Kinetic Equation, Mathematical Physics and Probability

In memory of Stefan Adams to whom the conference is dedicated

BCAM-Basque Center for Applied Mathematics

Mazarredo 14, Bilbao, Basque Country Spain

17th-21st June, 2024

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PROGRAM

June 17, Monday	June 18, Tuesday	June 19, Wednesday	June 20, Thursday	June 21, Friday
	9h-10h20 E. Pulvirenti Lecture 1	9h-10h20 E. Pulvirenti Lecture 2	9h-10h20 S. Mischler Lecture 3	9h-9h55 S. Jansen
9h50-10h Presentation	**10h20-10h45 Coffee break**	**10h20-10h45 Coffee break**	**10h20-10h45 Coffee break**	**9h55-10h20 Coffee break**
10h-10h55 A. Guillin	10h45-11h15 C. Sánchez	10h45-11h40 A. Bianchi	10h45-11h15 T. Normand	10h20-10h50 S. Farhat
10h55-11h20 Coffee break	11h20 -11h50 M. Ben Said	11h45-12h40 L. Miclo	11h20-11h50 P. Gervais	10h55-11h25 S. Schraven
11h20-12h15 G. dos Reis	11h55-12h50 A. Frouvelle	**FREE**	11h55-12h50 M. Escobedo	11h30-12h25 C. Landim
LUNCH	**LUNCH**	**FREE**	**LUNCH**	**LUNCH**
14h -15h20: S. Mischler Lecture 1	14h30-15h50 S. Mischler Lecture 2	**FREE**	14h30-15h50 E. Pulvirenti Lecture 3	
15h20h-15h45: Coffee break	**15h50-16h15 Coffee break**		**15h50-16h15 Coffee break**	
15h45-16h15 M. André	16h15-17h10 O. Mohsen		16h15-17h10 P.A. Zitt	
16h20-16h50 L. Journel	17h15-18h10 A. Trescases		17h15-18h10 TBD	
16h55-17h50 M. Bonnefont				
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Book of Abstracts

June 13, 2024

Abstract

Originally used to describe gas dynamics, kinetic equations are today used in many fields of sciences (physics, chemistry, biology, materials sciences, epidemiology, etc.). Their study involves a wide mathematical palette ranging from modeling to theoretical study through numerical analysis. One of the main challenges is the study of the dynamic properties of these equations, particularly in a long time. Obtaining precise quantitative information is also a current challenge in theory. Recently significant progress has been made in the study of these models using tools from many areas of mathematics: PDE, Mathematical Physics, and Probability. This conference aims to bring together global experts on these themes to take stock of the latest advances and bring new issues to the surface. This is also the closing conference of the ANR QuAMProcs project (https://quamprocs.math.u-bordeaux.fr/projet/)

Contents

1 Minicourses

Stéphane Mischler, CEREMADE, Université Paris Dauphine-PSL & IUF

Title: *On the kinetic Fokker-Planck equation in a domain*
Schedule:
June 17, 14h-15h20
June 18, 14h30-15h50
June 20, 9h-10h20
Abstract: My lecture splits into three parts:

1. *Parabolic equations and the De Giorgi-Nash-Moser approach*. In a first talk, I will present some classical techniques for establishing gain of integrability estimates (first De Giorgi lemma) for solutions to parabolic equations with possibly rough coefficients. I will also discuss the link with regularity estimates (and the second De Giorgi Lemma), the Harnack estimate, the ultracontractivity property and other related results.

2. *Kinetic Fokker-Planck equation*. In a second talk, I will explain how the same kind of results can be generalized to kinetic Fokker-Planck equations set in the whole space and how these ones are useful for tackling existence, uniqueness and longtime behavior issues for related nonlinear and linear PDEs.

3. *Kinetic Fokker-Planck equation in a domain*. The last talk will be focused on the case of the kinetic Fokker-Planck equation set in a domain with different kinds of boundary conditions. I will show how the above techniques can be generalized to that framework and how one can establish the ultracontractivity of the associated semigroup and get a quantitative analysis of its longtime behaviour.

