A Proposal for a Dissertation on

Integration of *hp*-Adaptivity with a Two Grid Solver: Applications to Electromagnetics.

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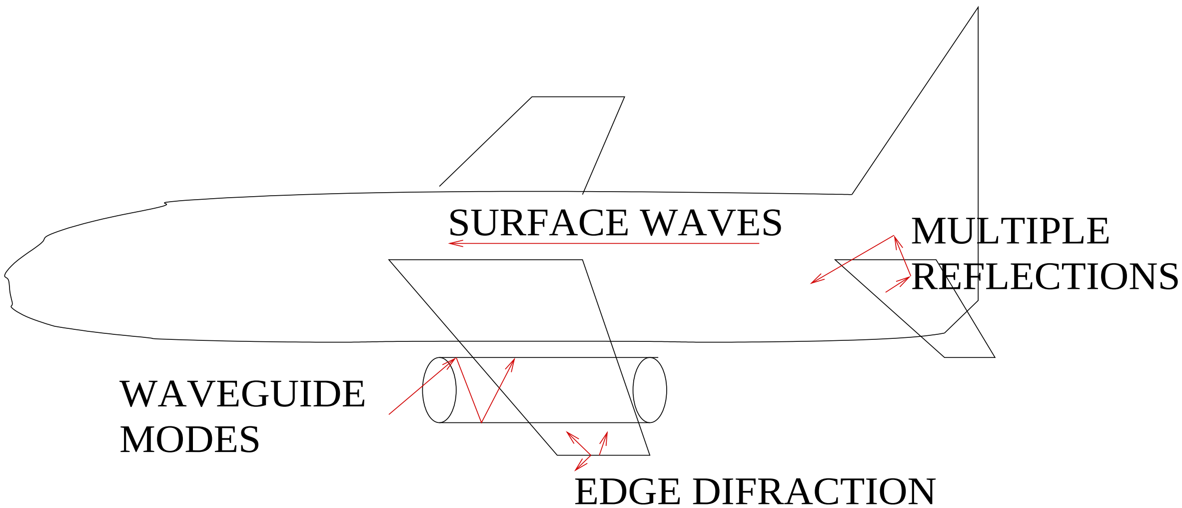
OUTLINE

1. **Introduction.**
   - **Motivation.**
   - **Maxwell's Equations.**
   - ***hp*-Finite Elements.**
   - **Literature Review.**
2. **Objectives.**
3. **Preliminary Work.**
4. **Integration of a Two Grid Solver with the Automatic *hp*-adaptivity for Maxwell's Equations.**
5. **Proposed Research.**

1. INTRODUCTION: MOTIVATION

Radar Cross Section (RCS) Analysis

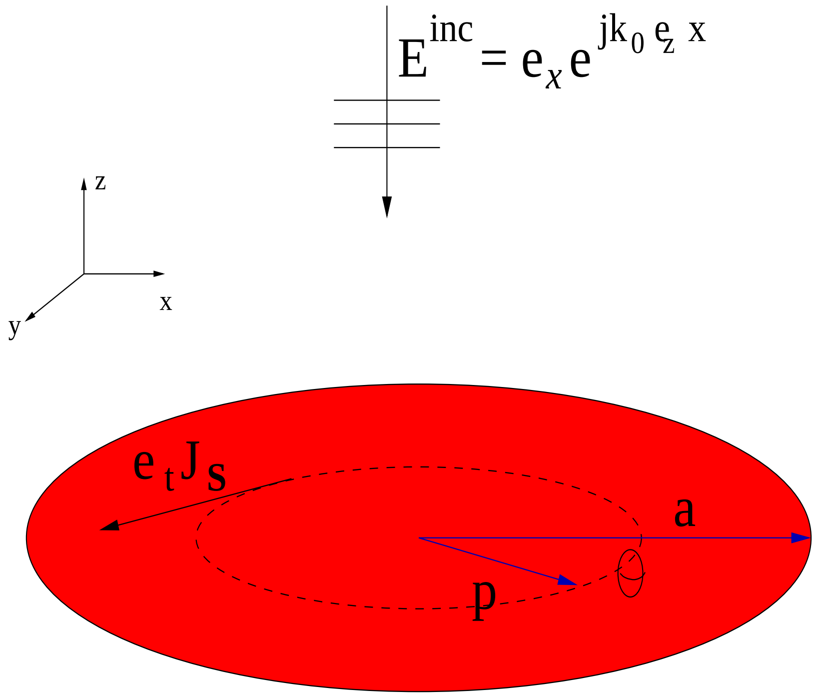


$$\textbf{RCS}=4\pi\frac{\text{Power scattered to receiver per unit solid angle}}{\text{Incident power density}} = \lim_{r\to\infty} 4\pi r^2 \frac{|E^s|}{|E^i|}.$$

Goal: Determine the RCS of a plane.

1. INTRODUCTION: MOTIVATION

Scattering on a perfect electric conductor (PEC) disk, ka=0.5



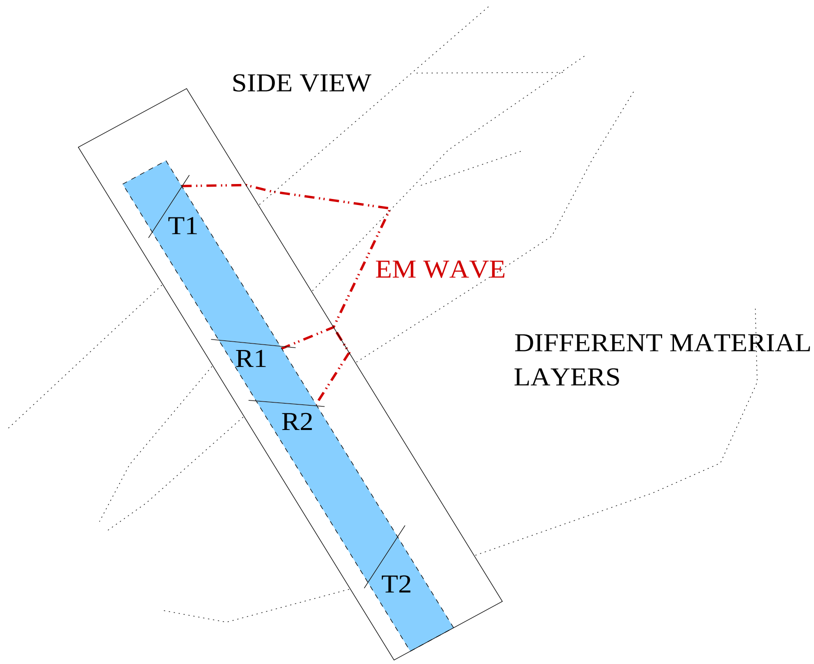
$e_t J_S = f(\theta)g(p)$ **where f is a regular function, and g has an edge singularity of the type** $g(p) = [1 - (p/a)^2]^{-1/2}$.

Goal: Determine the scattered electric field.

Ref: Zdunek, Rachowicz

1. INTRODUCTION: MOTIVATION

Modeling of a Logging While Drilling (LWD) electromagnetic measuring device



Goal: Determine EM field at the receiver antennas.

1. INTRODUCTION: MAXWELL'S EQUATIONS

Time Harmonic Maxwell's Equations:

$$\nabla \times \mathbf{E} = -j\mu\omega\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \sigma\mathbf{E} + \mathbf{J}^{imp}$$

Reduced Wave Equation:

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega\boldsymbol{J}^{imp} \,,$$

Boundary Conditions (BC):

- **Dirichlet BC at a PEC surface:**

$$\mathbf{n} \times \boldsymbol{E}^s = -\mathbf{n} \times \boldsymbol{E}^{inc} \qquad \mathbf{n} \times \boldsymbol{E} = 0$$

- **Neumann continuity BC at a material interface:**

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E}^s = -\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E}^{inc} \qquad \mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega\mathbf{J}_S^{imp}$$

- **Silver Müller radiation condition at ∞:**

$$\mathbf{e}_r \times (\nabla \times \boldsymbol{E}^s) - jk_0 \times \boldsymbol{E}^s = O(r^{-2})$$

1. INTRODUCTION: *hp*-FINITE ELEMENTS

Exponential convergence rates

for a number of regular and SINGULAR problems

for optimal hp-grids

in the asymptotic range (theoretical and numerical results), and

in the pre-asymptotic range (numerical results).

Smaller dispersion (pollution) error

as *p* increases.

More geometrical details captured

as *h* decreases.

1. INTRODUCTION: *hp*-FINITE ELEMENTS

1. **PHLEX**, an advanced hp-adaptive finite element kernel trademark of COMCO Corp. It also includes an automatic adaptive procedure in both h and p, but decision between h- and p-refinement for a given element has to be performed manually.
2. **PolyFEM**, a trademark of Computer Aided Design Software, Corralville, Iowa.
3. **Pro/MECHANICA**, a trademark of Parametric Technology Corp., Waltham, Masachussets.
4. **STRESSCHECK**, a trademark of ESRD Res. Inc., St. Louis, Missouri.
5. **STRIPE**, developed by Flygtekniska Försöksanstalten, Bromma, Sweden.
6. **PZ**, an hp-adaptive FE environment developed by Philippe Devloo, Unicamp, Brasil.
7. **Philipp Frauenfelder** and Ch. Schwab are developing an hp-FE code for general elliptic 3D problems, Zürich, Switzerland.
8. **3Dhp90**, under development by Dr. Demkowicz and his team, Austin, Texas.

1. INTRODUCTION: *hp*-FINITE ELEMENTS

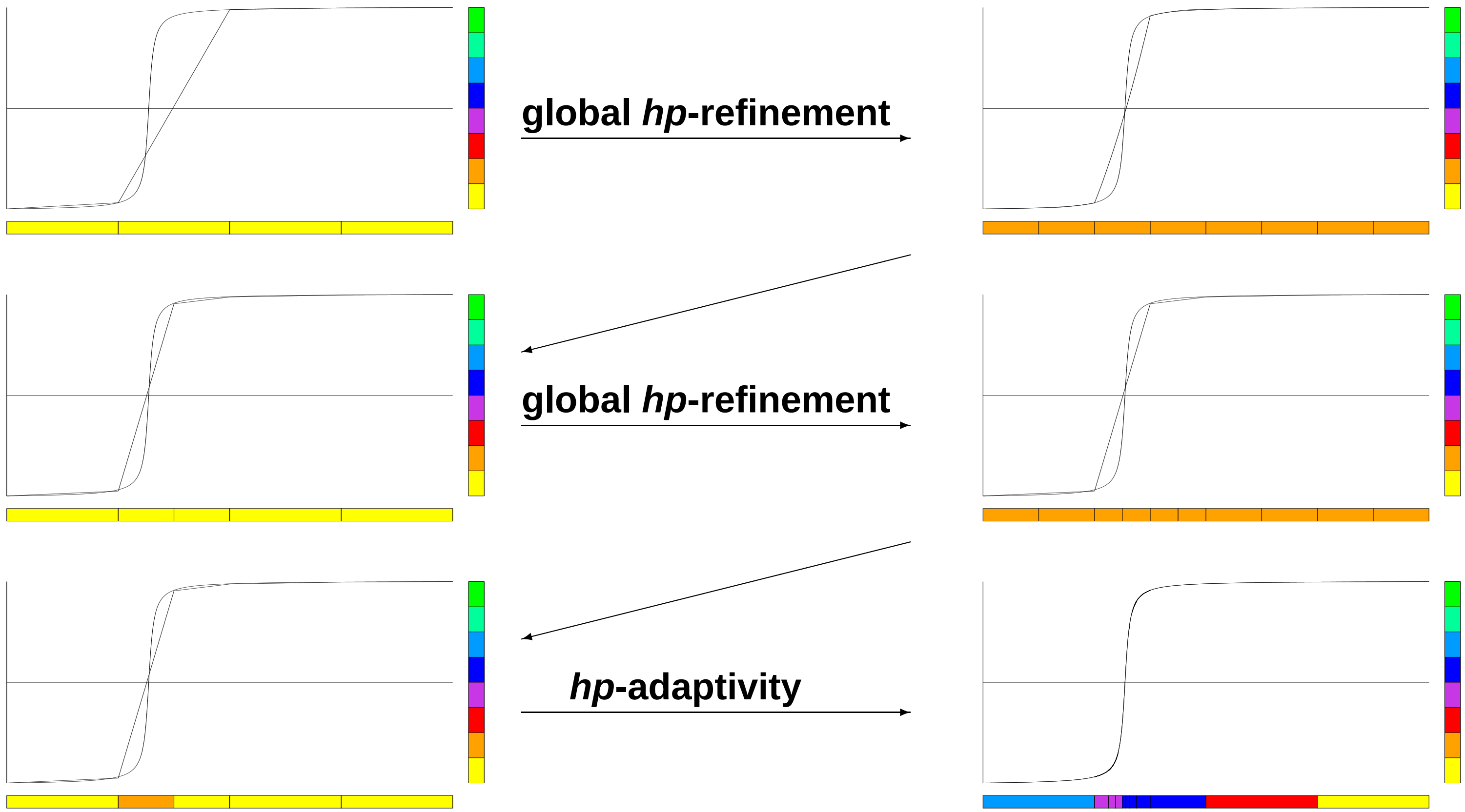
3Dhp90: main features

- Isoparametric hexahedras.
- Isotropic and anisotropic mesh refinements.
- Geometrical Modeling Package (GMP).
- New data structure in Fortran 90.
- Constrained information reconstructed (not stored).
- Two levels of logical operations:
  1. operations for nodes - problem independent.
  2. operations for nodal dof - problem dependent.
- Fully automatic hp-adaptive strategy.

—provides exponential convergence rates—

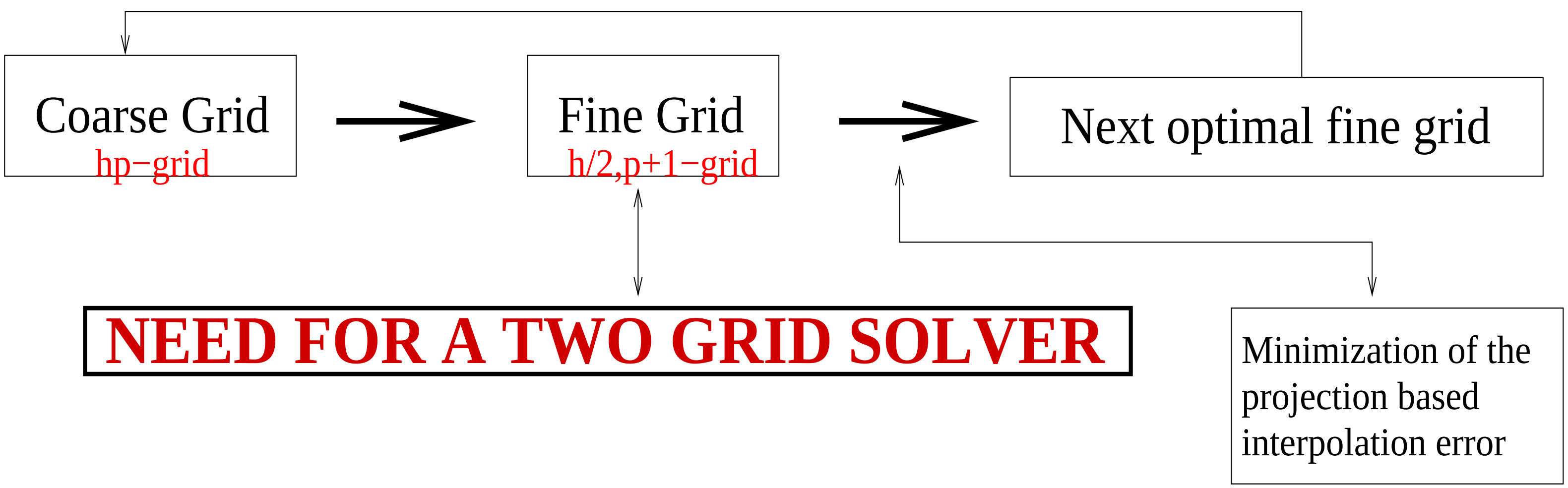
1. INTRODUCTION: *hp*-FINITE ELEMENTS

Fully automatic *hp*-adaptive strategy



1. INTRODUCTION: *hp*-FINITE ELEMENTS

Automatic *hp*-adaptivity delivers exponential convergence and enables solution of challenging EM problems



1. INTRODUCTION: Literature Review

- ***hp*-Finite Elements.**
- ***hp*-edge Finite Elements.**
- **Multigrid solver for self-adjoint and positive definite (spd) problems.**
- **Multigrid solver for electromagnetics.**

1. INTRODUCTION: Literature Review

hp-Finite Elements

- **Exponential convergence.**
  - **1D** - Gui and Babuska, 1986 -.
  - **2D and 3D** - Babuska and Guo, 1986, 1996 -.
- **Nearly singular problems (robustness)** - Babuska, Suri, 1992 -.
- **Data structures** - Demkowicz, Oden, Rachowicz, Hardy, 1989 - , - Demkowicz, Pardo, Rachowicz, 2002 - .
- ***A posteriori* error estimates** - Gui and Babuska, 1986 - , - Oden, Rachowicz, Demkowicz, Westermann, 1989 - , - Ainsworth and Oden, 2000 - , - Babuska and Strouboulis, 2001 - .
- ***hp-* adaptivity** - Rachowicz, Oden, Demkowicz, 1989 -, - Demkowicz, Rachowicz, Devloo, 2001 - .
- **Goal oriented *hp*-adaptivity** - Solin, Demkowicz, 2002 - , - Zdunek, Rachowicz, 2002 - .

1. INTRODUCTION: Literature Review

hp-edge Finite Elements

- $H(\text{curl})-$ **conforming families of FE** - Nedelec, 1980, 1986 -.
- hp**-extensions of Nedelec's elements** - Monk, 1994 -, - Demkowicz, Vardapetyan, 1998 -, - Demkowicz, 2000 -.
- **Pollution error estimates** - Ihlenburg, Babuska, 1997 -.
- **De Rham diagram for** hp**-edge FE** - Demkowicz, Monk, Vardapetyan, Rachowicz, 1999 -, - Demkowicz, Babuska, 2001 -, - Demkowicz, Babuska, Schoberl, Monk, 2003? -.
- **Infinite elements** - Cecot, Demkowicz, Rachowicz, 2000 -.
- **Implementation details**:
  - **2D** - Rachowicz, Demkowicz, 1998 -.
  - **3D** - Rachowicz, Demkowicz, 2002 - , - Zdunek, Rachowicz, 2002 -.

1. INTRODUCTION: Literature Review

Multigrid solver for spd problems

- **Origin of the method** - Fedorenko, 1961, 1964 -, - Brandt, 1973, 1977 -.
- **Symmetric and positive definite problems**
  - **h-FE** - Braess, Hackbusch, 1983 -, - Hackbusch, 1985 - , - Bramble, Pasciak and Xu, 1991 -, - Bramble, 1995- , - Smith, Bjorstad and Gropp, 1996 -.
  - **p-FE** - Babuska, Craig, Mandel, Pitkäranta, 1991 -, -Pavarino, 1994-, - Mandel, 1994, 1996 -.
  - **hp-FE** - Ainsworth, 1996 -.
  - **Mixed-FE** - Glowinski and Wheeler, 1990 -, - Wheeler and Yotov, 1998 -.
  - **Parallel implementations** - Cowsar, Wheeler, 1990 -, - Oden, Patra and Feng, 1993 -.
  - **Conjugate Gradient** - Shewchuck, 1994 -.

1. INTRODUCTION: Literature Review

Multigrid solver for electromagnetics

- **Multigrid for indefinite problems** - Cai, Widlund, 1992 -.
- **Multigrid for electromagnetics**
  - - Hiptmair, 1998 -
  - - Beck, Deuflhard, Hiptmair, 1999 -.
  - - Rachowicz, Demkowicz, Bajer, Walsh, 1999 -.
  - - Arnold, Falk, Winther, 2000 -.
  - - Haase, Kuhn, Langer, 2001 -.
  - - Gopalakrishnan, Pasciak, Demkowicz, 2002 -.

2. OBJECTIVES

- **Design, implement, and study (numerically and theoretically) a two grid solver for:**
  1. **2D and 3D real and complex valued elliptic problems.**
  2. **2D and 3D electromagnetic problems.**
- **Integrate the two grid solver with the hp-adaptive strategy.**
- **Solve the presented problems in RCS analysis and modeling of LWD electromagnetic measuring devices.**

3. PRELIMINARY WORK: Outline

Integration of *hp*-adaptivity with a two grid solver for spd problems

- **Formulation of the method.**
- **Implementation.**
- **Numerical results. Possibility of guiding hp-refinements with a partially converged solution.**
- **Conclusions.**

3.PRELIMINARY WORK:Formulation of the method

We seek x such that $Ax = b$. Consider the following iterative scheme:

$$r^{(n+1)} = [I - \alpha^{(n)} AS] r^{(n)}$$
$$x^{(n+1)} = [I - \alpha^{(n)} S] r^{(n)}$$

where S is a matrix, and $\alpha^{(n)}$ is a relaxation parameter. $\alpha^{(n)}$ *optimal* if:

$$\alpha^{(n)} = \arg\ \min \parallel x^{(n+1)} - x \parallel_A = \frac{(A^{-1} r^{(n)}, S r^{(n)})_A}{(S r^{(n)}, S r^{(n)})_A}$$

Then, we define our two grid solver as:

1 Iteration with $S = S_F = \sum A_i^{-1} \quad +$

1 Iteration with $S = S_C = P A^{-1} R$

3.PRELIMINARY WORK:Formulation of the method

Error reduction and stopping criteria

Let $e^{(n)} = x^{(n)} - x$ **the error at step** n, $\tilde{e}^{(n)} = [I - S_C A]e^{(n)} = [I - P_C]e^{(n)}$**. Then:**

$$\frac{\| e^{(n+1)} \|_A^2}{\| e^{(n)} \|_A^2} = 1 - \frac{| (\tilde{e}^{(n)}, S_F A \tilde{e}^{(n)})_A |^2}{\| \tilde{e}^{(n)} \|_A^2 \| S_F A \tilde{e}^{(n)} \|_A^2} = 1 - \frac{| (\tilde{e}^{(n)}, (P_C + S_F A)\tilde{e}^{(n)})_A |^2}{\| \tilde{e}^{(n)} \|_A^2 \| S_F A \tilde{e}^{(n)} \|_A^2}$$

Then:

$$\frac{\| e^{(n+1)} \|_A^2}{\| e^{(n)} \|_A^2} \leq \sup_e [1 - \frac{| (e, (P_C + S_F A)e)_A |^2}{\| e \|_A^2 \| S_F A e \|_A^2}] \leq C < 1$$

(Error Reduction)

For our stopping criteria, we want: Iterative Solver Error $\approx$ **Discretization Error. That is:**

$$\frac{\| e^{(n+1)} \|_A}{\| e^{(0)} \|_A} \leq 0.01$$

(Stopping Criteria)

3. PRELIMINARY WORK: Implementation

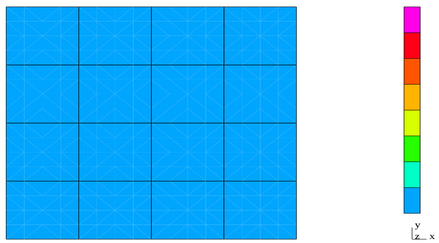
Assembling

- **Stiffness Matrix.**
- **Block Jacobi Smoother.**
- **Prolongation Operator.**
- **Restriction Operator.**

3. PRELIMINARY WORK: Implementation

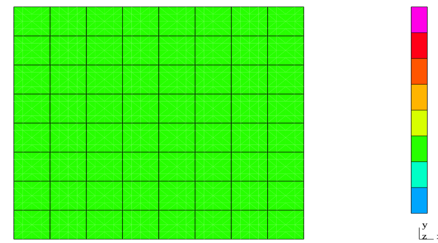
Selection of patches (for block Jacobi smoother)

Coarse Grid

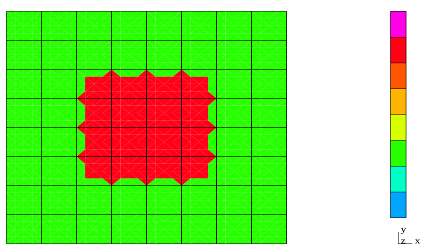


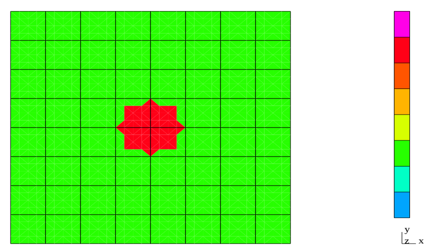
global *hp*-refinement →

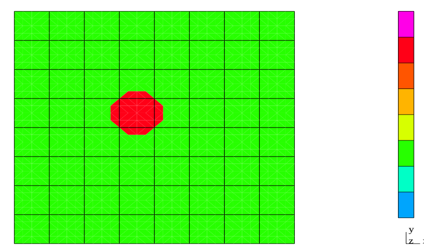
Fine Grid



Three examples of patches (blocks) for the Block Jacobi smoother:







Example 1: span of basis functions with support contained in the support of a coarse grid vertex node basis function.

Example 2: span of basis functions with support contained in the support of a fine grid vertex node basis function.

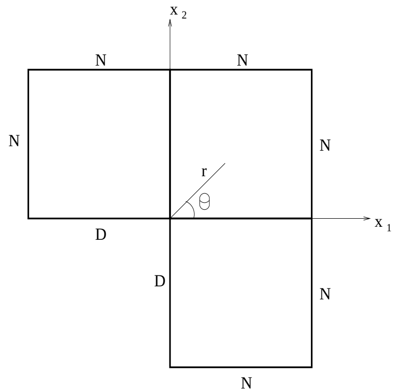
Example 3: span of basis functions corresponding to an element stiffness matrix.

4. PRELIMINARY WORK: Numerical Results

- Presentation of four examples.
- Importance of using automatic hp-adaptivity.
- Different sets of shape functions highly affect conditioning of stiffness matrix.
- Performance of different smoothers.
- Importance of optimal relaxation parameter.
- Error estimation.
- Possibility of guiding hp-refinements with a partially converged solution.

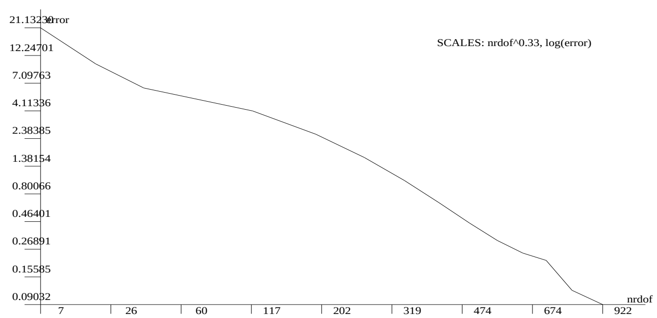
4. PRELIMINARY WORK: Numerical Results

L-shape domain example

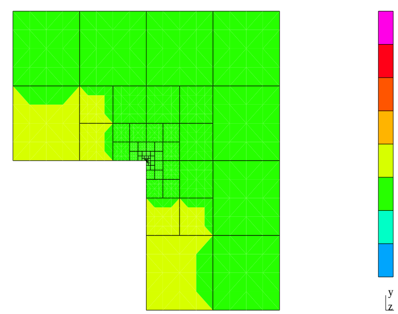


Equation: $-\Delta u = 0$
Boundary Conditions: N-Neumann, D-Dirichlet

Solution:
$u = r^{2/3} sin(2\theta/3 + \pi/3)$



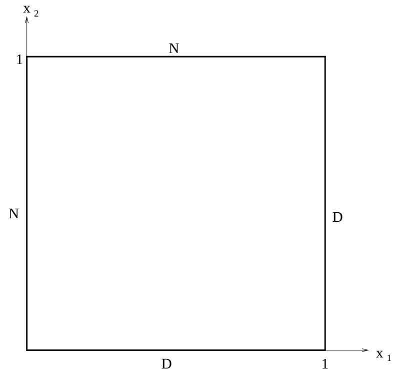
Convergence history
(tolerance error = 0.1 %)



Final hp**-grid**

4. PRELIMINARY WORK: Numerical Results

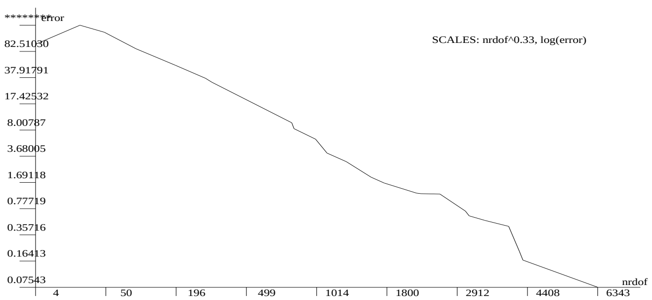
Shock like solution example



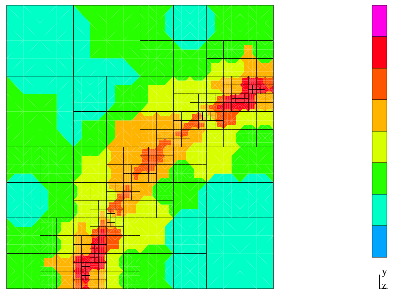
Equation: $-\Delta u = f$
Boundary Conditions: N - Neumann, D - Dirichlet

Solution:

$$u = arctan[60(r-1)]$$
$$r = \sqrt{(x-1.25)^2 + (y+0.25)^2}$$



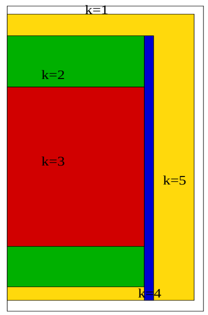
Convergence history
(tolerance error = 0.1 %)



Final hp-grid

4. PRELIMINARY WORK: Numerical Results

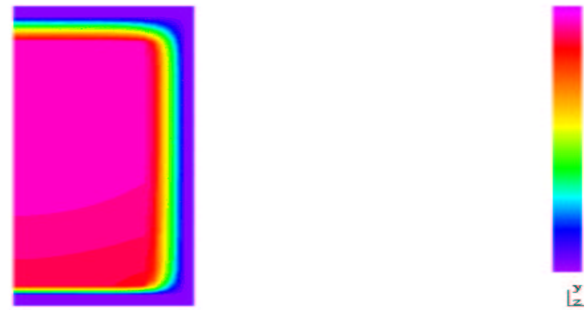
Isotropic heat conduction example



Equation: $\nabla(\mathrm{K}\nabla u) = f^{(k)}$

$K = K^{(k)} = K_x^{(k)}$

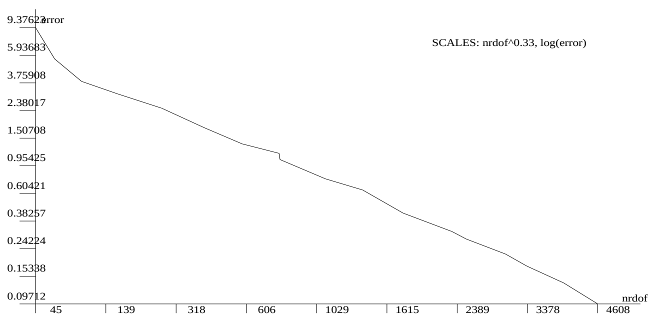
$K_x^{(k)} = (25,\ 7,\ 5,\ 0.2,\ 0.05)$



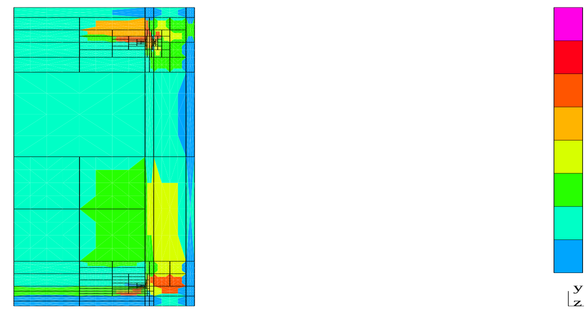
Solution: unknown

Boundary Conditions:

$K\nabla u \cdot n = g^{(i)} - \alpha^{(i)} u$



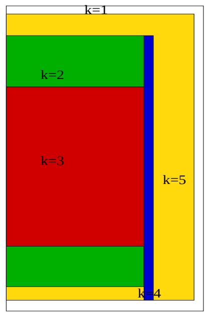
Convergence history
(tolerance error = 0.1 %)



Final hp**-grid**

4. PRELIMINARY WORK: Numerical Results

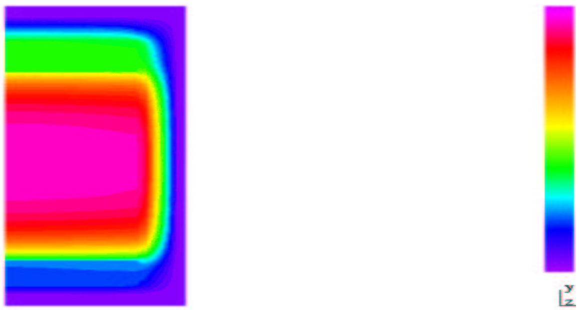
Orthotropic heat conduction example



Equation: $\nabla(\mathrm{K}\nabla u) = f^{(k)}$

$$\mathrm{K} = \mathrm{K}^{(k)} = \begin{bmatrix} K_x^{(k)} & 0 \\ 0 & K_y^{(k)} \end{bmatrix}$$

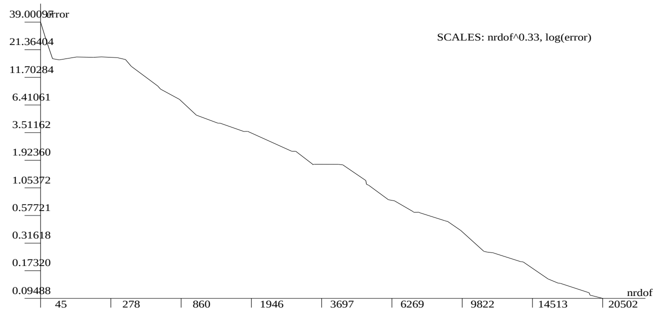
$$K_x^{(k)} = (25,\ 7,\ 5,\ 0.2,\ 0.05)$$
$$K_y^{(k)} = (25,\ 0.8,\ 0.0001,\ 0.2,\ 0.05)$$



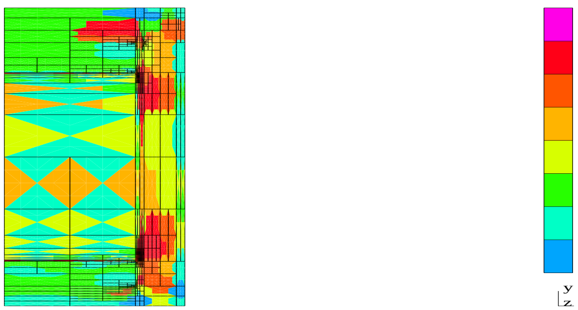
Solution: unknown

Boundary Conditions:

$$K^{(i)}\nabla u \cdot n = g^{(i)} - \alpha^{(i)} u$$



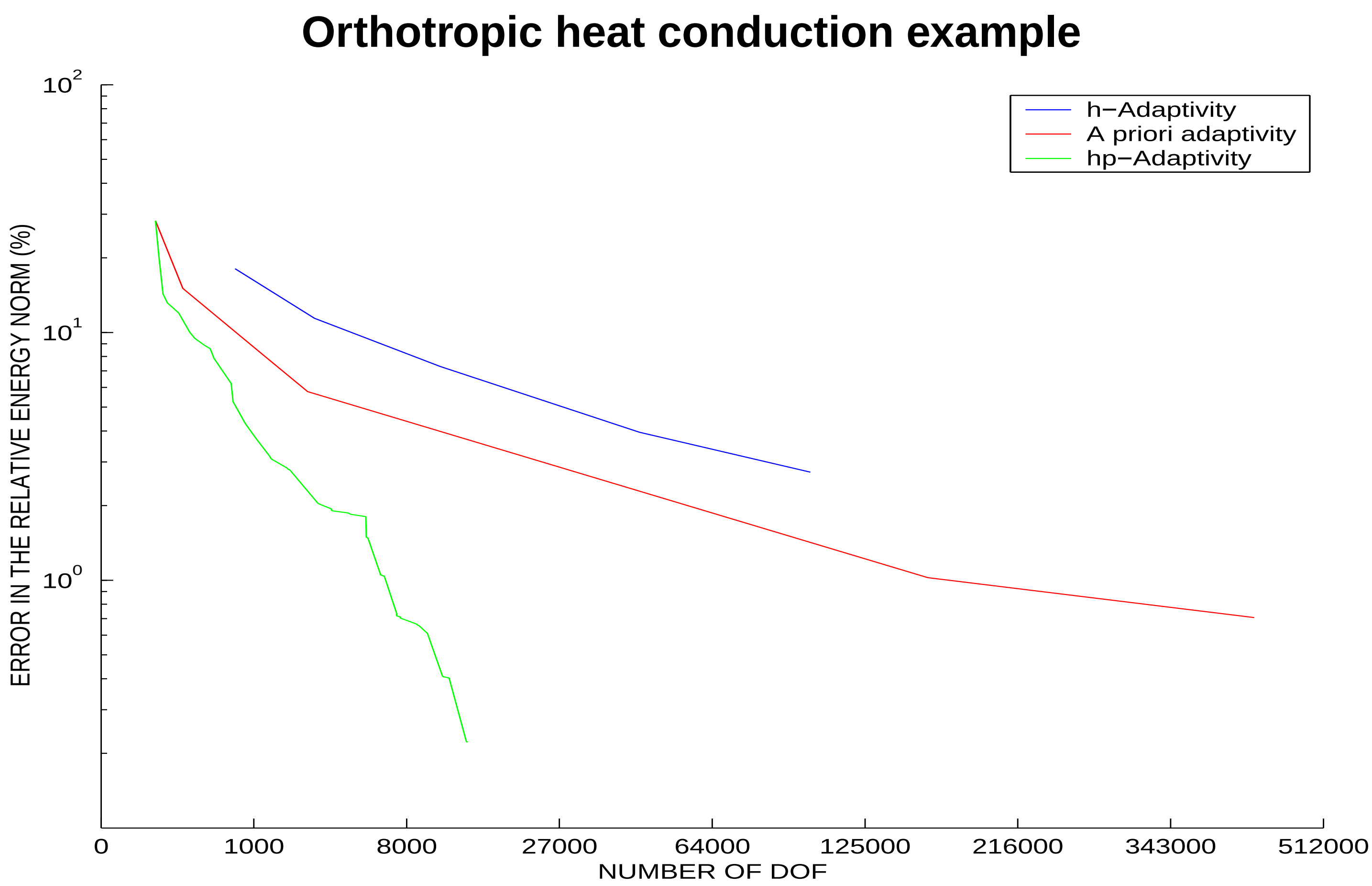
Convergence history
(tolerance error = 0.1 %)



Final hp grid

4. PRELIMINARY WORK: Numerical Results

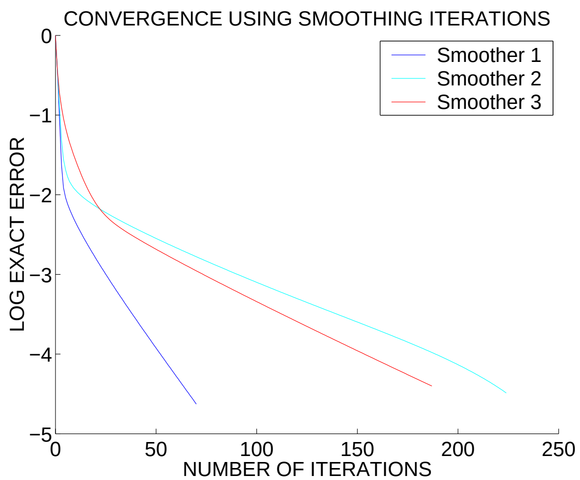
Convergence comparison

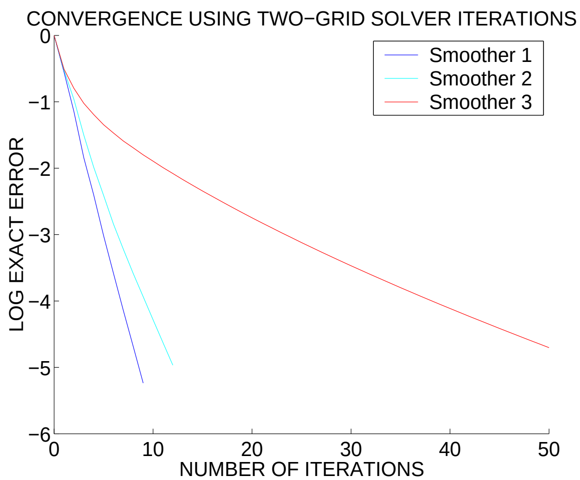


4. PRELIMINARY WORK: Numerical Results

Performance of different smoothers

L-shape domain example (11837 dof)





Smoother 1: requires 16 times more memory than stiffness matrix.

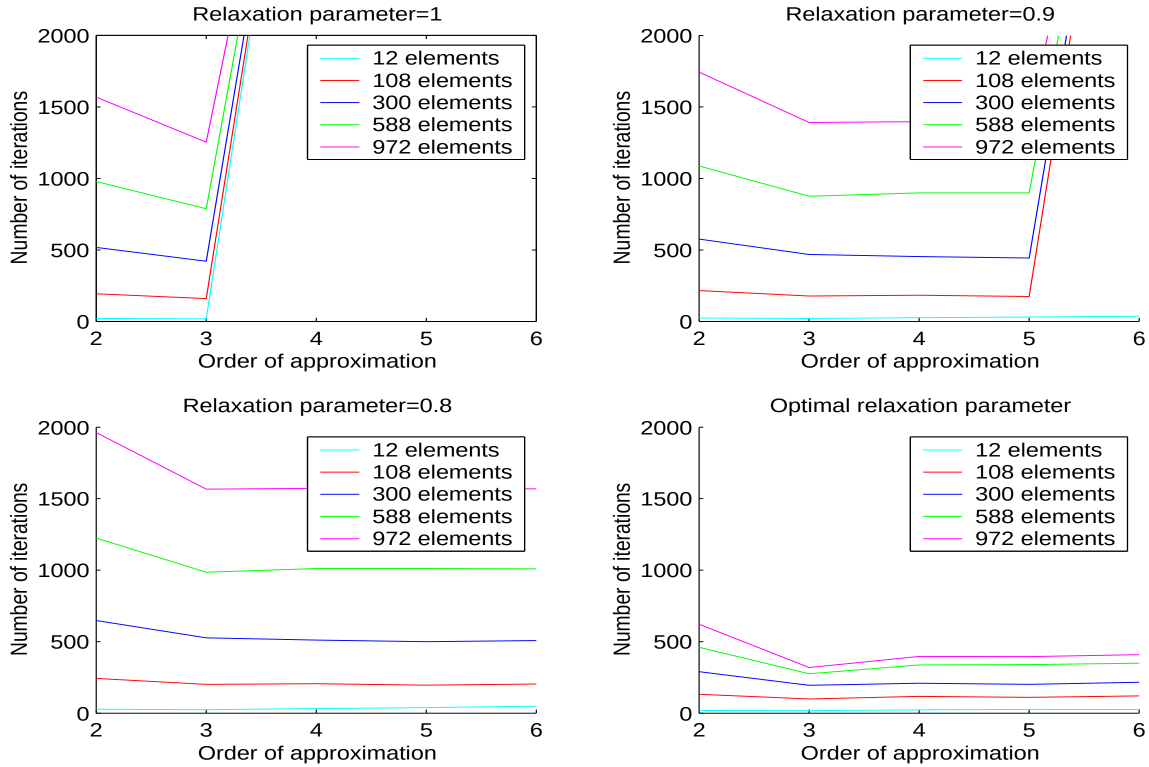
Smoother 2: requires 4 times more memory than stiffness matrix.

Smoother 3: requires as much memory as the stiffness matrix.

4. PRELIMINARY WORK: Numerical Results

Relaxation parameter

L-shape domain example (only smoothing operations)



Convergence or not, depends almost exclusively upon p.
***Convergence rate* of the method (provided that the method converges) depends almost exclusively upon h.**
The optimal relaxation guarantees faster convergence than any fixed relaxation parameter.

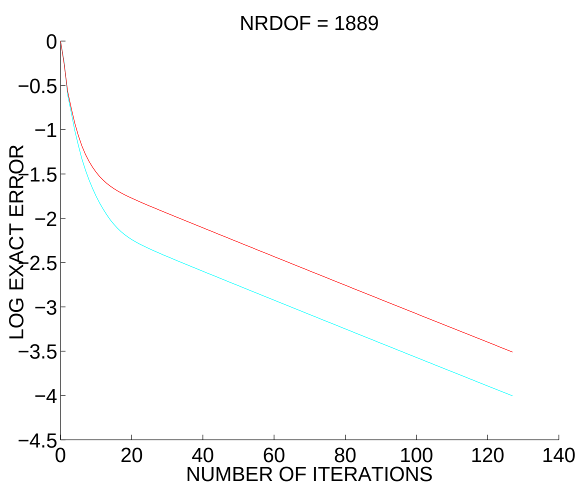
4. PRELIMINARY WORK: Numerical Results

Error Estimation

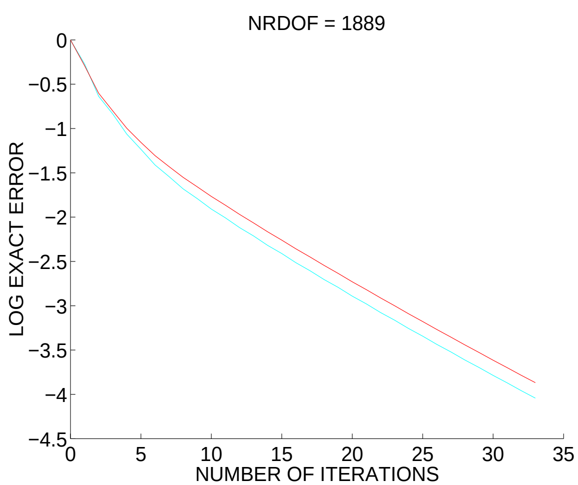
$$\frac{\| e^{(n)} \|_A}{\| e^{(0)} \|_A} = \frac{\| A^{-1} r^{(n)} \|_A}{\| A^{-1} r^{(0)} \|_A} \approx \frac{\| \alpha^{(n)} S r^{(n)} e^{(n)} \|_A}{\| \alpha^{(0)} S r^{(0)} e^{(0)} \|_A}$$ **(Error Estimate)**

L-shape domain (1889 dof)

Smoothing iterations only



Two grid solver iterations

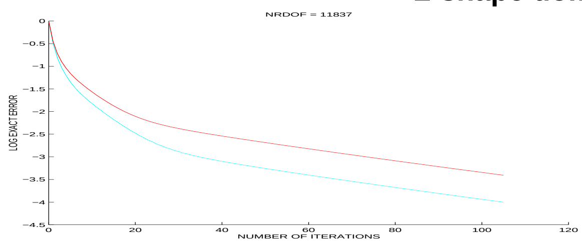


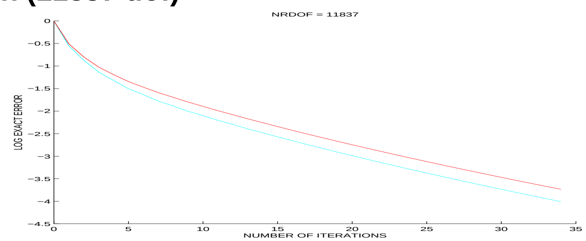
Comparing the exact error vs an estimate to the error.

4. PRELIMINARY WORK: Numerical Results

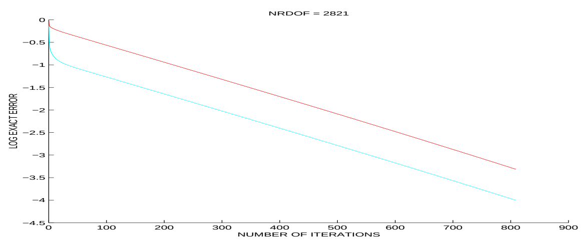
Error Estimation

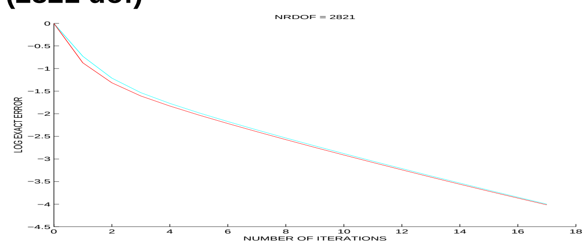
L-shape domain (11837 dof)



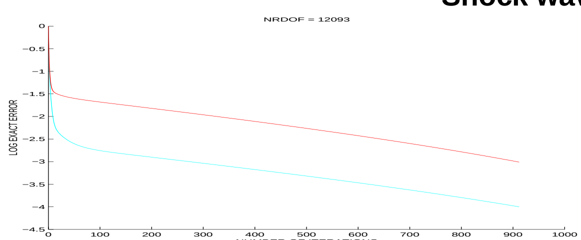


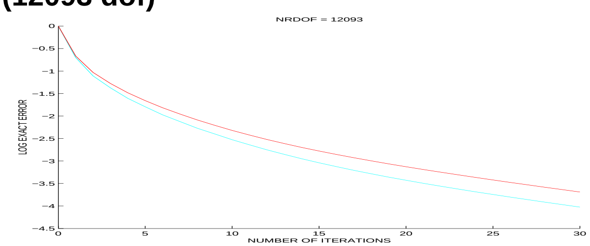
Shock wave (2821 dof)





Shock wave (12093 dof)



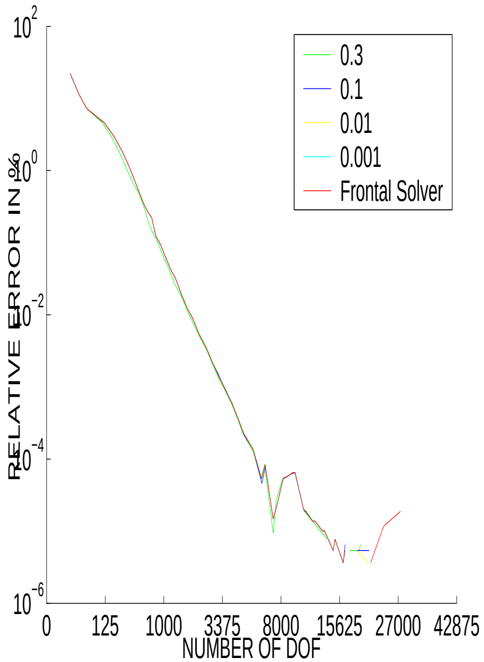


Comparing the exact error vs an estimate to the error.

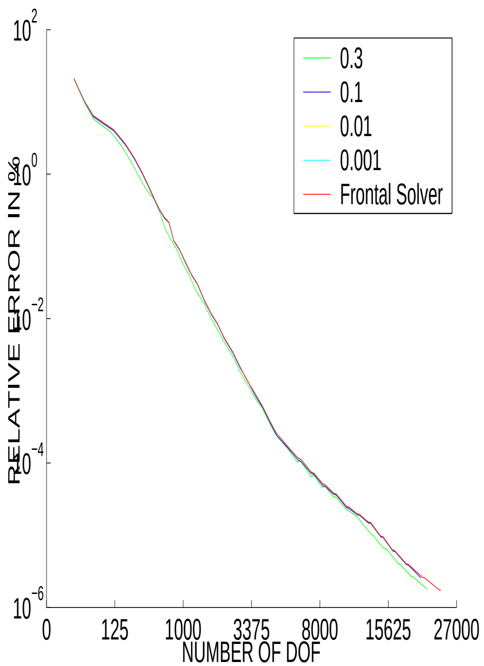
4. PRELIMINARY WORK: Numerical Results

Guiding automatic hp-refinements

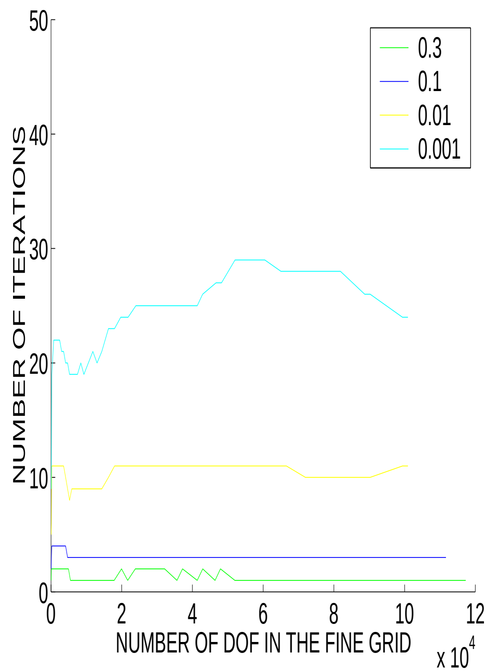
L-shape domain problem. Guiding hp-refinements with a partially converged solution.



Energy error estimate



Discretization error estimate

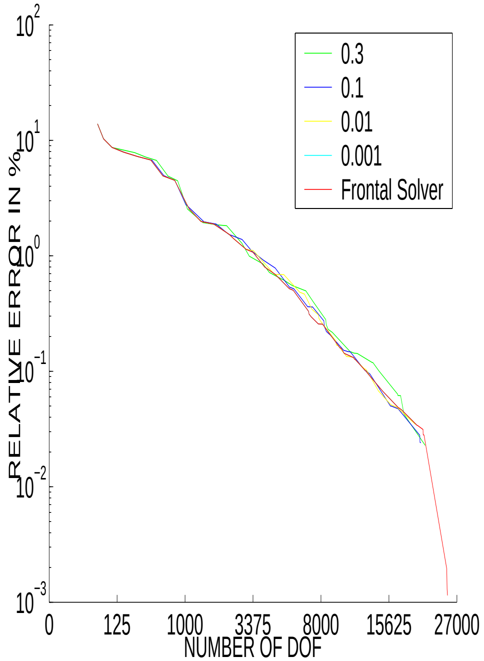


Number of iterations

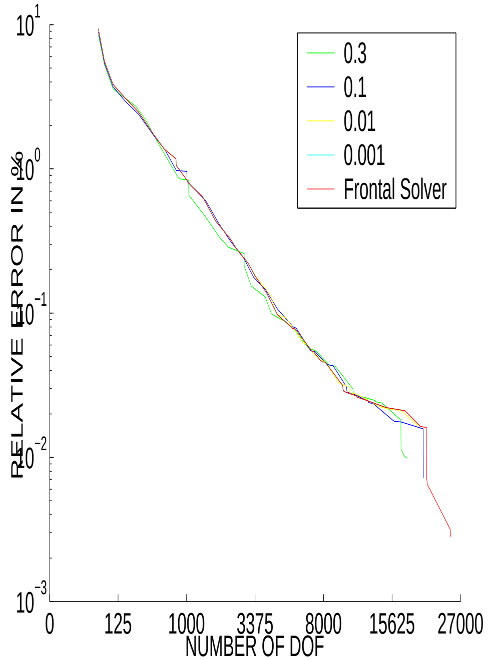
4. PRELIMINARY WORK: Numerical Results

Guiding automatic hp-refinements

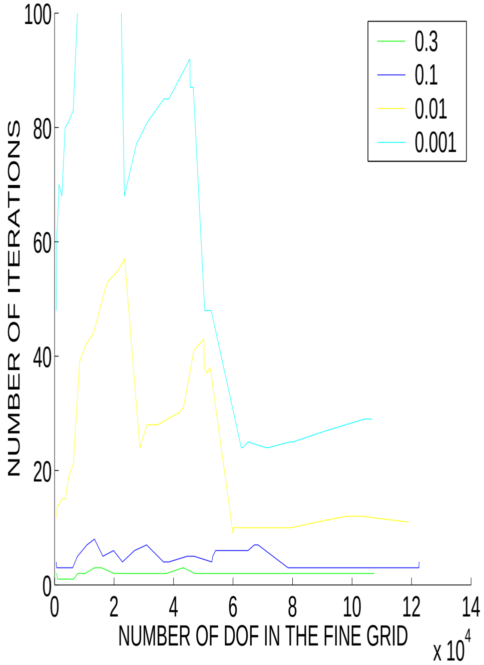
Isotropic heat conduction. Guiding hp-refinements with a partially converged solution.



Energy error estimate



Discretization error estimate

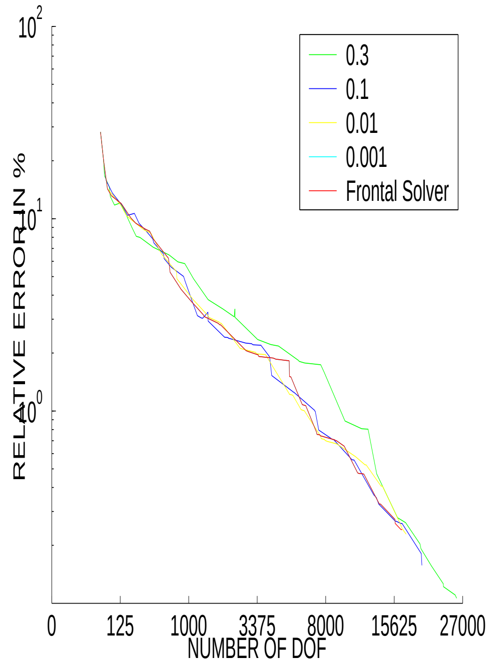


Number of iterations

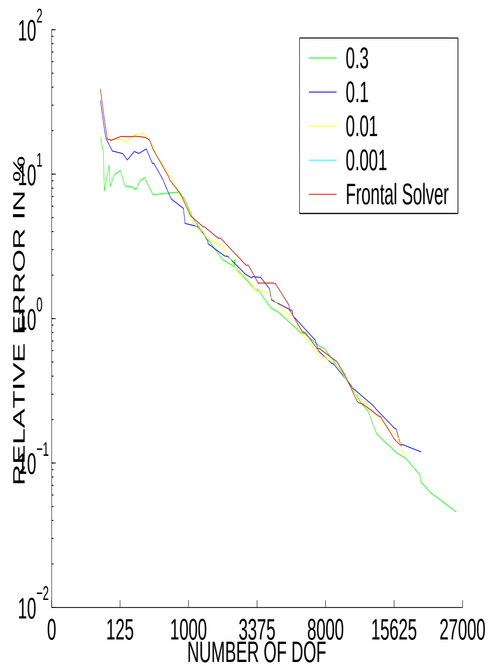
4. PRELIMINARY WORK: Numerical Results

Guiding automatic hp-refinements

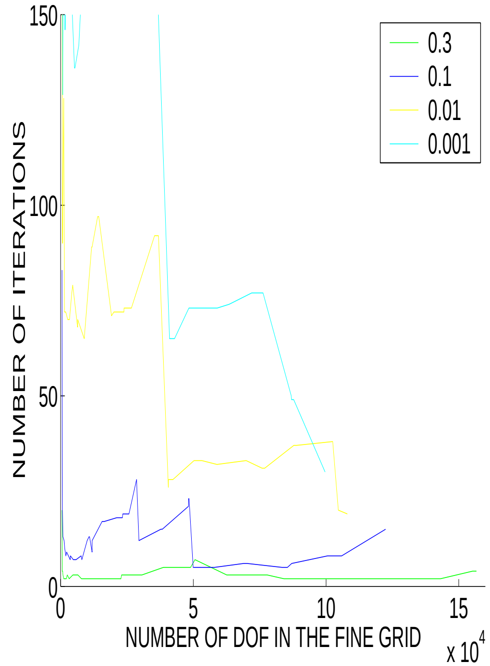
Orthotropic heat conduction. Guiding hp-refinements with a partially converged solution.



Energy error estimate



Discretization error estimate



Number of iterations

4. PRELIMINARY WORK: Conclusions

- **Assembling reduces logical complexity.**
- **Jacobi patches defined by d.o.f. associated to elements.**
- **Optimal relaxation parameter needed for hp-meshes.**
- **Two grid solver outperforms smoothing iterations only.**
- **Guiding hp-refinements is possible with only partially converged solutions.**
- **Number of iterations needed for guiding hp-refinements at a level below 5 per mesh.**

5. MAXWELL'S EQUATIONS for hp-FEM

Variational formulation

The reduced wave equation in Ω,

$$\nabla \times \left(\frac{1}{\mu}\nabla \times E\right) - (\omega^2\epsilon - j\omega\sigma)E = -j\omega J^{imp} ,$$

A variational formulation

$$\begin{cases} \text{Find } \mathrm{E} \in H_D(\mathrm{curl};\Omega) \text{ such that} \\ \int_\Omega \frac{1}{\mu}(\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{F})dx - \int_\Omega (\omega^2\epsilon - j\omega\sigma)E \cdot \bar{\mathrm{F}}dx = \\ \qquad -j\omega\left\{\int_\Omega J^{imp} \cdot \bar{F}dx + \int_{\Gamma_2} J_S^{imp} \cdot \bar{F}dS\right\} \quad \text{for all } F \in H_D(\mathrm{curl};\Omega) . \end{cases}$$

A regularized variational formulation (using *Lagrange multipliers*):

$$\begin{cases} \text{Find } E \in H_D(\mathrm{curl};\Omega), p \in H_D^1(\Omega) \text{ such that} \\ \int_\Omega \frac{1}{\mu}(\nabla \times E)(\nabla \times \bar{F})dx - \int_\Omega (\omega^2\epsilon - j\omega\sigma)E \cdot \bar{F}dx - \int_\Omega (\omega^2\epsilon - j\omega\sigma)\nabla p \cdot \bar{F}dx = \\ \qquad -j\omega\left\{\int_\Omega J^{imp} \cdot \bar{F}dx + \int_{\Gamma_2} J_S^{imp} \cdot \bar{F}dS\right\} \quad \forall F \in H_D(\mathrm{curl};\Omega) \\ -\int_\Omega (\omega^2\epsilon - j\omega\sigma)E \cdot \nabla\bar{q}\, dx = -j\omega\left\{\int_\Omega J^{imp} \cdot \nabla\bar{q}\, dx + \int_{\Gamma_2} J_S^{imp} \cdot \nabla\bar{q}dS\right\} \quad \forall q \in H_D^1(\Omega) . \end{cases}$$

5. MAXWELL'S EQUATIONS and hp-FEM

De Rham diagram

De Rham diagram is critical to the theory of FE discretizations of Maxwell's equations.

$$
\begin{array}{ccccccccccc}
\mathbb{R} & \longrightarrow & W & \xrightarrow{\nabla} & Q & \xrightarrow{\nabla\times} & V & \xrightarrow{\nabla\circ} & L^2 & \longrightarrow & 0 \\
\downarrow id & & \downarrow \Pi & & \downarrow \Pi^{\mathrm{curl}} & & \downarrow \Pi^{\mathrm{div}} & & \downarrow P & & \\
\mathbb{R} & \longrightarrow & W^{\mathrm{p}} & \xrightarrow{\nabla} & \mathrm{Q}^{\mathrm{p}} & \xrightarrow{\nabla\times} & \mathrm{V}^{\mathrm{p}} & \xrightarrow{\nabla\circ} & W^{\mathrm{p}-1} & \longrightarrow & 0\,.
\end{array}
$$

This diagram relates two exact sequences of spaces, on both continuous and discrete levels, and corresponding interpolation operators.

5. MAXWELL'S EQUATIONS and hp-FEM

A two grid solver for discretization of Maxwell's equations using hp-FE

Helmholtz decomposition:

$$H_D(\mathrm{curl};\Omega) = (Ker(\mathrm{curl})) \oplus (Ker(\mathrm{curl}))^{\perp}$$

We define the following subspaces (T =grid, K =element, v =vertex, e =edge):

$\Omega^v_{k,i} = \mathrm{int}(\bigcup\{\bar{K} \in T_k : v_{k,i} \in \partial K\})$; $\Omega^e_{k,i} = \mathrm{int}(\bigcup\{\bar{K} \in T_k : e_{k,i} \in \partial K\})$ — *Domain decomposition*

$M^v_{k,i} = \{u \in M_k : \mathrm{supp}(u) \subset \Omega^v_{k,i}\}$; $M^e_{k,i} = \{u \in M_k : \mathrm{supp(u)} \subset \Omega^e_{k,i}\}$ — *Nedelec's elements decomposition*

$W^v_{k,i} = \{u \in W_k : \mathrm{supp}(u) \subset \Omega^v_{k,i}\}$; $W^e_{k,i} = \{u \in W_k : \mathrm{supp}(u) \subset \Omega^e_{k,i}\} = \emptyset$ — *Polynomial spaces decomposition*

Hiptmair proposed the following decomposition of M_k:

$$M_k = \sum_e M^e_{k,i} + \sum_v \nabla W^v_{k,i}$$

Arnold *et. al* proposed the following decomposition of M_k:

$$M_k = \sum_v M^v_{k,i}$$

6. PROPOSED RESEARCH

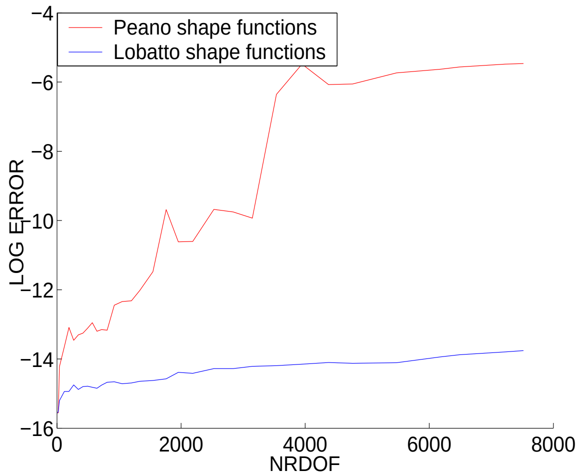
- Design and implement a 3D version of the two grid solver for elliptic problems.
- Study numerically the 3D version of the two grid solver.
- Design, study and implement a two grid solver for 2D Maxwell's equations.
- Design, study and implement a two grid solver for 3D Maxwell's equations.
- Study and design an error estimator for a two grid solver for Maxwell's equations.
- Study performance of different smoothers (in context of the two grid solver) for Maxwell's equations.
- Design, study, and implement a flexible CG/GMRES method that is suitable to accelerate the two grid solver for Maxwell's equations.
- Develop a convergence theory for all algorithms mentioned above.
- Apply the *hp*-adaptive strategy (possibly goal-oriented) combined with the two grid solver in order to solve a number of problems related to radar cross section (RCS) analysis and modeling of LWD electromagnetic measuring devices.

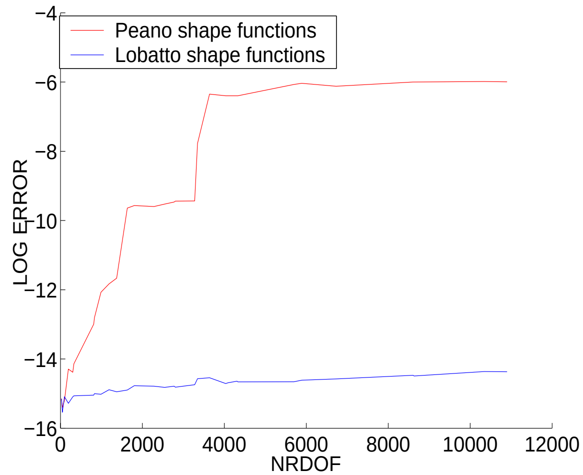
6. PROPOSED RESEARCH

- **Area A:** Design and study a two grid solver for 2D and 3D spd problems and Maxwell's equations, an adequate error estimator and smoother, and develop a convergence theory.
- **Area B:** Design and implement a two grid solver for 2D and 3D spd problems and Maxwell's equations, an adequate error estimator and three different smoothers. Study numerically these algorithms, including convergence properties, importance of shape functions and relaxation parameter, acceleration produced by CG/GMRES algorithm, accuracy of the error estimate, and possibility of guiding hp-refinements with partially converged solutions.
- **Area C:** Create a two grid solver integrated with hp-adaptivity for electromagnetic applications. Apply the hp-adaptive strategy (possibly goal-oriented) combined with the two grid solver in order to solve a number of problems related to radar cross section (RCS) analysis and modeling of LWD electromagnetic measuring devices.

4. PRELIMINARY WORK: Numerical Results

Sensitivity of the solution with respect to the selection of shape functions





Difference between the frontal solver and superLU solutions, measured in the relative (with respect to the energy norm of the solution) energy norm for the L-shape (left) and shock (right) problems

4. PRELIMINARY WORK: Numerical Results

Two grid solver vs smoother

Example	Nr of dof	1 - 1	3 - 1	Only Smoothing
L-shape	1889	13 (24)	14 (25)	34 (96)
L-shape	11837	12 (24)	13 (24)	18 (74)
Shock	2821	5 (11)	6 (11)	478 (732)
Shock	12093	8 (19)	9 (20)	326 (908)
Shock	34389	12 (28)	13 (30)	18 (257)

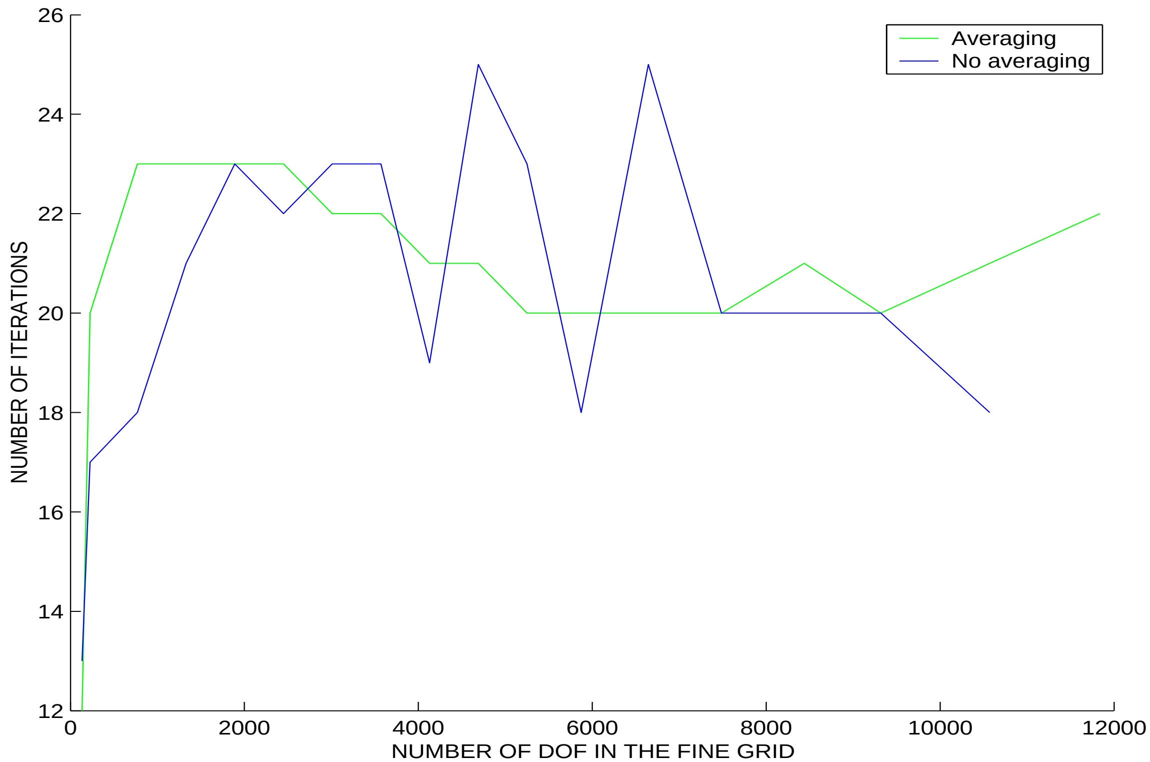
Number of iterations needed for relative EXACT ERROR $\leq$ 0.01 (0.001).

Example	Nr of dof	1 - 1	3 - 1	Only Smoothing
L-shape	1889	11 (22)	12 (23)	14 (65)
L-shape	11837	9 (21)	10 (22)	13 (35)
Shock	2821	6 (11)	6 (11)	295 (556)
Shock	12093	7 (15)	7 (17)	9 (274)
Shock	34389	9 (23)	10 (24)	12 (33)

Number of iterations needed for relative ERROR ESTIMATE $\leq$ 0.01 (0.001).

4. PRELIMINARY WORK: Numerical Results

Importance (or not) of averaging operator



L-shape domain example