BCAM Presentation

Inversion of Resistivity and Multiphysics Measurements.

Part I: Main Idea and Library Design.

D. Pardo, C. Torres-Verdín, M. J. Nam, V. M. Calo

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basque center for applied mathematics

Basque Center for Applied Mathematics (BCAM)

Promoting Technological Advances Through Mathematics

OVERVIEW

1. Motivation and Objectives: Joint Multi-Physics Inversion

2. Main Idea for the Inversion

3. Mathematical Formulation

4. Method

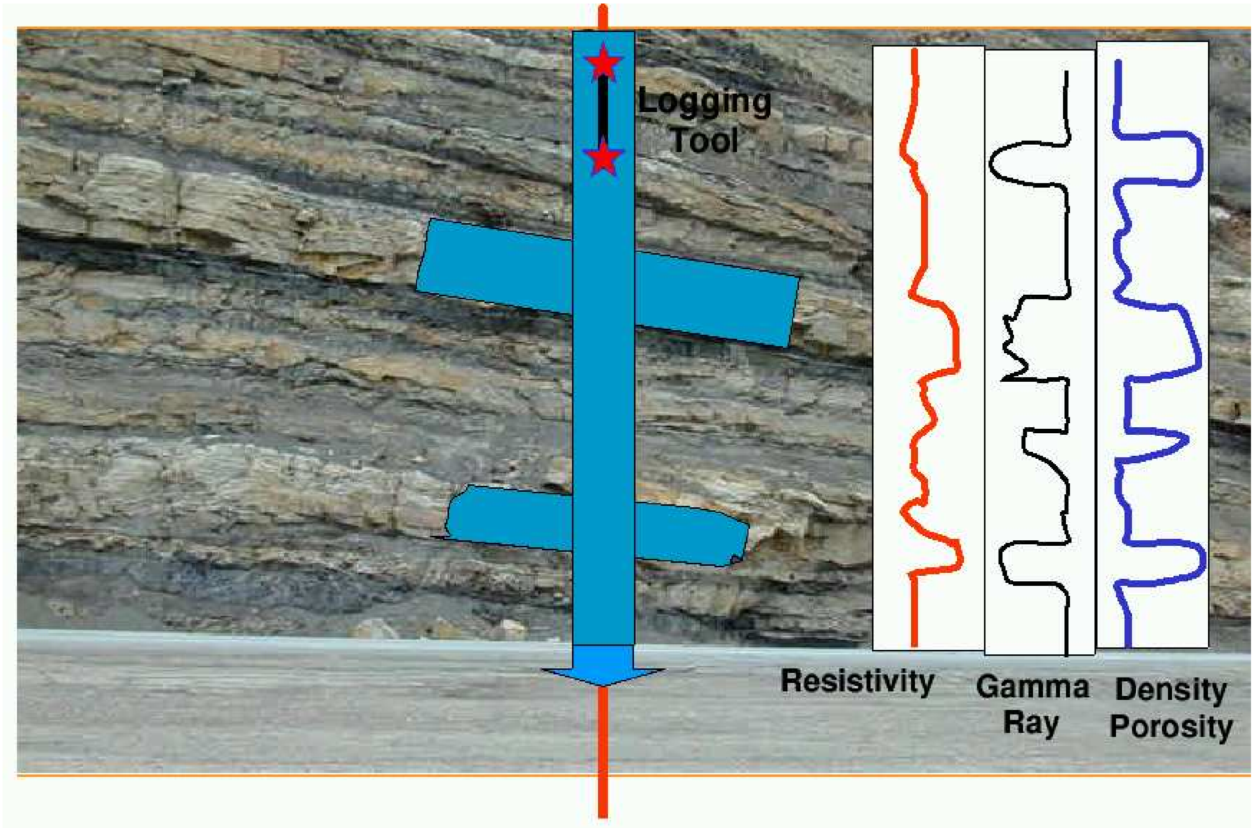
   - h-Adaptive Newton's Method (Inverse Problem), and
   - Parallel Self-Adaptive Goal-Oriented hp-Finite Element Method (Direct Problem)

5. Implementation

6. Conclusions and Future Work

MOTIVATION AND OBJECTIVES

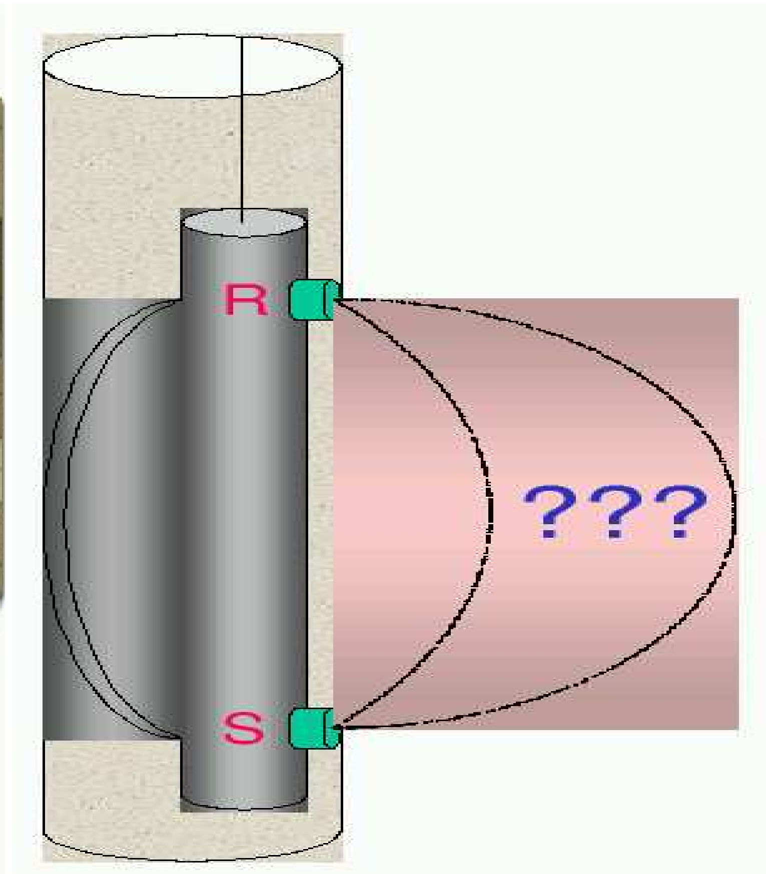
Multiphysics Logging Measurements



OBJECTIVES: To determine payzones (porosity), amount of oil/gas (saturation), and ability to extract oil/gas (permeability).

MOTIVATION AND OBJECTIVES

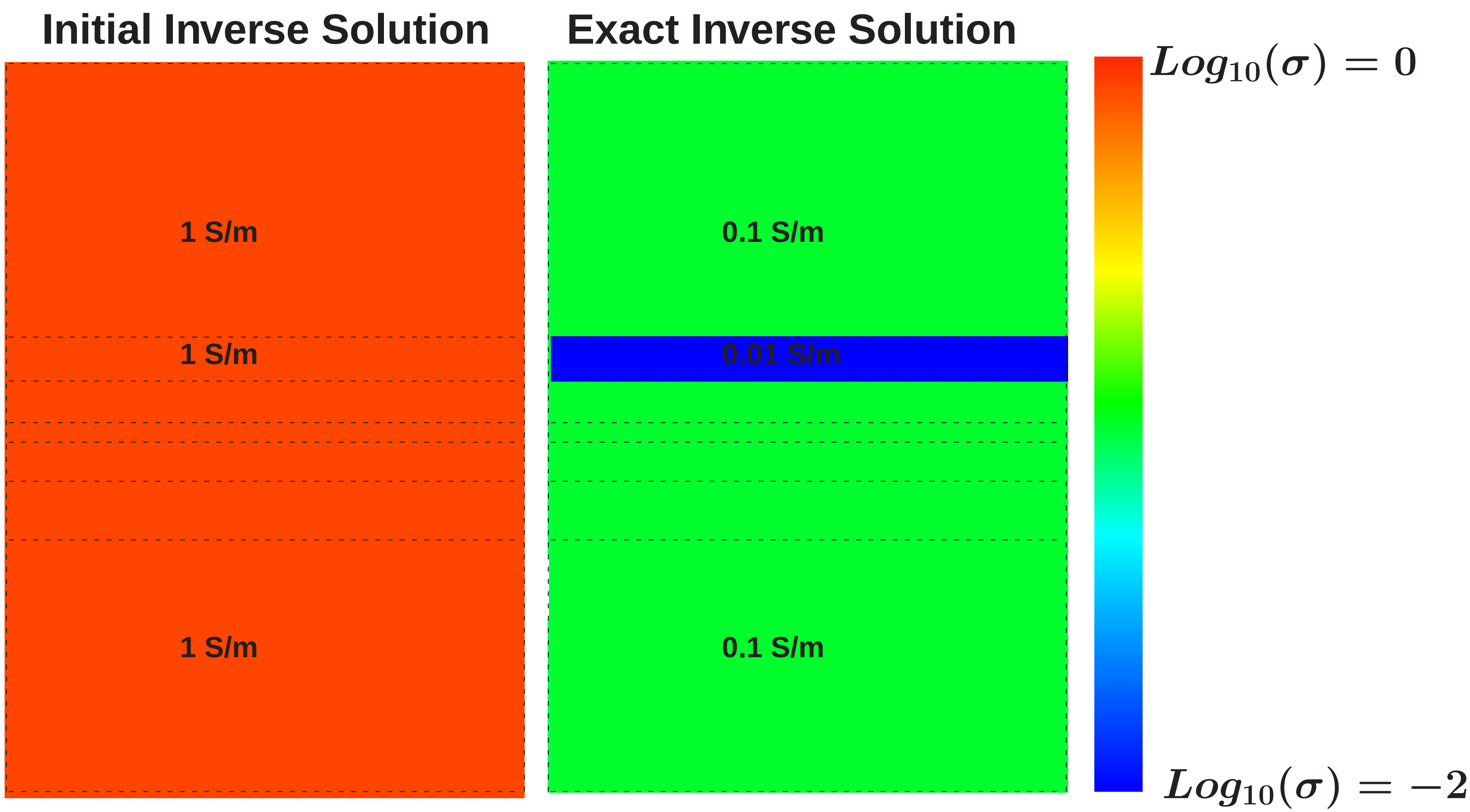
Main Objective: To Solve a Multiphysics Inverse Problem



Given multi-frequency electromagnetic, acoustic, and nuclear measurements, the objective is to determine porosity, saturation, and permeability distributions in the reservoir.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

1D Inversion (2D Forward) Example with 7 Parameters



15 Forward measurements and 7 unknowns (conductivities) for 1D layers with different thicknesses. 2 MHz. 20 m x 40 m domain.