Elena Pulvirenti, Delft Institute of Applied Mathematics (DIAM), TU Delft

Title: *Metastability of interacting particle systems: from discrete to continuum models*
Schedule:
June 18, 9h-10h20
June 19, 9h-10h20
June 20, 14h30-15h50
Abstract: The seemingly stable behaviour of a system which, after a long time and upon reaching a critical configuration suddenly transitions to its true stable state is called *metastability*. The distinguishing feature of metastability is that a dynamical system exhibits a very different behaviour on different time scales: while the system quickly reaches a metastable state, it takes a very long time for it to transition to the stable one.

- *Lecture 1.* I will start by heuristically explaining the phenomenon of metastability and its universality. Then, I will explain how one can model this behaviour in the setting of Markov processes and how one can study it mathematically with the help of *potential theory*. This approach, initiated around 2000, interprets the metastability phenomenon as a sequence of visits by the path to different metastable sets and focuses on the precise analysis of the respective hitting probabilities and hitting times of these sets. The key

to estimating these mean hitting times is to express them in terms of two fundamental objects from potential theory: the *capacity* and the *equilibrium potential*. Estimates on the former can be obtained with the help of well-known variational principles (such as the Dirichlet and Thomson principles), while estimates on the latter are generally more involved. I will conclude by applying these methods to two simple examples: the *random walk* and the *Curie-Weiss* model for ferromagnets.

- *Lecture 2*. I will discuss the metastable behaviour of three types of mean-field-like spin systems with *random couplings*: the disordered Curie-Weiss model, a general spin system with inhomogeneous bond disorder and the dilute Potts model. This class of models comprises, for example, both the Ising model and the Potts model on inhomogeneous dense random graphs. I will then present quantitative estimates of metastability in large volumes at fixed temperatures when these systems evolve according to a *Glauber dynamics*, i.e. where spins flip with Metropolis rates. A first result identifies conditions ensuring that with high probability the system behaves like the corresponding system where the random couplings are replaced by their averages. More precisely, we prove that the metastability of the former system is implied with high probability by the metastability of the latter. Moreover, I will show relevant metastable hitting times of the two systems and find the asymptotic tail behaviour and the moments of their ratio. Based on joint works in collaboration with Anton Bovier (Bonn), Johan Dubbeldam (Delft), Frank den Hollander (Leiden), Vicente Lenz (Delft), Saeda Marello (Bonn) and Martin Slowik (Mannheim).

- *Lecture 3*. I will introduce a model of fluids in the continuum called the *Widom-Rowlinson* model. The energy of a particle configuration is determined by its halo, given by the union of unit discs centred at the positions of the particles. I will show that this model has a dual representation and exhibits a 'gas-liquid' phase transition.
  I will then introduce a dynamic version of the Widom-Rowlinson model, where particles are randomly created and annihilated inside a torus as if the system were in contact with an infinite reservoir of particles. I will then discuss the metastable behaviour of the WR model, when the dynamics starts from an empty torus. In particular, I will focus on the first time when the torus is fully covered by unit discs, which can be viewed as the crossover time from a 'gas phase' to a 'liquid phase'.
  I will show how in order to achieve the transition from empty to full, the system needs to create a sufficiently large droplet, called *critical droplet*, which triggers the crossover. It turns out that the critical droplet is close to a disc of a certain deterministic radius, with a boundary that is random and consists of a large number of unit discs that stick out by a small distance. A precise analysis of the *surface fluctuations* allows us to derive both a volume term and a surface term in the asymptotics of the average crossover time.
  Based on joint works in collaboration with Frank den Hollander (Leiden), Sabine Jansen (Munich) and Roman Kotecky (Prague).

