VI EIEC, Chiclana, Spain

An hp Fourier Finite Element (FFE) Framework with Electromagnetics and Multiphysics Applications

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basque center for applied mathematics

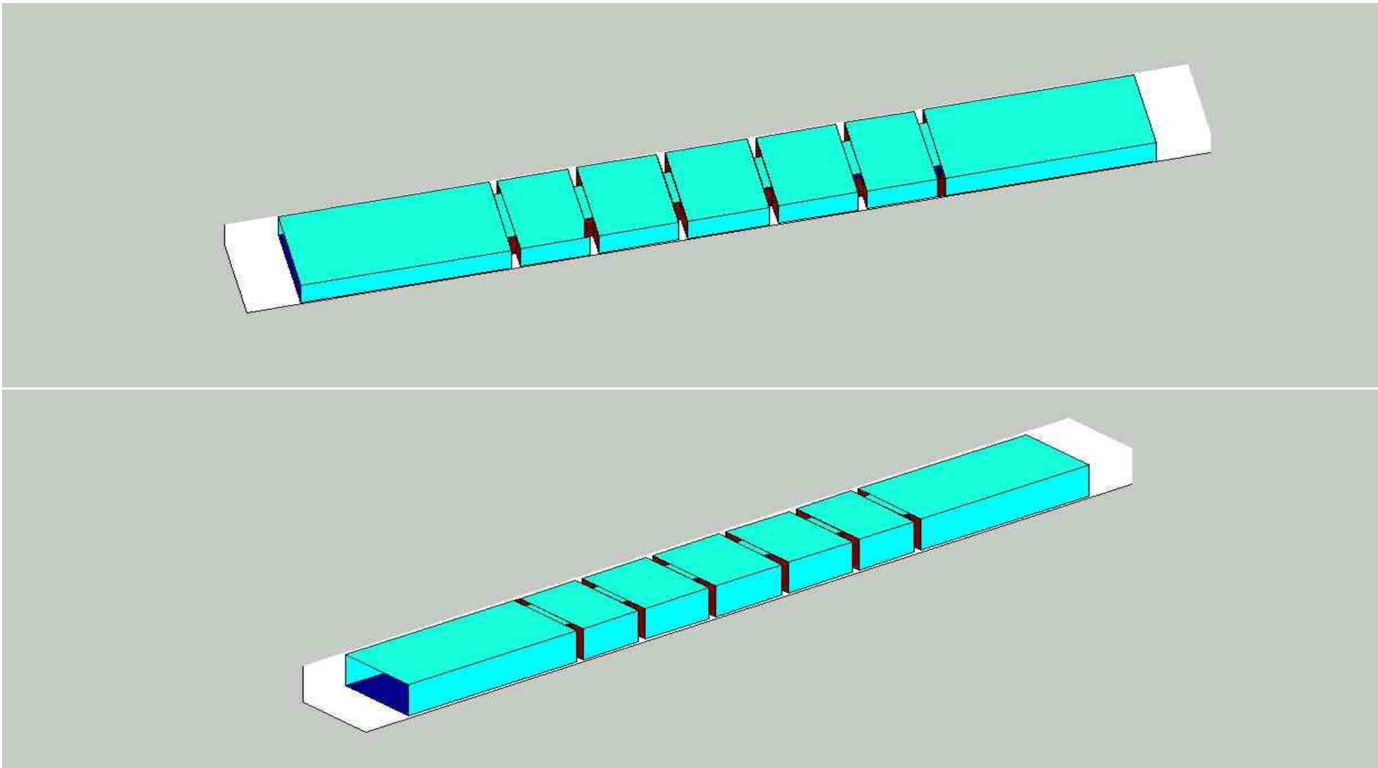
Basque Center for Applied Mathematics (BCAM)
Promoting Technological Advances Through Mathematics

OVERVIEW

1. **Motivation: Waveguide Design and Oil-Industry Applications.**
2. **Method:**
   - Fourier finite element (FFE) method.
   - Parallel implementation.
   - Multi-physics framework.
3. **Numerical Simulations:**
   - 2D resistivity logging measurements.
   - Marine controlled source electromagnetic (CSEM) measurements.
   - 3D resistivity logging measurements.
4. **Conclusions and Future Work.**

MOTIVATION (WAVEGUIDES)

Waveguide Design



Goal: Determine electric field intensity at the ports.

MOTIVATION (OIL-INDUSTRY)

Seismic Measurements

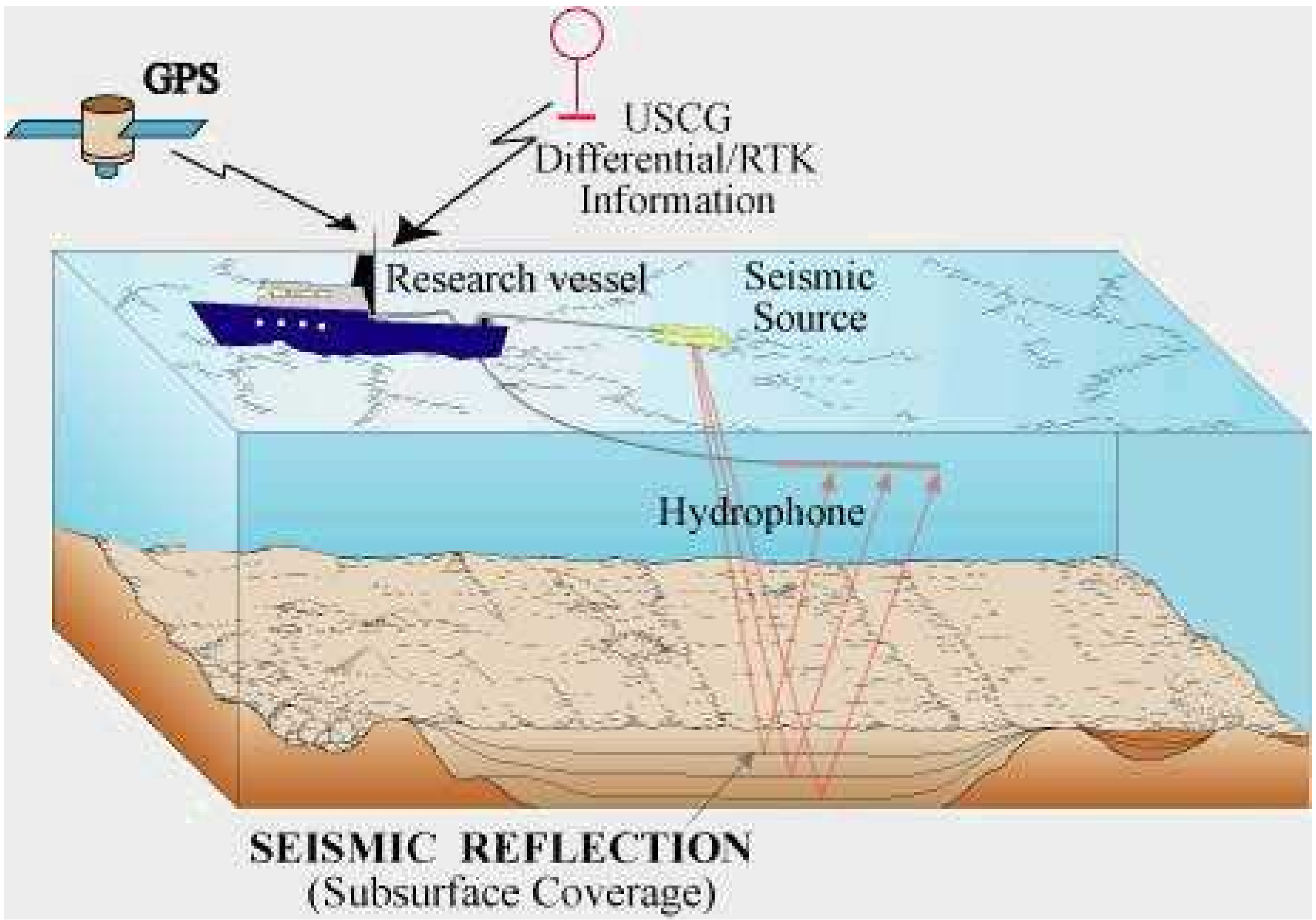


Figure from the USGS Science Center for Coastal and Marine Geology

MOTIVATION (OIL-INDUSTRY)

Marine Controlled-Source Electromagnetics (CSEM)

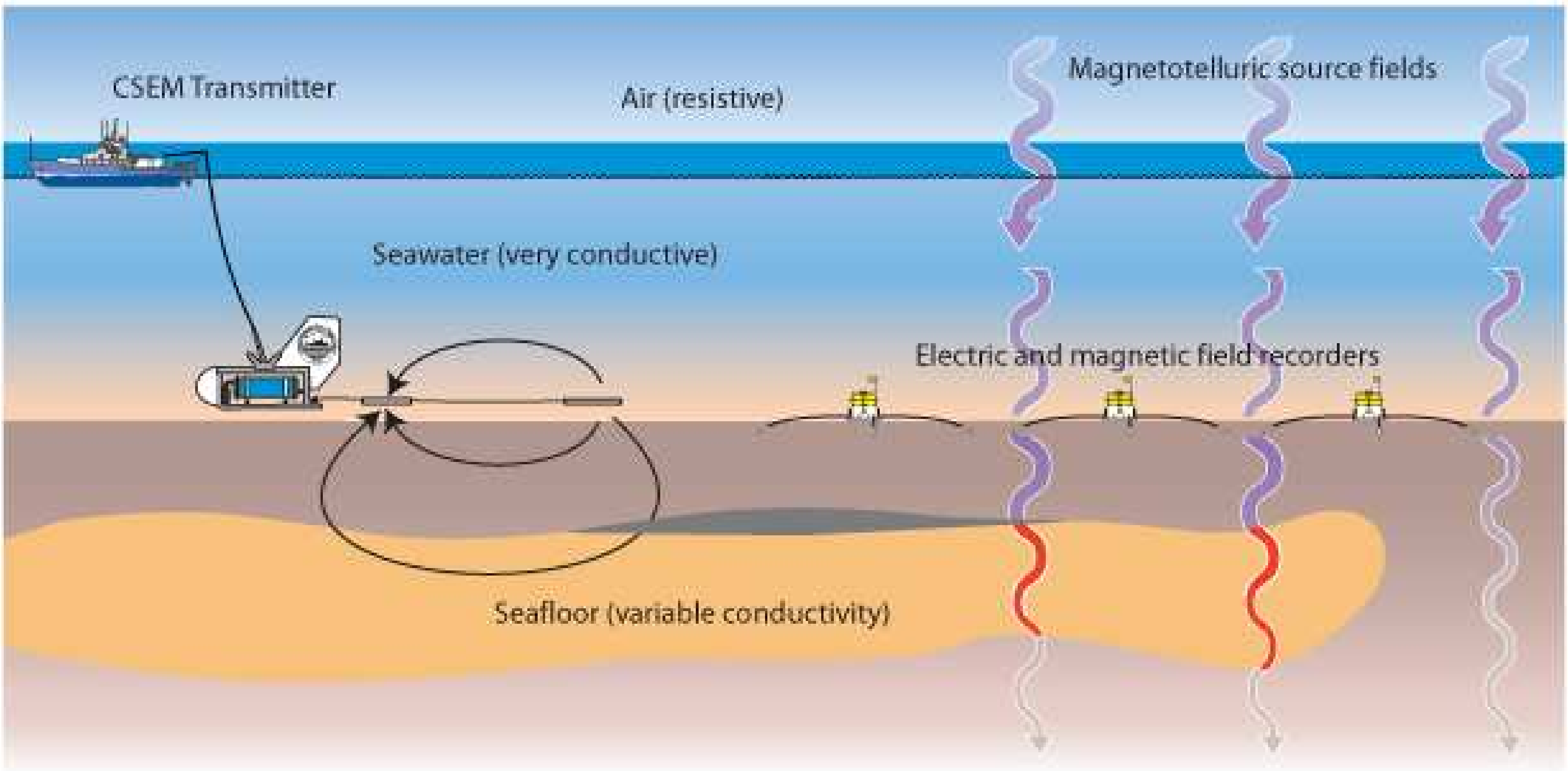
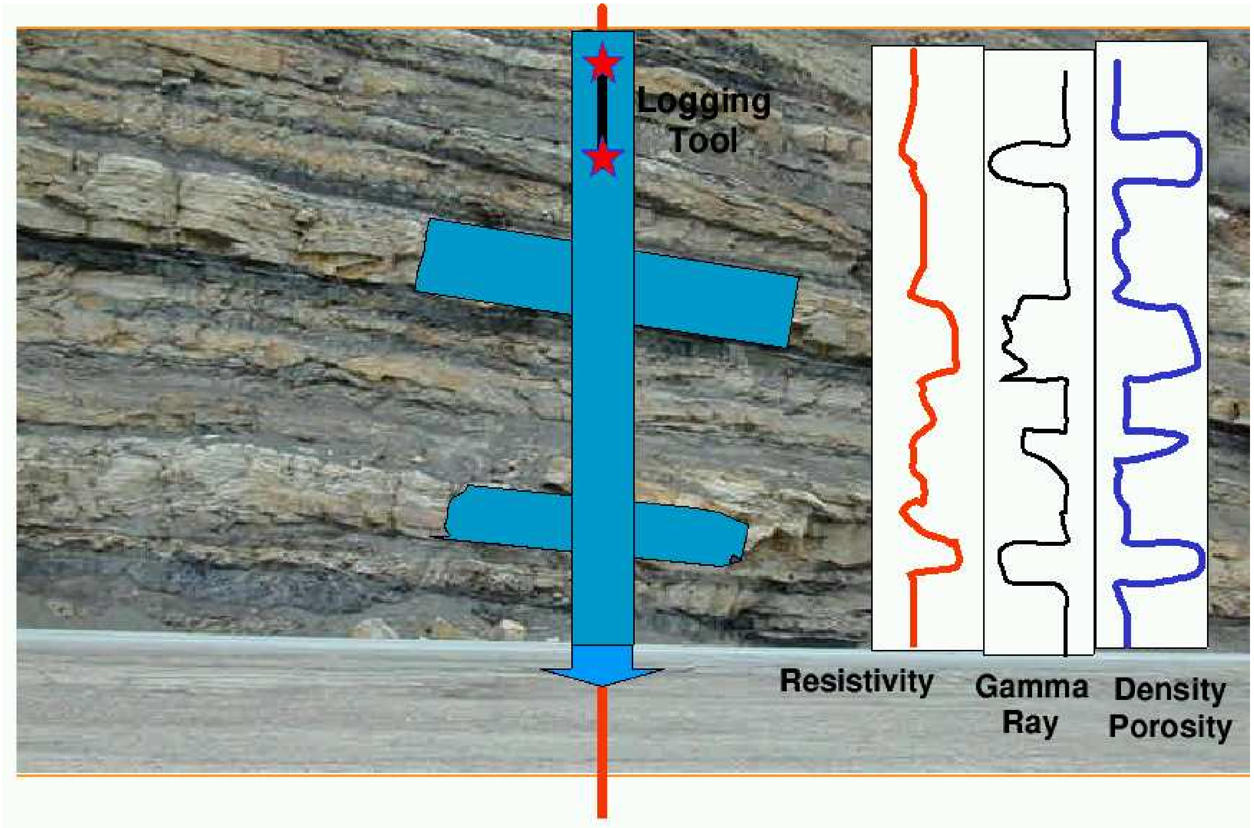


Figure from the UCSD Institute of Oceanography

MOTIVATION (OIL-INDUSTRY)

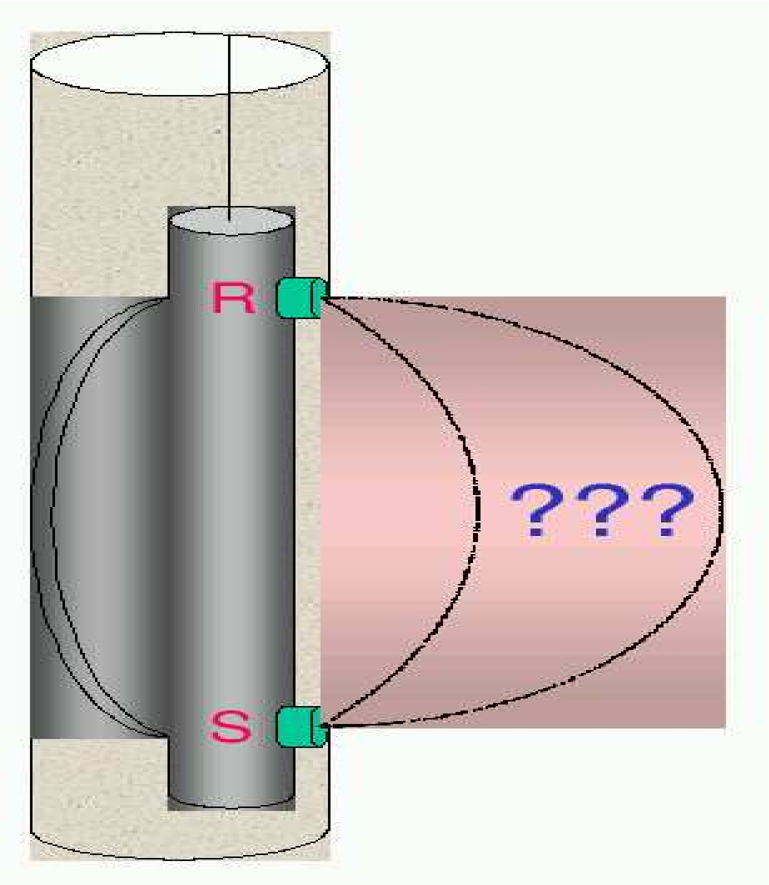
Multiphysics Logging Measurements



OBJECTIVES: To determine payzones (porosity), amount of oil/gas (saturation), and ability to extract oil/gas (permeability).

MOTIVATION (OIL-INDUSTRY)

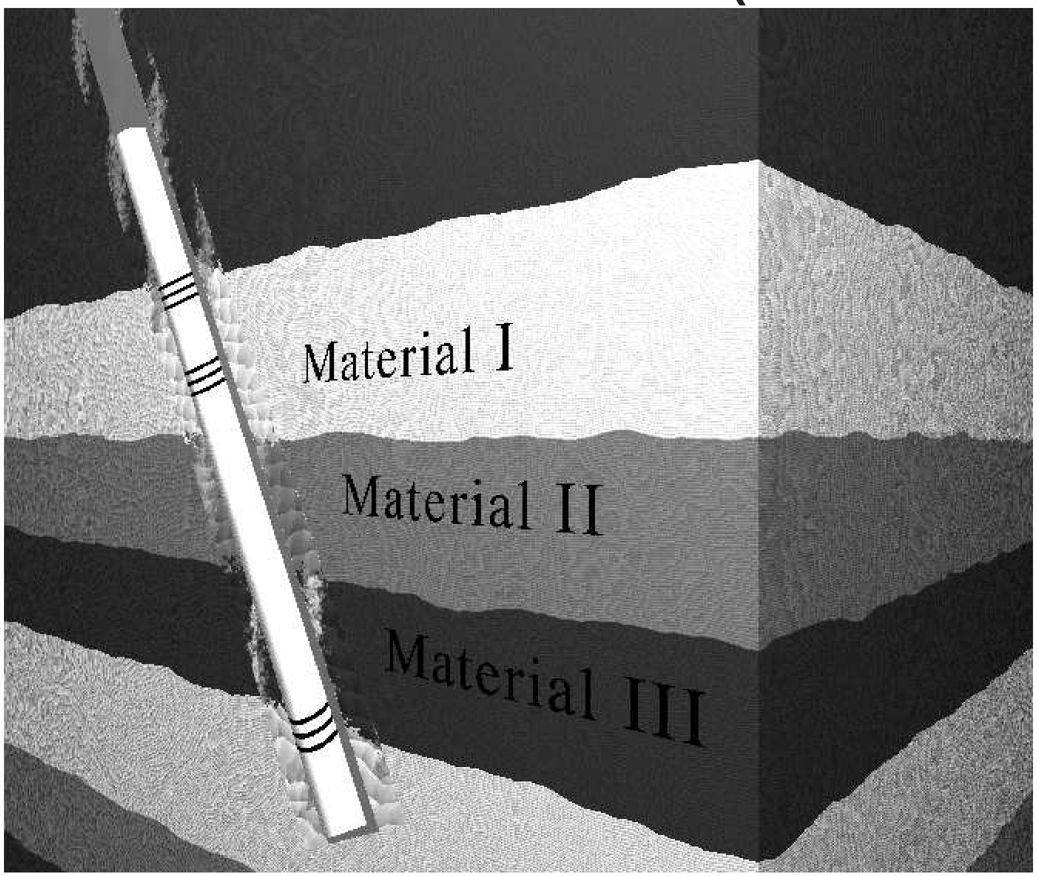
Main Objective: To Solve a Multiphysics Inverse Problem



Given multi-frequency electromagnetic, acoustic, and nuclear measurements, the objective is to determine porosity, saturation, and permeability distributions in the reservoir.

FOURIER FINITE ELEMENT METHOD

Deviated Wells (Forward Problem)

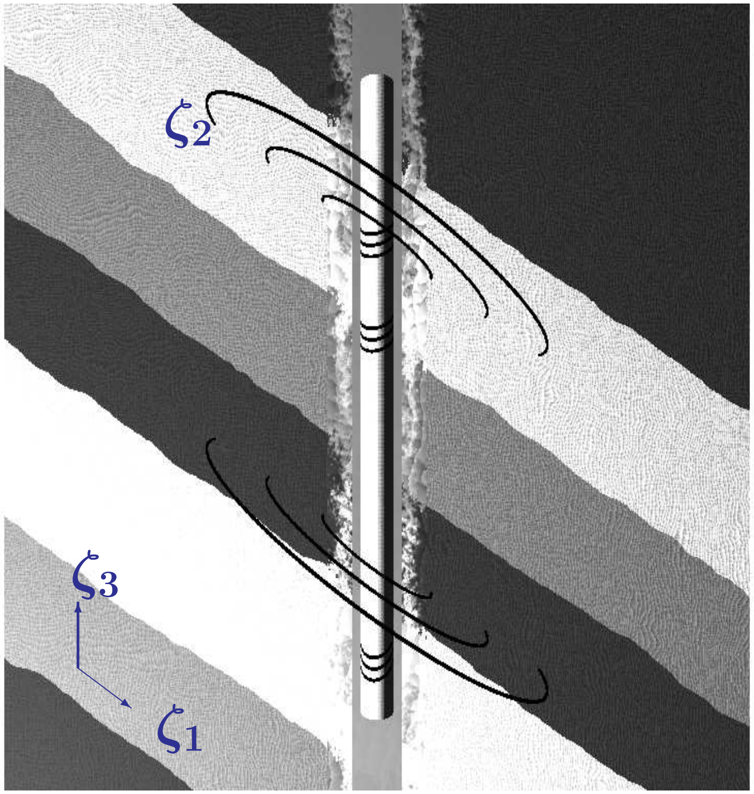


- Dip Angle
- Invasion
- Anisotropy
- Triaxial Induction
- Eccentricity
- Laterolog
- Through-Casing
- Induction-LWD
- Induction-Wireline
- Inverse Problems
- Multi-Physics

Objective: Find solution at the receiver antennas.

FOURIER FINITE ELEMENT METHOD

Non-Orthogonal System of Coordinates



Material coefficients are constant with respect to the quasi-azimuthal direction ζ_2

Fourier Series Expansion in ζ_2

DC Problems: $-\nabla\sigma\nabla u = f$

$$u(\zeta_1,\zeta_2,\zeta_3) = \sum_{l=-\infty}^{l=\infty} u_l(\zeta_1,\zeta_3)e^{jl\zeta_2}$$

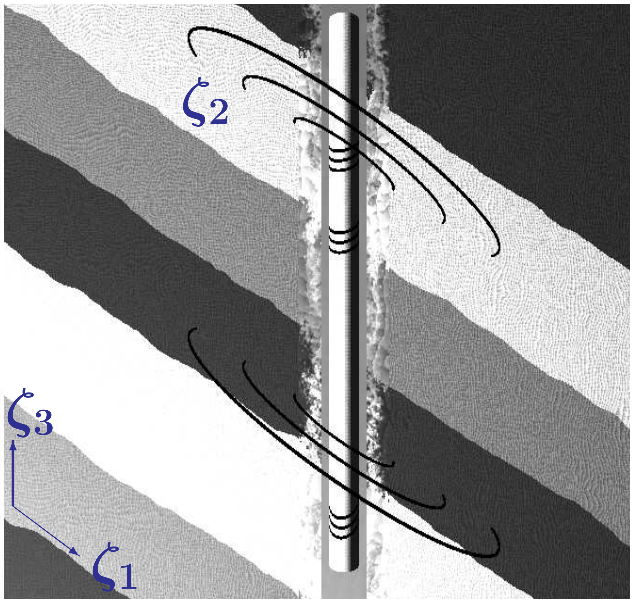
$$\sigma(\zeta_1,\zeta_2,\zeta_3) = \sum_{m=-\infty}^{m=\infty} \sigma_m(\zeta_1,\zeta_3)e^{jm\zeta_2}$$

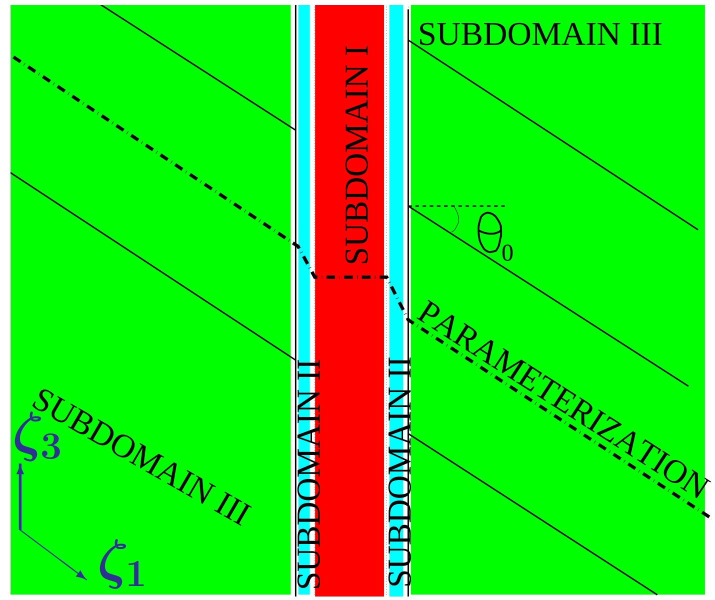
$$f(\zeta_1,\zeta_2,\zeta_3) = \sum_{n=-\infty}^{n=\infty} f_n(\zeta_1,\zeta_3)e^{jn\zeta_2}$$

Fourier modes $e^{jl\zeta_2}$ are orthogonal high-order basis functions that are (almost) invariant with respect to the gradient operator.

FOURIER FINITE ELEMENT METHOD

Cartesian system of coordinates: $\mathrm{x} = (x_1, x_2, x_3)$.
New non-orthogonal system of coordinates: $\zeta = (\zeta_1, \zeta_2, \zeta_3)$.