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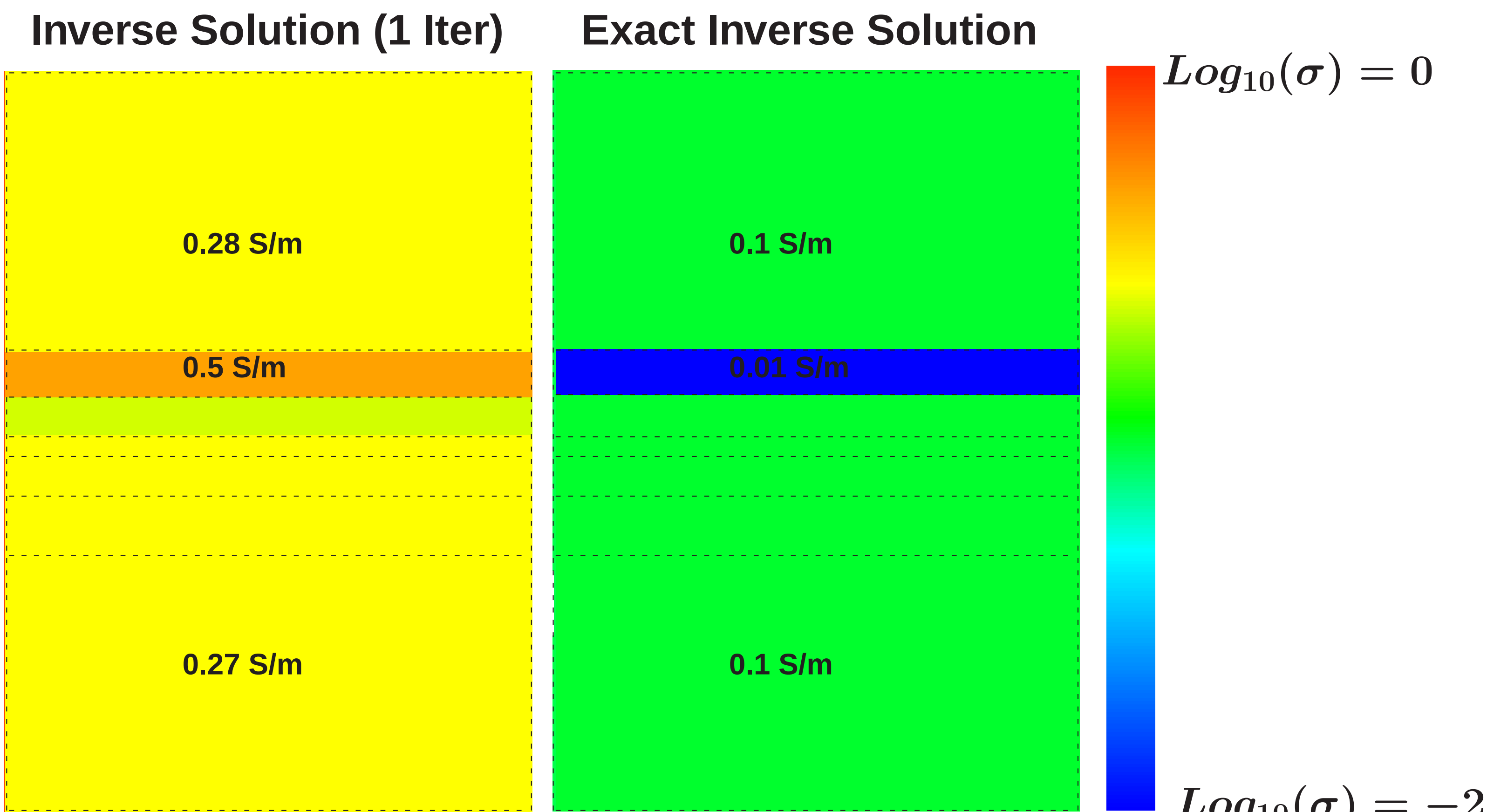
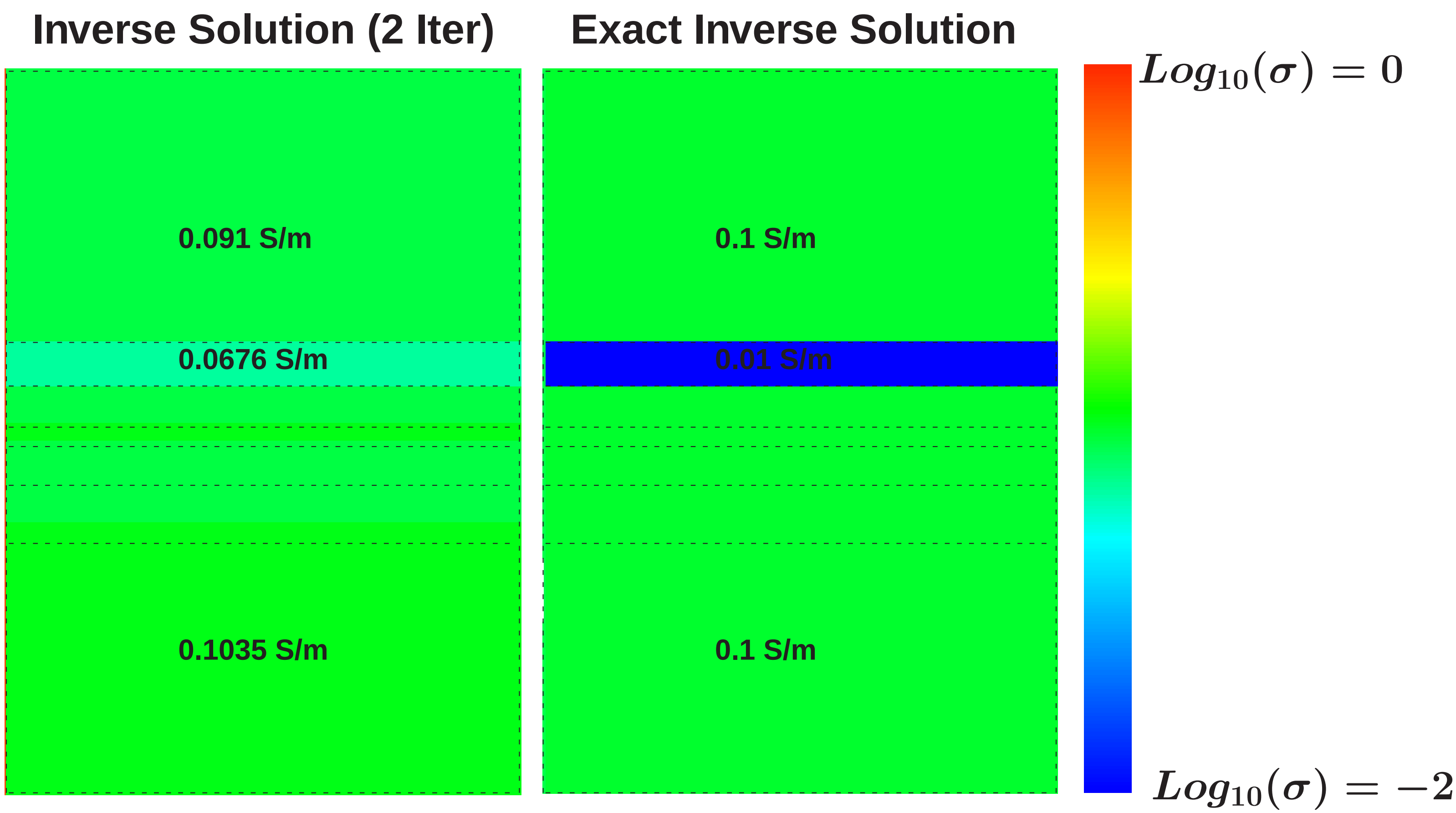


No regularization. No a priori information. Overdetermined problem.
Inversion solution at the first iteration.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

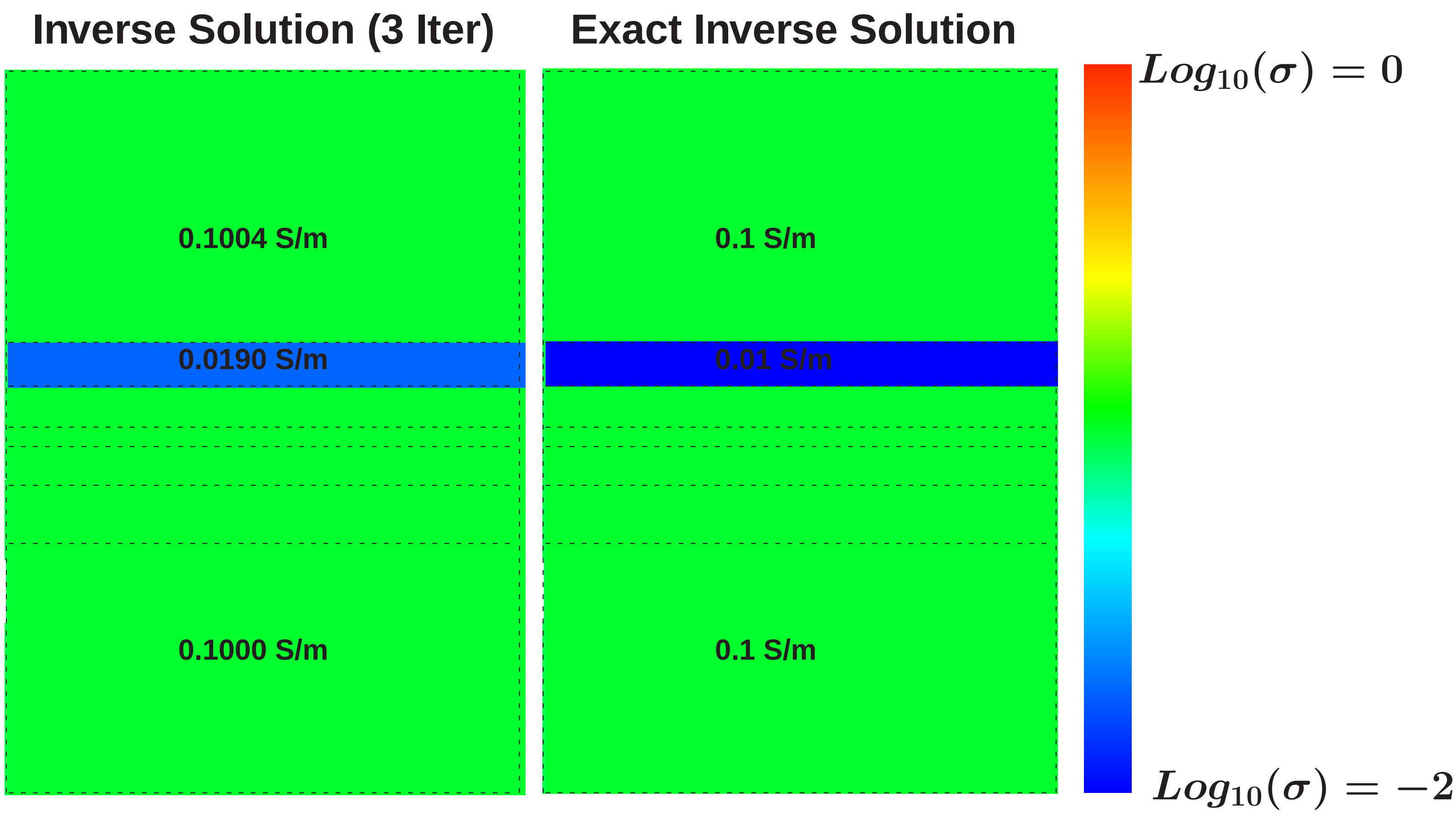
1D Inversion (2D Forward) Example with 7 Parameters



No regularization. No a priori information. Overdetermined problem. Inversion solution at the second iteration.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

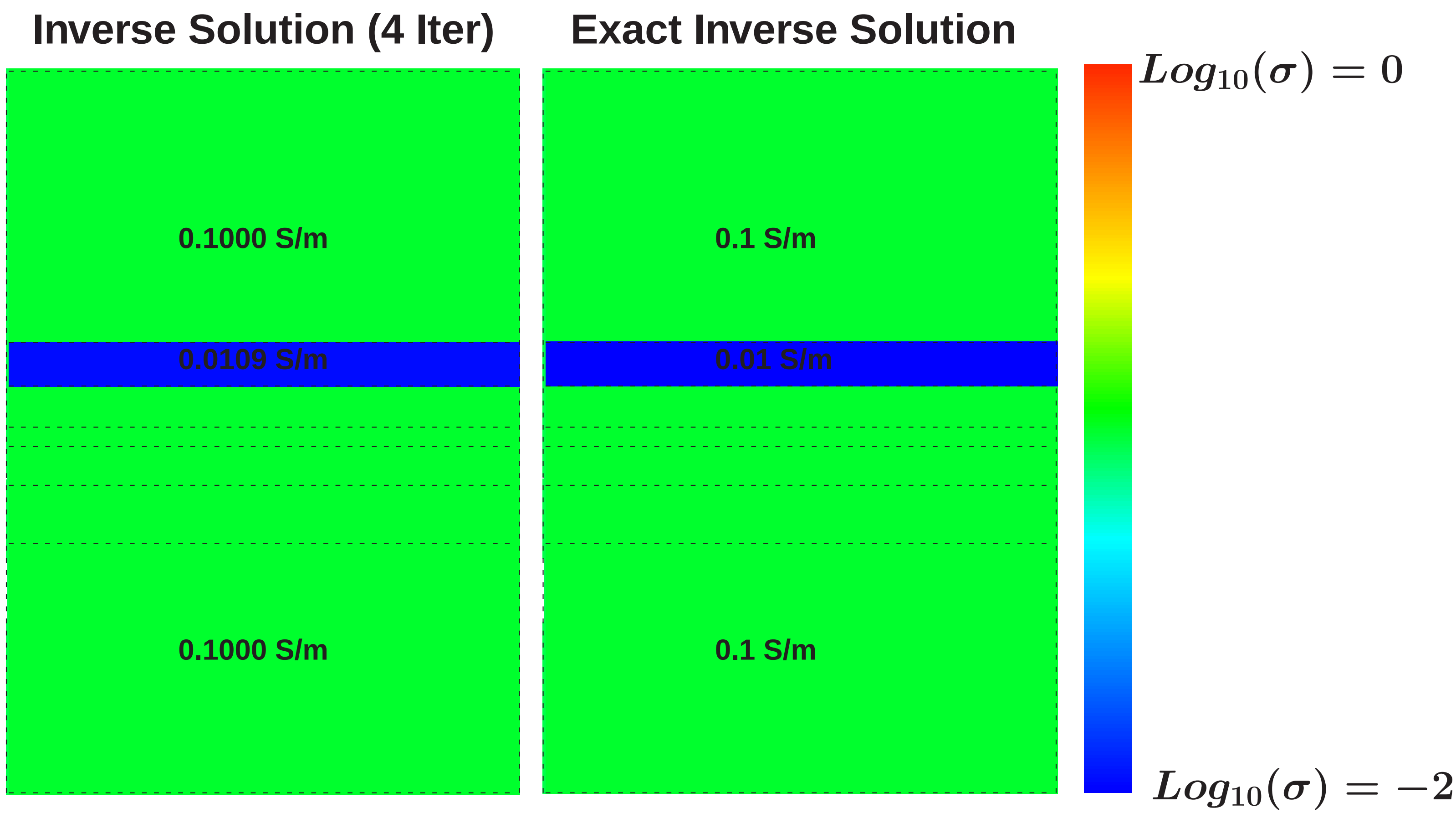
1D Inversion (2D Forward) Example with 7 Parameters



No regularization. No a priori information. Overdetermined problem.
Inversion solution at the third iteration.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