2 Scientific Talks

2.1 *June 17, Monday*

Arnaud Guillin, Université Clermont-Auvergne

Title: *Convergence to equilibrium of some non reversible process via Lift/Collapse and Flow Poincaré inequality*
Schedule: 10h-10h55
Abstract: Convergence to equilibrium of the underdamped Langevin process or of some piecewise deterministic sampler has attracted many interesting methodologies starting from the works of Herau and then Villani on hypocercivity and of Dolbeault-Mouhaut-Schmeiser. If quantitative rates can be achieved, they often are not of the correct order being of the order of the spectral gap of the invariant measures. Recently Albritton-Armstrong-Mourrat-Novack have derived a variational approach which has enabled Cao-Lu-Wang or Lu-Wang to obtain correct order wrt the spectral gap or dimension dependance. By using the concept of lift/collapse processes, we present here a general methodology which allows for quite diverse collapse process (overdamped Langevin process, sticky Brownian motion, ...) and prove convergence to equilibrium, via a novel Flow Poincaré inequality, of underdamped Langevin process, PDMP sampler or jamming particles.
(In collaboration with Andreas Eberle, Leo Hahn, Francis Loerler, Manon Michel)

Goncalo dos Reis, University of Edinburgh

Title: *High order splitting methods for stochastic differential equations*
Schedule: 11h20-12h15
Abstract: In this talk, we will discuss how ideas from rough path theory can be leveraged to develop high order numerical methods for SDEs. To motivate our approach, we consider what happens when the Brownian motion driving an SDE is replaced by a piecewise linear path. We show that this procedure transforms the SDE into a sequence of ODEs – which can then be discretized using an appropriate ODE solver. Moreover, to achieve a high accuracy, we construct these piecewise linear paths to match certain "iterated" integrals of the Brownian motion. At the same time, the ODE sequences obtained from this path-based approach can be interpreted as a splitting method, which neatly connects our work to the existing literature. For example, we show that the well-known Strang splitting falls under this framework and can be modified to give an improved convergence rate. We will conclude the talk with a couple of examples, demonstrating the flexibility and convergence properties of our methodology.
(joint work with James Foster and Calum Strange).

See https://arxiv.org/abs/2210.17543 and https://epubs.siam.org/doi/10.1137/23M155147X)

Morgan André, Universidade de São Paulo

Title: *A Quasi-Stationary Approach to Metastability in a System of Spiking Neurons with Synaptic Plasticity*
Schedule: 15h45-16h15
Abstract: After reviewing the behavioral studies of working memory, we argue that metastable states constitute candidates for the type of transient information storage required by working

memory. We then present a simple stochastic model of neural network whose synapses exhibit short-term facilitation. The metastable properties of this system are studied quantitatively, through the analysis of its quasi-stationary distribution. We will present both analytical and computational results illustrating the usefulness of the theory of quasi-stationary distributions in the study of metastable dynamics.

Lucas Journel, Laboratoire Jacques-Louis Lions, Sorbonne Université, Paris

Title: *The sampling of singular Gibbs measure*
Schedule: 16h20-16h50
Abstract: In this talk, we will be interested in sampling the Gibbs measure

$$\mu(dx) = e^{-\beta U(x)}/Z dx,$$

for singular potential $U : \mathbb{R}^d \to \mathbb{R}+$. I will present how to derive a 'Talay-Tubaro' expansion of the invariant measure for some numerical scheme of the Langevin Process

$$\begin{cases} dX_t = Y_t dt \\ dY_t = -\nabla U(X_t)dt - \gamma Y_t dt + \sqrt{2\gamma\beta^{-1}} dB_t \,, \end{cases}$$

for a class of potential containing the case of n particles with pairwise interaction through the Lennard-Jones potential, in some confining potential. This involves studying the long-term behavior of the semi-group of the Langevin process in weighted Sobolev spaces.

Michel Bonnefont, Université Bordeaux

Title: *Intertwinning and Poincaré inequalities on domains*
Schedule: 16h55-17h50
Abstract: In this talk, I will discuss a duality method to obtain explicit constants for (log-concave) measures on domains in $\mathbb{R}^d$. The method is based on intetwinning between gradients and the generators on functions and on gradients. This is a joint work with Aldéric Joulin.

2.2 *June 18, Tuesday*

Claudia Fonte Sanchez, INRIA Grenoble

Title: *On the voltage-conductance kinetic equation*
Schedule: 10h45-11h15
Abstract: This talk delves into the non-linear kinetic Voltage-Conductance equation, a crucial mathematical model for understanding neuronal activity. We obtain two key results in a framework suitable for weak interactions. First, we establish the existence of solutions. Second, we prove linear asymptotic exponential stability of the steady state. This stability result builds upon a recent estimate by Dou (2023) and offers a constructive approach. Both results are based in a fundamental way on some ultracontractivity property of the flow associated to the linear of the Voltage-Conductance equation.

Mona Ben Said, Faculty of Sciences of Monastir (Tunisia)

Title: *Compactness of the Resolvent for Degenerate Kramers-Fokker-Planck Operators*
Schedule: 11h20-11h50
Abstract: In this talk, I will discuss recent contributions to the spectral theory of Kramers-Fokker-Planck operators with a degenerate potential, in that I will present sufficient conditions for the compactness of the resolvent, a fundamental problem related to the Helffer-Nier conjecture, which links the compactness of the resolvent of the Kramers-Fokker-Planck operator with that of the associated Witten Laplacian on zero-forms.

Amic Frouvelle, Ceremade (Université Paris Dauphine - PSL)

Title: *Hypocoercivity in a model of aligning self-propelled particles*
Schedule: 11h55-12h50
Abstract: This talk is about a joint work with Emeric Bouin. We are interested in the long-time behavior of the kinetic Vicsek equation with phase transition, in its spatially localized version. The method follows the now-classic approach of modifying Sobolev-type norms by adding cross-terms, linked to commutators between the different operators appearing in the kinetic equation. However, the fact that the velocity space is the sphere adds significant subtleties and requires to develop an adapted algebraic framework of operators. Taking advantage of this new framework, we manage to perform an approach à la Hérau to show the nonlinear stability.

Omar Mohsen, Paris-Saclay University

Title: *On maximally hypoelliptic differential operators*
Schedule: 16h15-17h10
Abstract: The class of Maximally hypoelliptic differential operators is a large class of differential operators that contains elliptic operators as well as Hörmander's sum of squares operators. I will present our work where we define a principal symbol generalising the classical principal symbol for elliptic operators. I will then state our main theorem showing that maximal hypoellipticity is equivalent to invertibility of our principal symbol, thus generalising the main regularity theorem for elliptic operators and confirming a conjecture of Helffer and Nourrigat. This is joint work with Androulidakis and Yuncken.

Ariane Trescases, Université Paul Sabatier, Toulouse

Title: *Long-time behaviour of Keller-Segel type systems modeling local sensing*
Schedule: 17h15-18h10
Abstract: We consider a class of cross-diffusion systems modeling chemotactic aggregation with local sensing. We will see that while reminiscent of the classical (minimal) Keller-Segel system, which may exhibit blow-up in finite time, this class of system typically possesses global-in-time solutions. Using entropy and duality methods, we discuss their rich long-time behaviour. We will also discuss possible extensions in presence of nonlinear self-diffusion.

2.3 *June 19, Wednesday*

Alessandra Bianchi, Università degli Studi di Padova

Title: *Mixing cutoff for simple random walks on the Chung-Lu directed graph*
Schedule: 10h45-11h40
Abstract: In this talk, we consider a simple random walk defined on a Chung-Lu directed graph, an inhomogeneous random network that extends the Erdos Renyi random digraph by including edges independently according to given Bernoulli laws, so that the average degrees are fixed. In this non-reversible setting, our focus is on the convergence toward the equilibrium of the dynamics.

In particular, under the assumption that the average degree grows logarithmically in the size n of the graph (weakly dense regime), we establish a cutoff phenomenon at the entropic time of order $\log(n)/\log\log(n)$. Moreover, we prove that on a precise window, the cutoff profile converges to the Gaussian tail function. This is qualitatively similar to what was proved in a series of works by Bordenave, Caputo, Salez for the directed configuration model, where degrees are deterministically fixed. In terms of statistical ensembles, our analysis provides an extension of these cutoff results from a hard to a soft-constrained model.

Laurent Miclo, CNRS, Toulouse School of Economics and Institut de Mathématiques de Toulouse

Title: *On fraudulent stochastic algorithms*
Schedule: 11h45-12h40
Abstract: We introduce and analyse the almost sure convergence of a stochastic algorithm for the global minimization of Morse functions on compact Riemannian manifolds.

This diffusion process is called fraudulent because it requires the knowledge of minimal value of the function. Its investigation is nevertheless important, since in particular it appears as the limit behavior of non-fraudulent and time-inhomogeneous swarm mean-field algorithms for global optimization or in stochastic gradient descent algorithms in overparametrized learning applications. The talk is based on collaborations with Benaïm, Bolte and Villeneuve.

2.4 *June 20, Thursday*

Thomas Normand, Université de Nantes

Title: *One-well metastability for an inelastic linear Boltzmann operator*
Schedule: 10h45-11h15
Abstract: We consider an inhomogeneous linear Boltzmann equation in a low temperature regime, in the presence of an external force deriving from a single-well potential and with a collision operator featuring multiple conservation laws.

We start by giving a description of the purely imaginary spectrum of the associated operator.

We then go further and provide a hypocoercive result on the spectrum with real part smaller than h.

It enables us to obtain some information on the long time behavior of the solutions and in particular to show the existence of metastable states.

Pierre Gervais, Université de Lille

Title: *Hydrodynamic limit of elastic kinetic equations by a spectral approach*
Schedule: 11h20-11h50
Abstract: Among the 23 problems listed by D. Hilbert during the International Congress of Mathematicians in 1900, the 6th one concerns the derivation of macroscopic descriptions of fluids from their microscopic descriptions. One possible strategy involves going through an intermediate level of description called mesoscopic, or kinetic, such as the Boltzmann or Landau models. This is referred to as the problem of hydrodynamic limits.

In the early 1990s, C. Bardos, F. Golse, and D. Levermore proved that one could formally derive the Navier-Stokes equations from kinetic equations conserving mass, velocity, and energy, and dissipating entropy, and the specific cases of the Boltzmann and Landau equations were gradually and independently addressed over the following three decades, despite their common structure.

The work on hydrodynamic limits is partly constrained by tools dating back to the early days of Boltzmann theory in the 1960s, allowing only for solutions satisfying a very restrictive integrability assumption, but also by results established using non-constructive arguments. In the case of Cauchy theories of kinetic equations, these restrictions have been lifted thanks to modern tools of "enlargement theory" and hypocoercivity methods developed from the 2000s onwards, notably by C. Mouhot, S. Mischler, and M. Gualdani.

In this talk, I present a collaboration with Bertrand Lods in which we have, on the one hand, considered the question of hydrodynamic limit for a kinetic equation **under generic assumptions** close to those of Bardos-Golse-Levermore, thus unifying previous results, and, on the other hand, modernized the necessary spectral study using the new theories of enlargement and hypocoercivity, thus providing the first fully quantitative results of hydrodynamic limits.

Miguel Escobedo, University of the Basque Country UPV/EHU

Title: *Recent results on wave turbulence for the Schrödinger equation*
Schedule: 11h55-12h50
Abstract: Kinetic wave equations are obtained as mathematical description of large systems of weakly interacting nonlinear waves. They share some similarities, but also present strong differences with the classical kinetic equations for particles. I will present recent results for the case where the waves obey the cubic Schrödinger equation.

Pierre André Zitt, Laboratoire d'Analyse et de Mathématiques Appliquées, Université Paris Est-Marne la Vallée

Title: *Reducing variance by reweighting samples*
Schedule: 16h15-17h10
Abstract: We discuss a variance reduction method adapted to cases where the variable of interest is correlated with a variable whose distribution is exactly known. The method may be seen as a generalization of control variates and post-stratification. Joint work with M. Rousset and Y. Xu.

2.5 *June 21, Friday*

Sabine Jansen, LMU Munich

Title: *Duality, interacting particle systems, and infinite-dimensional orthogonal polynomials*
Schedule: 9h-9h55
Abstract: Studying the time-evolution of a many-particle system is a difficult task. For some interacting particle systems in $\mathbb{Z}^d$, duality and intertwining allow to map the time evolution of one- or two-point correlation functions of a many-particle system to the time evolution of a one- or two-particle system, a considerable simplification. Often duality functions are products of univariate orthogonal polynomials, one for each site of the lattice. In the talk I will explain how to generalize these dualities, and the algebraic approach with representations of Lie algebras, to particles in $\mathbb{R}^d$. This brings in Lévy point processes and infinite-dimensional orthogonal polynomials. Based on joint work with Simone Floreani, