# **Book of Abstracts**

# 2nd Bilbao Workshop on Fluid Dynamics

Organizing Committee: Arnab Roy & Arghir Zarnescu

Basque Center for Applied Mathematics (BCAM), Bilbao, November 3–5, 2025

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#### **Abstracts**

## Jone Apraiz (UPV/EHU)

Fluid-solid interaction inverse problems in one dimension

One of the reasons why the analysis and solution of inverse problems has increased a lot the last decades is that these studies are very important in many applications: elastography and medical imaging, seismology, potential theory, ion transport problems or chromatography and other similar fields. In this talk we will see some particular results we have obtained related to that area in the context of fluid dynamics. We will consider a one-dimensional fluid-solid interaction model governed by the Burgers equation with a time varying interface. This is a preliminary simplified version of other more complicated and more realistic models in higher dimensions. We will see the results we have obtained for the inverse problem of determining the shape of the interface from Dirichlet and Neumann data at one end point of the spatial interval. In particular, we will show uniqueness results and some conditional stability estimates. The main results that will be shown in this talk have been published in: *Inverse problems for one-dimensional fluid-solid interaction models*, Communications on Applied Mathematics and Computation, 2024.

## Jon Asier Bárcena (UPV/EHU)

Null controllability of a penalized Navier-Stokes system

In this talk we consider a Navier–Stokes system where the incompressibility condition is replaced by a penalization and prove its local null controllability. For that, we rely on the null controllability of the linearized system, that is, of the penalized Stokes system, which we also prove. All this is done with a uniform cost as the penalization parameter tends to 0. Finally, we present some open problems involving other kinds of penalizations.

#### Marco Bravin (University of Cantabria)

Non-scaling invariant boundary conditions for elliptic and parabolic PDE in a wedge

In this talk, I will focus my attention on two PDE on a wedge: the Stokes equation with Navier-slip boundary condition and the heat equation with Robin boundary condition. Both boundary conditions are scaling invariant and create difficulties in the study of a well-posedness result. The goal of this talk is, after presenting the two well-posedness results, to highlight the main differences between considering an elliptic or parabolic PDE in a wedge with non-scaling boundary condition. This is joint work with Gnann, Knüpfer, Masmoudi, Roodenburg, Sauer.

**References:** Bravin, M., Gnann, M. V., Knüpfer, H., Masmoudi, N., Roodenburg, F. B., & Sauer, J. (2024). Well-posedness of the Stokes equations on a wedge with Navier-slip boundary conditions. arXiv:2407.15517.

Bravin, M., Gnann, M. V., Knüpfer, H., Masmoudi, N., Roodenburg, F. B., & Sauer, J. (2025). Well-posedness and regularity of the heat equation with Robin boundary conditions in the two-dimensional wedge. *Comm. PDE*, 50(9), 1099–1134.

#### Miguel Escobedo (UPV/EHU)

Recent results on the mathematical description of a gas of bosons: the three waves equation

In a brief introduction I will present the mathematical description of a homogeneous, isotropic gas of bosons. I will motivate the three waves equation showing its relation with the Bose–

Einstein condensate. In the second part I will present some recent results on the existence and regularity of solutions to the three waves equation.

## Francesco Fanelli (BCAM)

The non-homogeneous Euler equations below the Lipschitz threshold

The incompressible Euler equations are well-known to be globally well-posed in the case d = 2, both in the strong solutions framework and in the Yudovich framework. No results of that kind are known for the non-homogeneous (density-dependent) incompressible Euler system.

In this talk, we show that both problems (global well-posedness and theory à la Yudovich for the density-dependent case) can be reduced to the study of a non-linear geometric quantity, which encodes the regularity of the velocity field along the level lines of the density. Such a geometric regularity places itself below the Lipschitz threshold.