Subdomain I ; **Subdomain II** ; **Subdomain III**

$$\begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 \end{cases} ; \quad \begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 + \tan \theta_0 \dfrac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \end{cases} ; \quad \begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 + \tan \theta_0 \zeta_1 \end{cases}$$

FOURIER FINITE ELEMENT METHOD

Final Variational Formulation

We define the Jacobian matrix $\mathcal{J} = \dfrac{\partial(x_1, x_2, x_3)}{\partial(\zeta_1, \zeta_2, \zeta_3)}$ **and its determinant** $|\mathcal{J}| = \det(\mathcal{J})$**.**

Variational formulation in the new system of coordinates:

$$\left\{ \begin{array}{l} \text{Find } u \in u_D + H_D^1(\Omega) \text{ such that:} \\ \left\langle \dfrac{\partial v}{\partial \zeta} \, , \, \tilde{\sigma} \dfrac{\partial u}{\partial \zeta} \right\rangle_{L^2(\Omega)} = \left\langle v \, , \, \tilde{f} \right\rangle_{L^2(\Omega)} \quad \forall v \in H_D^1(\Omega) \, , \end{array} \right.$$

where:

$$\tilde{\sigma} := \mathcal{J}^{-1} \sigma \mathcal{J}^{-1^T} |\mathcal{J}| \quad ; \quad \tilde{f} := f |\mathcal{J}| \, .$$

Same variational formulation with new materials and load data

FOURIER FINITE ELEMENT METHOD

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

Direct Current:

$$\begin{cases} \text{Find } u \in u_D + H^1_D(\Omega) \text{ such that:} \\ \displaystyle\sum_{n=k-2}^{n=k+2} \left\langle \left(\frac{\partial v}{\partial \zeta}\right)_k , \; \tilde{\sigma}_{k-n} \left(\frac{\partial u}{\partial \zeta}\right)_n \right\rangle_{L^2(\Omega_{2D})} = \left\langle v_k \, , \, \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})} \quad \forall v_k \end{cases}$$

Alternate Current:

$$\begin{cases} \text{Find } (\mathrm{E})_s \in H_{\Gamma_E}(\mathrm{curl};\Omega) \text{ such that:} \\ \displaystyle\sum_{n=s-2}^{n=s+2} \left\langle (\nabla^\zeta \times \mathrm{F})_s \, , \, (\tilde{\mu}^{-1})_{s-n} \, (\nabla^\zeta \times \mathrm{E})_l \right\rangle_{L^2(\Omega_{2D})} \\ \quad - \left\langle \mathrm{F}_s \, , \, (\widetilde{k^2})_{s-n} \mathrm{E}_l \right\rangle_{L^2(\Omega_{2D})} = -j\omega \left\langle \mathrm{F}_s \, , \, (\tilde{\mathrm{J}}^{imp})_s \right\rangle_{L^2(\Omega_{2D})} \quad \forall \, \mathrm{F}_s \end{cases}$$

FOURIER FINITE ELEMENT METHOD

Example (7 Fourier Modes)

$$\underbrace{\sum_{n=k-2}^{n=k+2} \left\langle \left(\frac{\partial v}{\partial \zeta}\right)_k , \tilde{\sigma}_{k-n} \left(\frac{\partial u}{\partial \zeta}\right)_n \right\rangle_{L^2(\Omega_{2D})}}_{(k,k-n,n)} = \left\langle v_k , \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})}$$

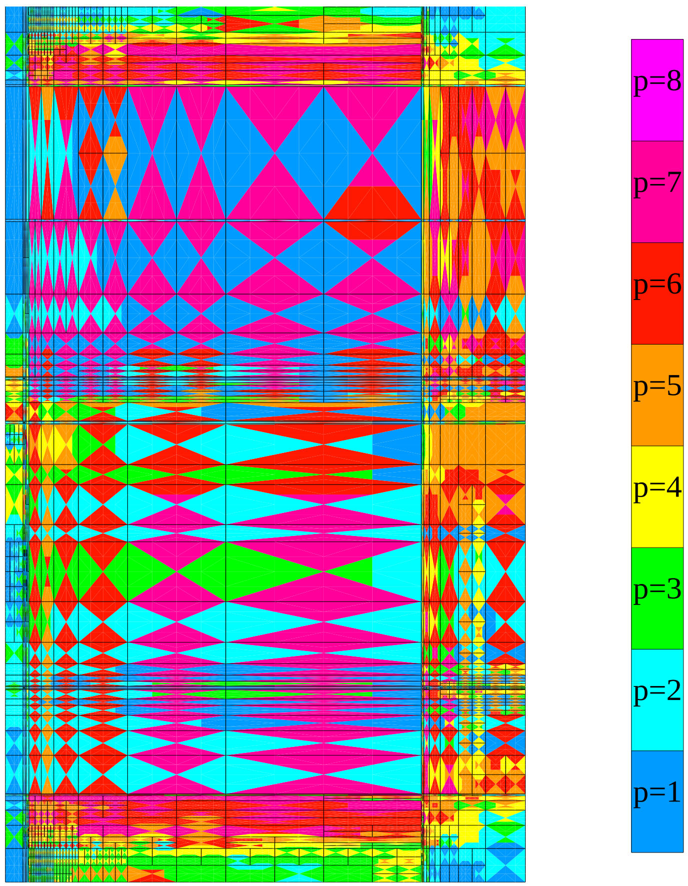
Stiffness Matrix:

$$\begin{pmatrix}
(-3,0,-3) & (-3,-1,-2) & (-3,-2,-1) & 0 & 0 & 0 & 0 \\
(-2,1,-3) & (-2,0,-2) & (-2,-1,-1) & (-2,-2,0) & 0 & 0 & 0 \\
(-1,2,-3) & (-1,1,-2) & (-1,0,-1) & (-1,-1,0) & (-1,-2,1) & 0 & 0 \\
0 & (0,2,-2) & (0,1,-1) & (0,0,0) & (0,-1,1) & (0,-2,2) & 0 \\
0 & 0 & (1,2,-1) & (1,1,0) & (1,0,1) & (1,-1,2) & (1,-2,3) \\
0 & 0 & 0 & (2,2,0) & (2,1,1) & (2,0,2) & (2,-1,3) \\
0 & 0 & 0 & 0 & (3,2,1) & (3,1,2) & (3,0,3)
\end{pmatrix}$$

FOURIER FINITE ELEMENT METHOD

A Self-Adaptive Goal-Oriented hp-FEM

Optimal 2D Grid
(Through Casing Resistivity Problem)



We vary locally the element size h and the polynomial order of approximation p throughout the grid.

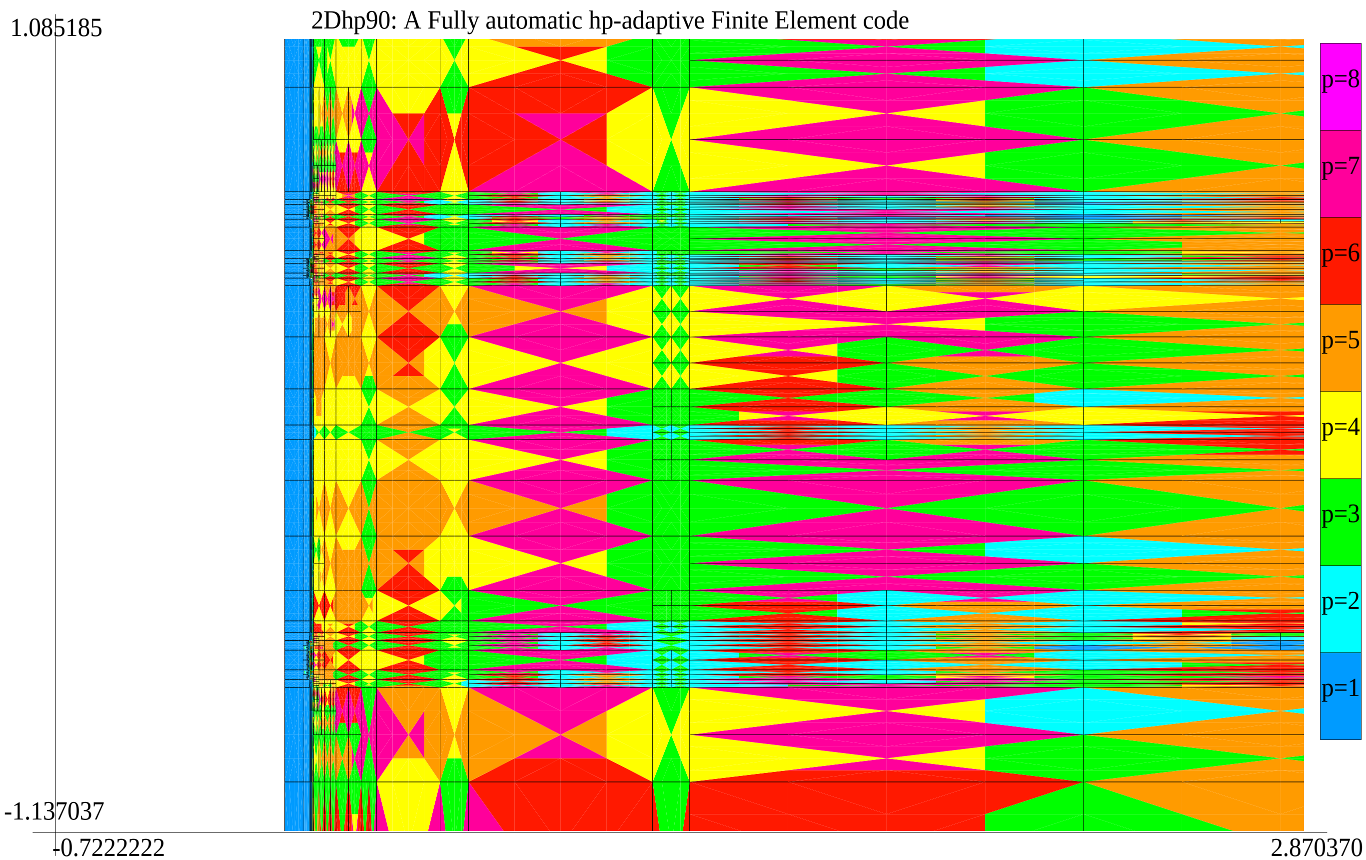
Optimal grids are automatically generated by the computer.

The self-adaptive goal-oriented hp-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.

FOURIER FINITE ELEMENT METHOD

Axisymmetric Logging-While-Drilling (LWD) Simulation

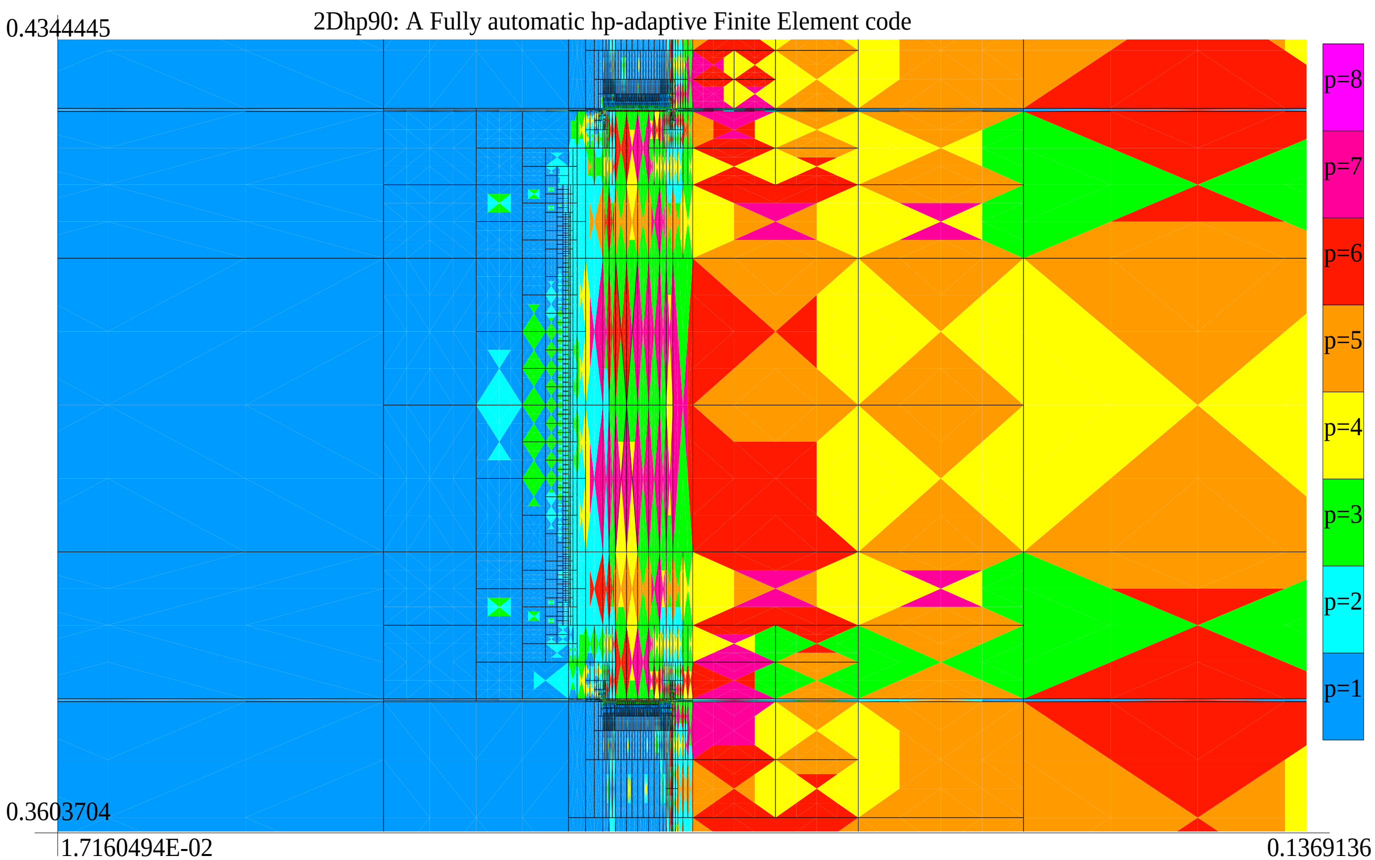
GOAL-ORIENTED HP-ADAPTIVITY (Quadrilateral Elements)



FOURIER FINITE ELEMENT METHOD

Axisymmetric Logging-While-Drilling (LWD) Simulation

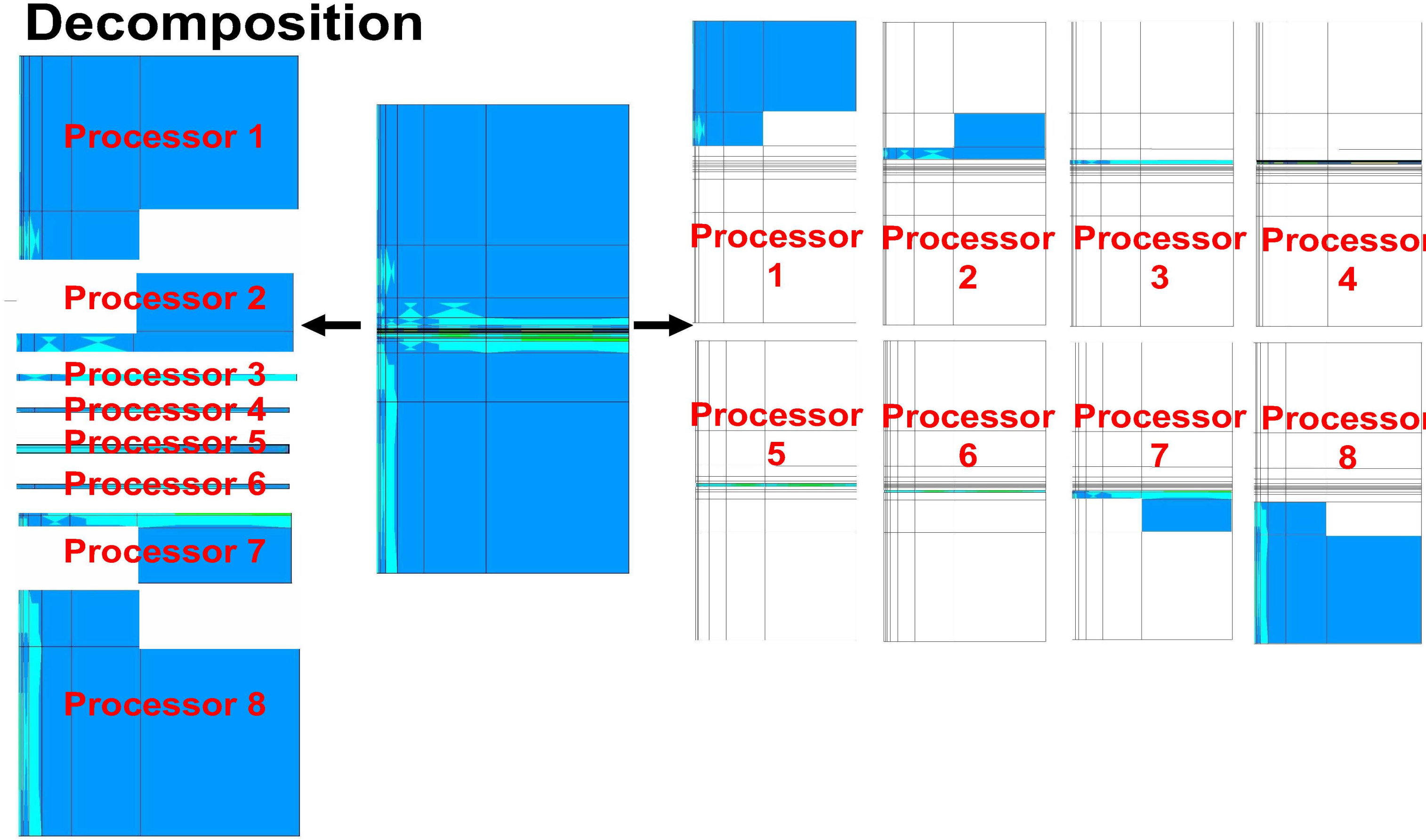
GOAL-ORIENTED HP-ADAPTIVITY (ZOOM TOWARDS FIRST RECEIVER ANTENNA)



PARALLEL IMPLEMENTATION

We Use Shared Domain Decomposition

Distributed Domain Decomposition **Shared Domain Decomposition**



MULTIPHYSICS FRAMEWORK

We are generating new data structures based on:

The addition of one new module/library for solving inverse problems.

The use of different number of equations for each element/node.

- **Enables to consider different physics for each element/node.**
- **Enables the use of different number of Fourier modes for each element/node.**

The combination of different types of elements used for each physics:

- **Continous H^1-elements (DC problems, elasticity, etc.)**
- **Nedelec (edge) $H(curl)$-elements (Electromagnetics).**
- **Raviart-Thomas (face) $H(div)$-elements (Fluid-dynamics)**
- **Discontinuous L^2-elements.**

MULTIPHYSICS FRAMEWORK

Final hp-grid and solution

Monopole source, open borehole setting:

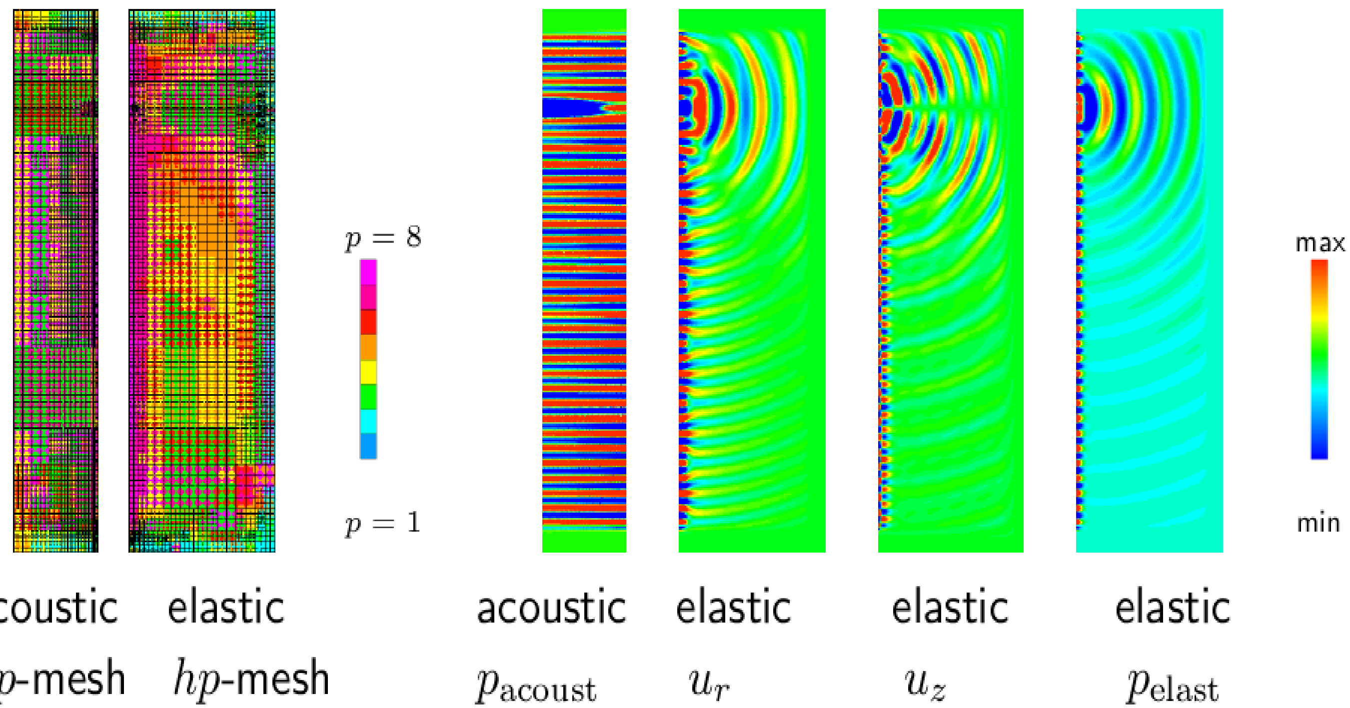
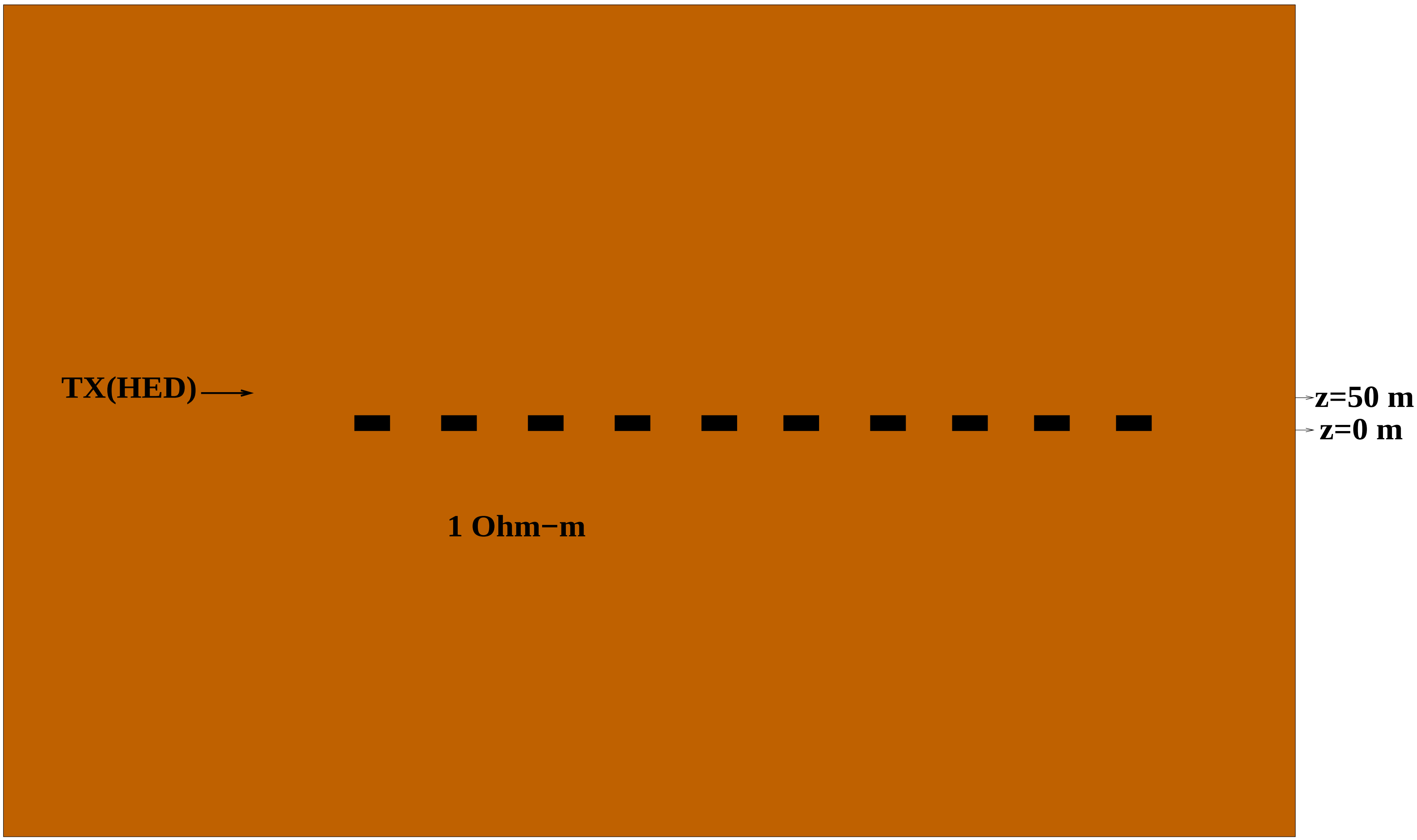


Figure: Frequency-domain solution at the center frequency of 8 kHz (acoustics subdomain scaled by a factor of 10 in radial direction; plotting ranges $[0.1\,\text{min}, 0.1\,\text{max}]$)

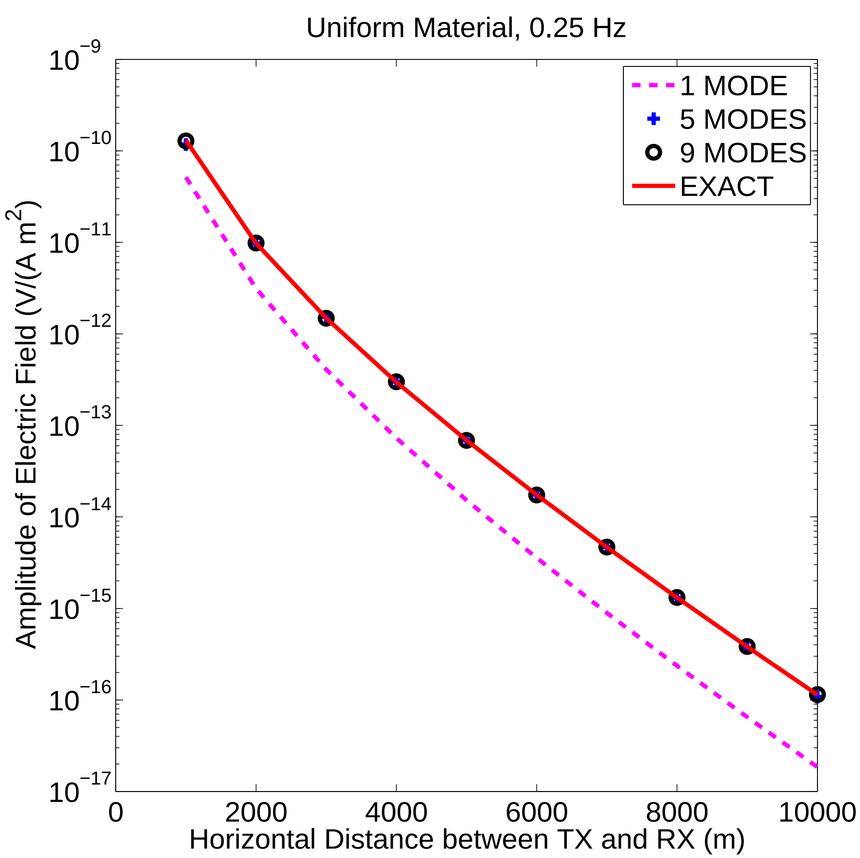
RESULTS: 2D MARINE CSEM

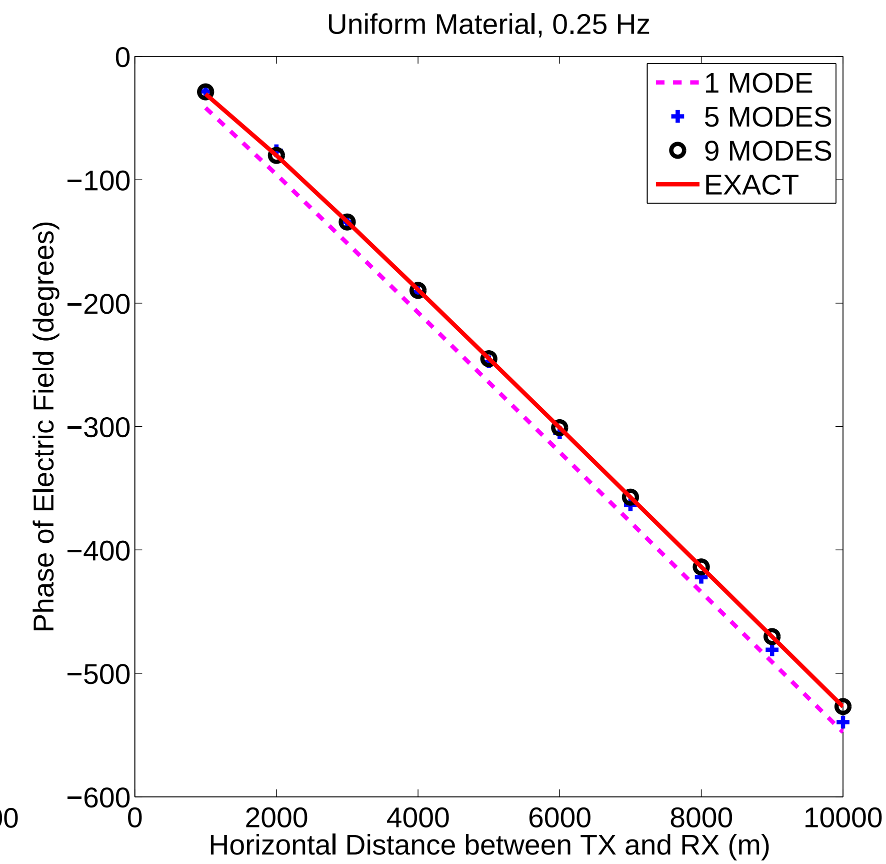
Model Problem I: UNIFORM FORMATION



RESULTS: 2D MARINE CSEM

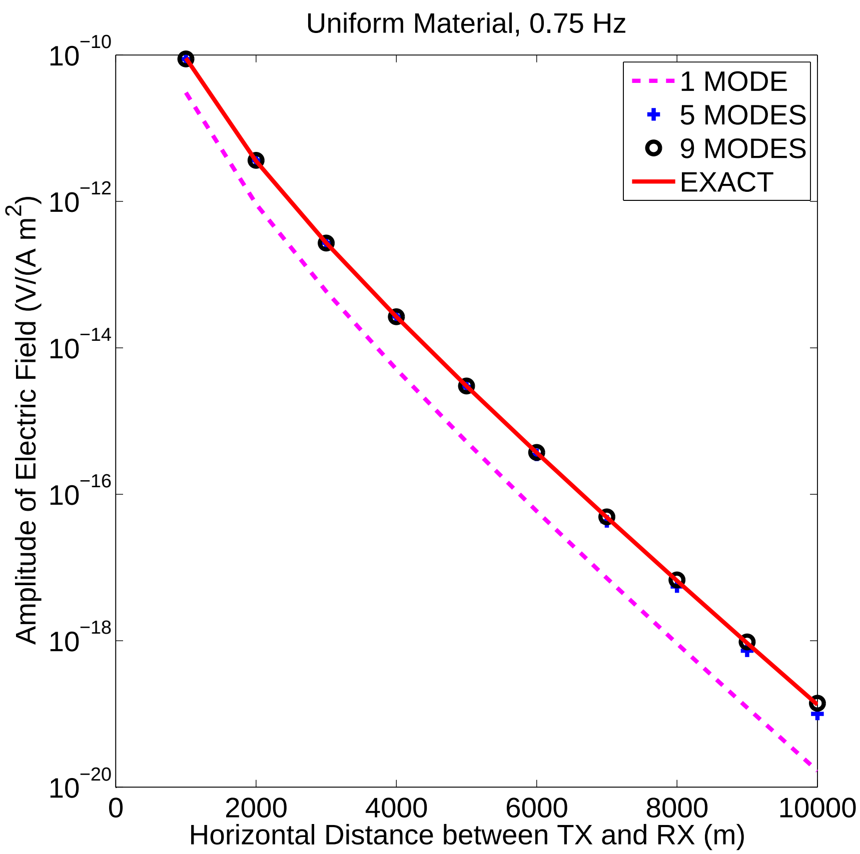
Model Problem I: UNIFORM FORMATION — 0.25 Hz —

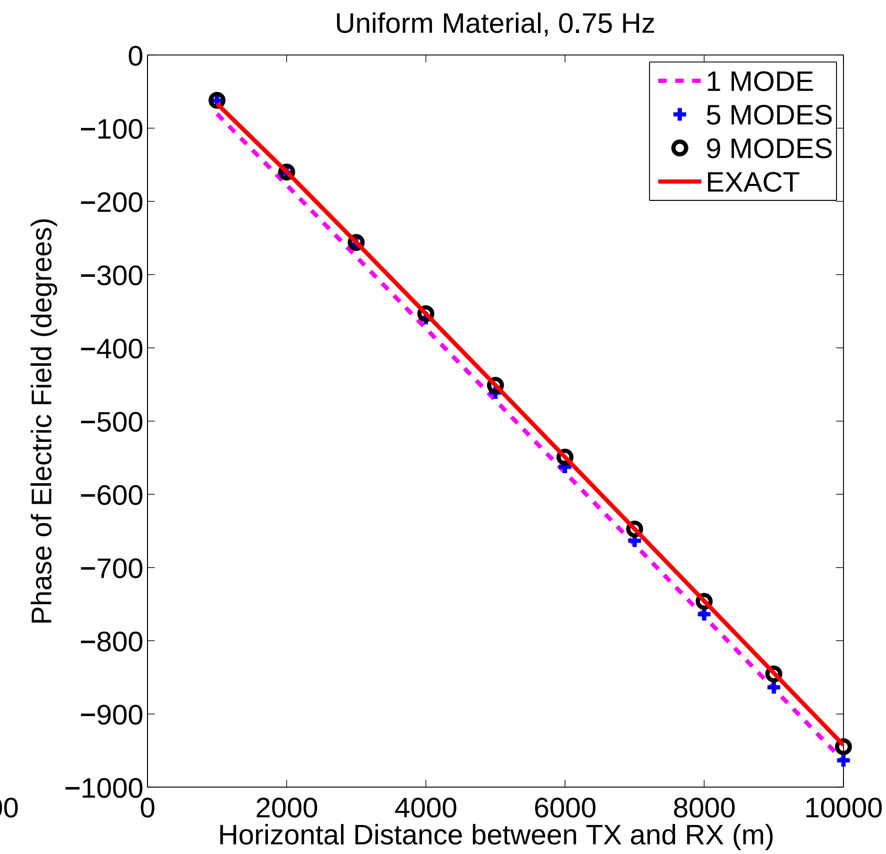




RESULTS: 2D MARINE CSEM

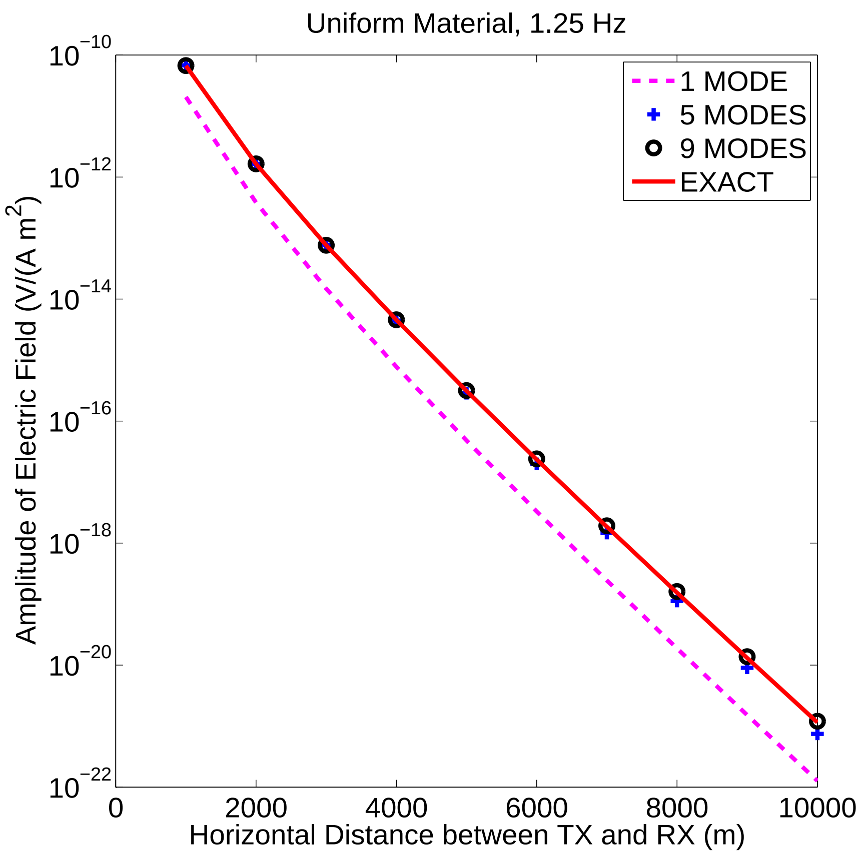
Model Problem I: UNIFORM FORMATION — 0.75 Hz —

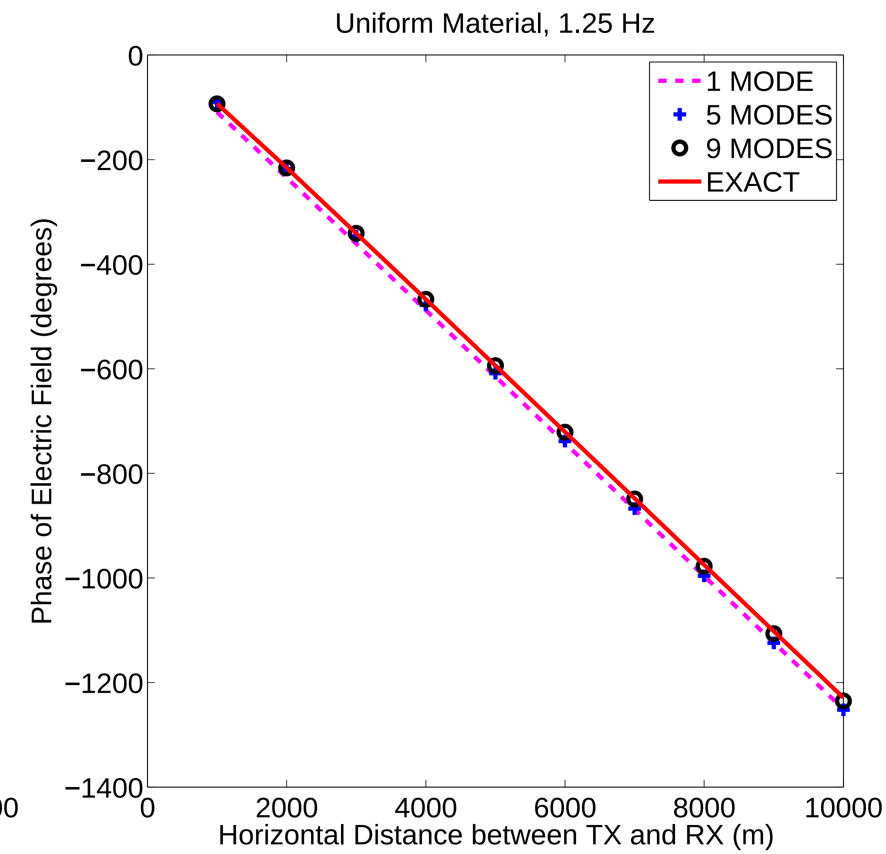




RESULTS: 2D MARINE CSEM

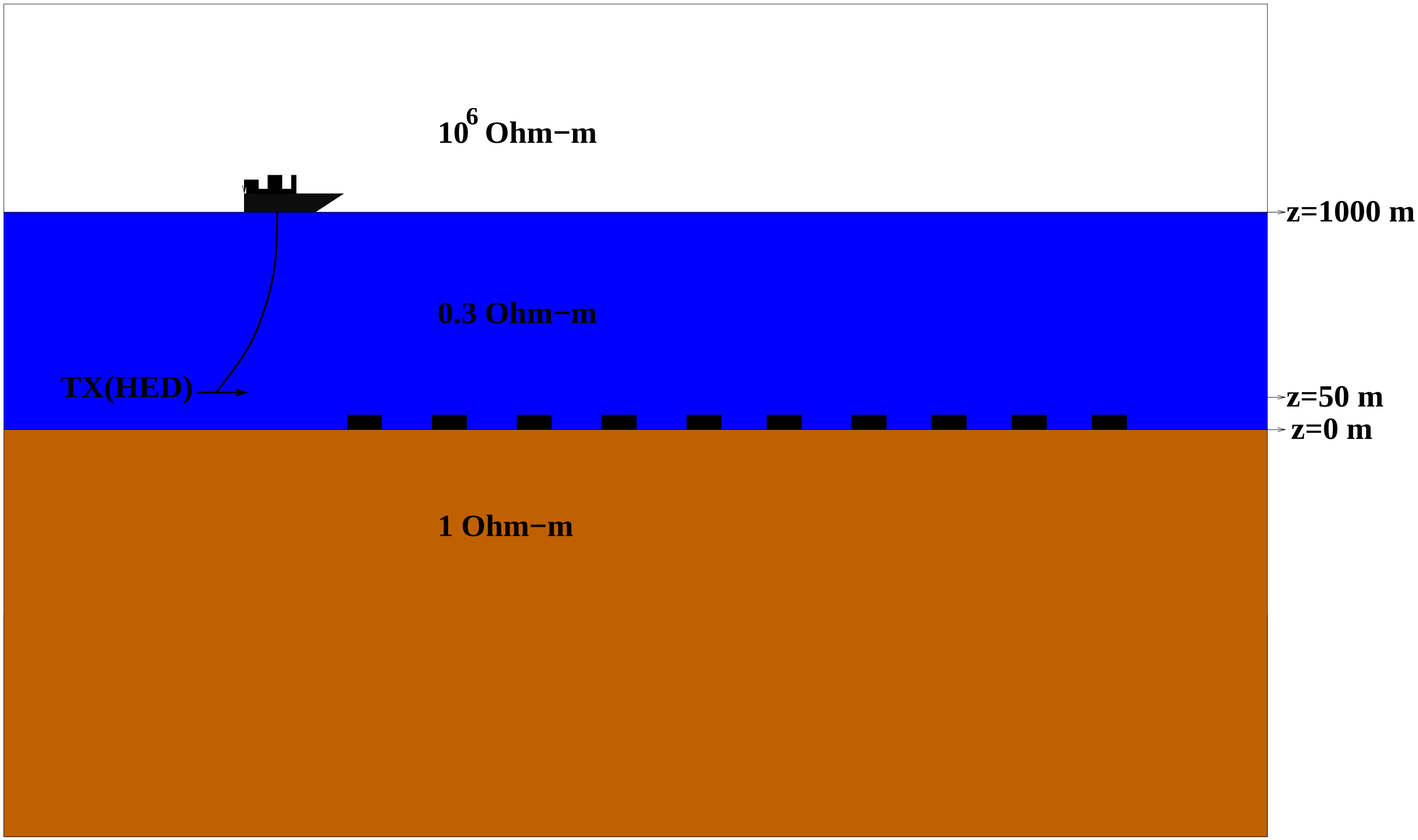
Model Problem I: UNIFORM FORMATION — 1.25 Hz —





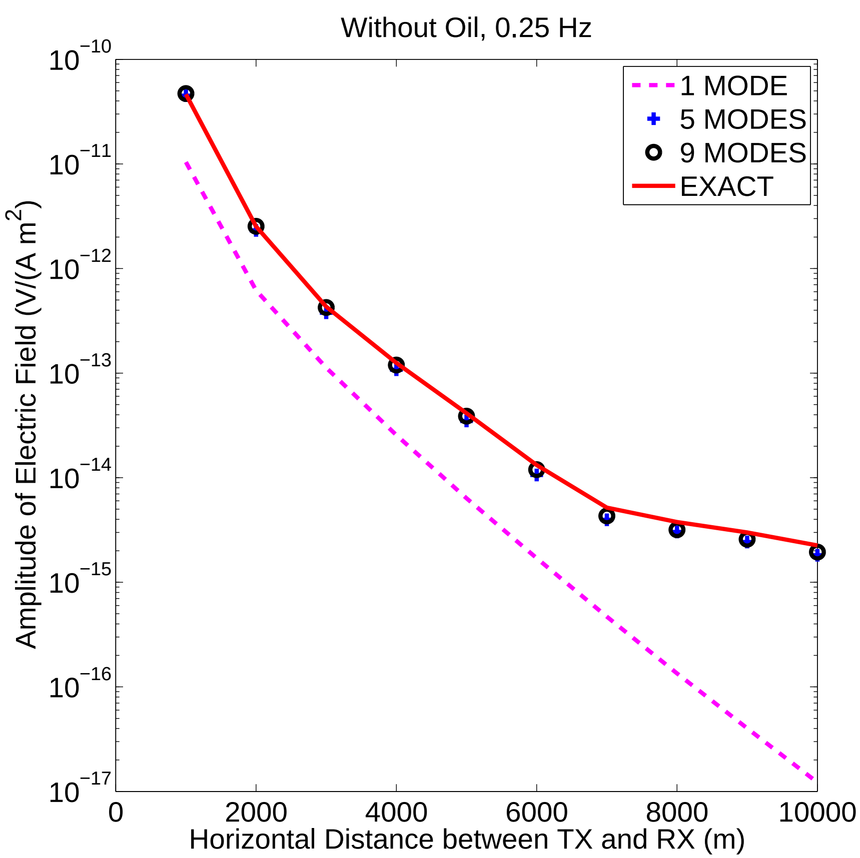
RESULTS: 2D MARINE CSEM

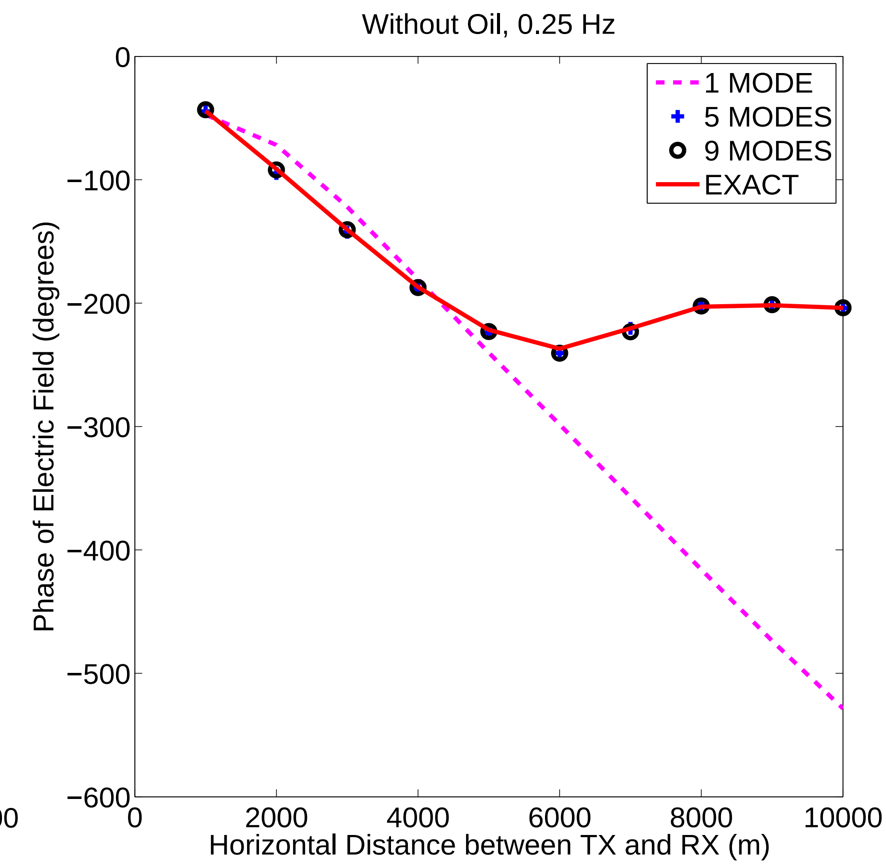
Model Problem II: CSEM SCENARIO WITHOUT OIL



RESULTS: 2D MARINE CSEM

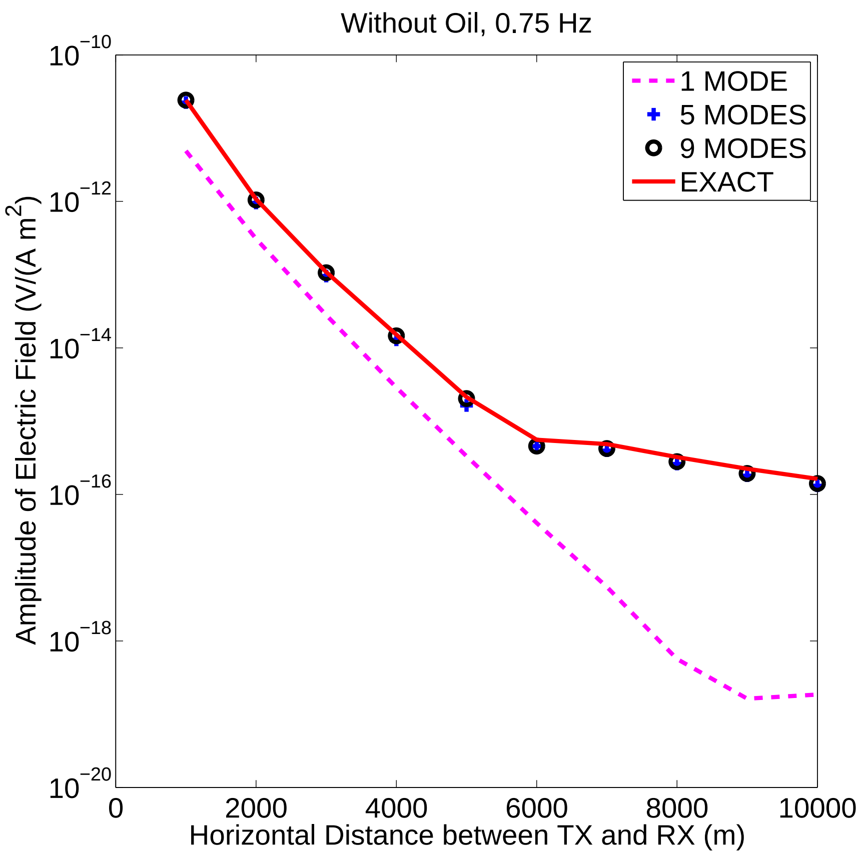
Model Problem II: WITHOUT OIL — 0.25 Hz —

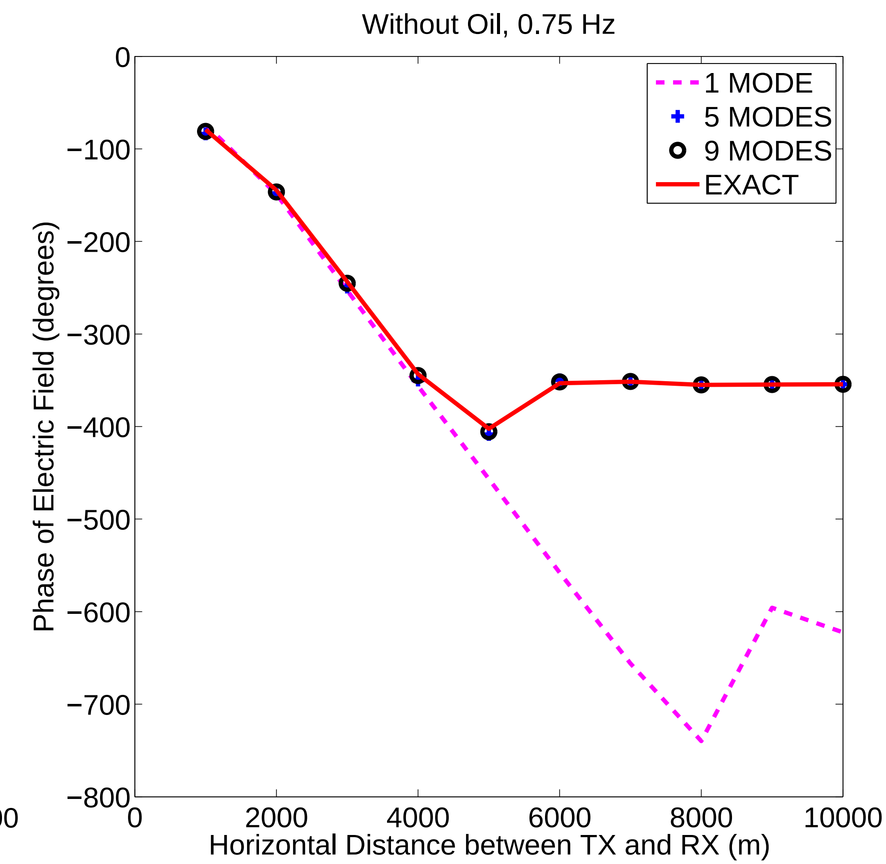




RESULTS: 2D MARINE CSEM

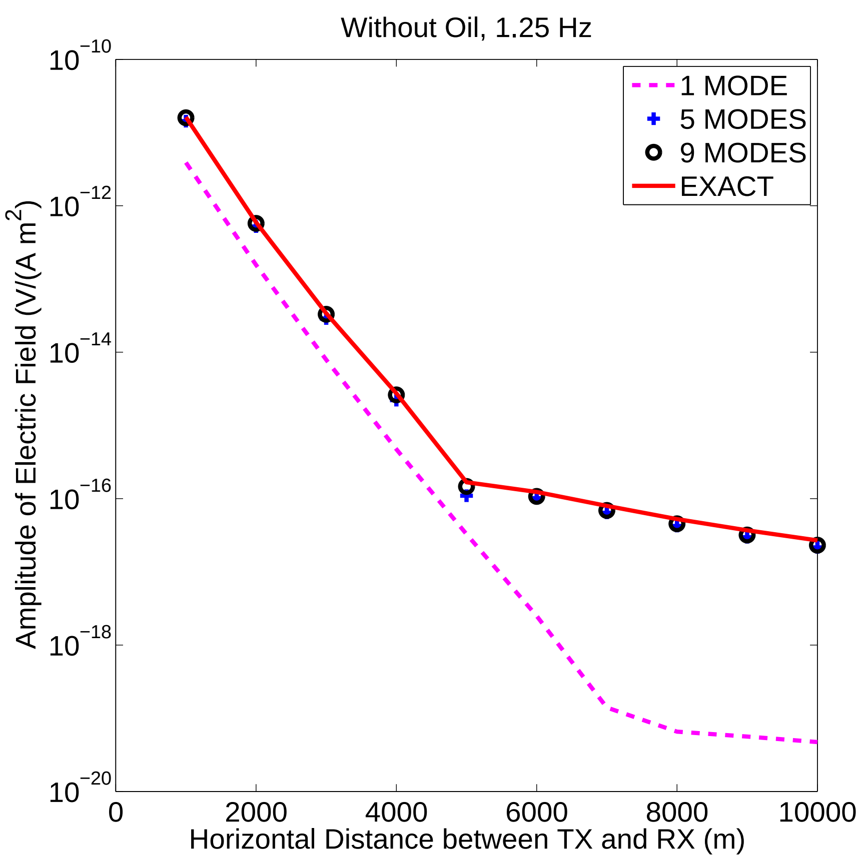
Model Problem II: WITHOUT OIL — 0.75 Hz —

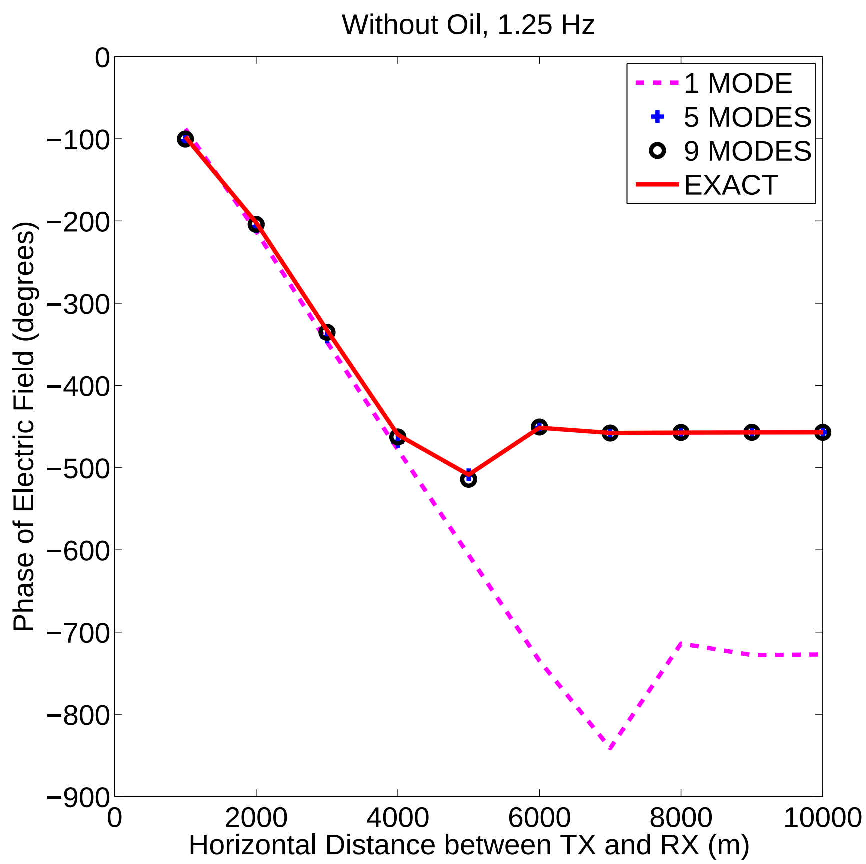




RESULTS: 2D MARINE CSEM

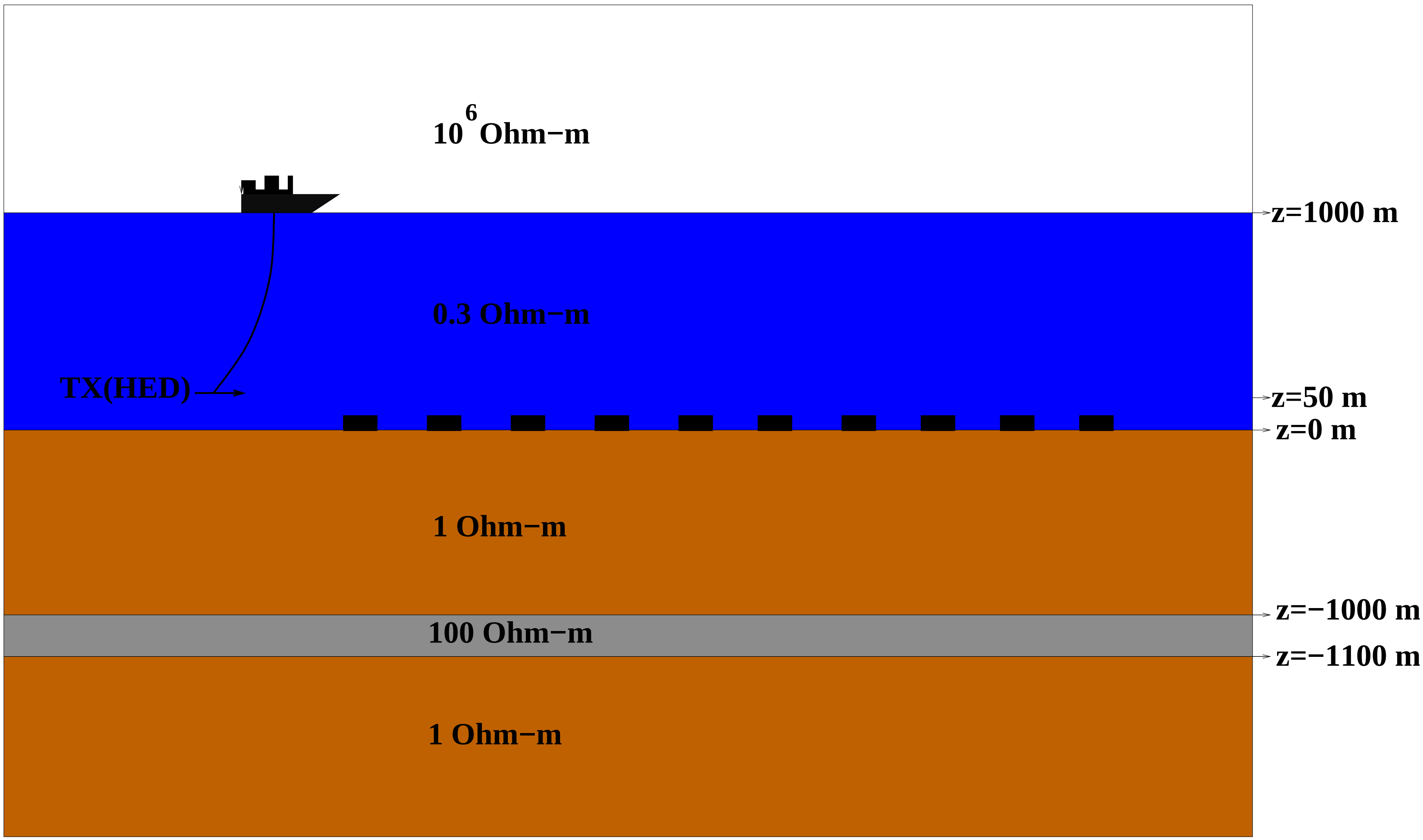
Model Problem II: WITHOUT OIL — 1.25 Hz —





