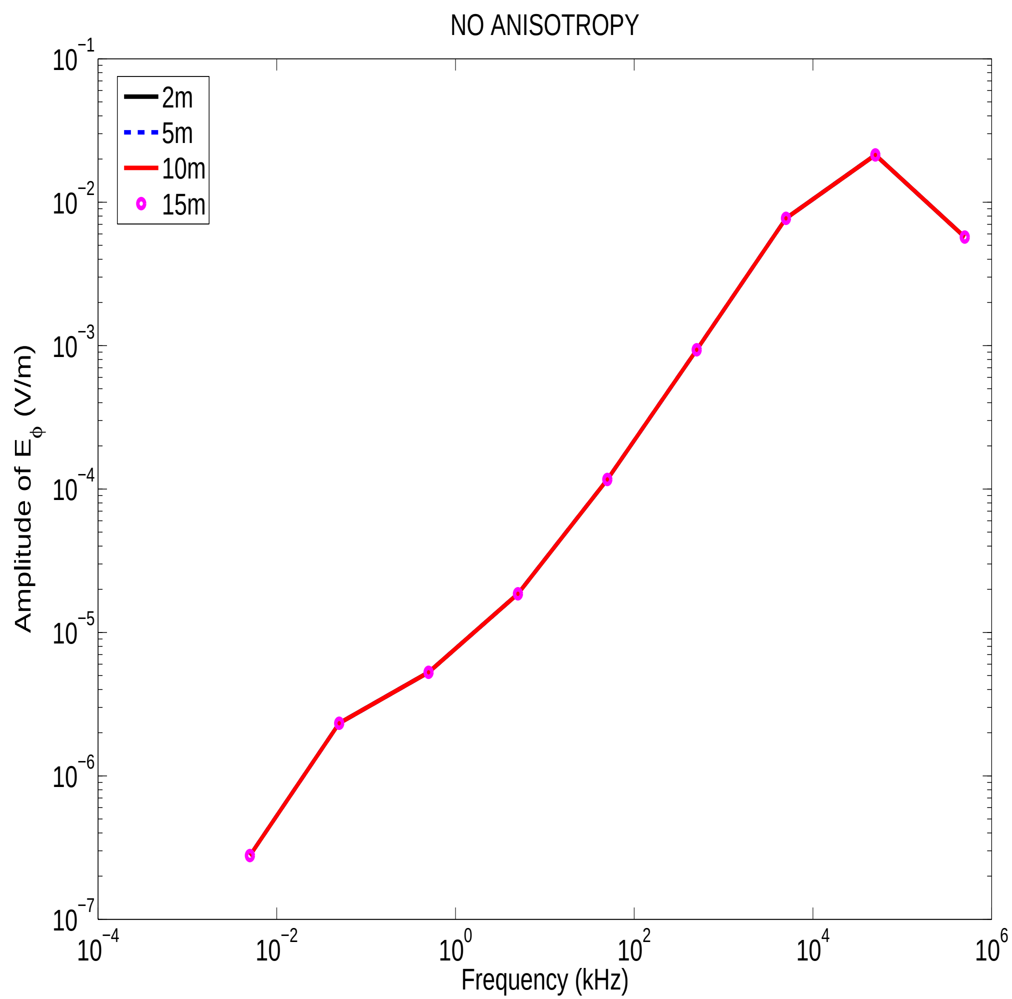
Axisymmetric 3D problem.

Seven different materials.