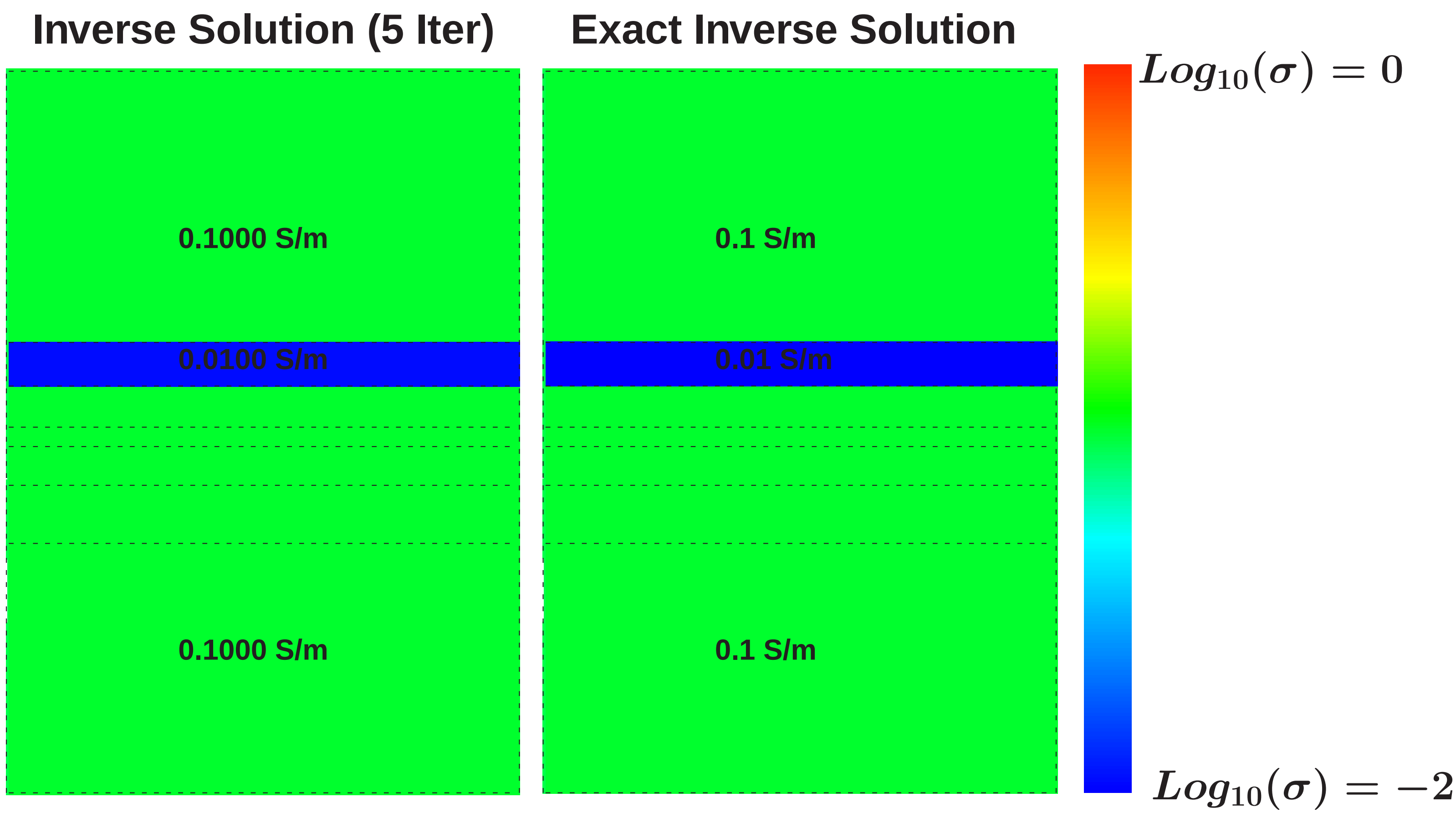
1D Inversion (2D Forward) Example with 7 Parameters



No regularization. No a priori information. Overdetermined problem.
Inversion solution at the fourth iteration.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

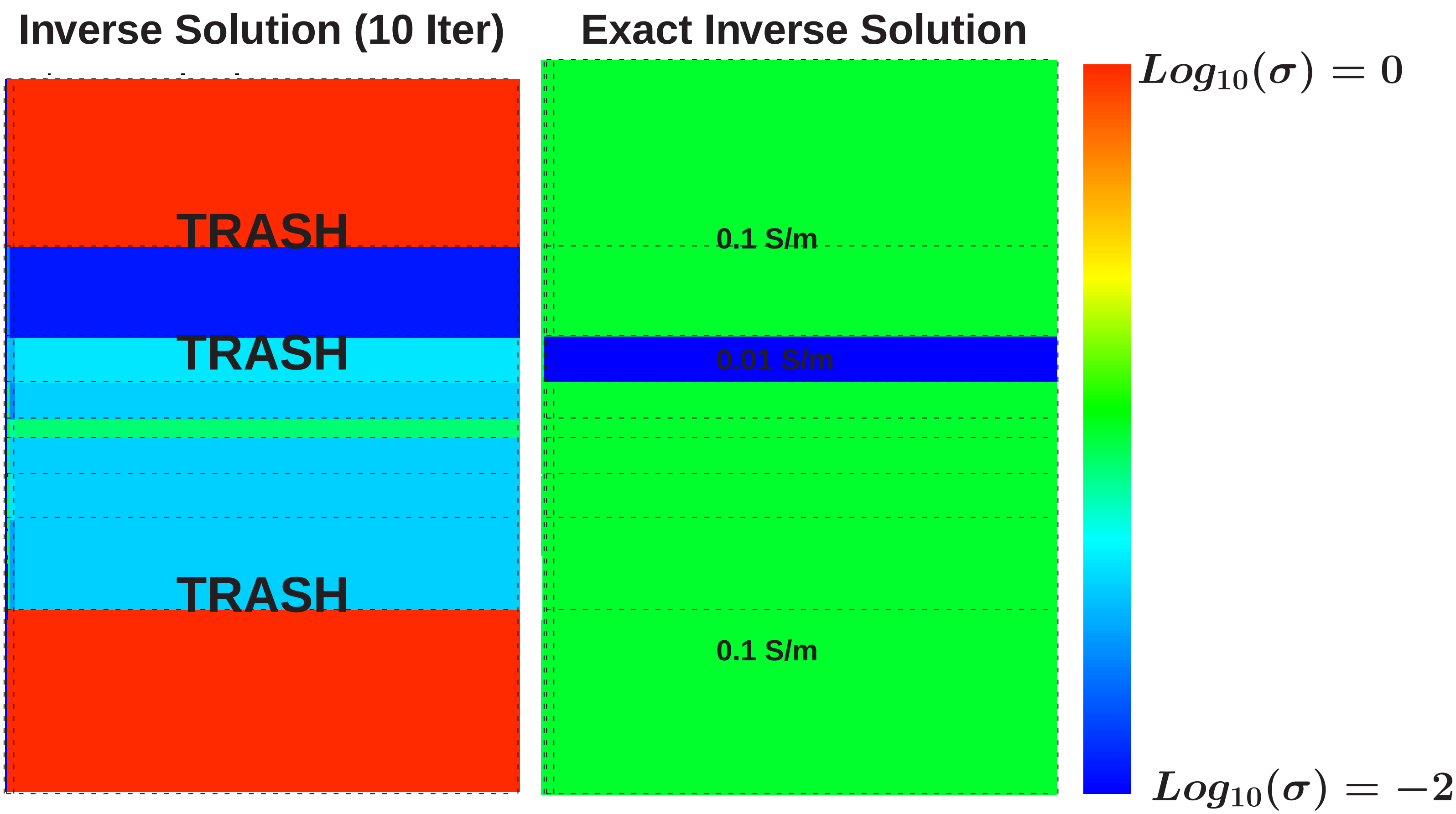
1D Inversion (2D Forward) Example with 7 Parameters



No regularization. No a priori information. Overdetermined problem. Inversion solution at the fifth iteration.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

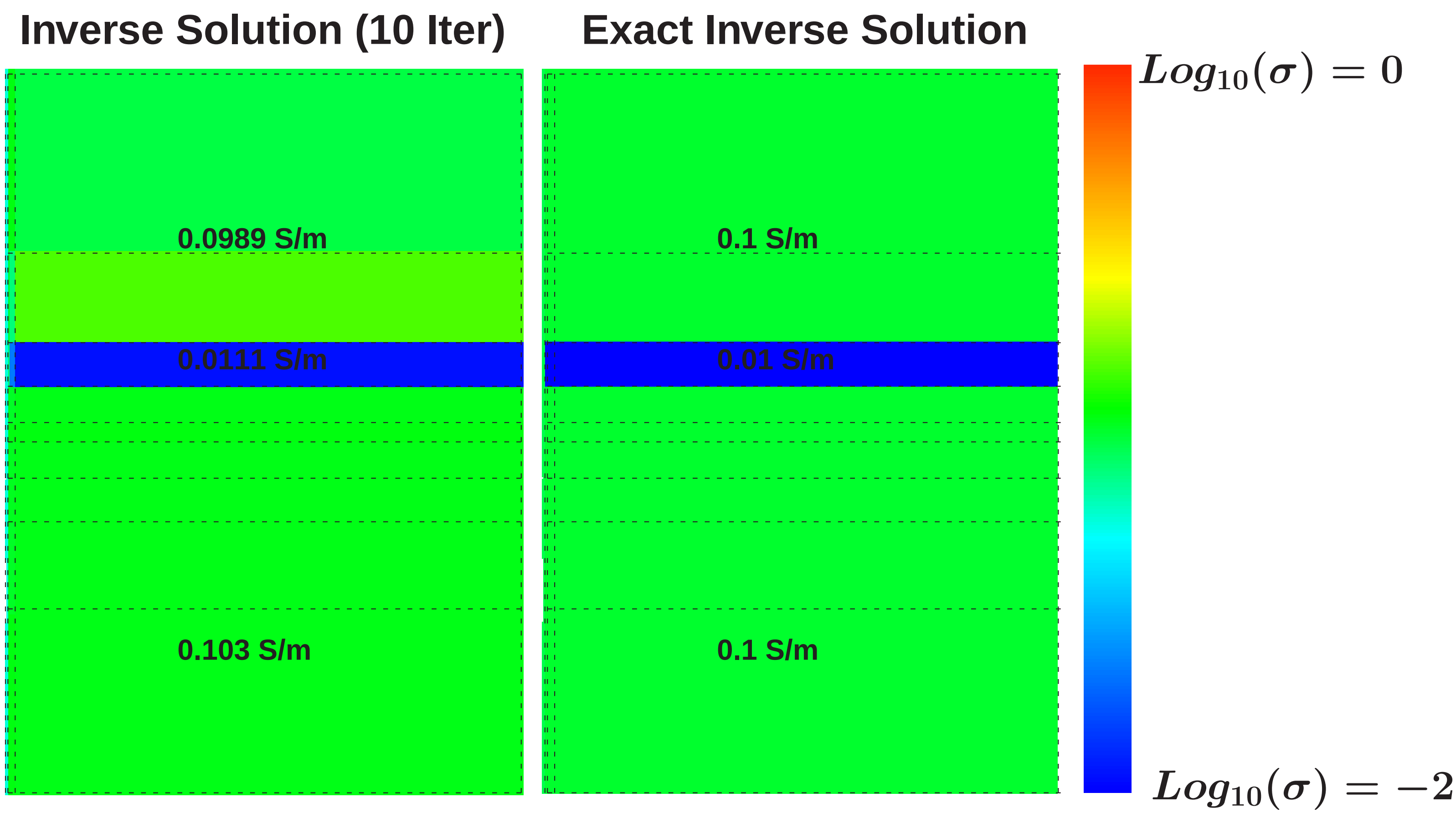
1D Inversion (2D Forward) Example with 27 Parameters



No regularization. As we increase the number of unknowns of the inverse problem, it becomes singular and unstable.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

1D Inversion (2D Forward) Example with 27 Parameters



With adjusted regularization. Assumed background conductivity: 0.1 S/m.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

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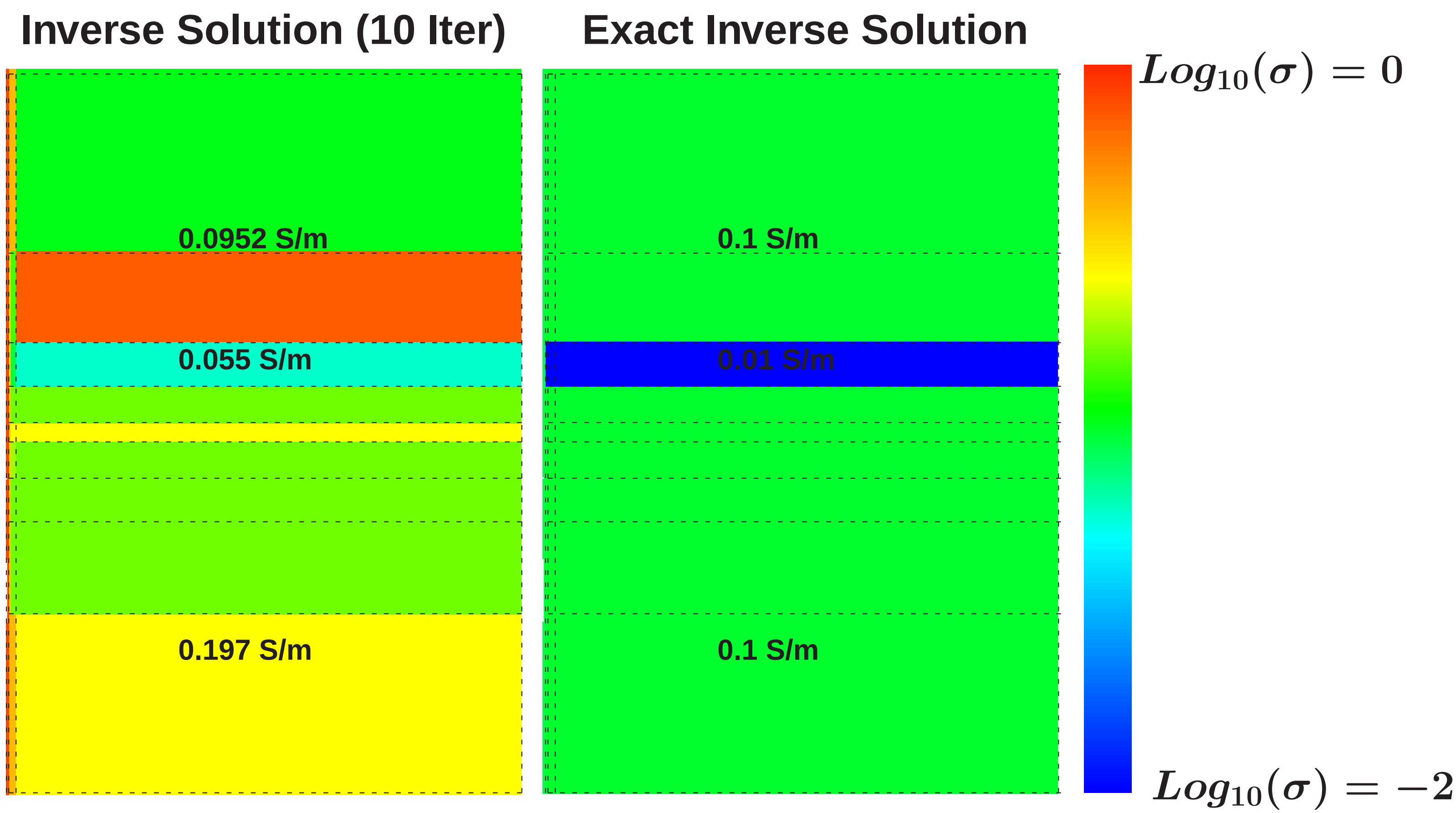
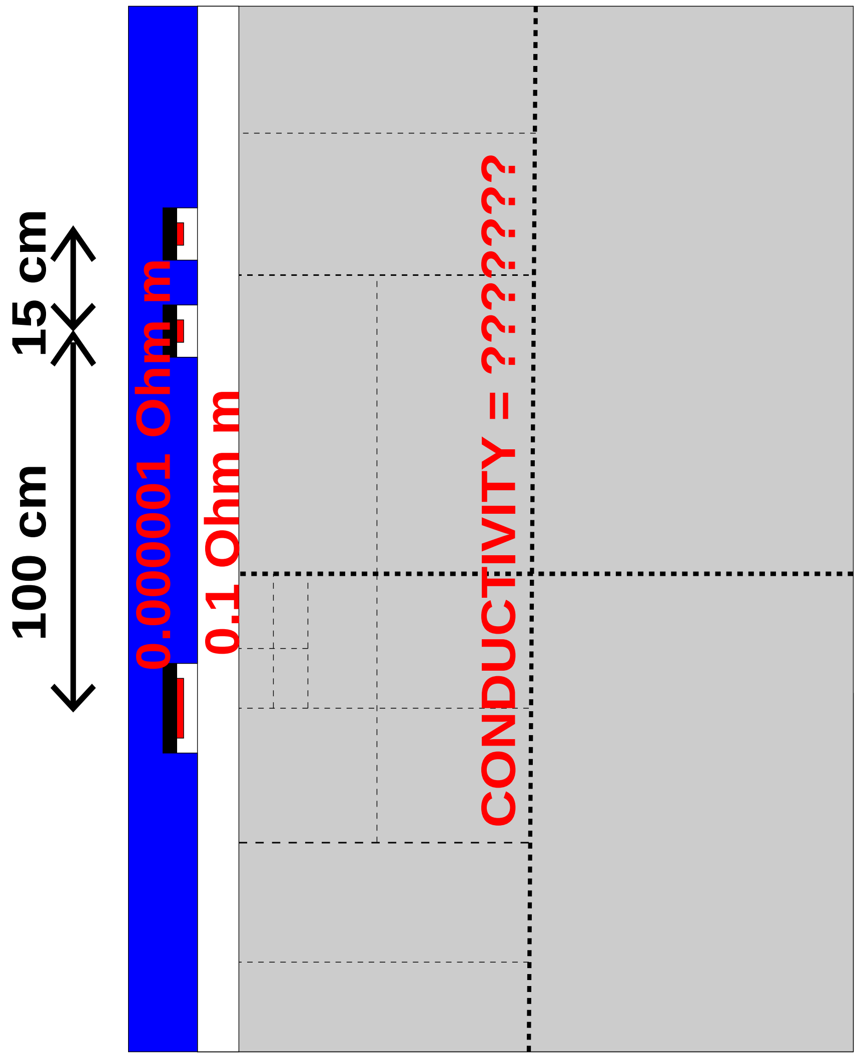