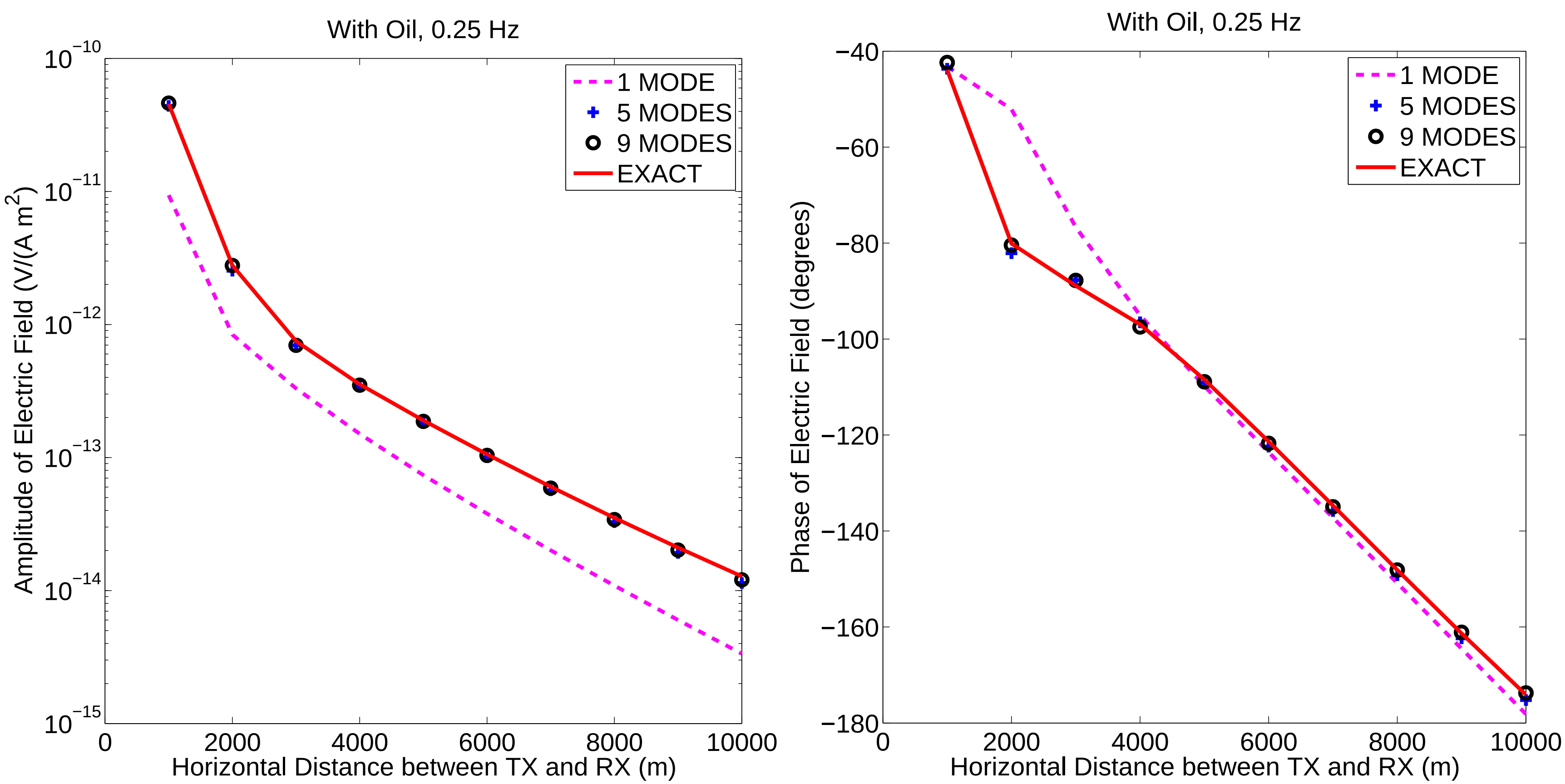
RESULTS: 2D MARINE CSEM

Model Problem III: CSEM SCENARIO WITH OIL



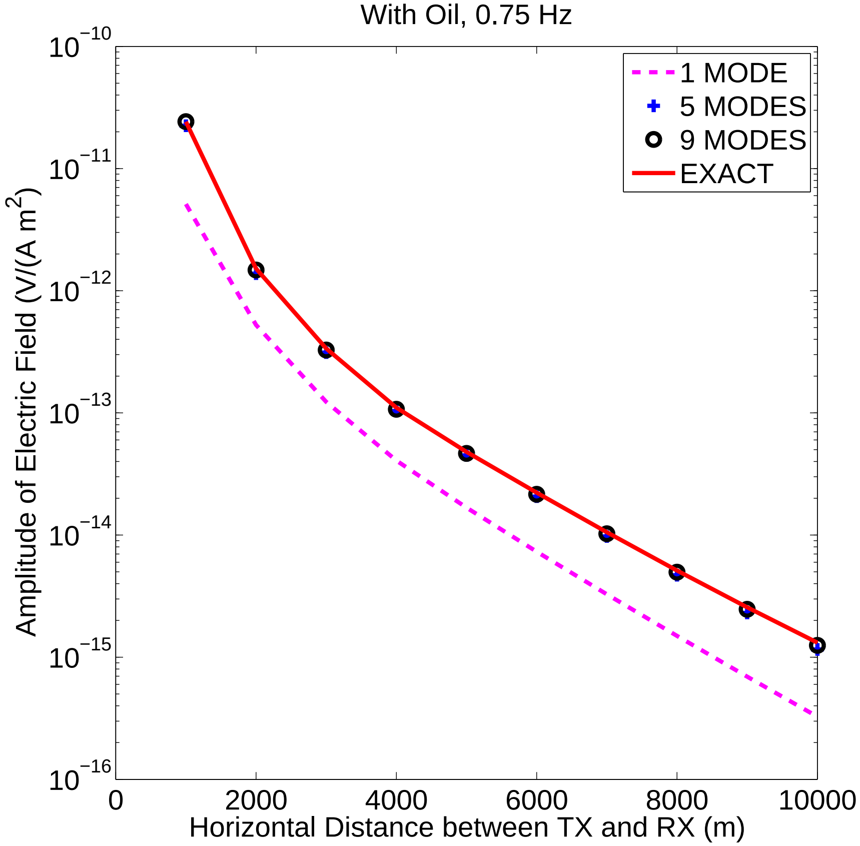
RESULTS: 2D MARINE CSEM

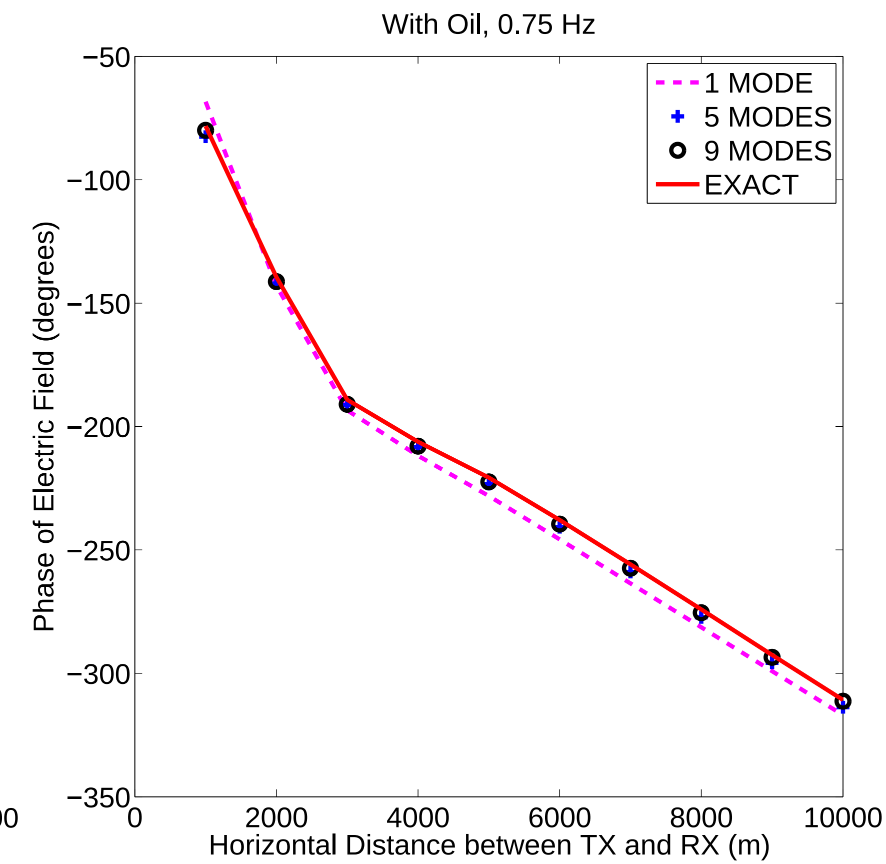
Model Problem III: WITH OIL — 0.25 Hz —



RESULTS: 2D MARINE CSEM

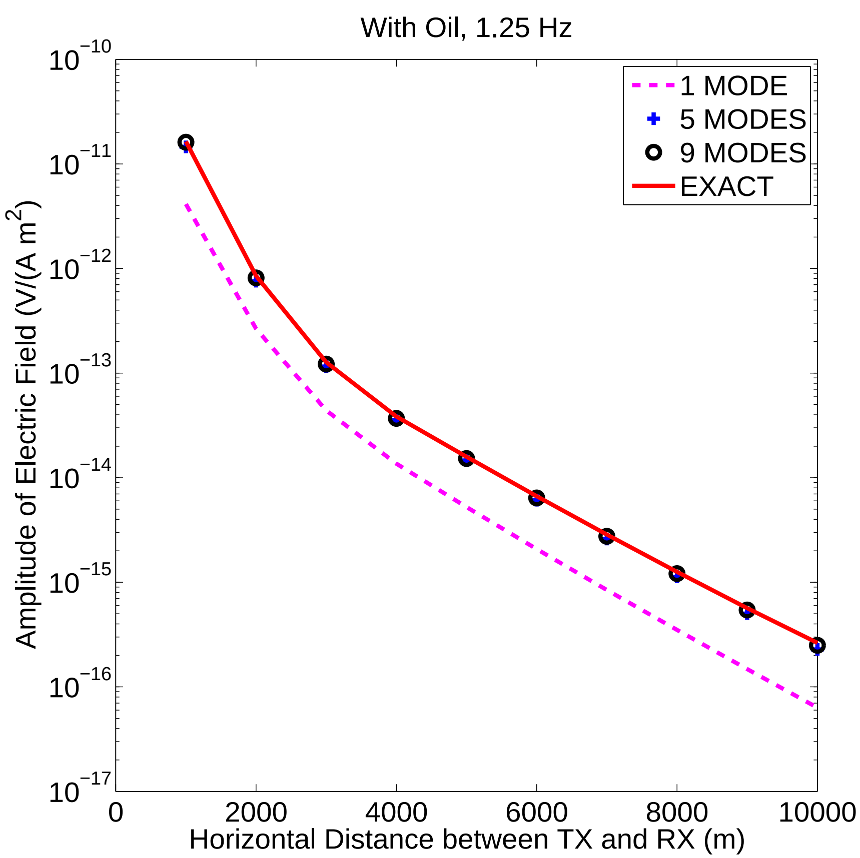
Model Problem III: WITH OIL — 0.75 Hz —

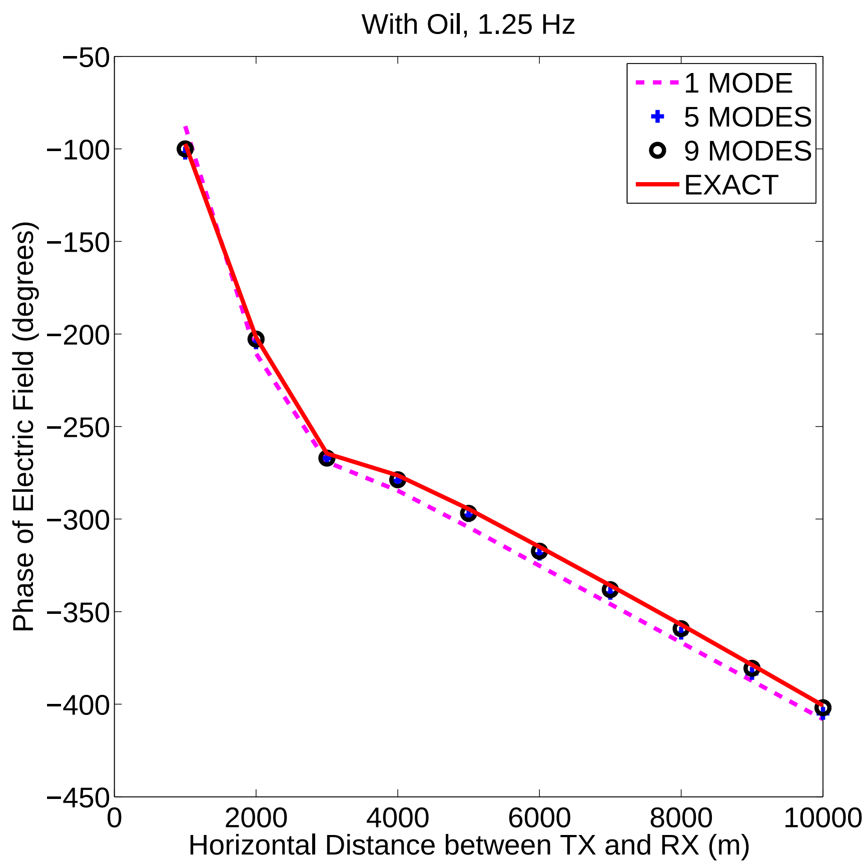




RESULTS: 2D MARINE CSEM

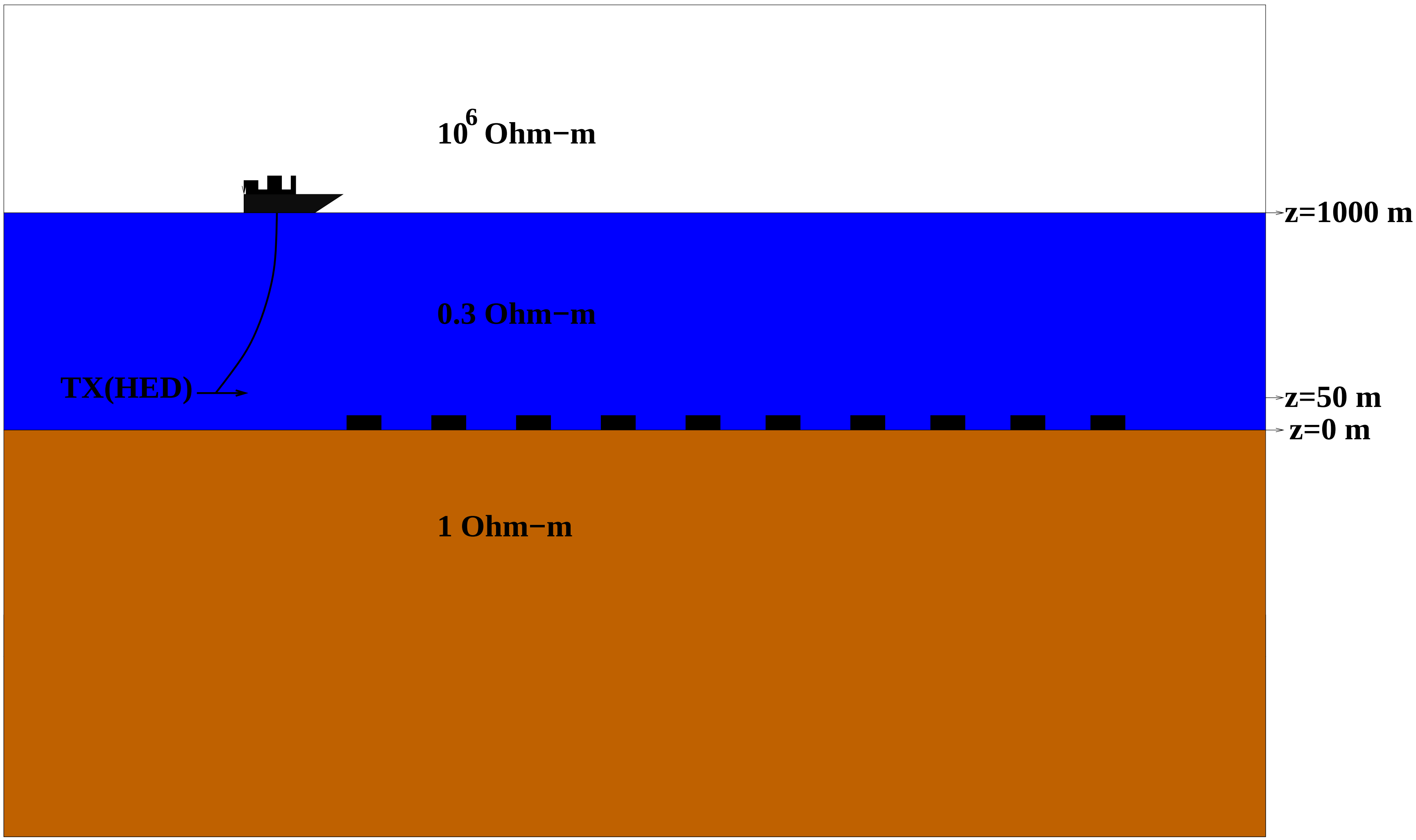
Model Problem III: WITH OIL — 1.25 Hz —





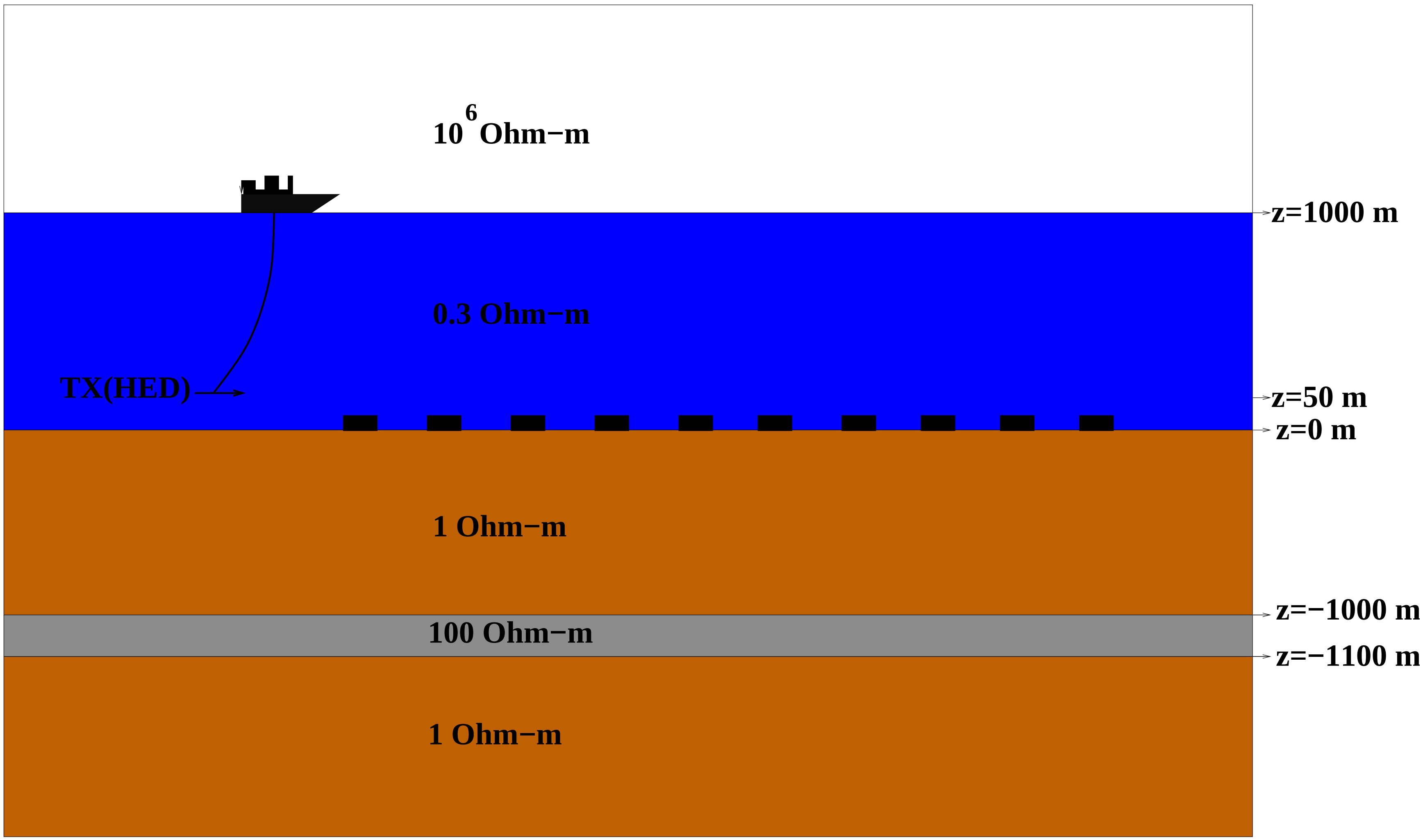
RESULTS: 2D MARINE CSEM

NO OIL



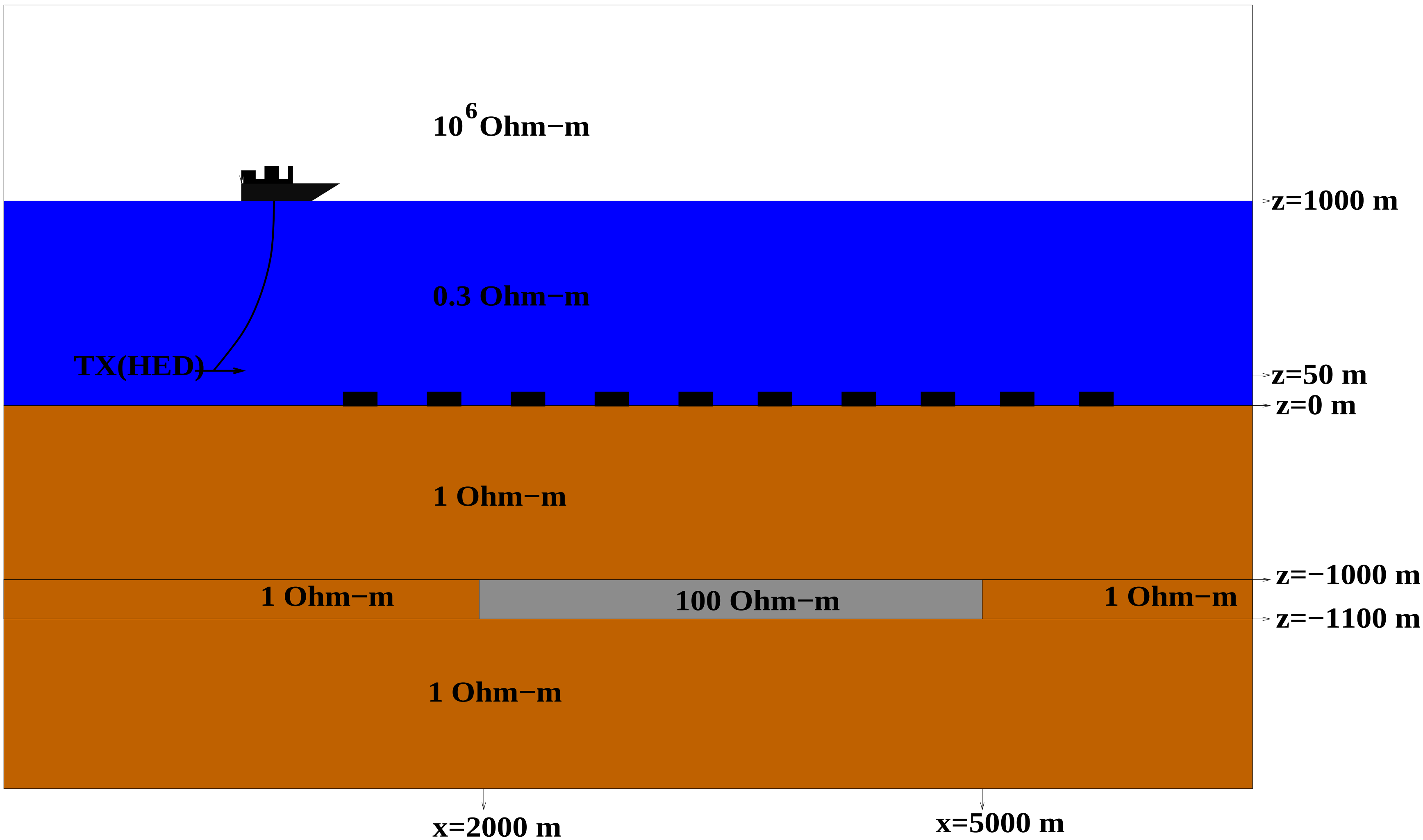
RESULTS: 2D MARINE CSEM

INFINITE LAYER OF OIL



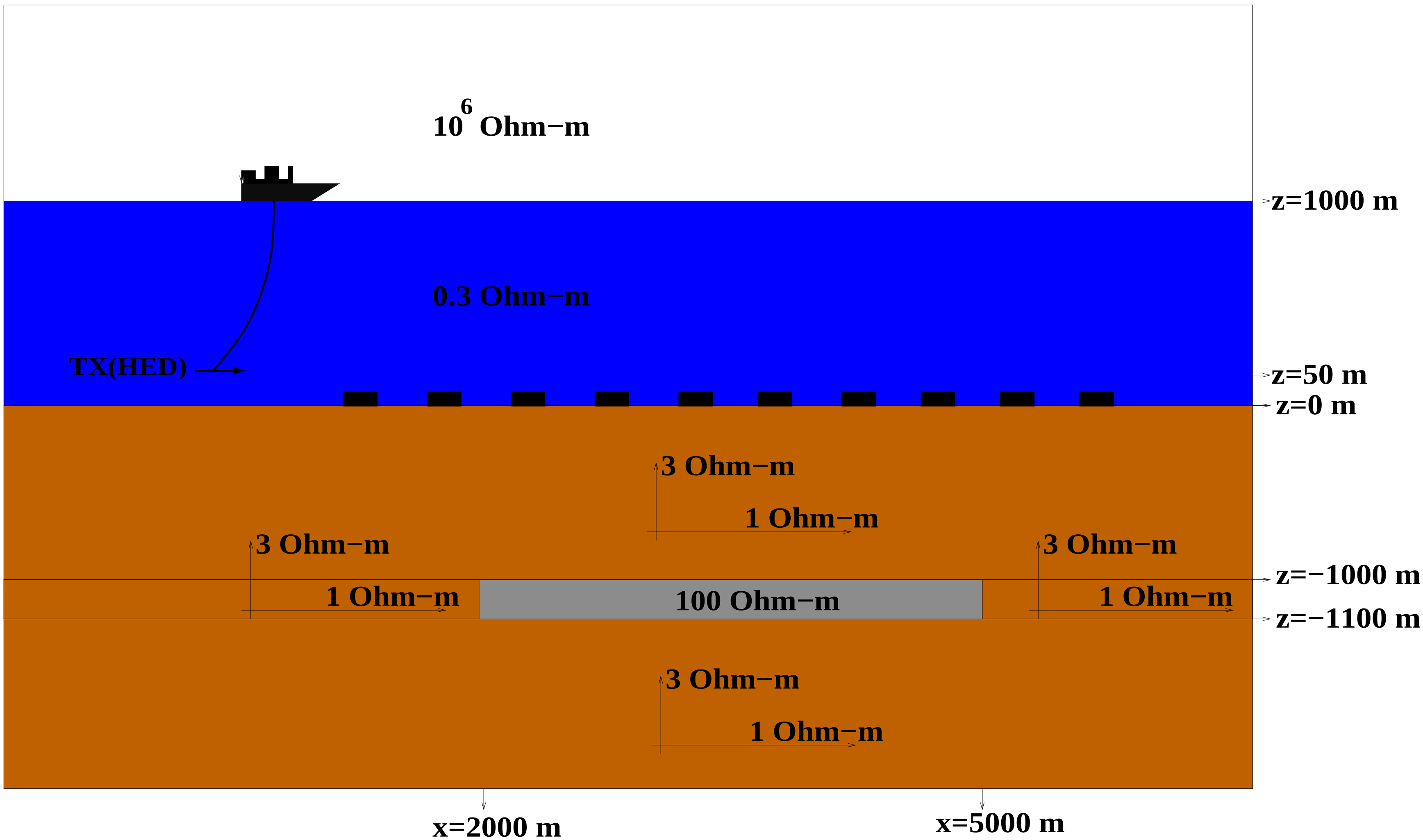
RESULTS: 2D MARINE CSEM

FINITE LAYER OF OIL



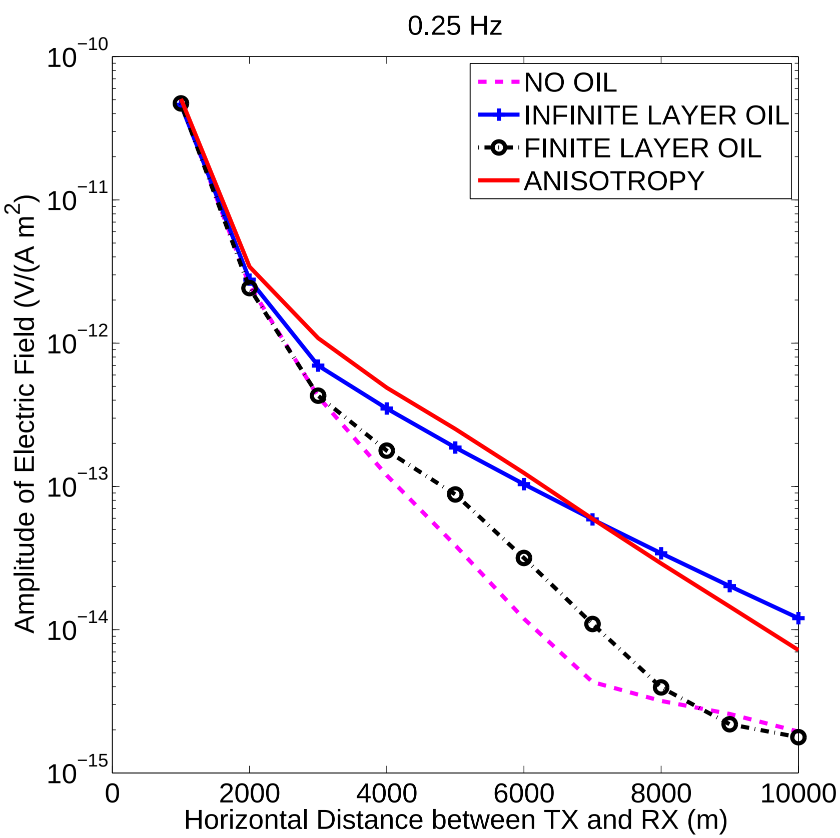
RESULTS: 2D MARINE CSEM

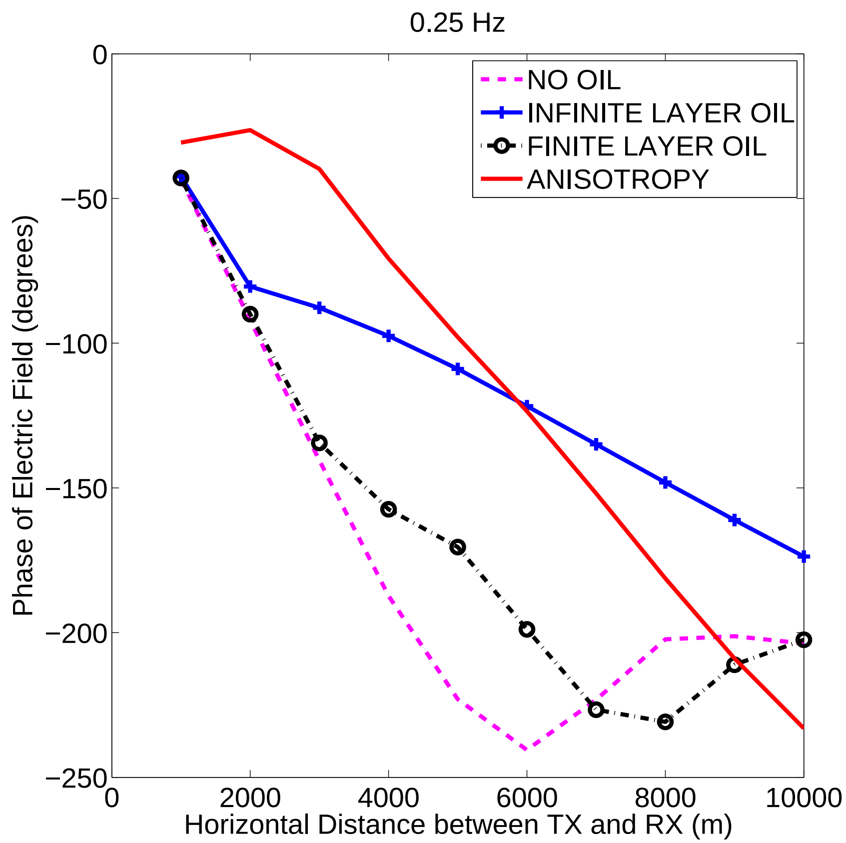
FINITE LAYER OF OIL + ANISOTROPY



RESULTS: 2D MARINE CSEM

Comparison — 0.25 Hz —

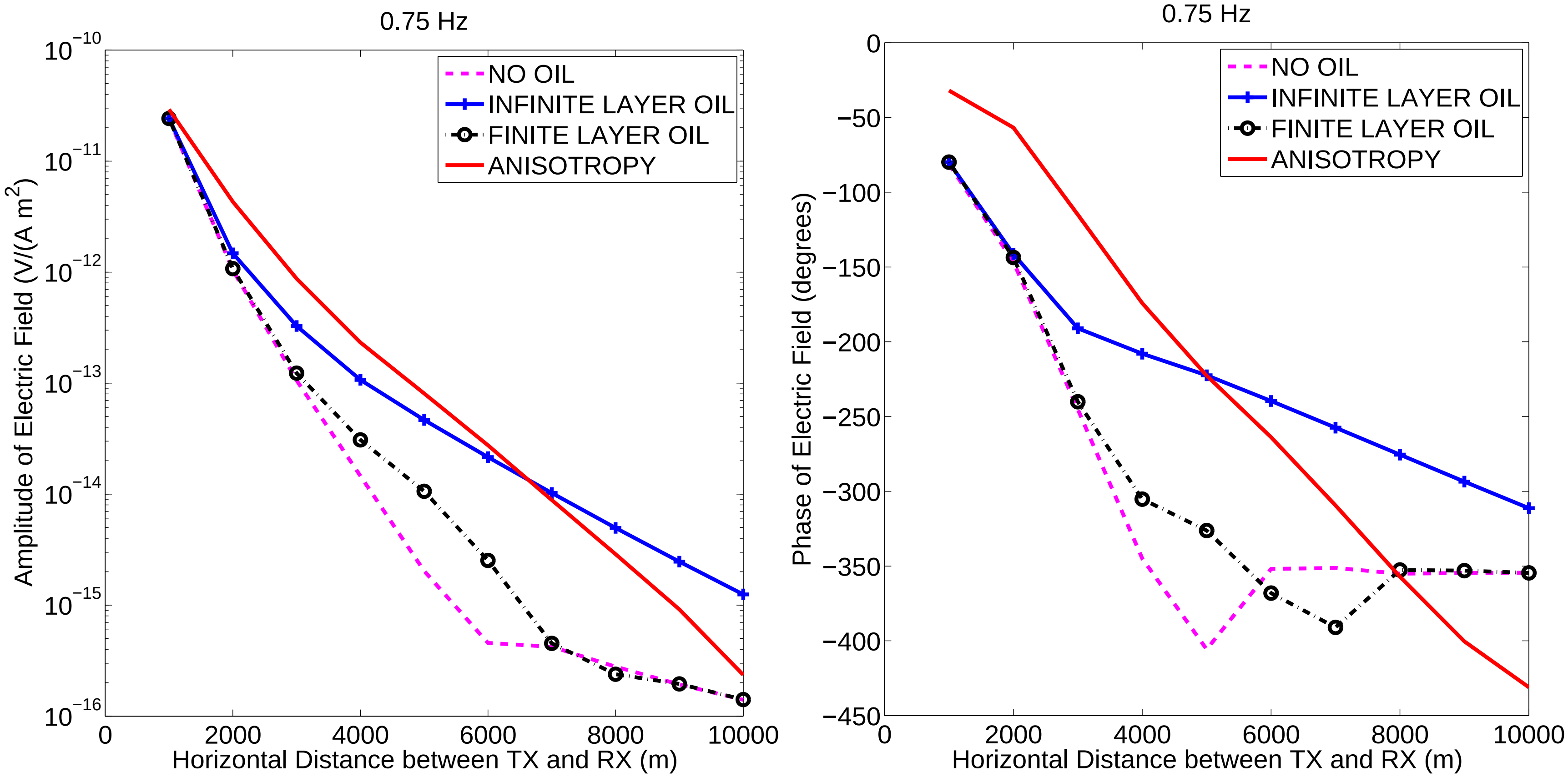




The finite layer of oil is clearly identified, and it is different from the solution for the infinite layer of oil. To consider anisotropy is essential.

RESULTS: 2D MARINE CSEM

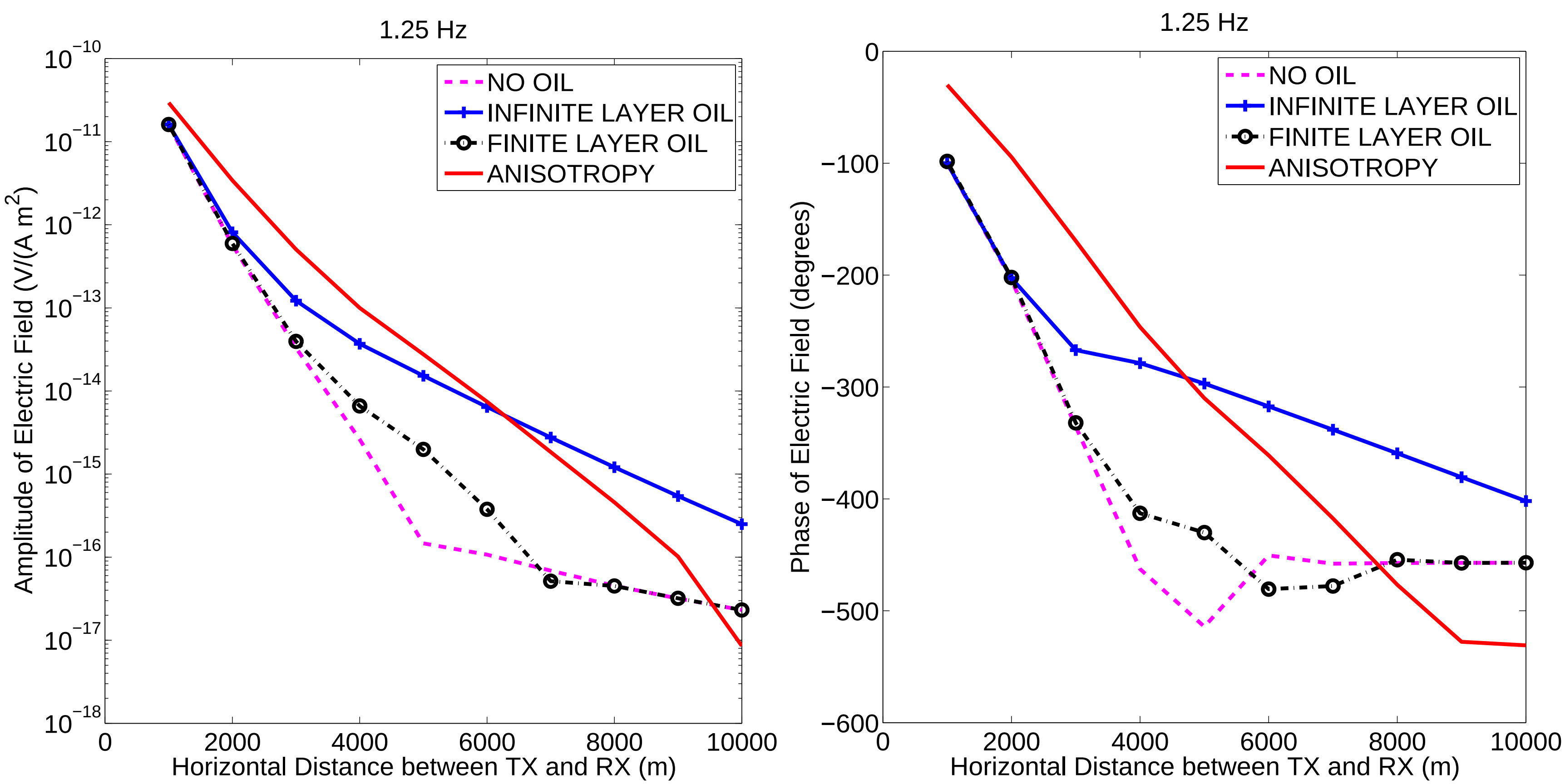
Comparison — 0.75 Hz —



As we increase the frequency, the effect of oil becomes more localized.

RESULTS: 2D MARINE CSEM

Comparison — 1.25 Hz —

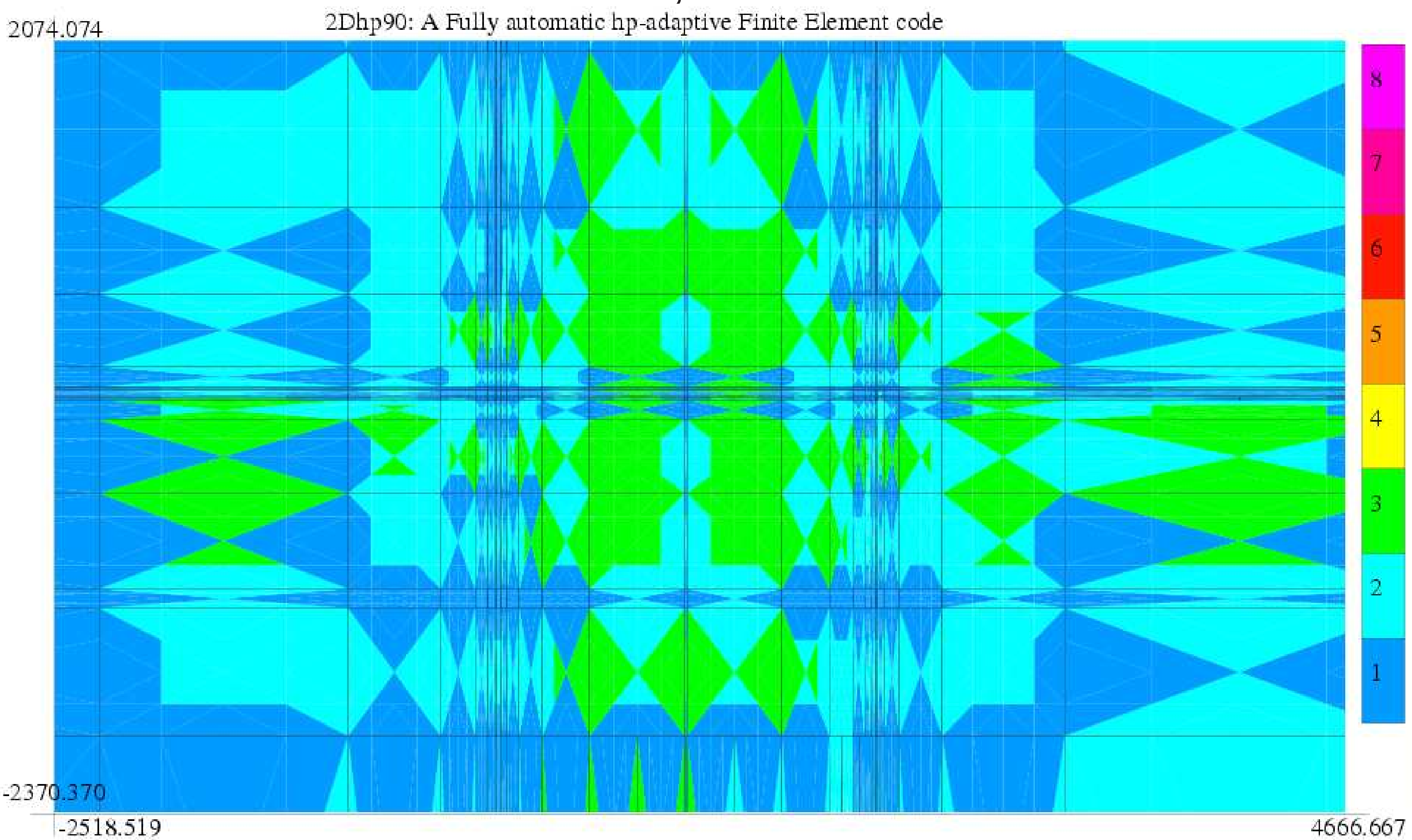


As we increase the frequency, the effect of oil becomes more localized.