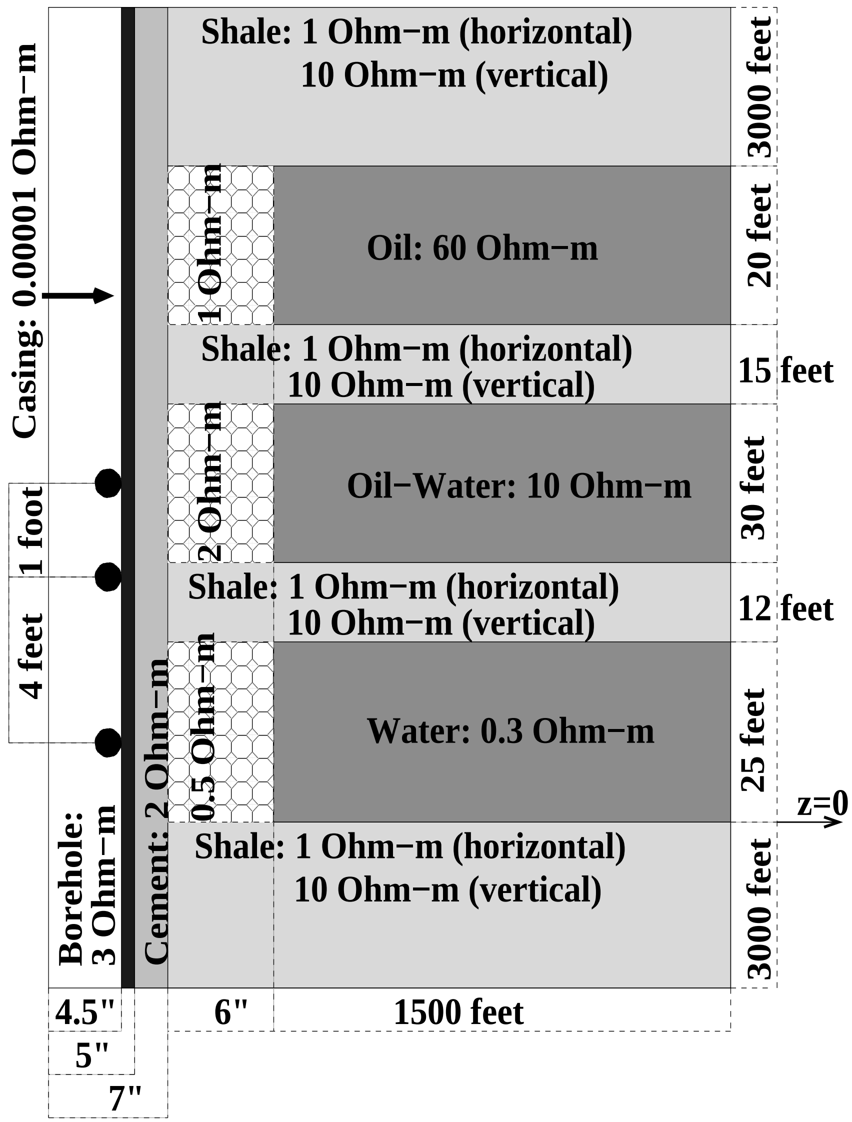
Through casing resistivity instrument.

Objective: Study the effect of invasion.