#### Enrique Fernández-Cara (University of Sevilla)

Some uniqueness results for inverse problems concerning Stokes and Navier-Stokes fluids

This talk deals with some geometric inverse problems for systems of the Navier–Stokes kind. The aim is to identify the shape of a solid immersed in a fluid using boundary or internal observations. To this purpose, we use analyticity in time and unique continuation properties. We obtain uniqueness in cases where the information is not complete (the initial data and even the coefficients in the equations are unknown). We also consider related reconstruction algorithms. The results have been obtained in collaboration with J. Apraiz, A. Doubova and M. Yamamoto.

#### Daniel Lear (University of Cantabria)

Traveling and rotating solutions close to shear flows

In this talk, we address the existence of rotating solutions arbitrarily close (in a suitable topology) to radial monotone decreasing vorticities for the 2D Euler equations. In a paper by Bedrossian, Coti-Zelati, and Vicol, it was shown that radial monotone decreasing vorticities are linearly stable. Our result demonstrates, however, that this stability can break down even under small perturbations. This phenomenon is closely related to the stability of shear flows and to the existence of stationary and traveling wave solutions in their vicinity.

#### Marta Lewicka (University of Pittsburgh)

Tug of war games for nonlinear PDEs

The lecture will present an overview of the probabilistic interpretation of the nonlinear potential theory, relying on the notion of tug-of-war games with noise, and the asymptotic expansions of averages. We will explore constructions pertaining to the p-Laplacian, the non-local setting, and the various boundary conditions.

#### Claudiu Mindrila (BCAM)

Weak solutions for incompressible fluids interacting with linear elastic shells and Navier's slip boundary conditions

We prove the existence of weak solutions for an incompressible viscous fluid of Navier–Stokes type interacting with a linear cylindrical elastic shell. The coupling between fluid and membrane is of

Navier's slip type, very relevant in modelling and numerics. We go beyond the usual existence results available in the no-slip case in 2 or 3 dimensions, and the existence of weak solutions in the slip case in 2D/1D (proved by Muha and Canic, JDE, 2016) and extend the results in a 3D/2D setting. The result features new estimates and compactness arguments. Joint work with Arnab Roy (BCAM and Ikerbasque).

## Nicolas Moreno (BCAM)

Fully Lagrangian Heterogeneous Multiscale Modelling of Complex Polymeric Materials

We propose a fully Lagrangian heterogeneous multiscale method (LHMM) to model non-Newtonian fluids where microscopic effects strongly influence macroscopic behavior. The LHMM discretizes the fluctuating Navier—Stokes equations using the Smoothed Dissipative Particle Dynamics (SDPD) method, allowing microscopic information derived on-the-fly to directly provide the stress tensor in the macroscale momentum balance. This eliminates the need for approximate constitutive relations, while naturally incorporating memory effects of the microscale into the continuum description. We validate the LHMM for Newtonian and complex fluids through benchmark flow configurations and demonstrate its effectiveness for polymer melts and adhesive materials.

#### Rafael Granero-Belinchón (University of Cantabria)

TBA

Abstract to be announced.

# Thomas Richter (University of Magdeburg)

Vibration induced propulsion of rigid bodies in an incompressible fluid

We study the motion of a rigid body that is immersed in a fluid and exposed to a periodic force that has average zero. We investigate the fundamental question of whether it is possible to give the body a non-zero average velocity even though the acting forces add up to zero. We report on analytical results from our collaboration partner Paolo Galdi who studies this problem from the theoretical point of view. Then, we describe efficient numerical approaches to experimentally study the coupled problem. We discuss acceleration possibilities for identifying truly periodic solutions and the effects that lead to as large as possible net motion of the body.

#### Diego Araújo de Souza (University of Sevilla)

On the control of some PDEs from fluid mechanics

The Navier–Stokes equations and their variants have been studied for many years and their understanding is very relevant from both mathematical and physical viewpoints. On the other hand, the control of PDEs has brought a lot of attention in the last few decades, motivated by its relevant role in applications. In this talk we present some recent results dealing with the control of systems of the Navier–Stokes kind.