RESULTS: 2D MARINE CSEM

0.75 Hz (FINITE LAYER OF OIL)

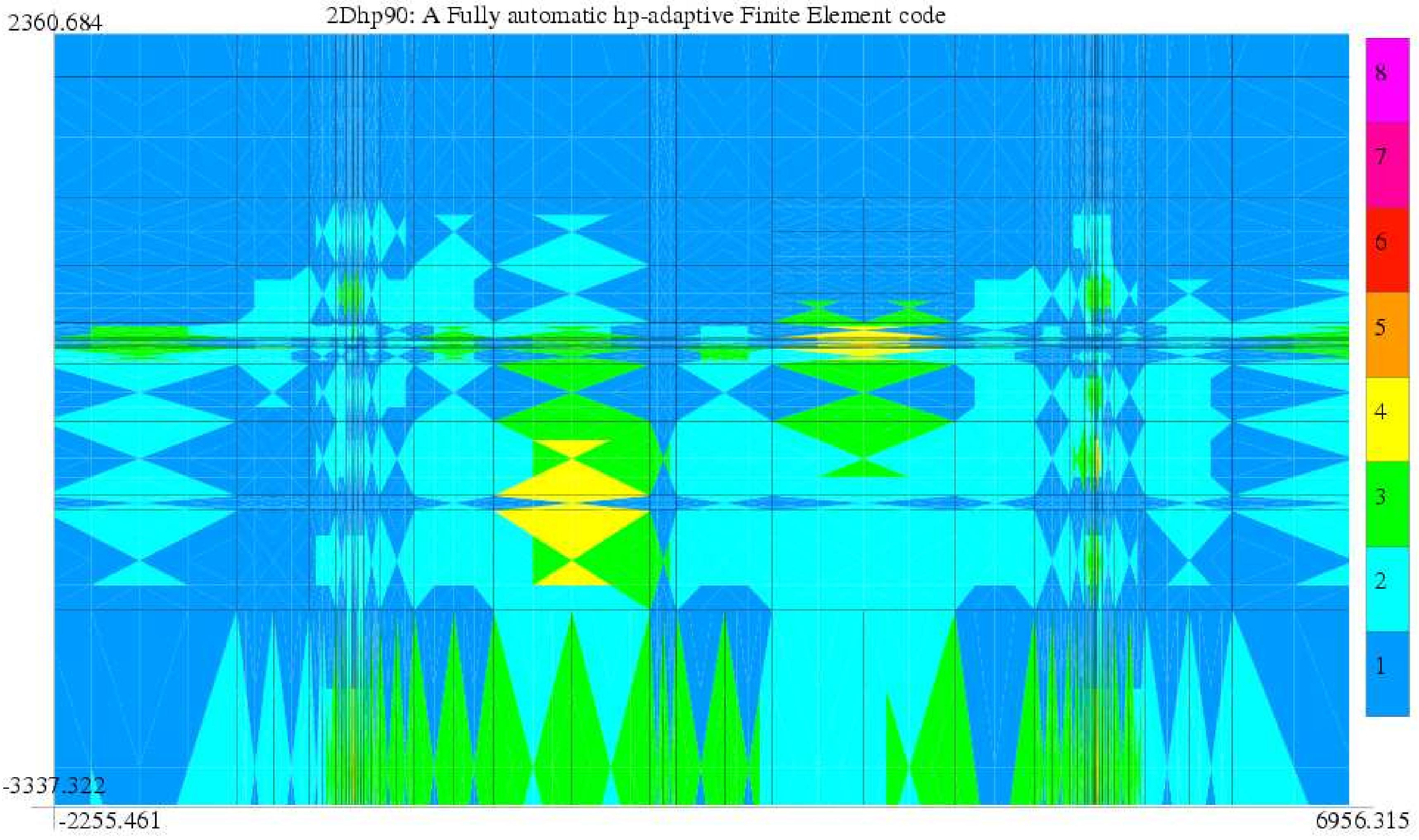
TX: x = 0 m ; RX: x = 2000 m.



RESULTS: 2D MARINE CSEM

0.75 Hz (FINITE LAYER OF OIL)

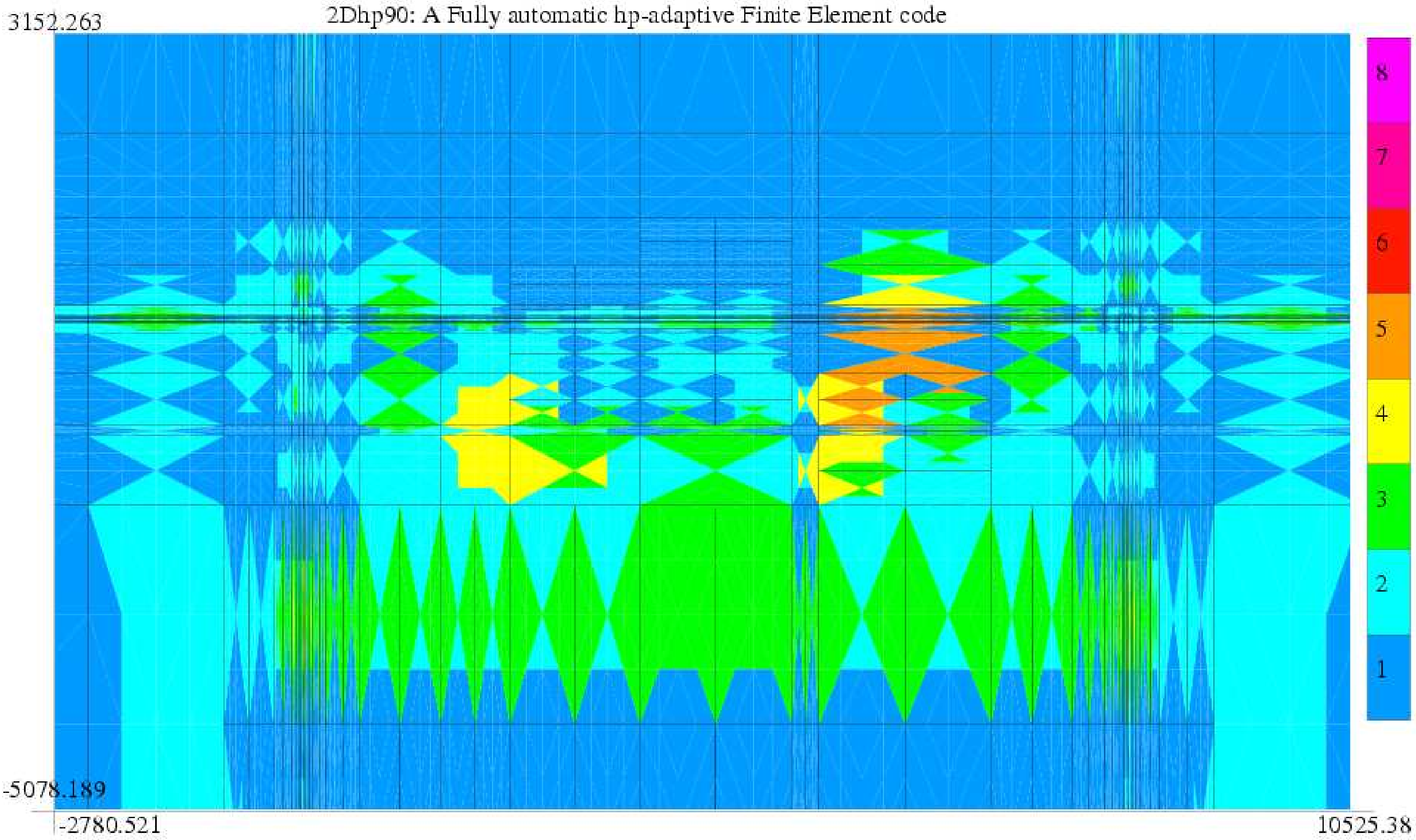
TX: x = 0 m ; RX: x = 5000 m.



RESULTS: 2D MARINE CSEM

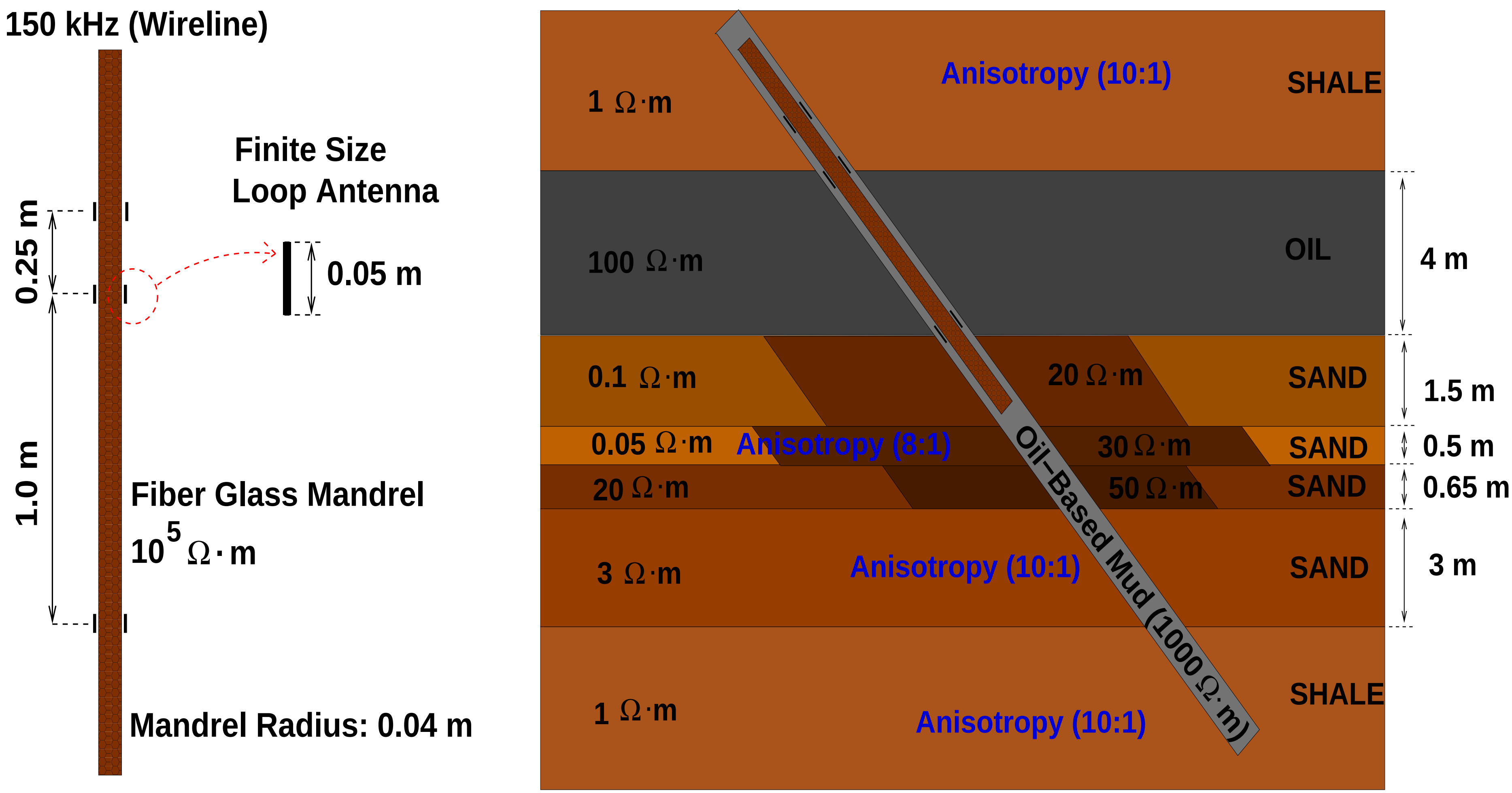
0.75 Hz (FINITE LAYER OF OIL)

TX: x = 0 m ; RX: x = 8000 m.



RESULTS: 3D RESISTIVITY LOGGING

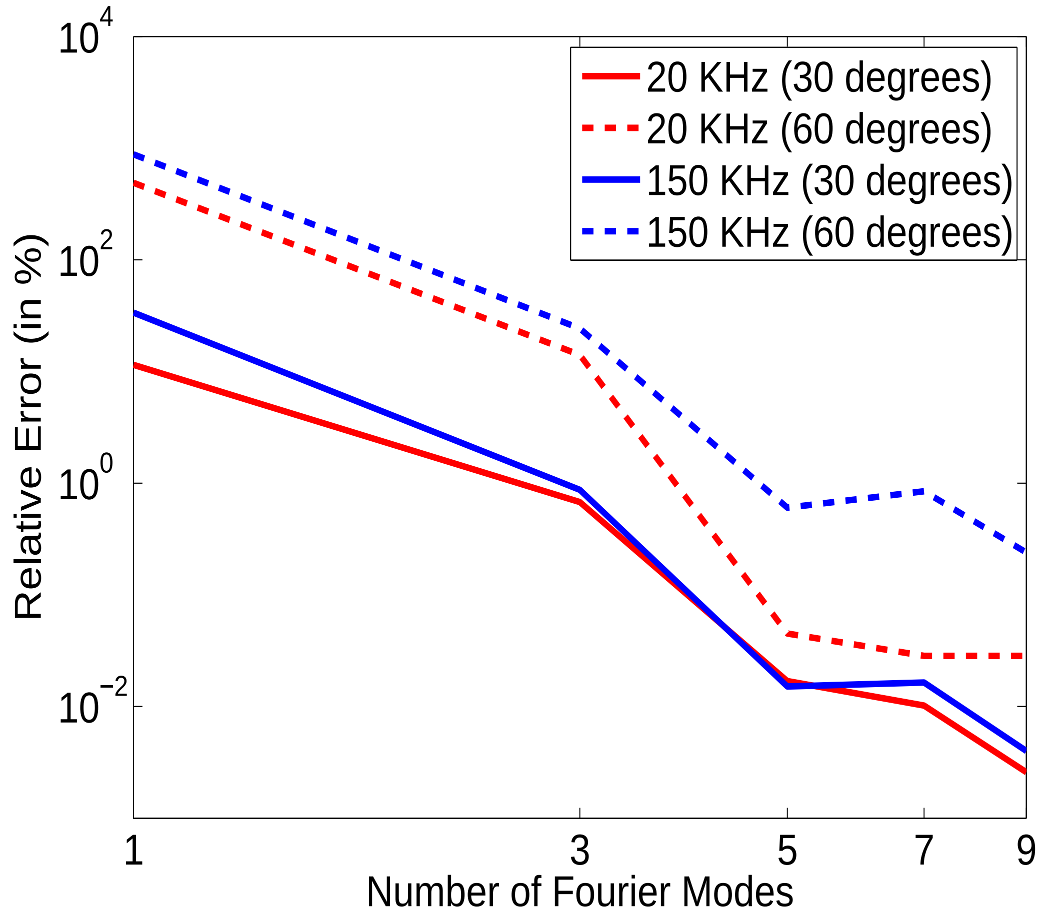
Model Problem



RESULTS: 3D RESISTIVITY LOGGING

Verification

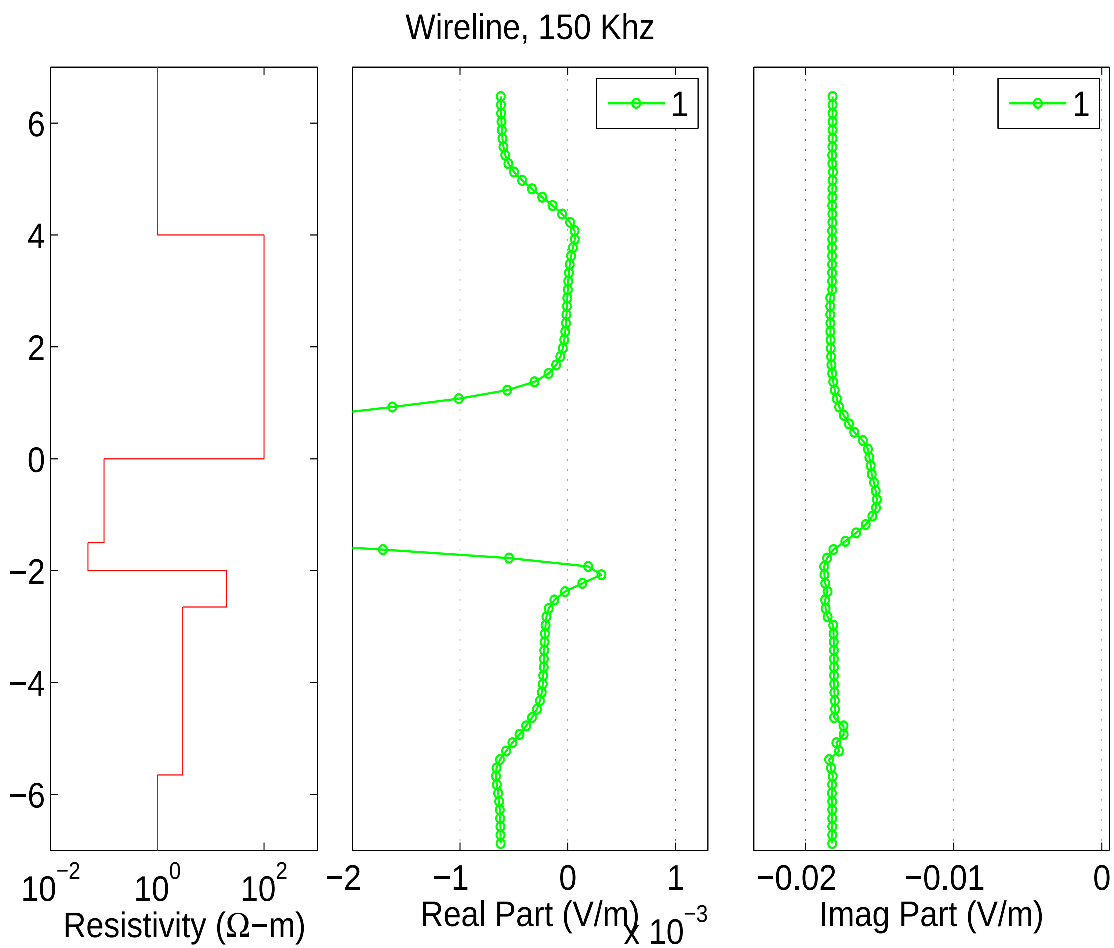
Logging Instrument in a Homogeneous Formation



RESULTS: 3D RESISTIVITY LOGGING

Verification

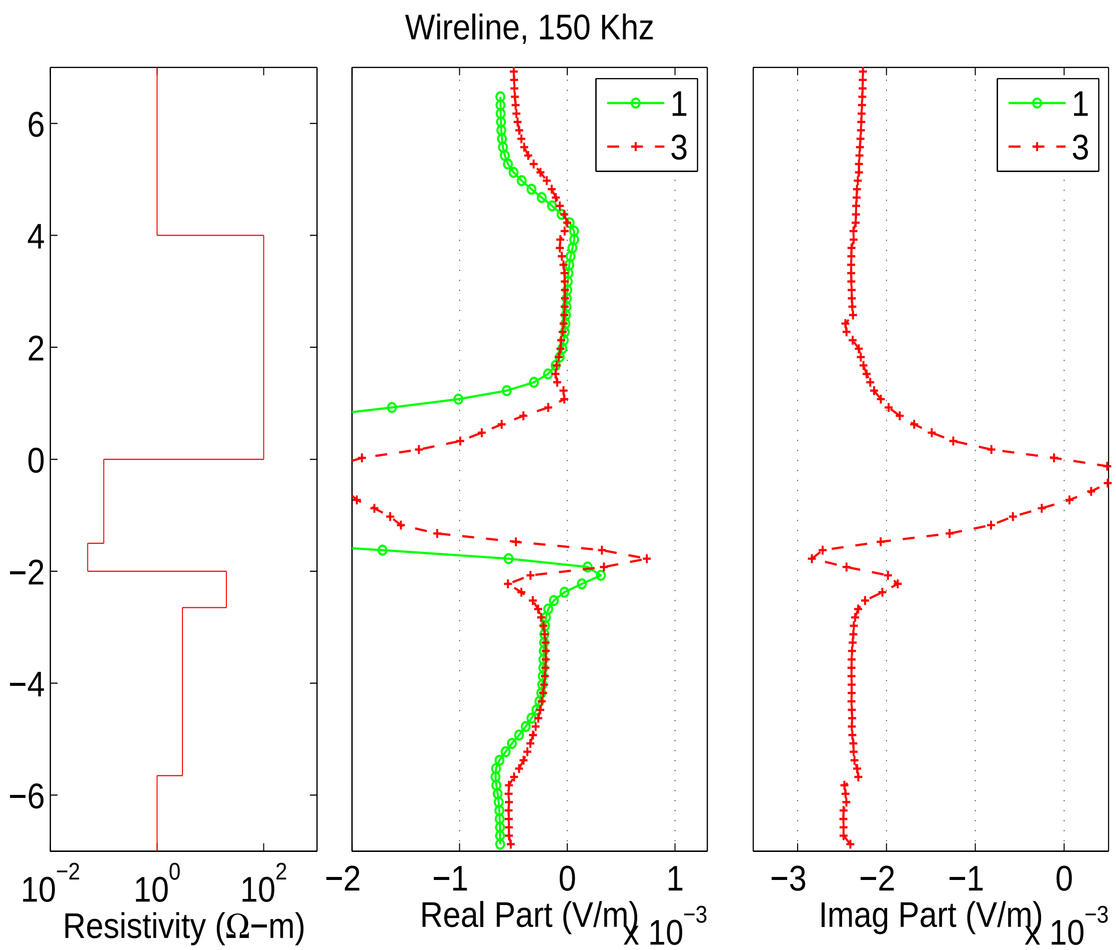
Logging Instrument in a Homogeneous Formation



RESULTS: 3D RESISTIVITY LOGGING

Verification

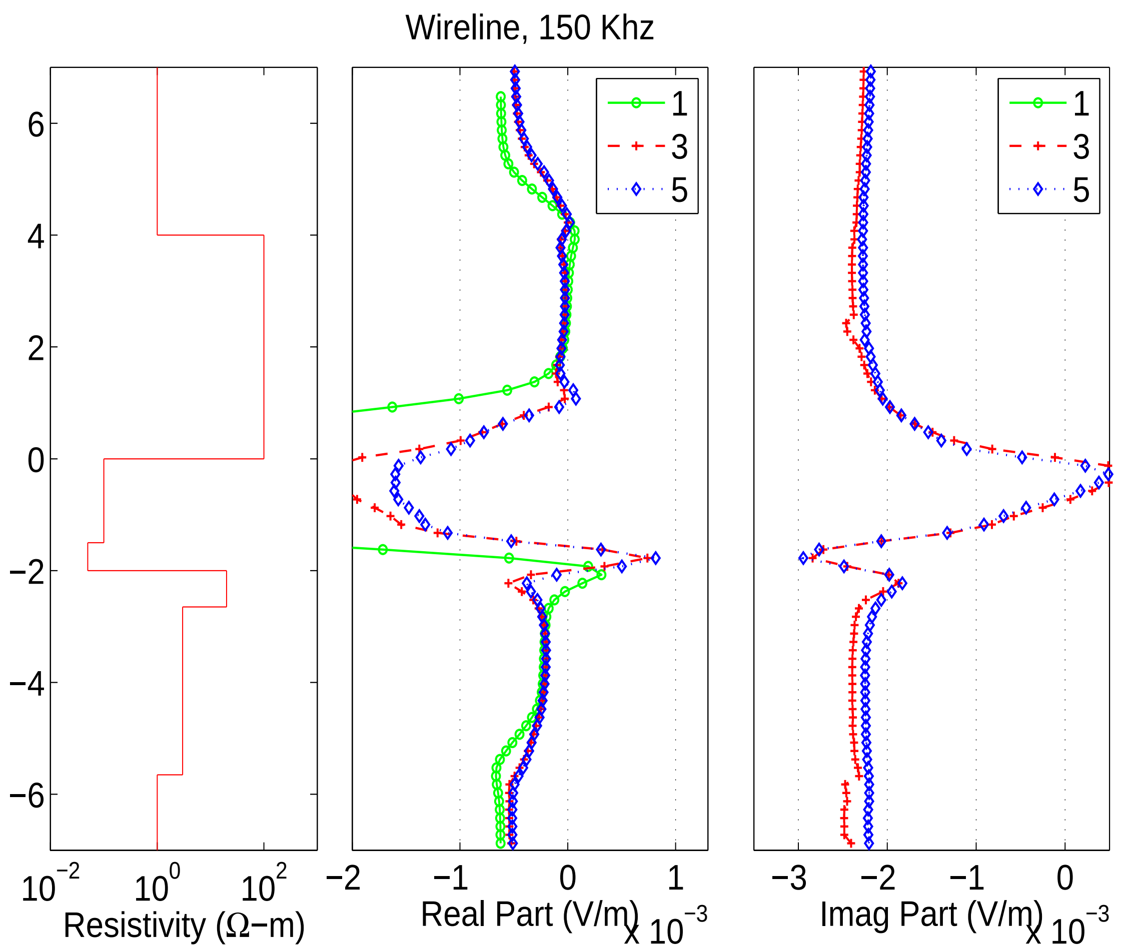
Logging Instrument in a Homogeneous Formation



RESULTS: 3D RESISTIVITY LOGGING

Verification

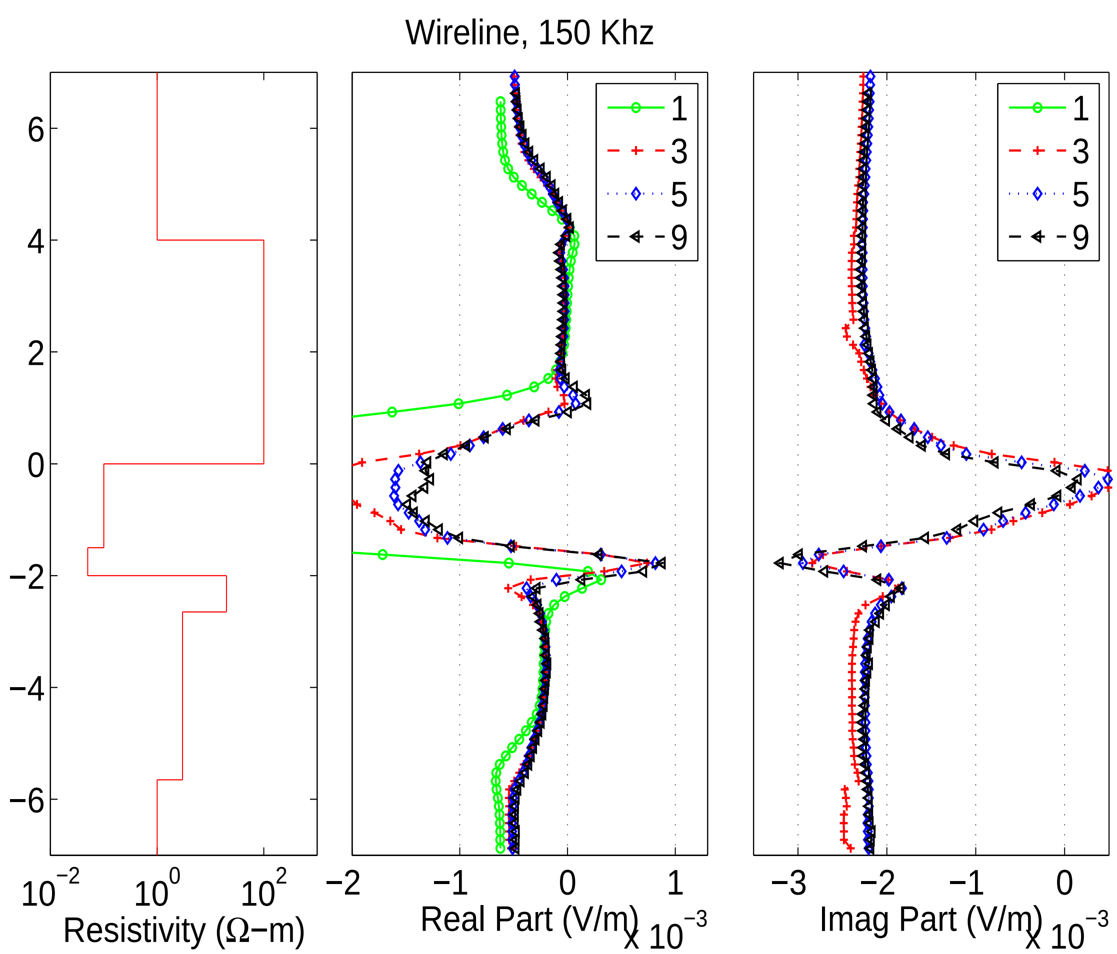
Logging Instrument in a Homogeneous Formation



RESULTS: 3D RESISTIVITY LOGGING

Verification

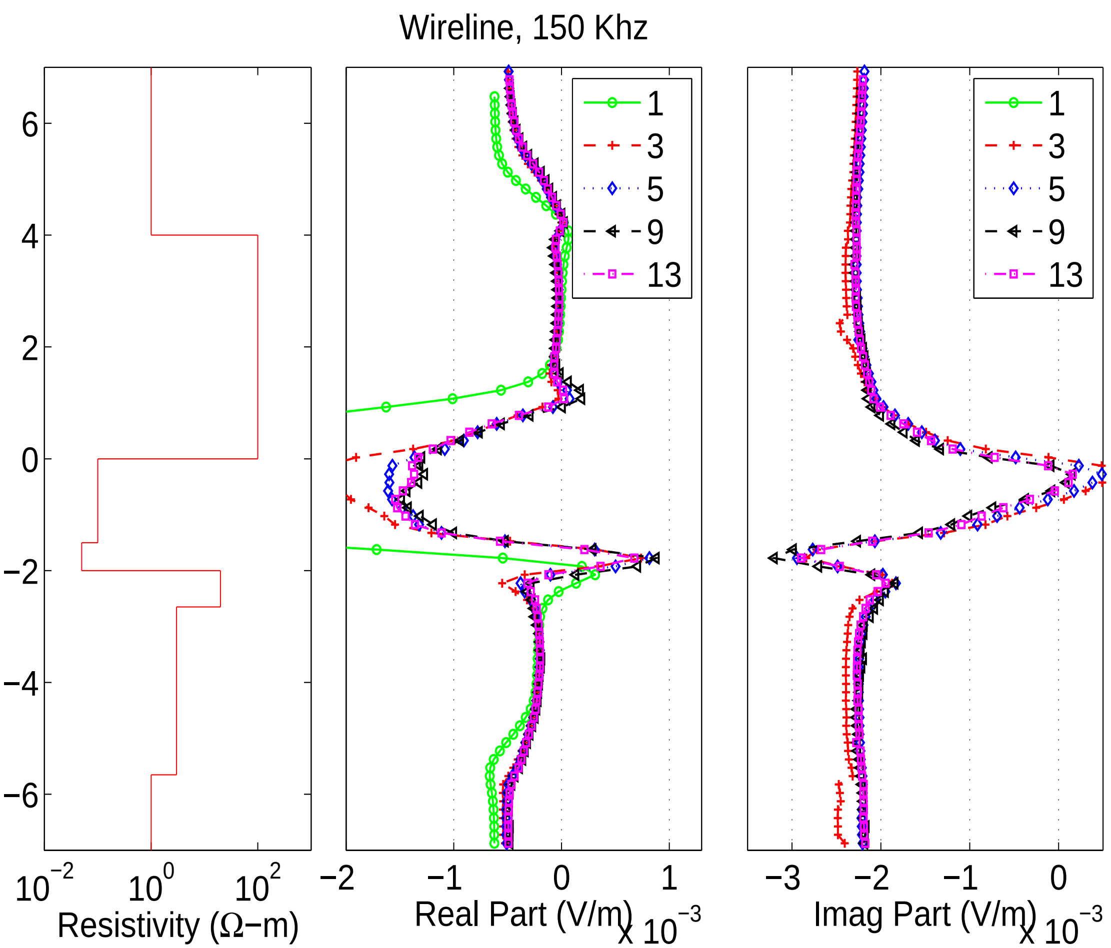
Logging Instrument in a Homogeneous Formation



RESULTS: 3D RESISTIVITY LOGGING

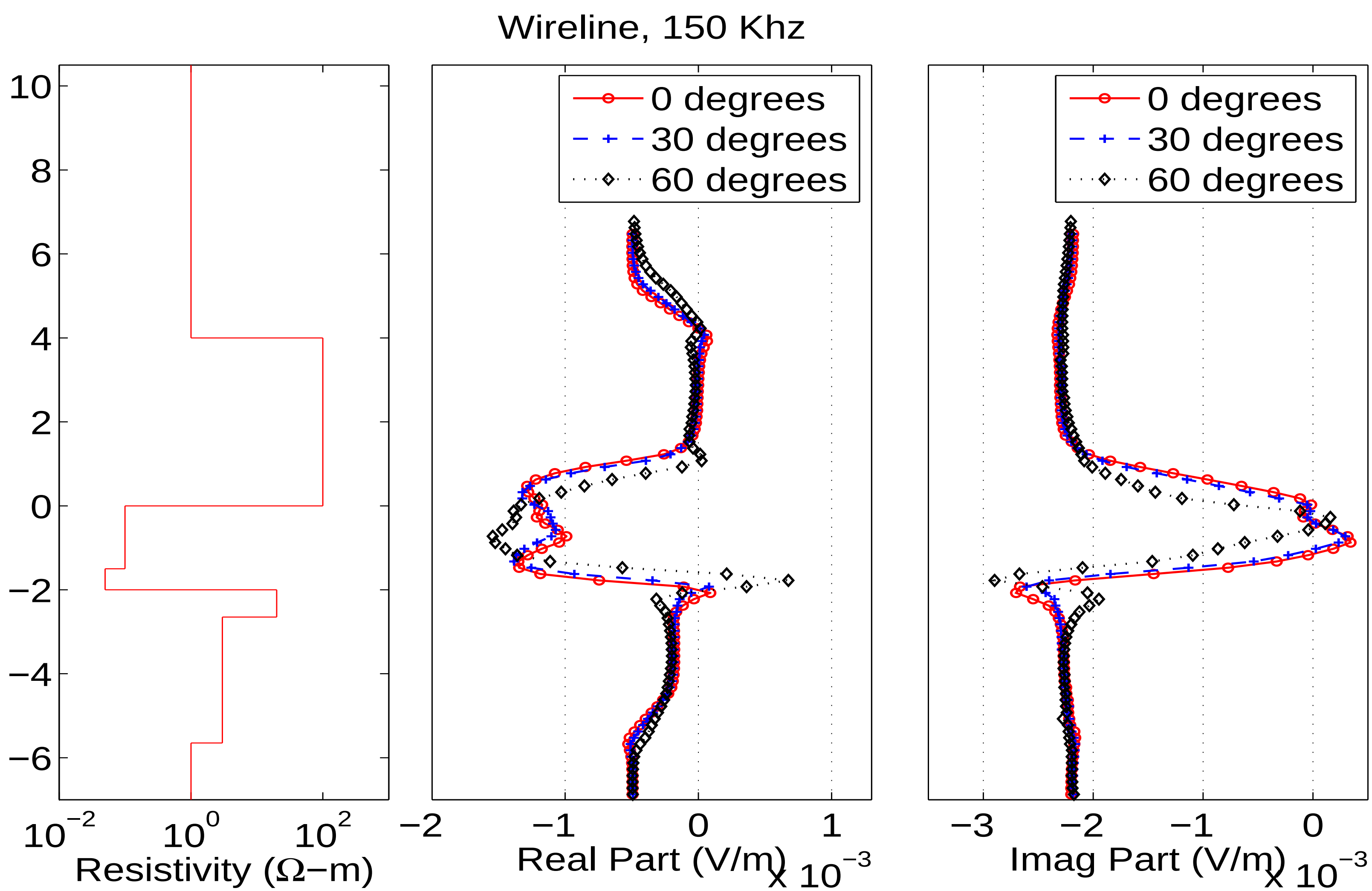
Verification

Logging Instrument in a Homogeneous Formation



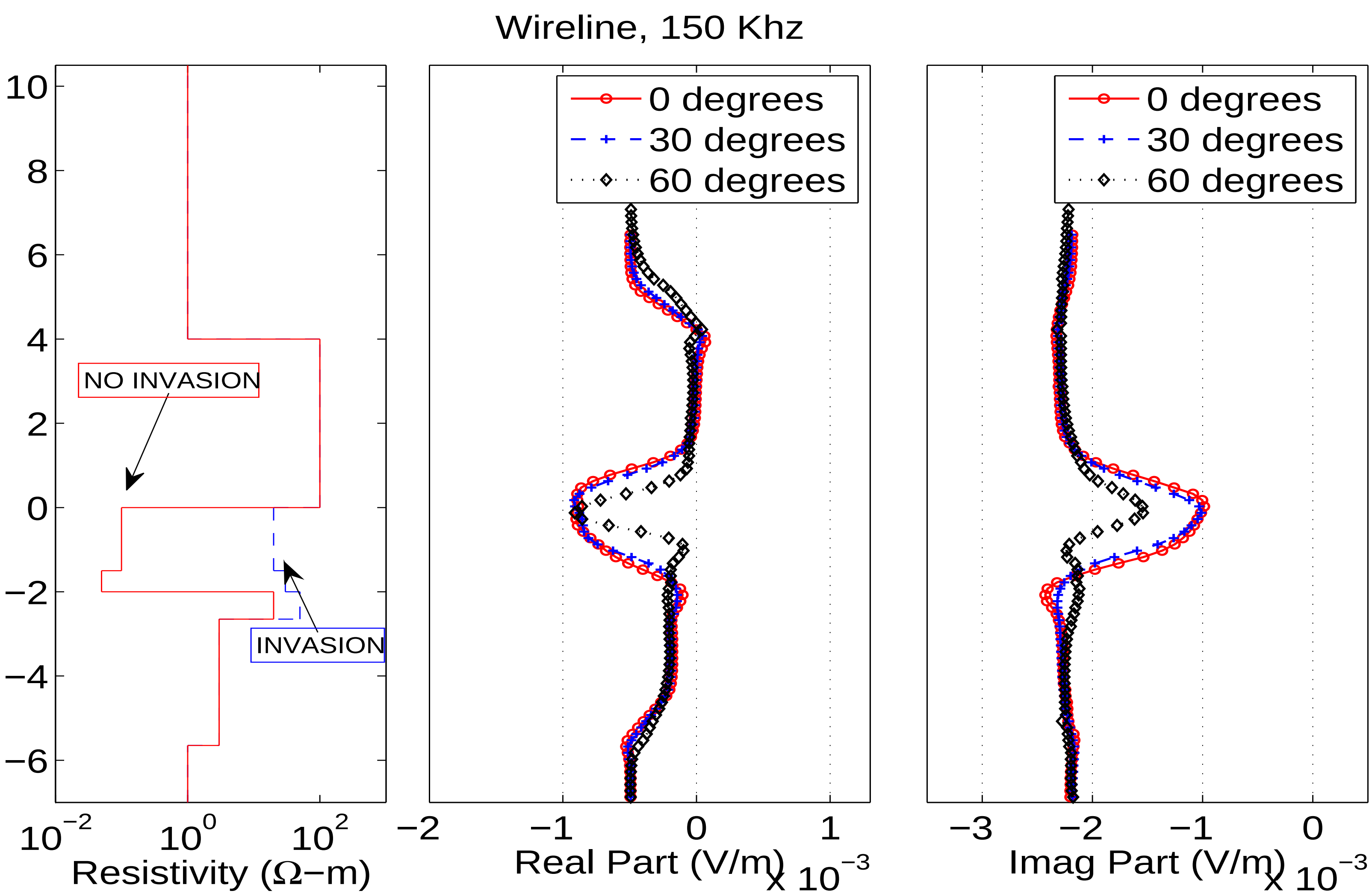
RESULTS: 3D RESISTIVITY LOGGING

Dip Angle



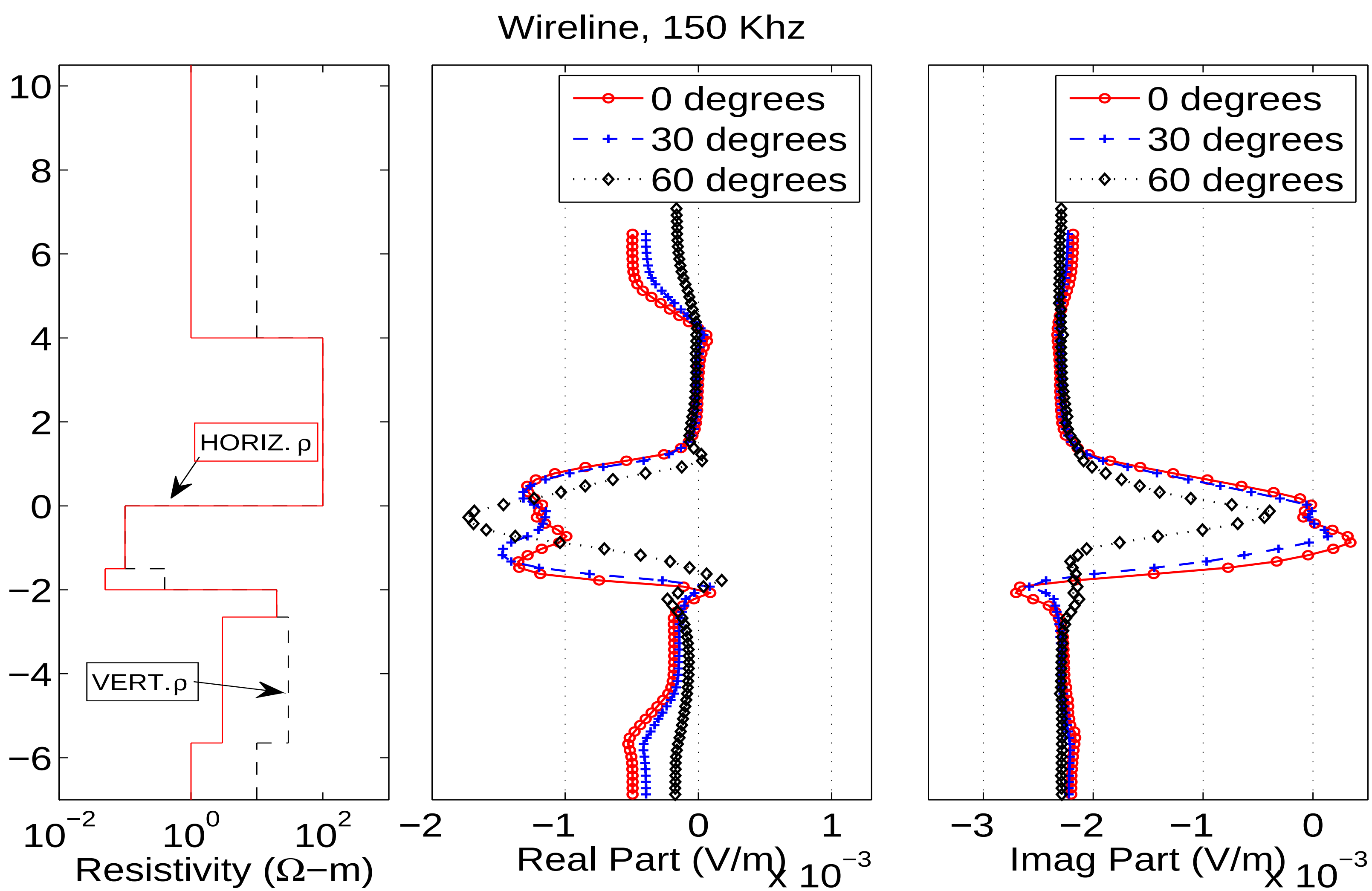
RESULTS: 3D RESISTIVITY LOGGING

Dip Angle + Invasion



RESULTS: 3D RESISTIVITY LOGGING

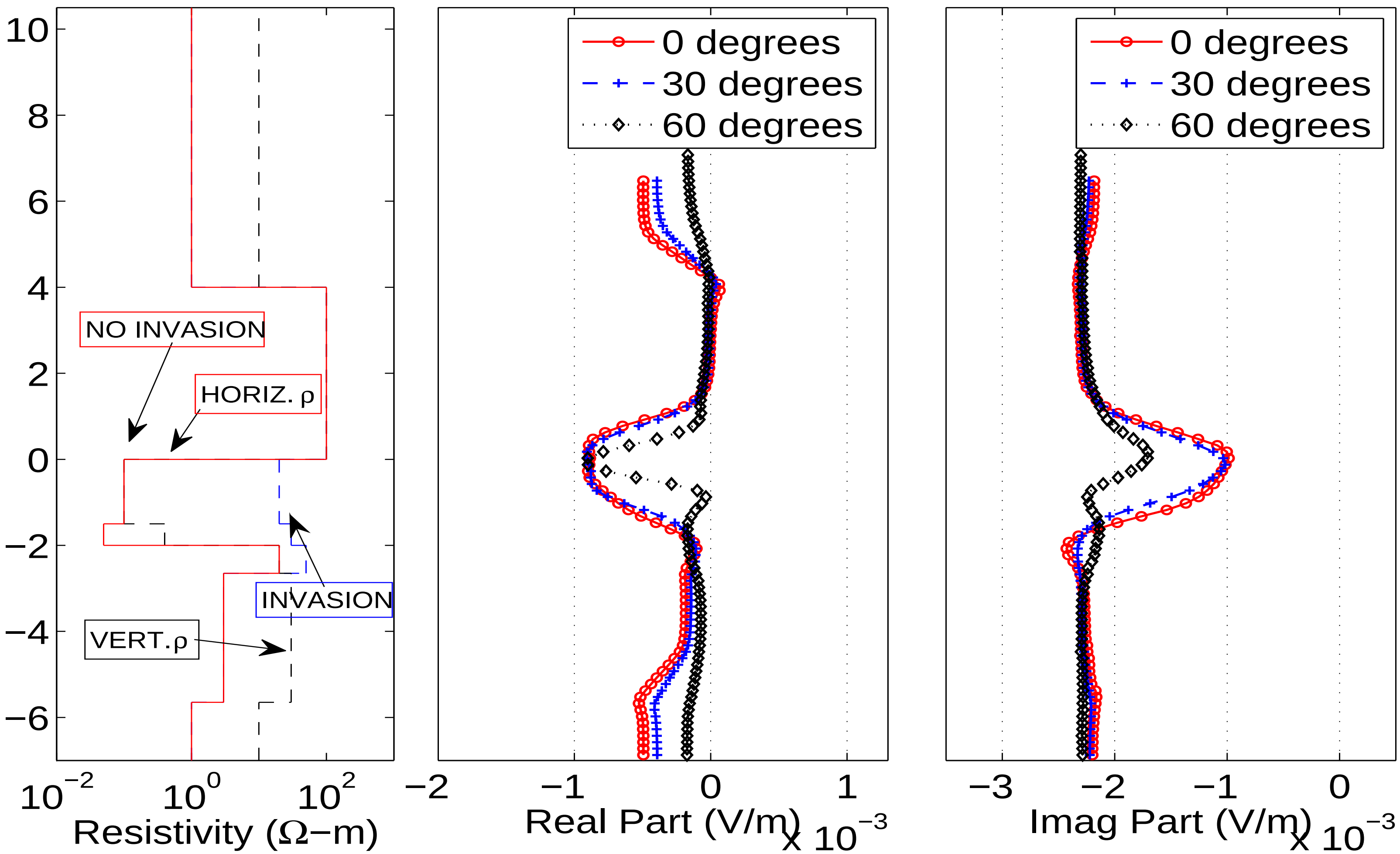
Dip Angle + Anisotropy



RESULTS: 3D RESISTIVITY LOGGING

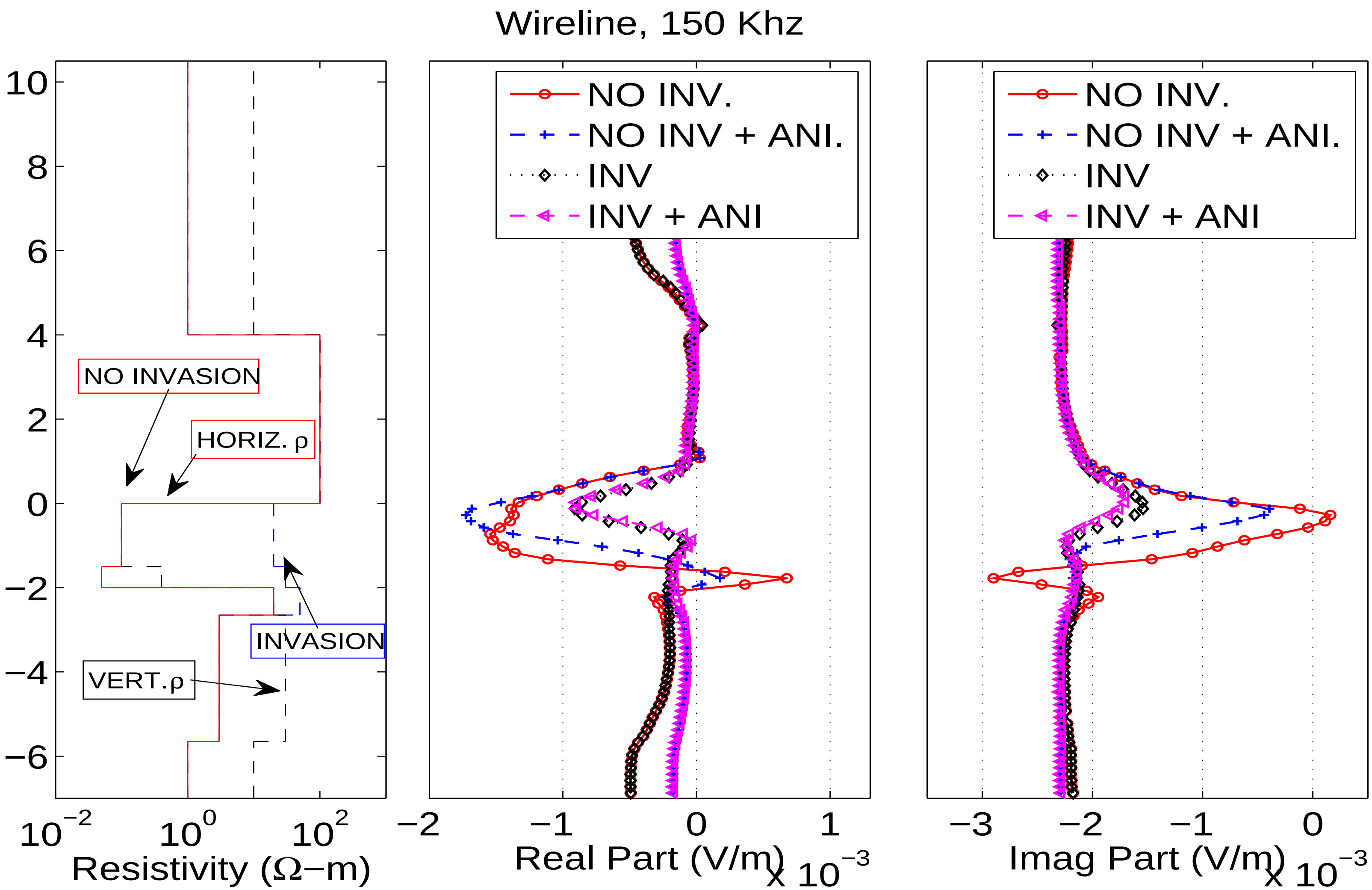
Dip Angle + Invasion + Anisotropy

Wireline, 150 Khz



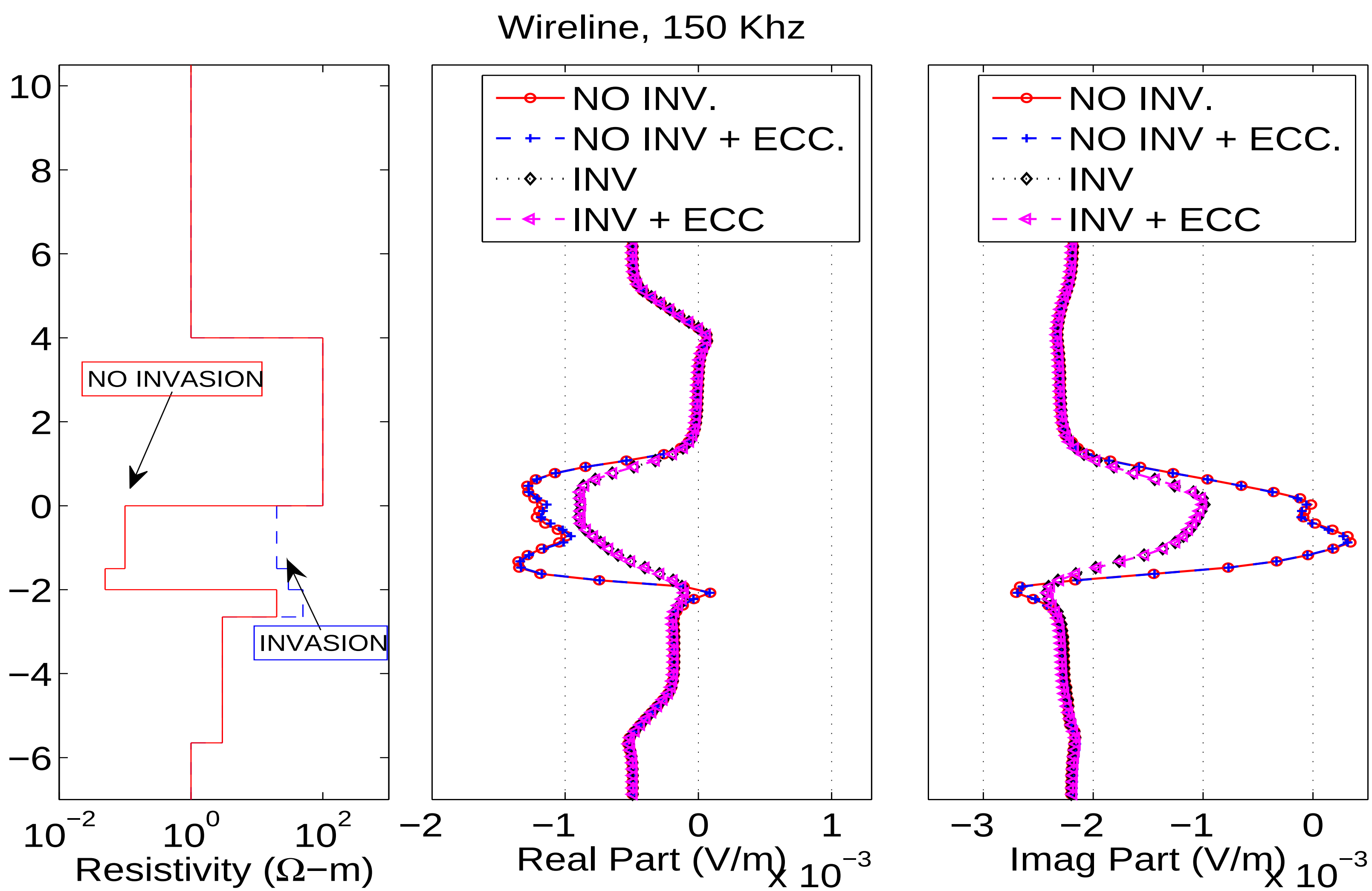
RESULTS: 3D RESISTIVITY LOGGING

60-Degree Deviated Well



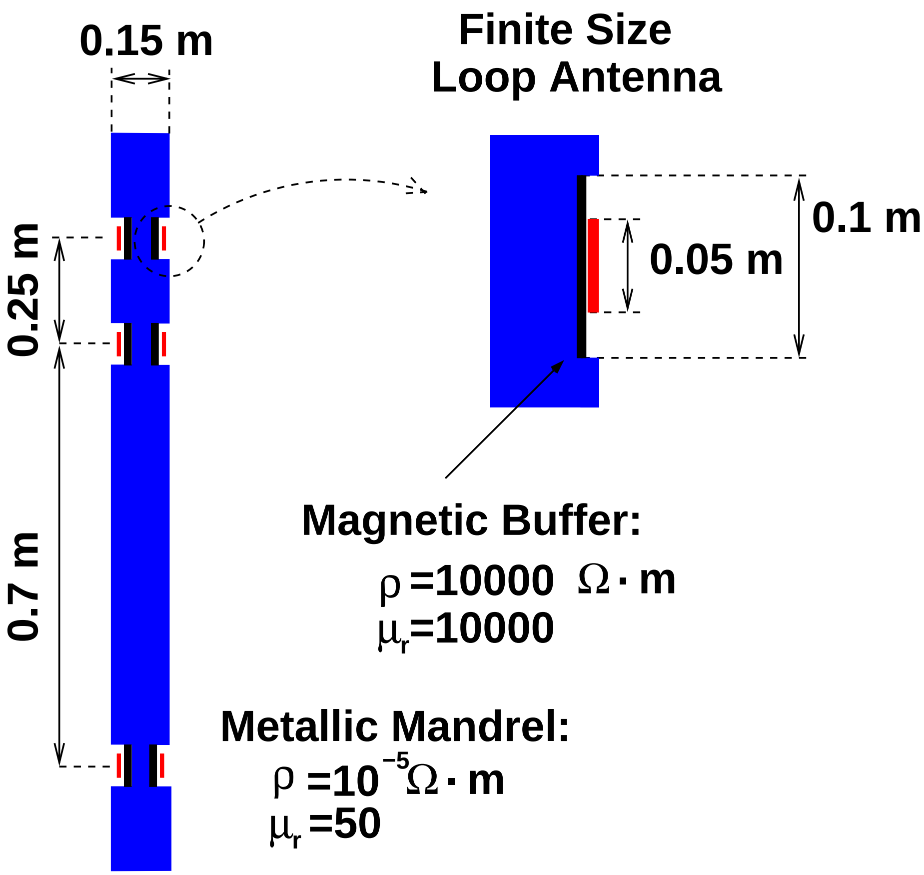
RESULTS: 3D RESISTIVITY LOGGING

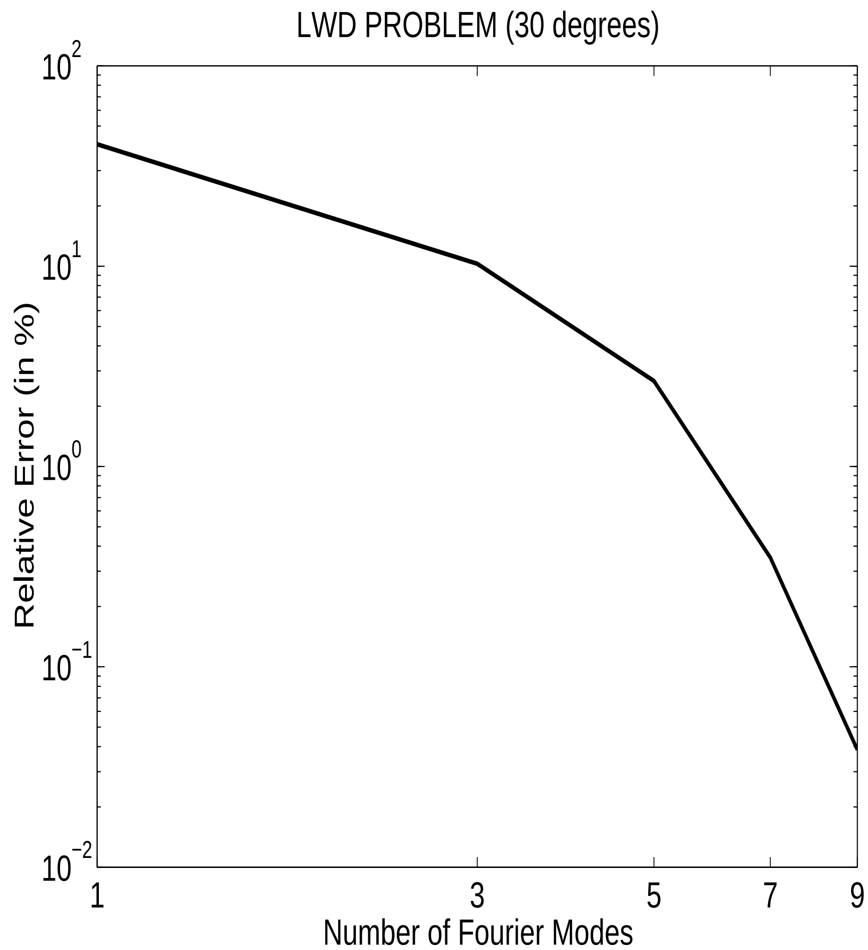
Vertical Well with 0.03 m Eccentricity



RESULTS: 3D RESISTIVITY LOGGING

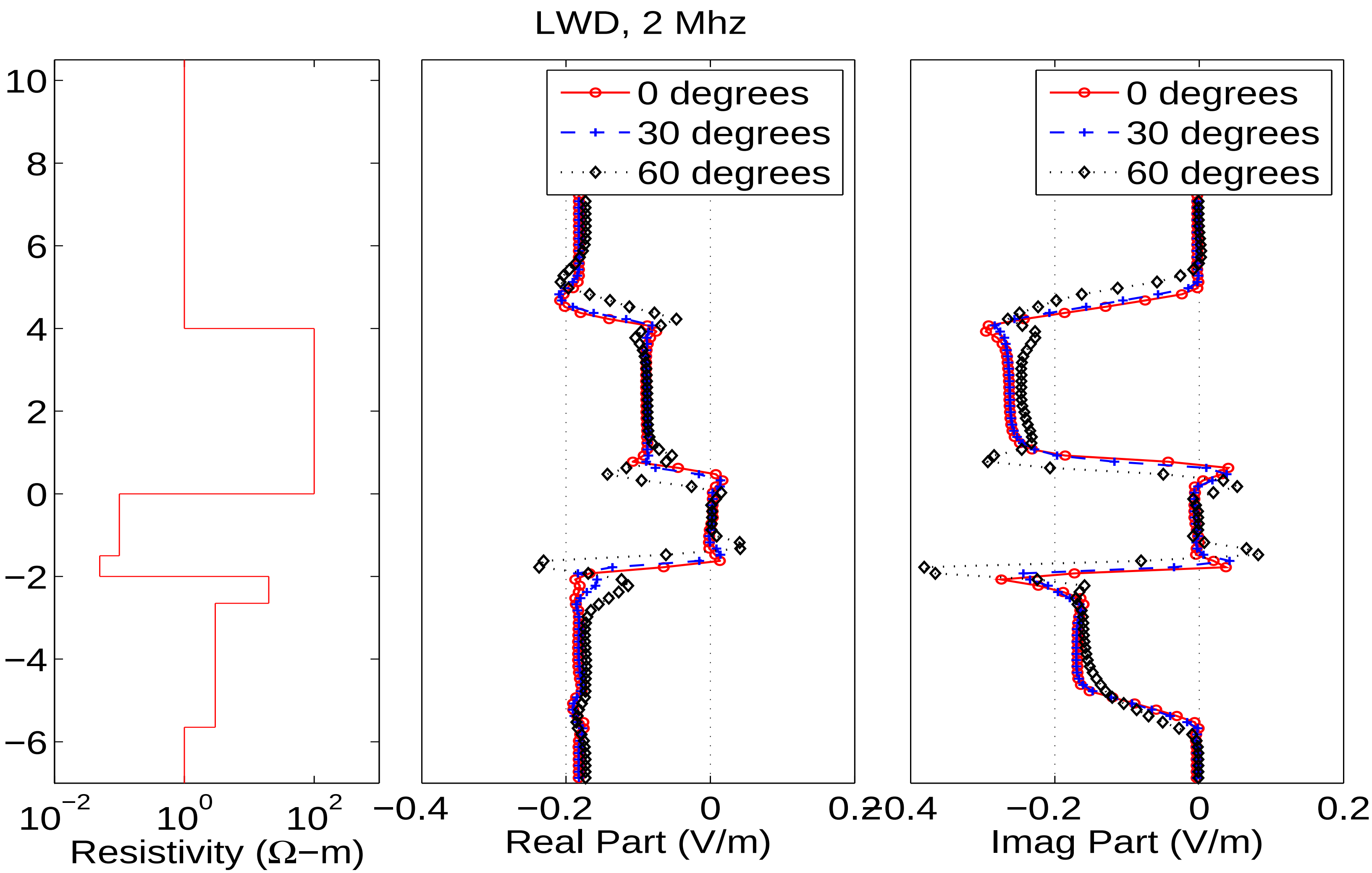
Model Problem and Verification





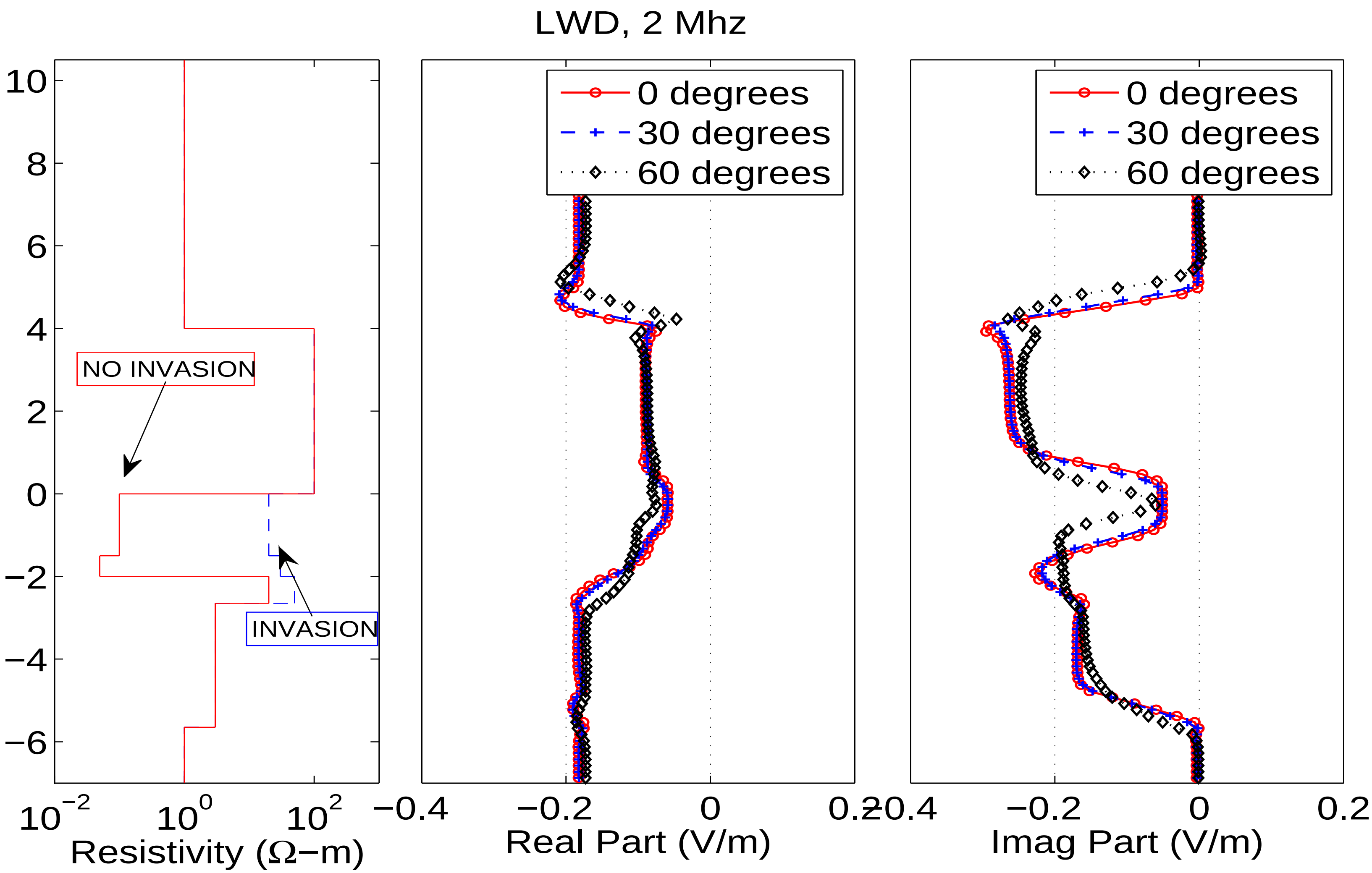
RESULTS: 3D RESISTIVITY LOGGING

Dip Angle



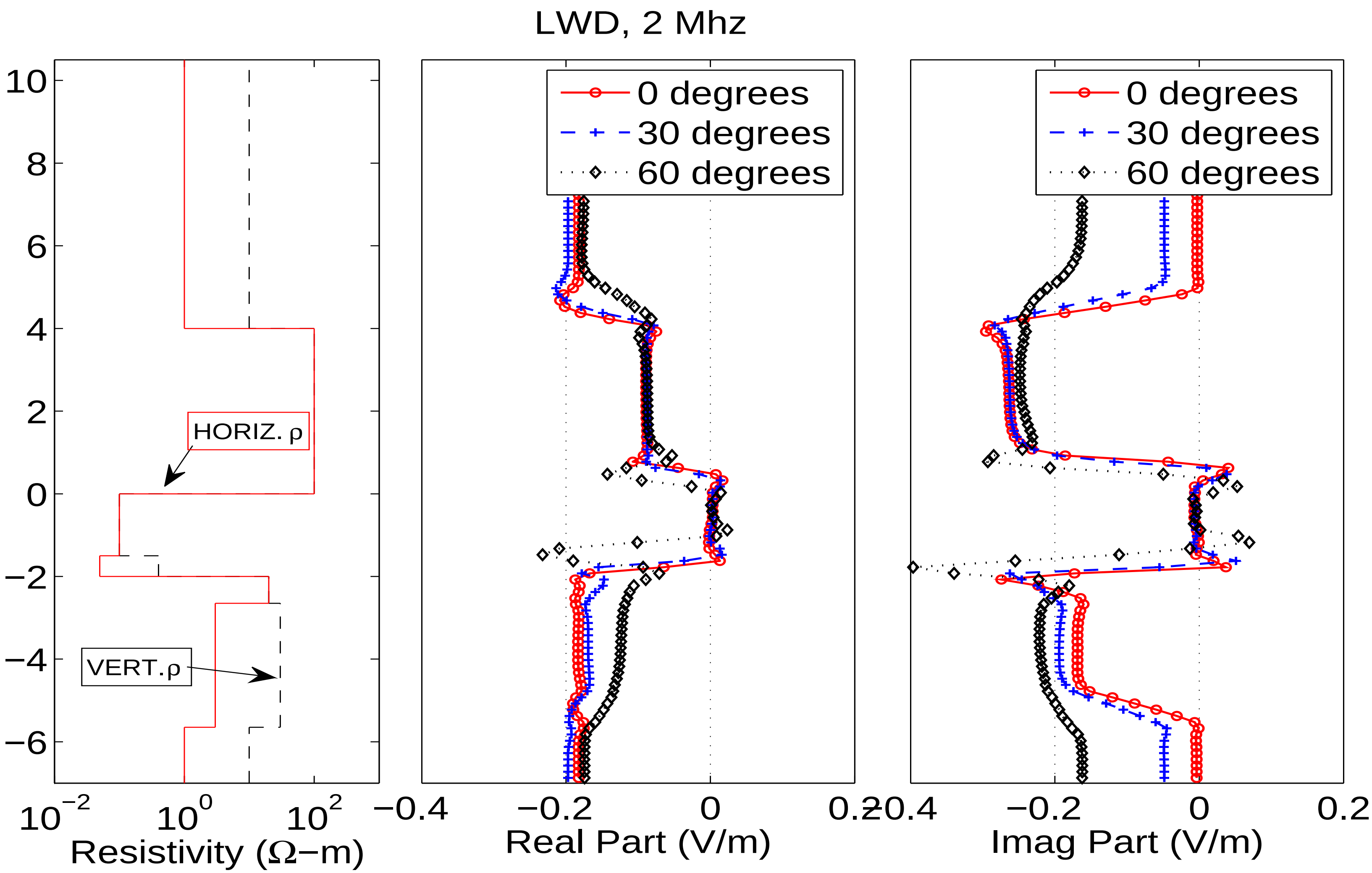
RESULTS: 3D RESISTIVITY LOGGING

Dip Angle + Invasion



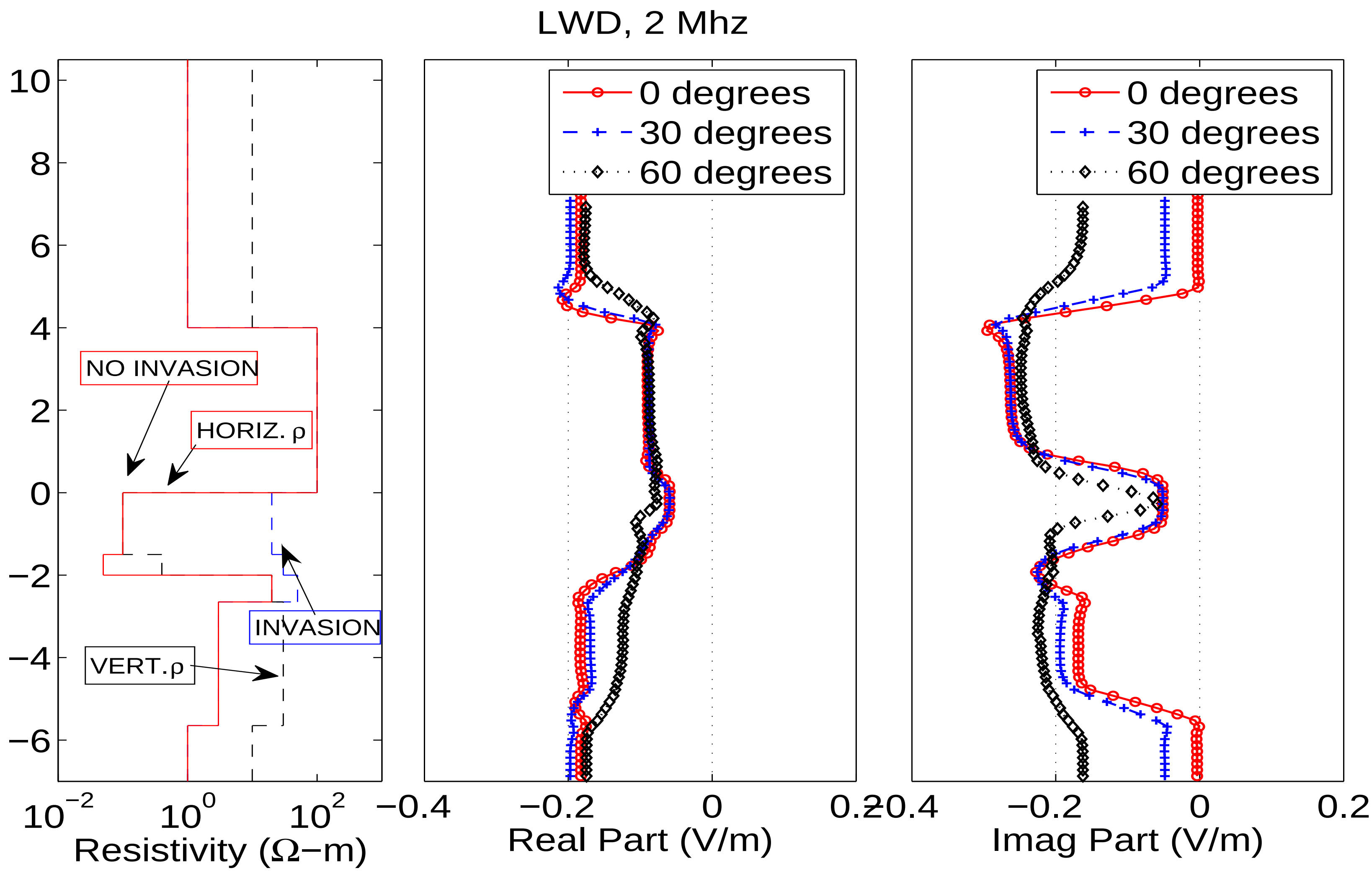
RESULTS: 3D RESISTIVITY LOGGING

Dip Angle + Anisotropy



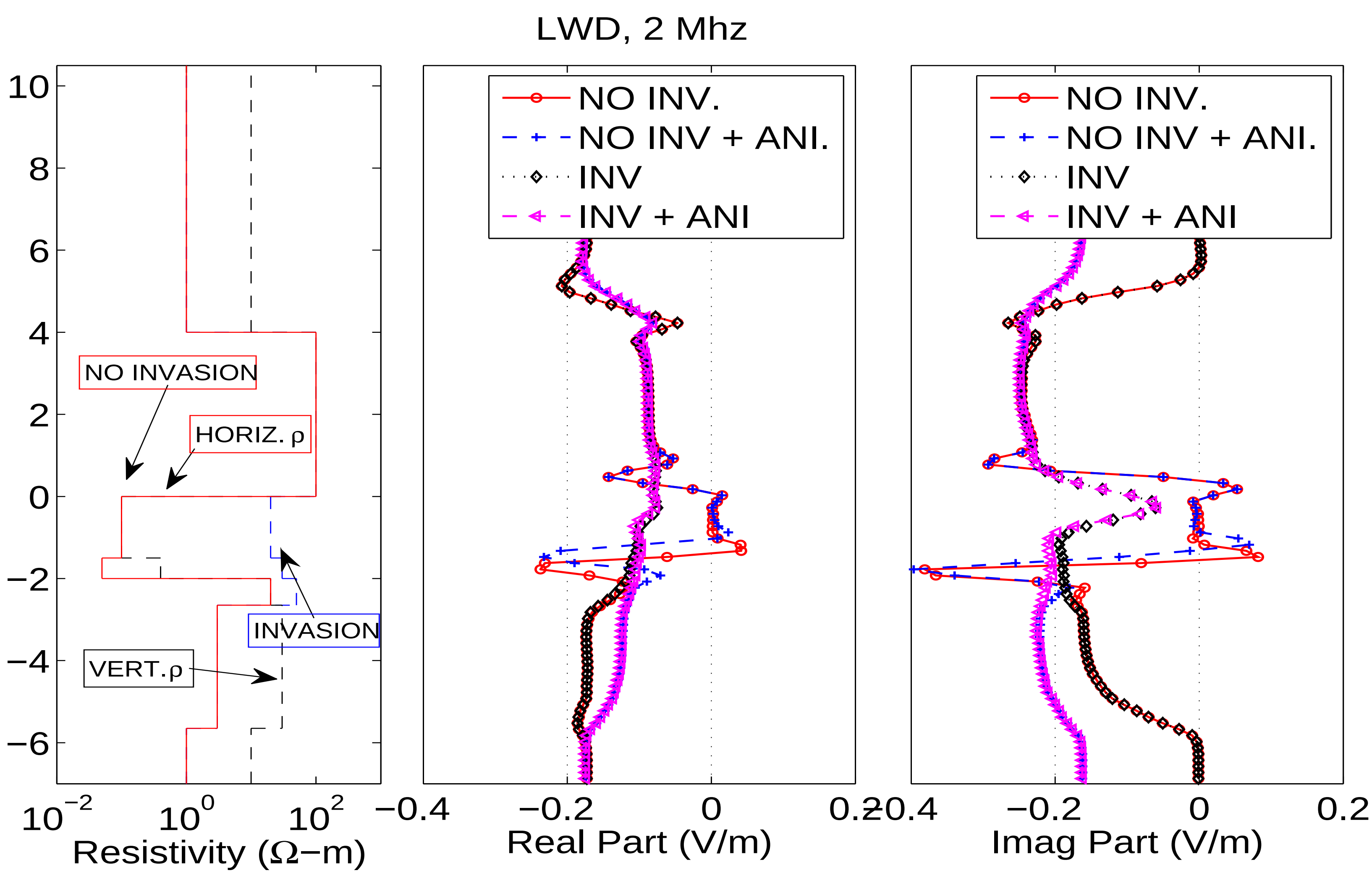
RESULTS: 3D RESISTIVITY LOGGING

Dip Angle + Invasion + Anisotropy



RESULTS: 3D RESISTIVITY LOGGING

60-Degree Deviated Well



CONCLUSIONS AND FUTURE WORK

- **A Fourier-Finite-Element method provides a suitable formulation for simulation of resistivity geophysical applications.**
- **Goal-oriented refinements are essential in marine CSEM geophysical applications due to the dissipative nature of the earth.**
- **A parallel implementation based on a shared domain-decomposition is simple and provides additional performance for a moderate number of processors.**
- **We are developing a multiphysics framework for the joint-inversion of multiphysics measurements.**
- **We are looking for Ph.D. students and postdoctoral fellows to further develop this software and work on the joint-inversion of multiphysics measurements.**

Basque Center for Applied Mathematics (BCAM)