SIMULATION OF LOGGING INSTRUMENTS



SIMULATION OF LOGGING INSTRUMENTS



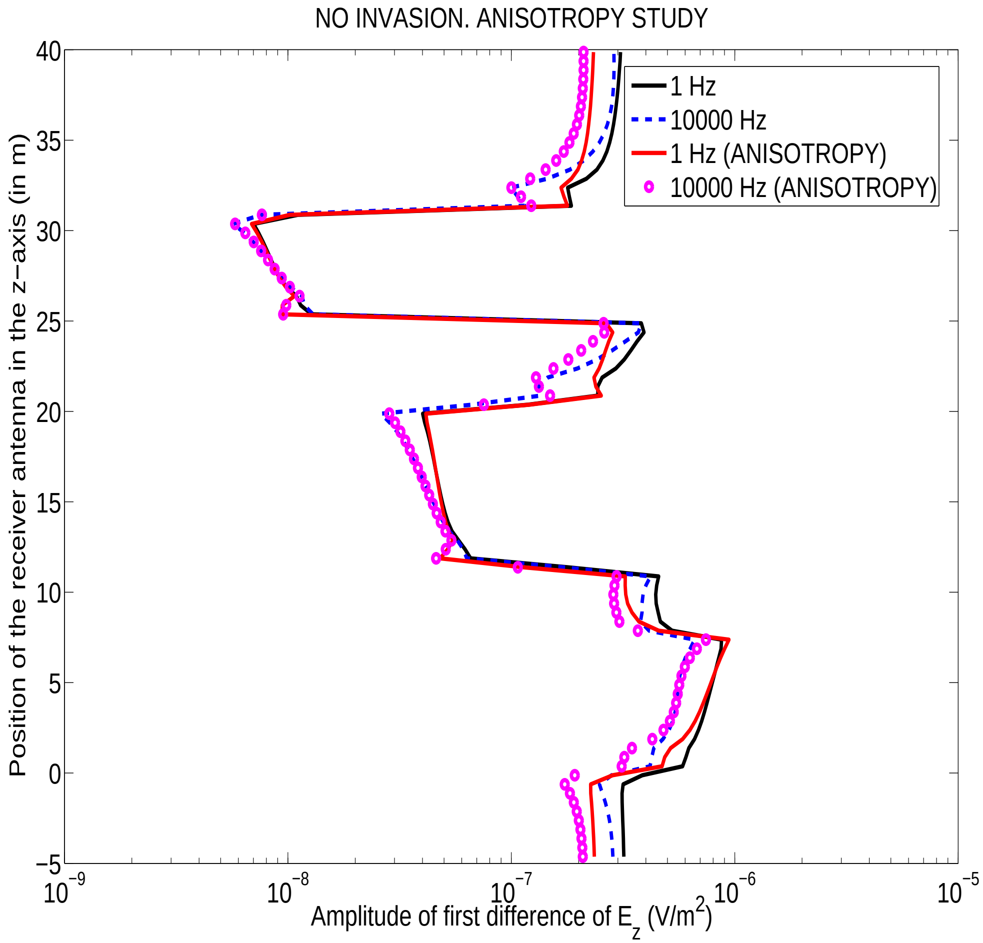
Axisymmetric 3D problem.

Seven different materials with high contrast on resistivity.

Through casing resistivity instrument.

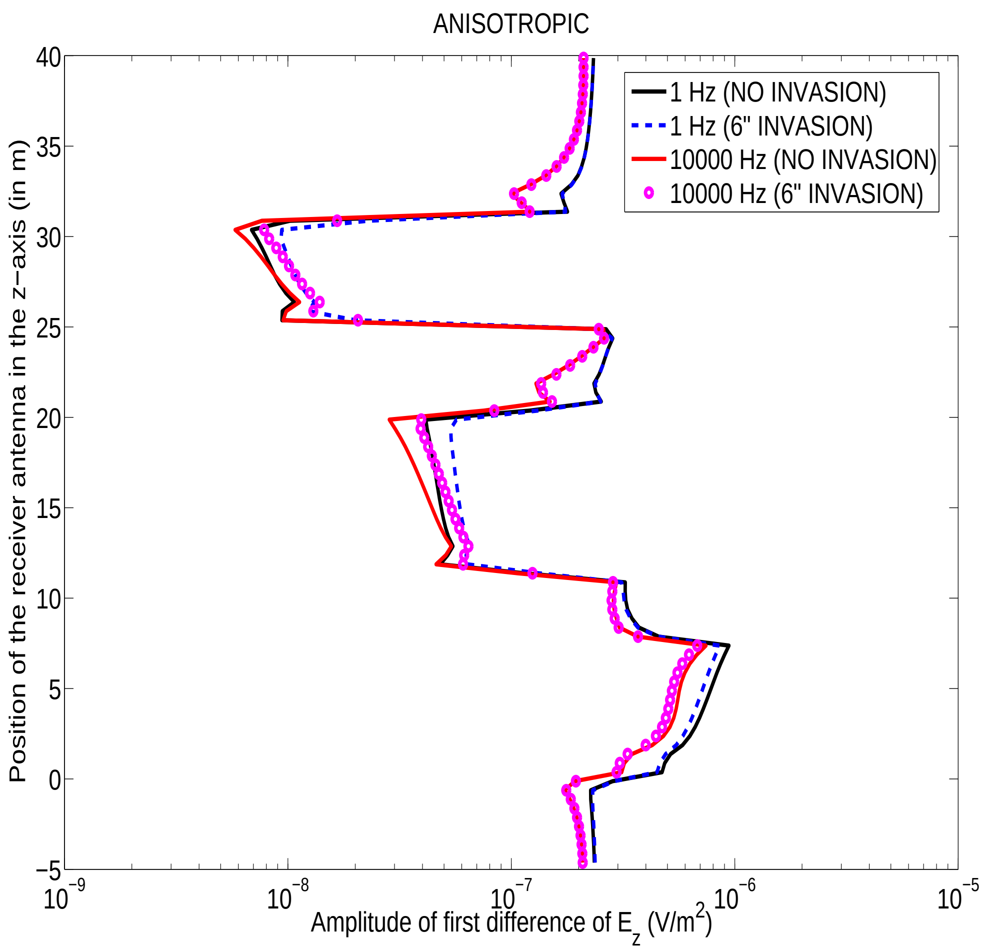
Objective: Study the effect of invasion and anisotropy THROUGH CASING.

SIMULATION OF LOGGING INSTRUMENTS



Study of anisotropy and frequency effects require from high accuracy simulations

SIMULATION OF LOGGING INSTRUMENTS



Variations due to invasion are below 20%.

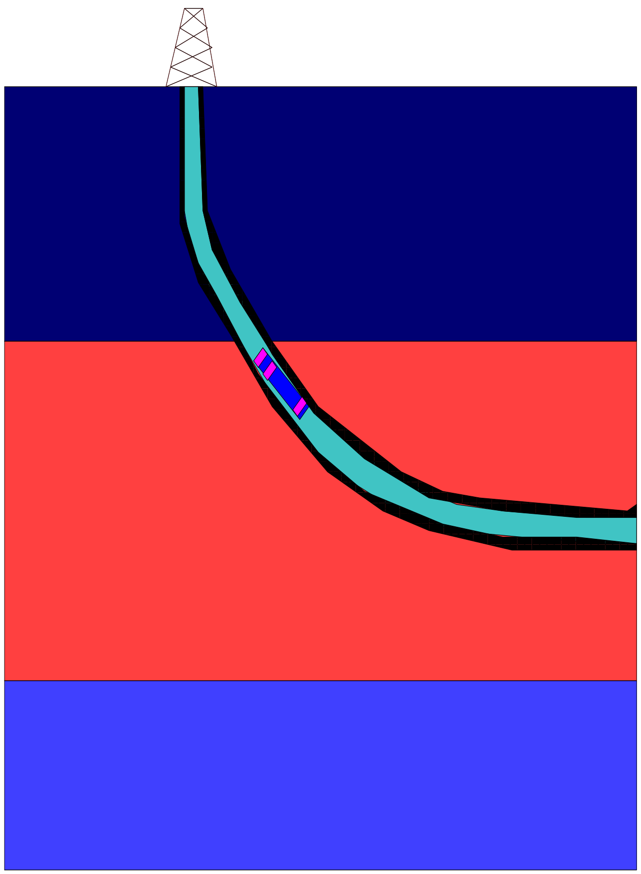
CONCLUSIONS AND FUTURE WORK

- **The self-adaptive goal-oriented hp-adaptive strategy converges exponentially in terms of a user-prescribed quantity of interest vs. the CPU time.**

- **It is possible to simulate a variety of EM logging instruments by using the self-adaptive goal-oriented hp-FEM.**
  - **The software can be utilized to simulate ALL axisymmetric resistivity logging instruments in possibly cased wells.**
  - **Furthermore, by using 2Dhp90, we can accurately describe the effect of water/oil-based mud invasion, anisotropy, magnetic buffers, etc.**

FUTURE WORK

Simulation of 3D Resistivity Logging Problems



- **PROJECT I: Simulate 3D DC and AC Resistivity Logging Problems.**
  - **Main challenge: To Perform Fast Large Computations.**
  - **Expected results: Similar results as in 2D.**
- **PROJECT II: Invert 2D Multi-Physic Problems.**
  - **Main challenge: To deal with different physics.**
  - **Expected results: Similar results as in 2D.**

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Schlumberger
Agip Eni
STATOIL
HALLIBURTON®
TOTAL