With adjusted regularization. Assumed background conductivity: 0.2 S/m.

MOTIVATION AND MAIN IDEA FOR THE INVERSION



Conductivities of borehole and logging instrument are known a priori.

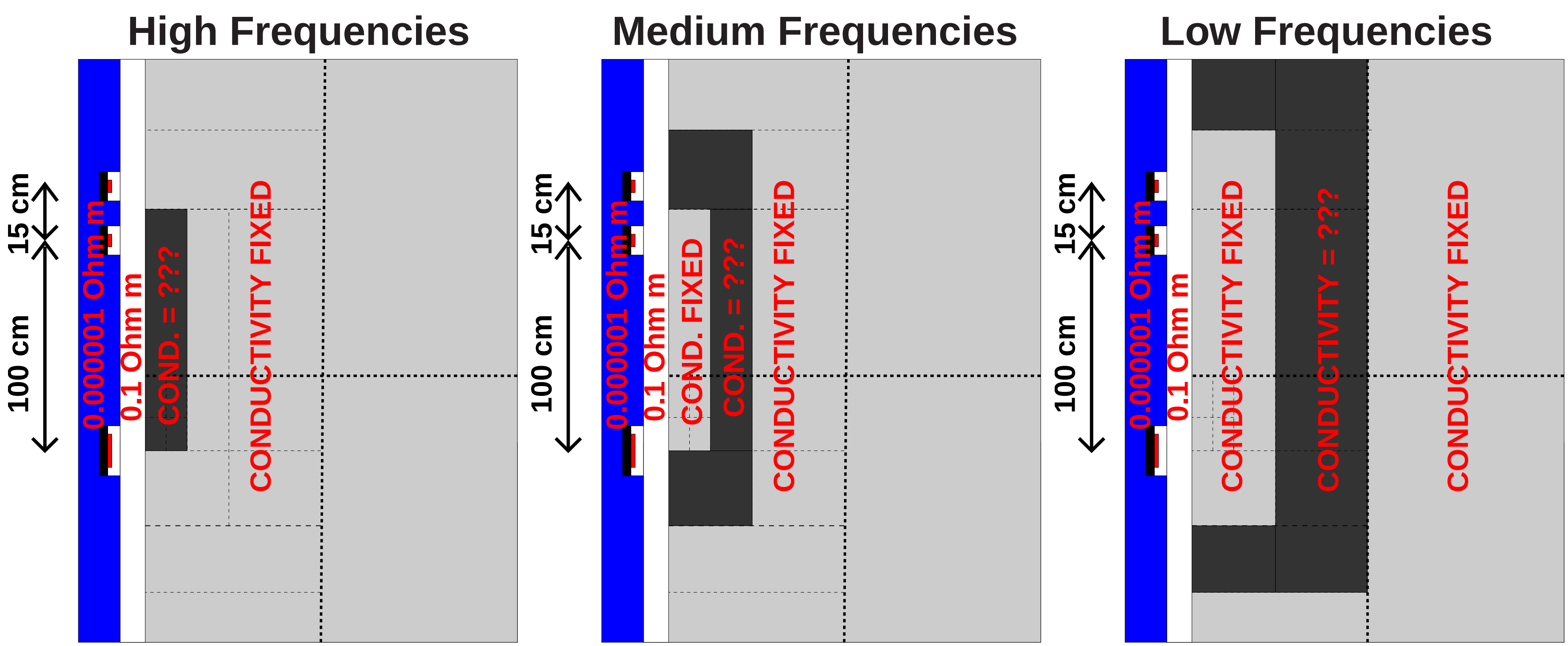
The conductivities of the formation are the unknowns (*parameter or model space*) of the inverse problem.

Grids for the inverse problem are different from grids for the forward problems.

We employ h-adaptive inverse grids and hp-adaptive forward grids.

MOTIVATION AND MAIN IDEA FOR THE INVERSION

Progressive (Gauss-Seidel Type) Inversion



Measurements are classified according to the support of their sensitivity functions

MOTIVATION AND MAIN IDEA FOR THE INVERSION

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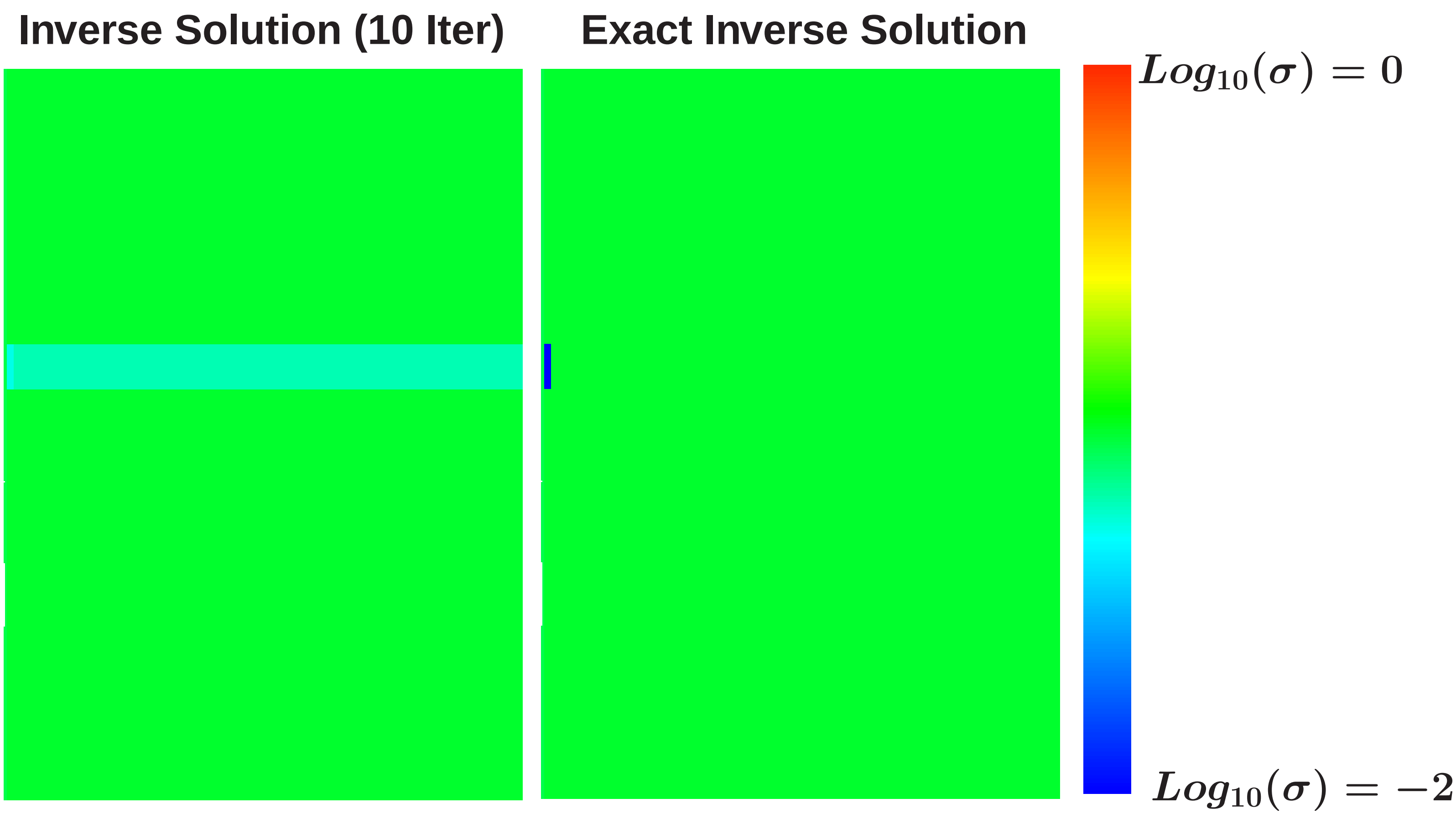
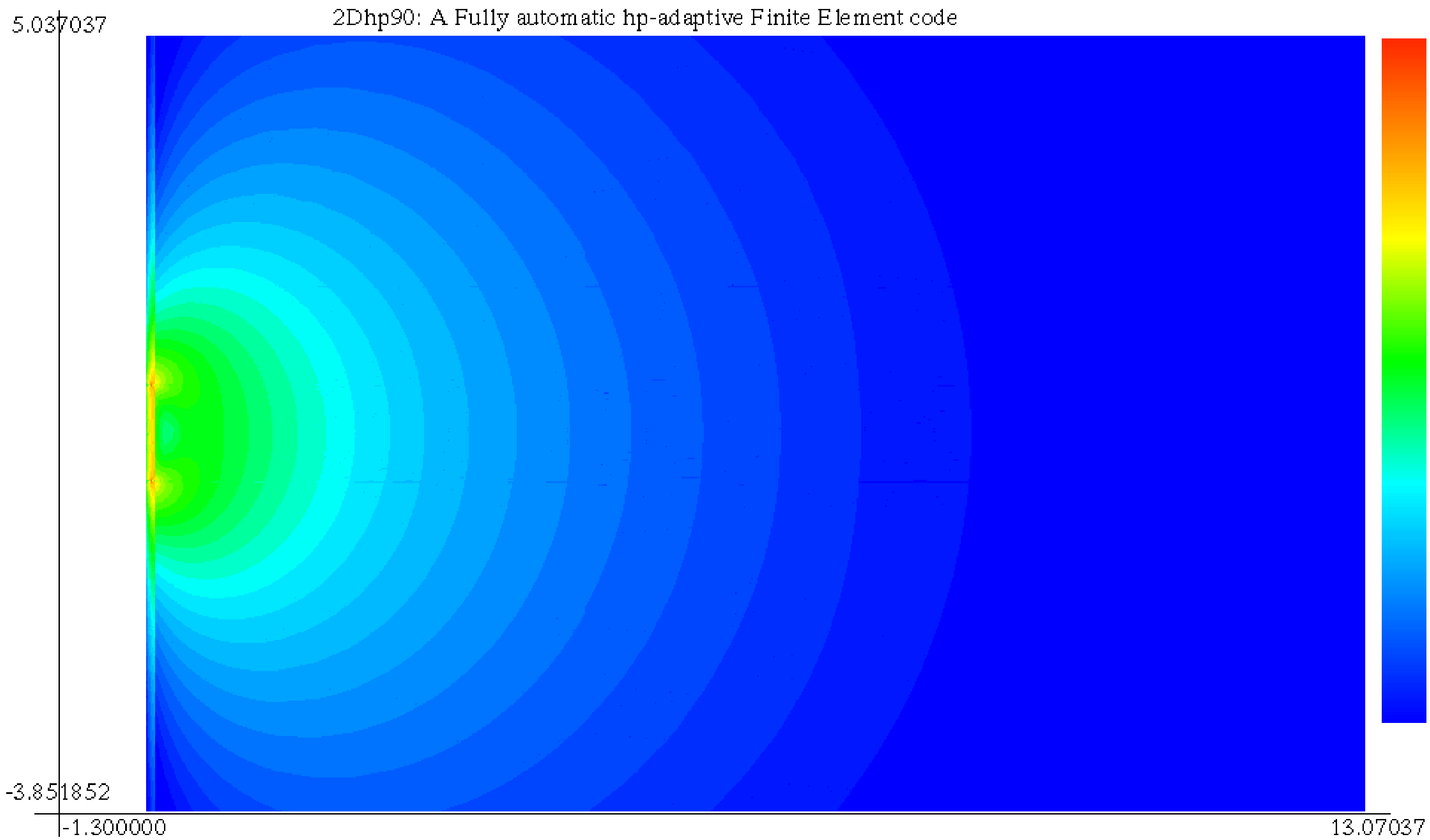


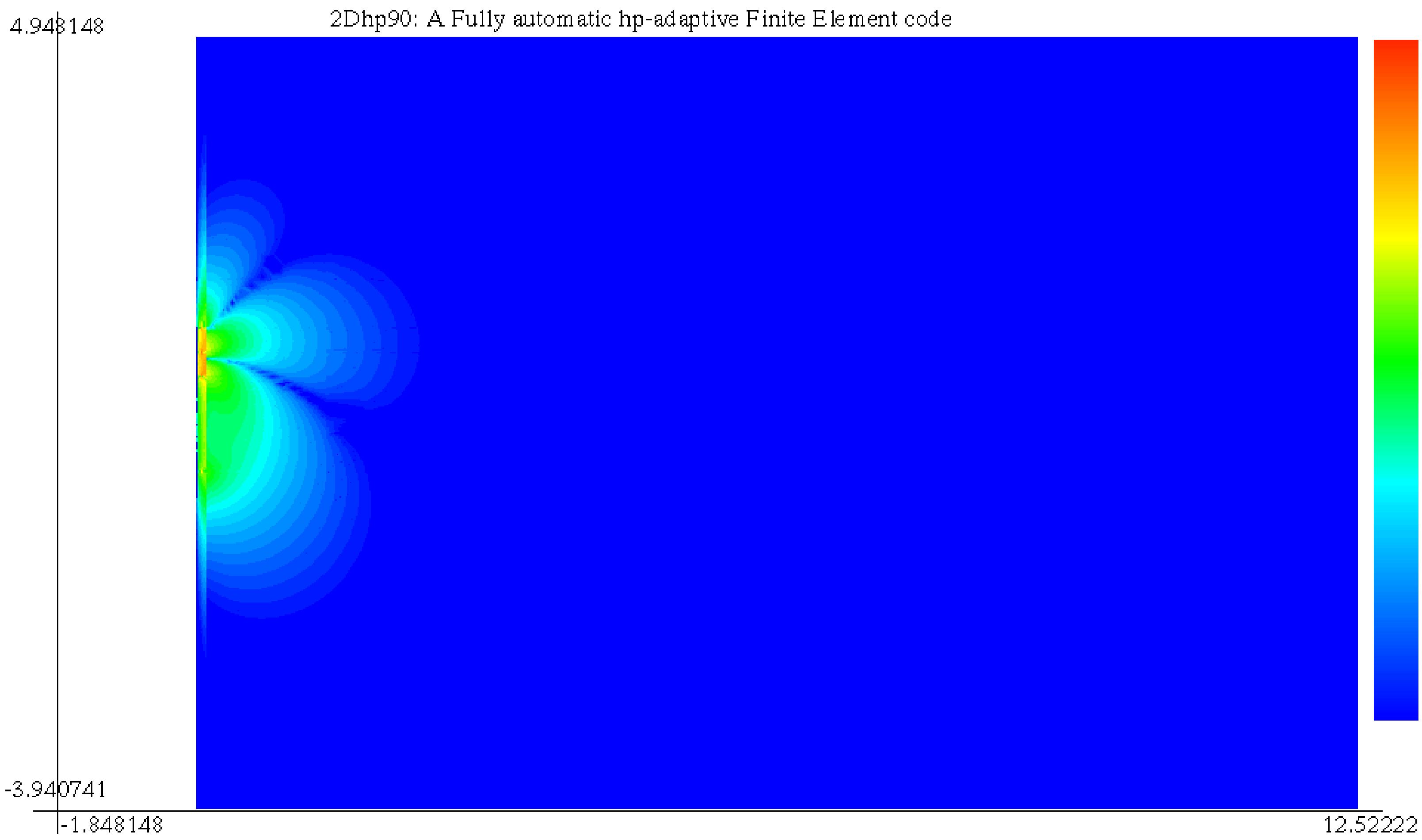
No regularization. Two-step 1D inversions. Miss-fit due to lack of agreement with sensitivity functions.

MOTIVATION AND MAIN IDEA FOR THE INVERSION



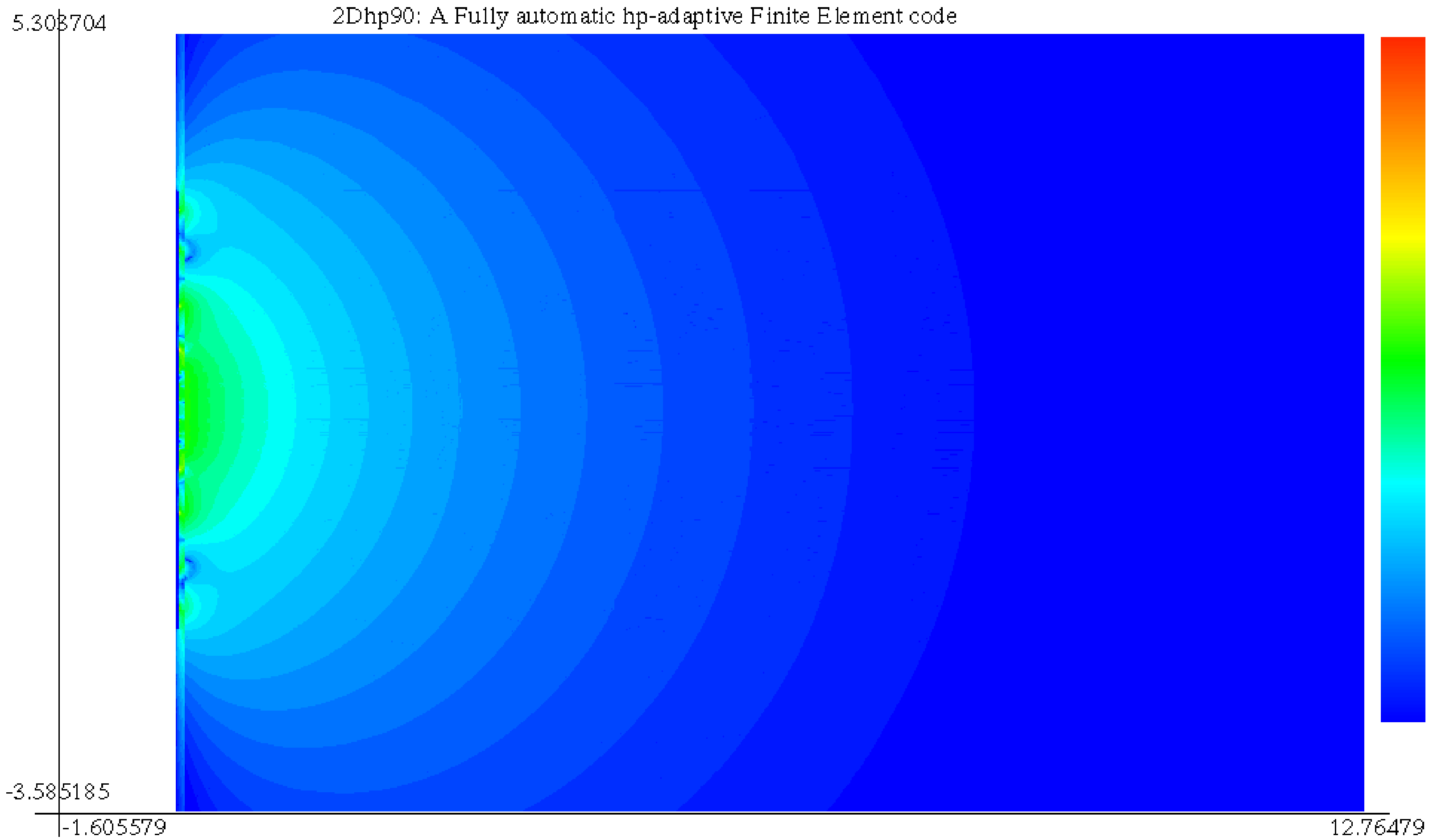
Sensitivity Function: One TX, one RX

MOTIVATION AND MAIN IDEA FOR THE INVERSION



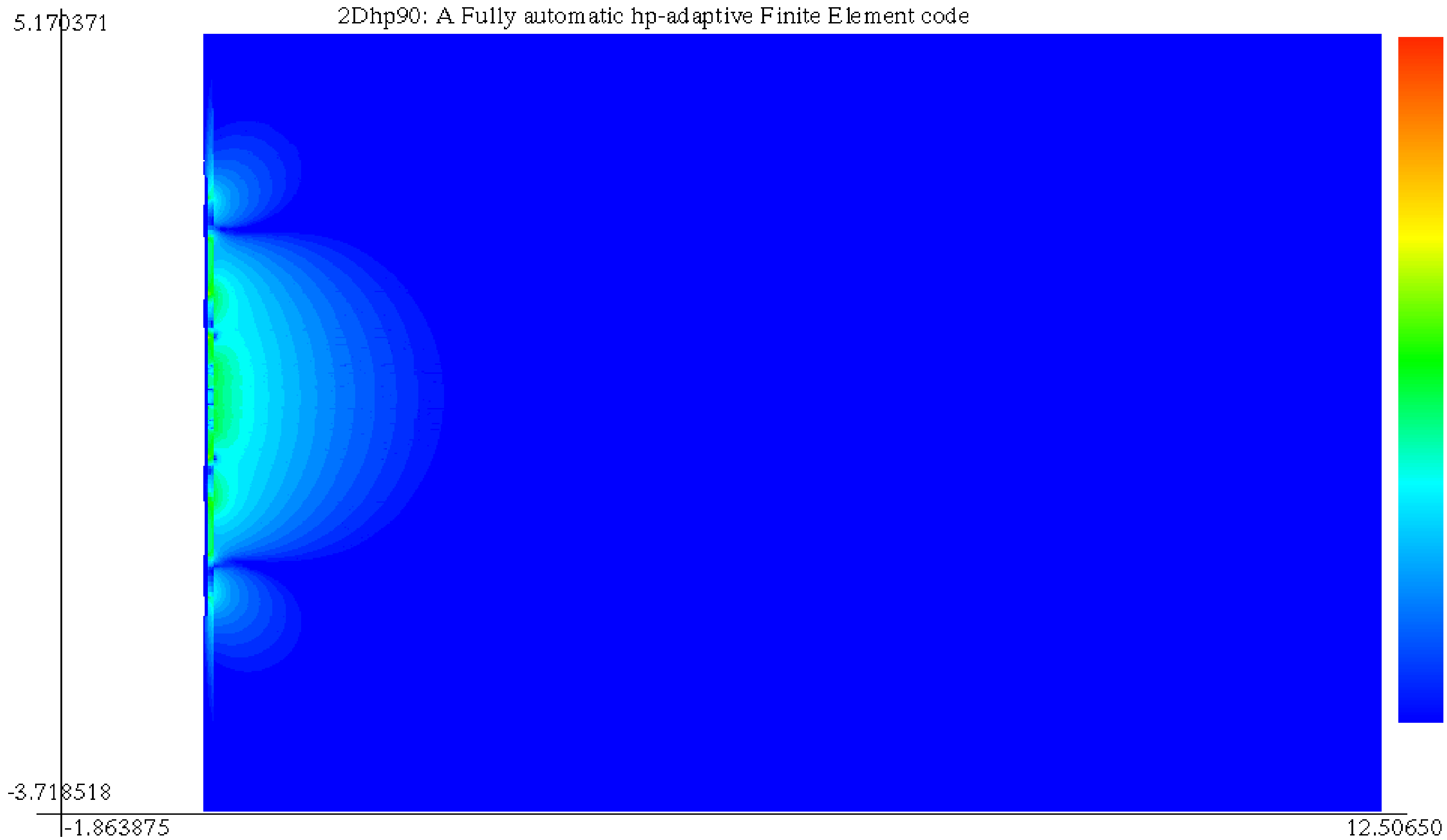
Sensitivity Function: One TX, three RXs

MOTIVATION AND MAIN IDEA FOR THE INVERSION



Sensitivity Function: Dual Laterolog (LLd — deep-sensing mode —)

MOTIVATION AND MAIN IDEA FOR THE INVERSION



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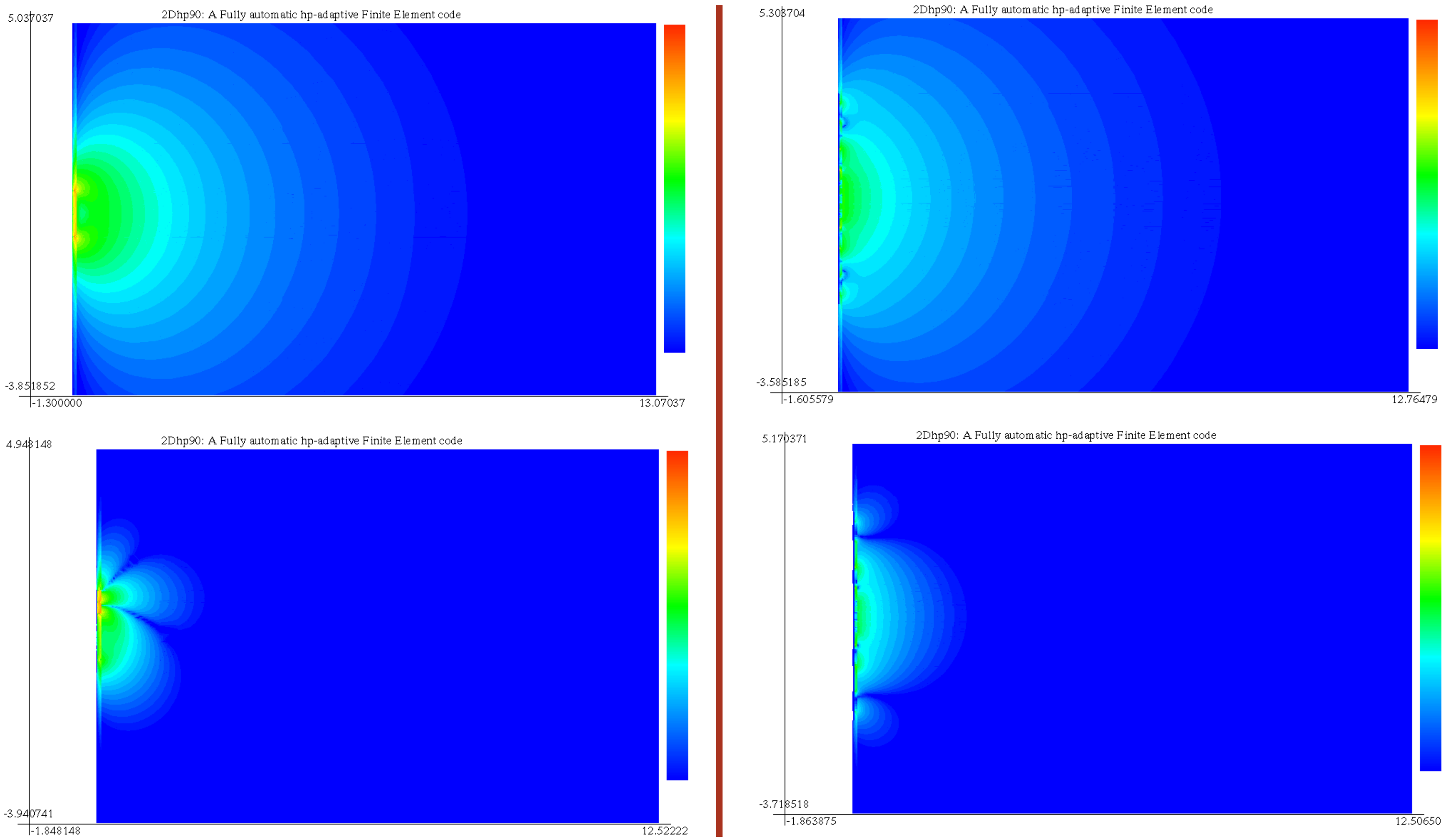
MOTIVATION AND MAIN IDEA FOR THE INVERSION

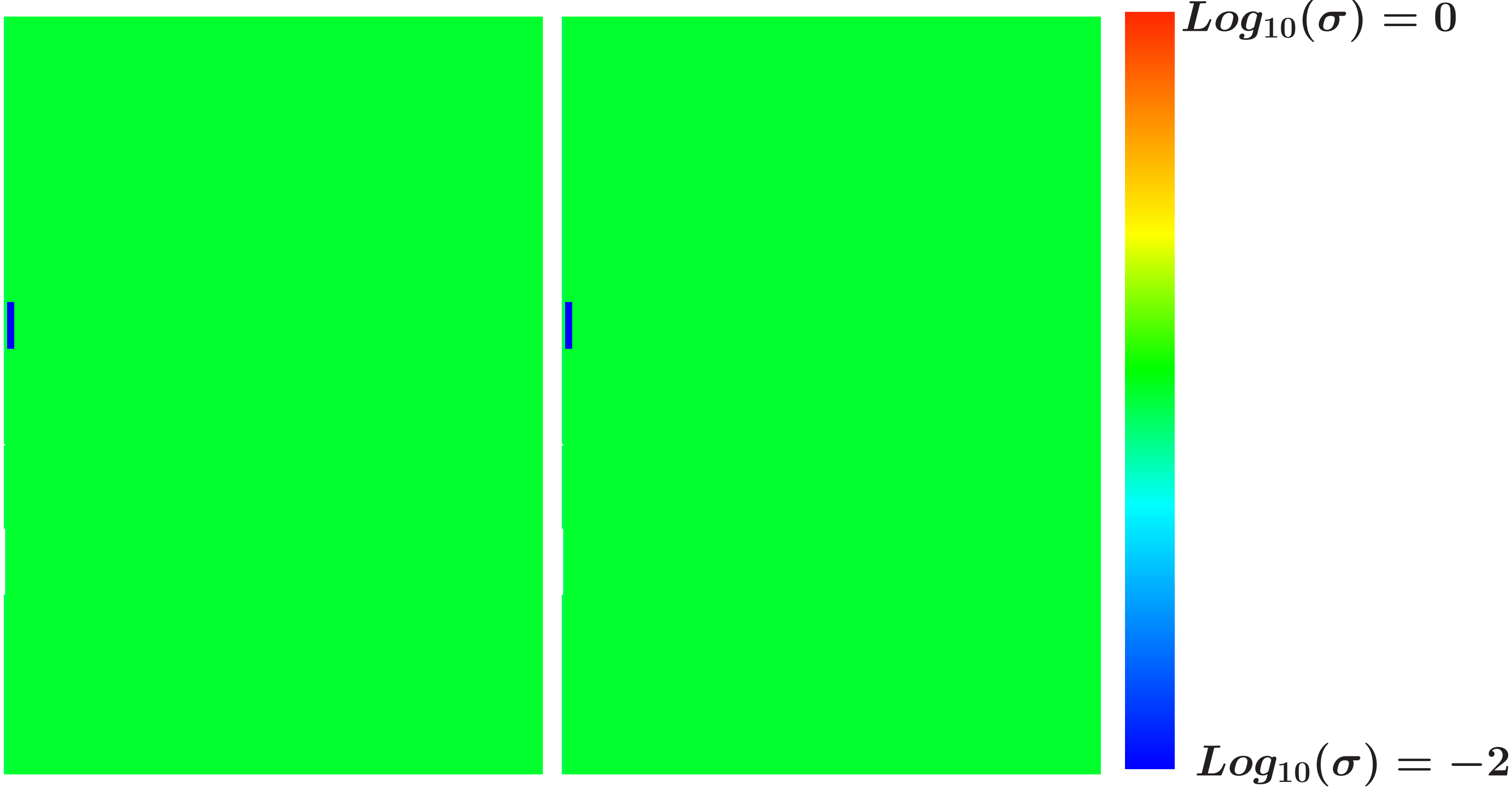


Different logging instruments provide different sensitivity functions

MOTIVATION AND MAIN IDEA FOR THE INVERSION

1D Inversion (2D Forward) Example with 27 Parameters

Inverse Solution (10 Iter) **Exact Inverse Solution**

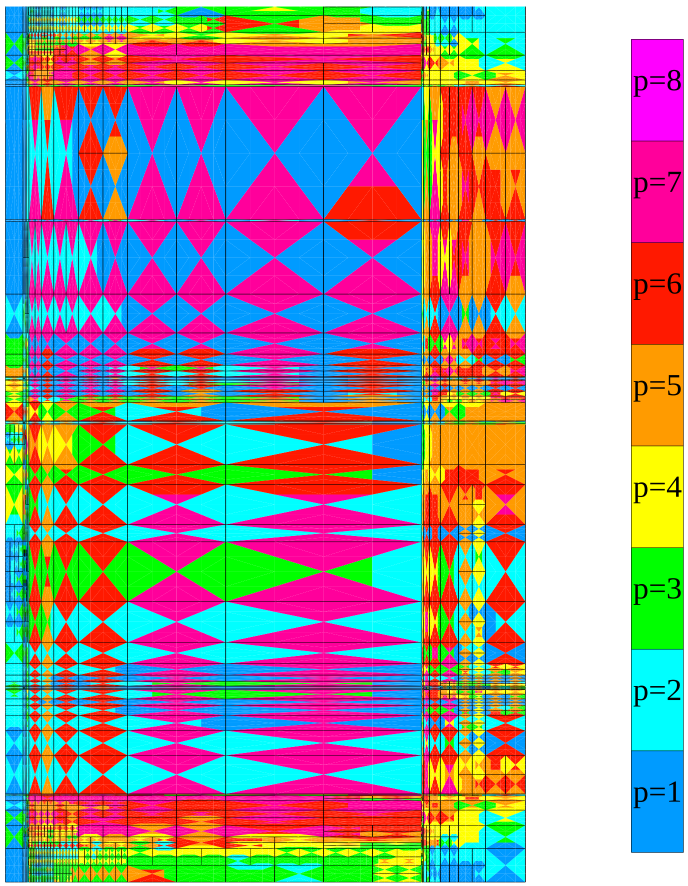


No regularization. Inversion with a subset of the 27 inverse parameters selected based on the sensitivity functions.

METHOD FOR THE FORWARD PROBLEM

A Self-Adaptive Goal-Oriented hp-FEM

Optimal 2D Grid
(Through Casing Resistivity Problem)



We vary locally the element size h and the polynomial order of approximation p throughout the grid.

Optimal grids are automatically generated by the computer.

The self-adaptive goal-oriented hp-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.

FORMULATION OF THE EM FORWARD PROBLEM

Variational Formulation (DC)

Notation:

$$B(u, v; \sigma) = < \nabla v, \sigma \nabla u >_{L^2(\Omega)}$$ **(bilinear in u and v)**

$$F_i(v) = < v, f_i >_{L^2(\Omega)} + < v, g_i >_{L^2(\partial\Omega)}$$ **(linear in v)**

$$L_i(u) = < l_i, u >_{L^2(\Omega)} + < h_i, u >_{L^2(\partial\Omega)}$$ **(linear in u)**

Direct Problem (homogeneous Dirichlet BC's):

$$\begin{cases} \text{Find } \hat{u}_i \text{ in V such that:} \\ B(\hat{u}_i, v; \sigma) = F_i(v) \quad \forall v \in V \end{cases}$$

Dual (Adjoint) Problem:

$$\begin{cases} \text{Find } \hat{v}_i \text{ in V such that:} \\ B(u, \hat{v}_i; \sigma) = L_i(u) \quad \forall u \in V \end{cases}$$

FORMULATION OF THE EM FORWARD PROBLEM

Variational Formulation (AC)

Notation:

$$B(\mathrm{E},\mathrm{F};\sigma) =< \nabla\times\mathrm{F}, \mu^{-1}\nabla\times\mathrm{E} >_{L^2(\Omega)} - < \mathrm{F}, (\omega^2\epsilon - j\omega\sigma)\mathrm{E} >_{L^2(\Omega)}$$

$$F_i(\mathrm{F}) = -j\omega < \mathrm{F}, \mathrm{J}_i^{imp} >_{L^2(\Omega)} + j\omega < \mathrm{F}, \mathrm{J}_{S,i}^{imp} >_{L^2(\partial\Omega)}$$

$$L_i(\mathrm{E}) =< \mathrm{J}_i^{adj}, \mathrm{E} >_{L^2(\Omega)} + < \mathrm{J}_{S,i}^{adj}, \mathrm{E} >_{L^2(\partial\Omega)}$$

Direct Problem (homogeneous Dirichlet BC's):

$$\begin{cases} \text{Find } \hat{\mathrm{E}}_i \text{ in W such that:} \\ B(\hat{\mathrm{E}}_i,\mathrm{F};\sigma) = F_i(\mathrm{F}) \quad \forall \mathrm{F} \in W \end{cases}$$

Dual (Adjoint) Problem:

$$\begin{cases} \text{Find } \hat{\mathrm{F}}_i \text{ in W such that:} \\ B(\mathrm{E},\hat{\mathrm{F}}_i;\sigma) = L_i(\mathrm{E}) \quad \forall \mathrm{E} \in W \end{cases}$$

FORMULATION OF THE INVERSE PROBLEM

Constrained Nonlinear Optimization Problem

Cost Functional:

$$\begin{cases} \text{Find } \boldsymbol{\sigma} > 0 \text{ such that it minimizes } C_\beta(\boldsymbol{\sigma}) \text{, where:} \\ C_\beta(\boldsymbol{\sigma}) = ||W_m(L(\hat{u}_{\boldsymbol{\sigma}}) - M)||^2_{l_2} + \beta||R(\boldsymbol{\sigma} - \boldsymbol{\sigma}_0)||^2_{L_2} \,, \end{cases}$$

where

M_i denotes the i-th measurement, $M = (M_1, ..., M_n)$

L_i is the i-th quantity of interest, $L = (L_1, ..., L_n)$

$$||M||^2_{l_2} = \sum_{i=1}^{n} M_i^2 \quad ; \quad ||R(\boldsymbol{\sigma} - \boldsymbol{\sigma}_0)||^2_{L_2} = \int (R(\boldsymbol{\sigma} - \boldsymbol{\sigma}_0))^2$$

β is the relaxation parameter, σ_0 is given, W_m are weights

Main objective (inversion problem): Find $\hat{\sigma} = \min_{\boldsymbol{\sigma} > 0} C_\beta(\boldsymbol{\sigma})$

METHOD FOR THE INVERSE PROBLEM

Solving a Constrained Nonlinear Optimization Problem

We select the following deterministic iterative method:

$$\boldsymbol{\sigma}^{(n+1)} = \boldsymbol{\sigma}^{(n)} + \alpha^{(n)} \delta \boldsymbol{\sigma}^{(n)}$$

- **How to find a search direction $\delta\boldsymbol{\sigma}^{(n)}$?**
  - We will employ a change of coordinates and a truncated Taylor's series expansion.
- **How to determine the step size $\alpha^{(n)}$?**
  - Either with a fixed size or using an approximation for computing $L(\boldsymbol{\sigma}^{(n)} + \alpha^{(n)} \delta \boldsymbol{\sigma}^{(n)})$.
- **How to guarantee that the nonlinear constraints will be satisfied?**
  - Imposing the Karush-Kuhn-Tucker (KKT) conditions or with a penalization method, or via a change of variables.

METHOD FOR THE INVERSE PROBLEM

Search Direction Method

Change of coordinates:

$$h(s) = \sigma \quad => \textbf{ Goal: Find } \hat{s} = \min_{h(s)>0} C_\beta(s)$$

Taylor's series expansion:

A) $C_\beta(s + \delta s) \approx C_\beta(s) + \delta s \nabla C_\beta(s) + 0.5 \delta s^2 H_{C_\beta}(s)$

B) $L(s + \delta s) \approx L(s) + \delta s \nabla L(s) \; , \; R(s + \delta s) = R(s) + \delta s \nabla R(s)$

Expansion A) leads to the Newton-Raphson method.

Expansion B) leads to the Gauss-Newton method.

Expansion A) with $H_{C_\beta} = I$ leads to the steepest descent method.

Higher-order expansions require from higher-order derivatives.

METHOD (COMPUTATION OF JACOBIAN)

Computation of Jacobian Matrix

Using the Fréchet Derivative:

$$\frac{\partial L_i(\hat{u}_i)}{\partial s_j} = B\left(\frac{\partial \hat{u}_i}{\partial s_j}, \hat{v}_i, h(s)\right) + B\left(\hat{u}_i, \frac{\partial \hat{v}_i}{\partial s_j}, h(s)\right) + B\left(\hat{u}_i, \hat{v}_i, \frac{\partial h(s)}{\partial s_j}\right)$$

$$\|$$

$$L_i\left(\frac{\partial \hat{u}_i}{\partial s_j}\right) = B\left(\frac{\partial \hat{u}_i}{\partial s_j}, \hat{v}_i, h(s)\right)$$

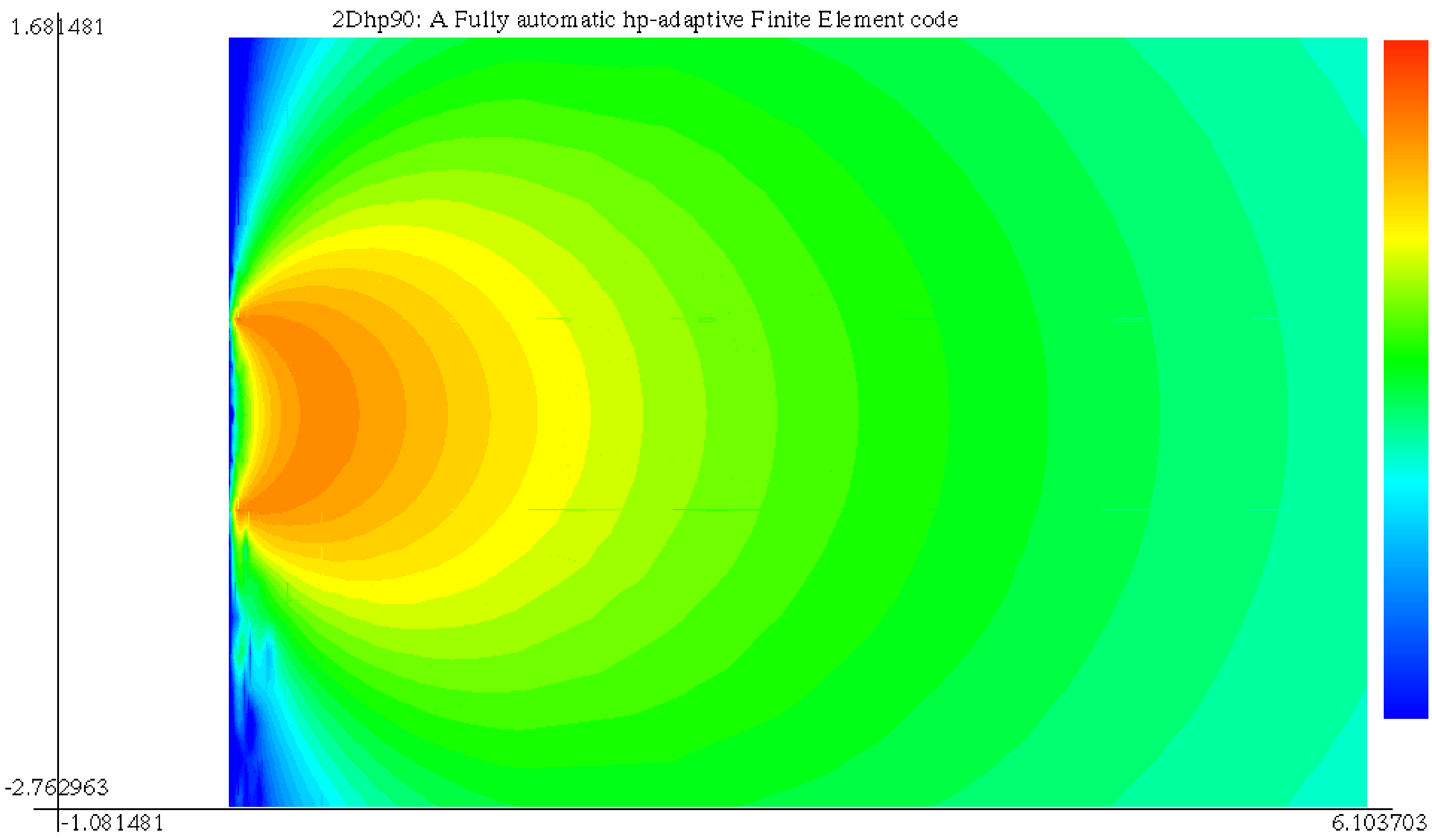
$$\|$$

$$F_i\left(\frac{\partial \hat{v}_i}{\partial s_j}\right) = B\left(\hat{u}_i, \frac{\partial \hat{v}_i}{\partial s_j}, h(s)\right)$$

Therefore, we conclude:

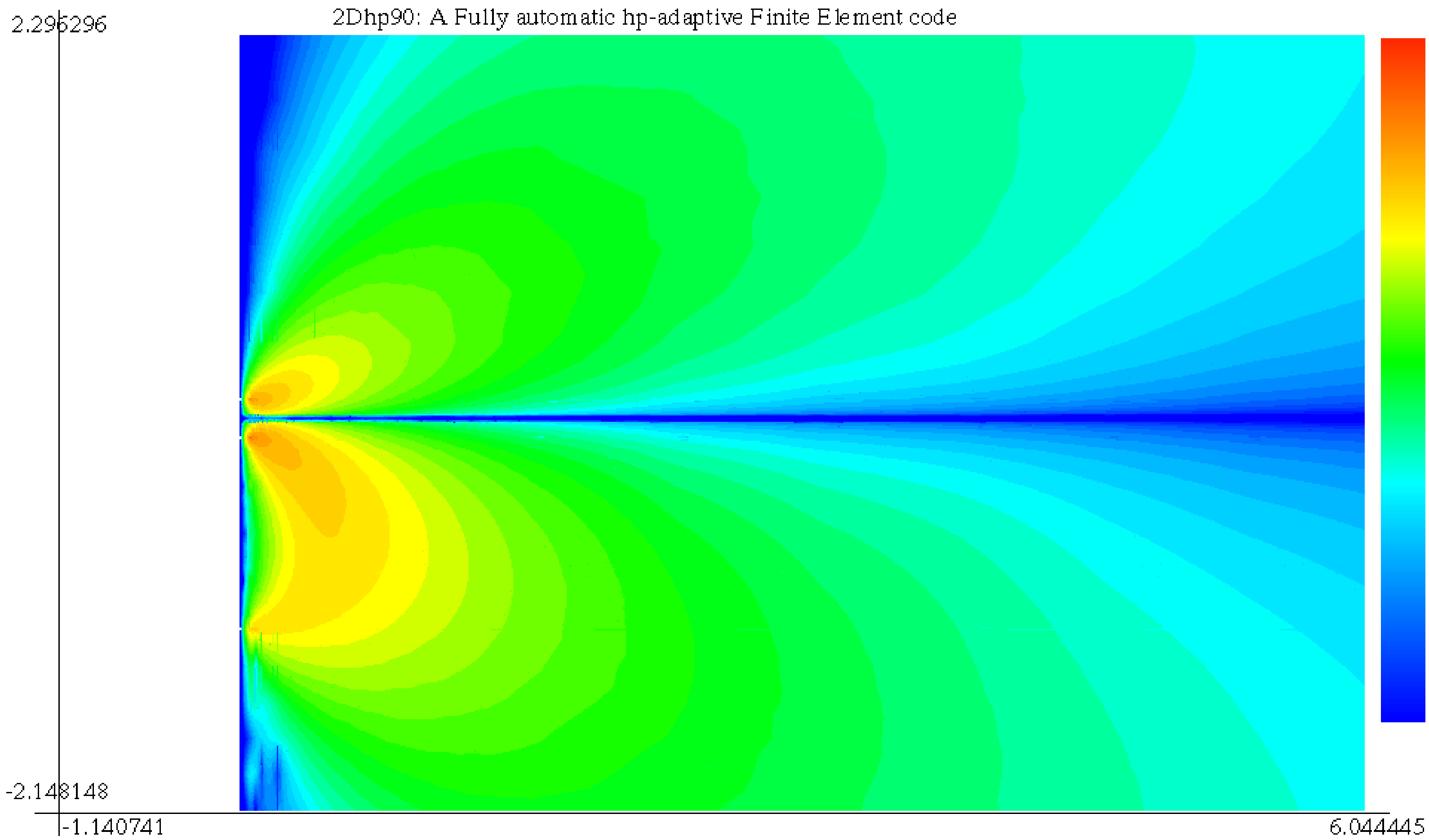
$$\textbf{Jacobian Matrix} = \frac{\partial L_i(\hat{u}_i)}{\partial s_j} = -B\left(\hat{u}_i, \hat{v}_i, \frac{\partial h(s)}{\partial s_j}\right)$$

METHOD (COMPUTATION OF JACOBIAN)



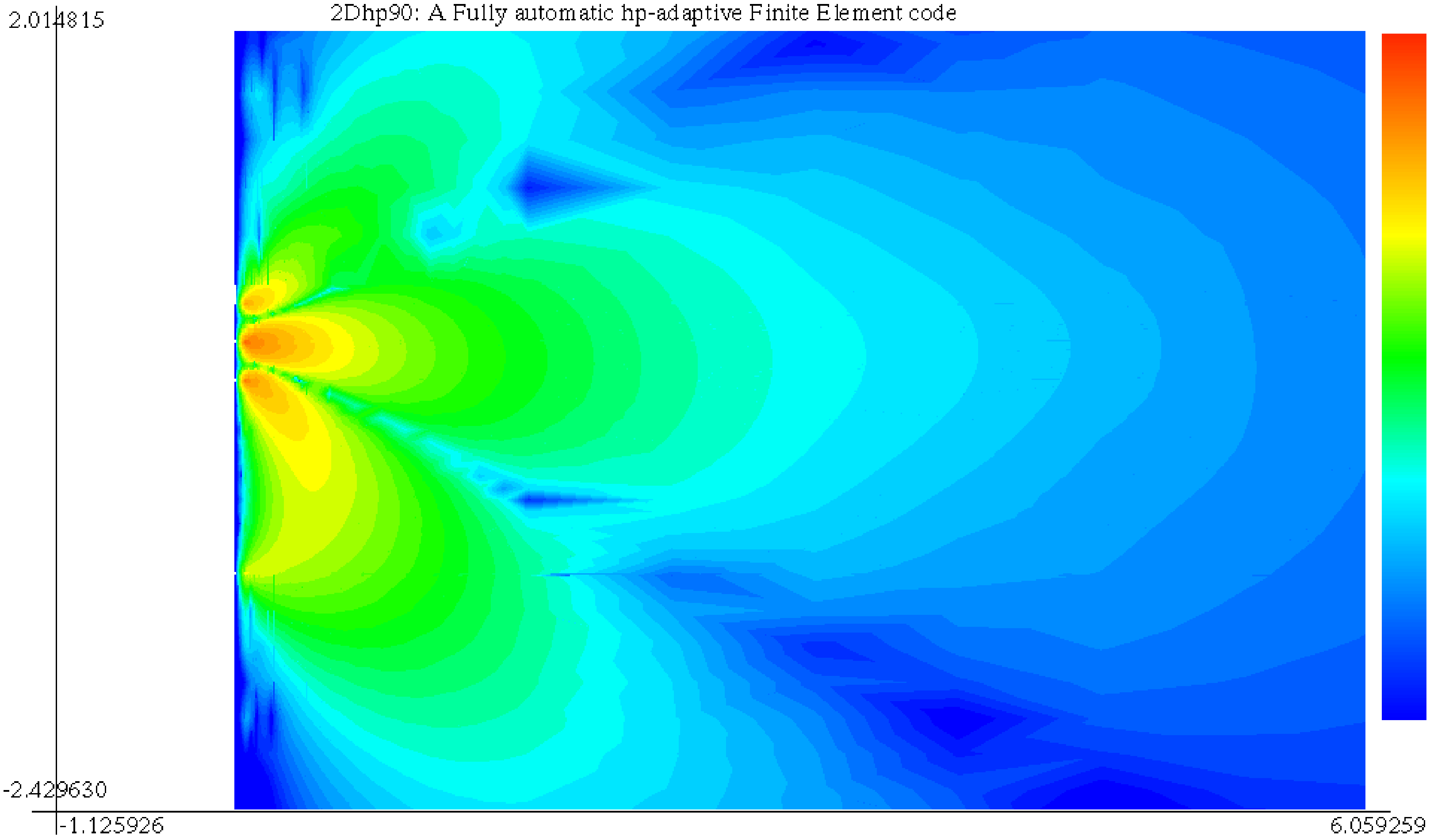
Jacobian Function: One TX, one RX

METHOD (COMPUTATION OF JACOBIAN)



Jacobian Function: One TX, two RXs

METHOD (COMPUTATION OF JACOBIAN)



Jacobian Function: One TX, three RXs

METHOD (COMPUTATION OF HESSIAN)

Computation of Hessian Matrix

Following a similar argument as for the Jacobian matrix, we obtain:

$$\frac{\partial^2 L_i(\hat{u}_i)}{\partial s_j \partial s_k} = -B\left(\frac{\partial \hat{u}_i}{\partial s_j}, \hat{v}_i, \frac{\partial h(s)}{\partial s_k}\right) - B\left(\hat{u}_i, \frac{\partial \hat{v}_i}{\partial s_j}, \frac{\partial h(s)}{\partial s_k}\right) - B\left(\hat{u}_i, \hat{v}_i, \frac{\partial^2 h(s)}{\partial s_j \partial s_k}\right)$$

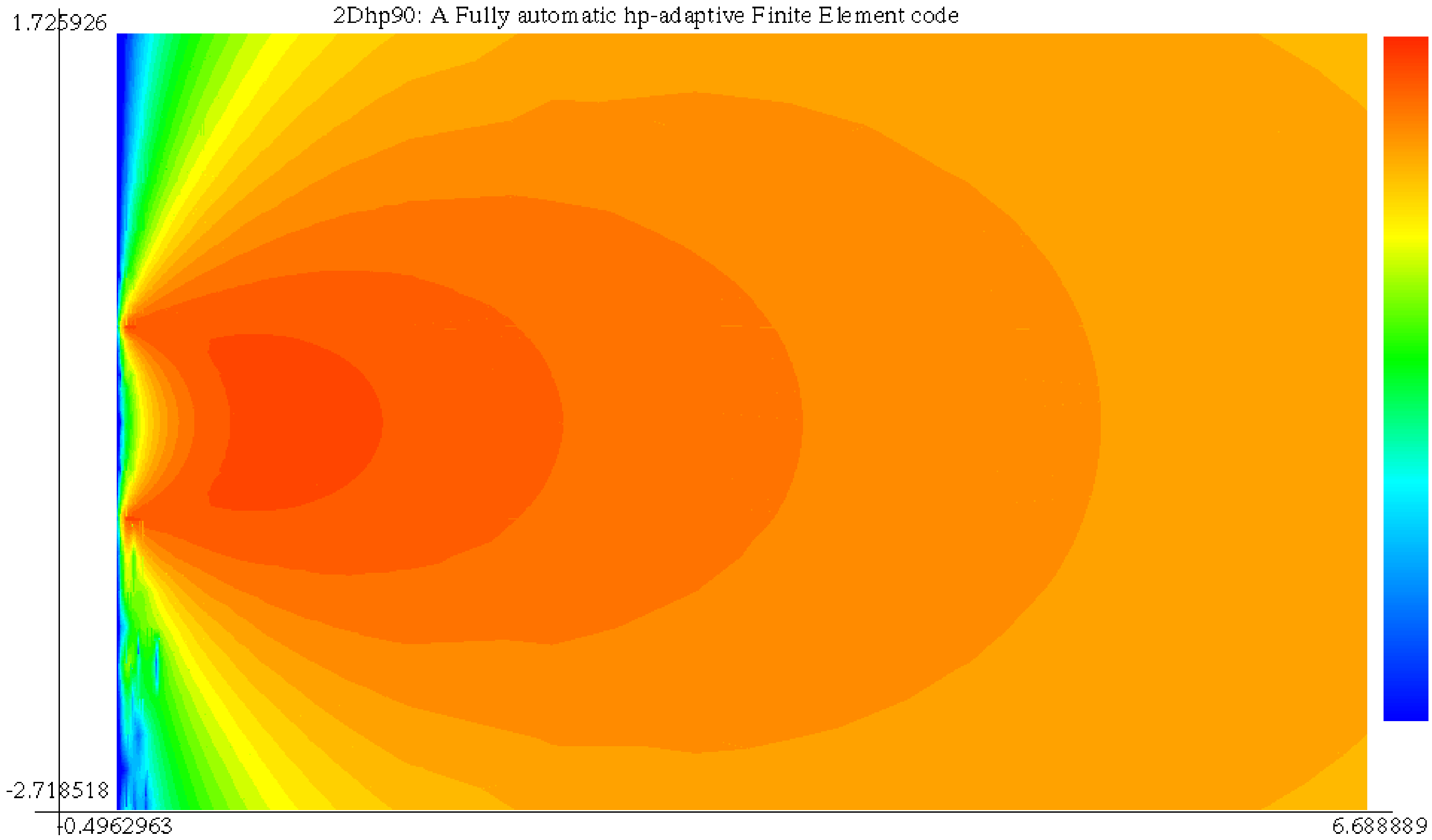
How do we compute $\frac{\partial \hat{u}_i}{\partial s_j}$ and $\frac{\partial \hat{v}_i}{\partial s_j}$?

Find $\frac{\partial \hat{u}_i}{\partial s_j}$ **such that:** $B\left(\frac{\partial \hat{u}_i}{\partial s_j}, v_i, h(s)\right) = -B\left(\hat{u}_i, v_i, \frac{\partial h(s)}{\partial s_j}\right) \quad \forall v_i$

Find $\frac{\partial \hat{v}_i}{\partial s_j}$ **such that:** $B\left(\frac{\partial \hat{v}_i}{\partial s_j}, u_i, h(s)\right) = -B\left(\hat{v}_i, u_i, \frac{\partial h(s)}{\partial s_j}\right) \quad \forall u_i$

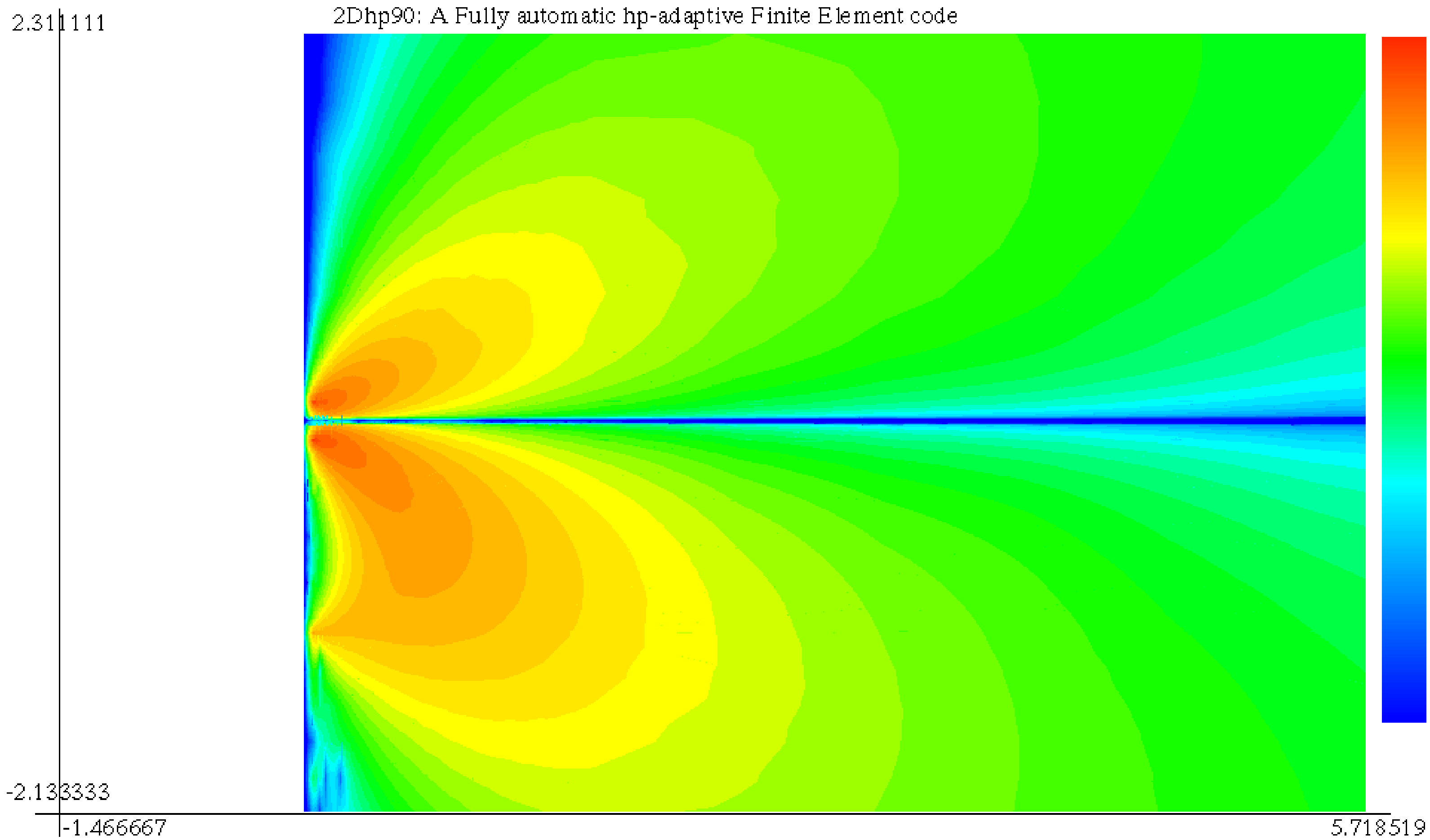
We can compute the Hessian matrix EXACTLY by just solving our original problem for different right-hand-sides, and performing additional integrations.

METHOD (COMPUTATION OF HESSIAN)



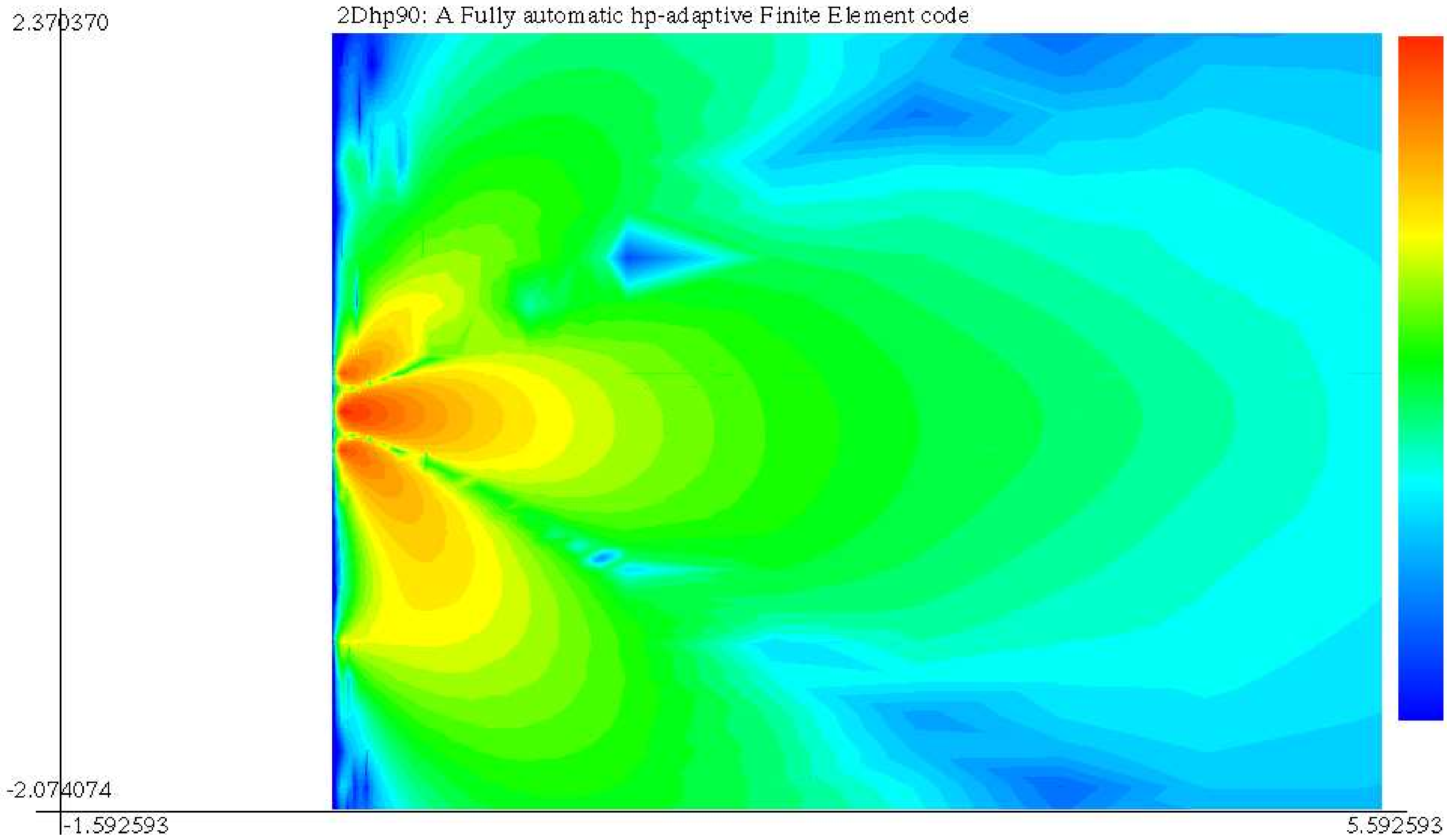
Hessian Function: One TX, one RX

METHOD (COMPUTATION OF HESSIAN)



Hessian Function: One TX, two RXs

METHOD (COMPUTATION OF HESSIAN)



Hessian Function: One TX, three RXs

METHOD (IMPLEMENTATION)

Main Implementation Features of the Inverse Library

- It consists of an additional independent module within the hp-Finite Element framework.
- It inherits all properties of the hp-Finite Element framework: parallel implementation, efficient forward solver, possibility of considering different logging instruments, frequencies, and/or physics, etc.
- It incorporates various inversion algorithms: Gauss-Newton, Newton-Raphson, arbitrary change of coordinates.
- The inverse grid is a subset of the forward grids.
- It enables the possibility of selecting during run-time the inverse elements and measurements that are going to participate in each step of the inversion procedure.

CONCLUSIONS AND FUTURE WORK

Conclusions

- **We propose to solve a joint multi-physics inverse problem based on solving several small "well-posed" problems, as opposed to one large "ill-posed" problem.**
- **We have finished with the implementation of phase I of a library that enables solution of inverse problems based on the above idea. Jacobian and Hessian matrix can be efficiently computed.**

Future Work

- **Expand the library to deal with real-life inverse problems.**
- **Perform additional numerical experimentation.**
- **Incorporate multi-physics measurements.**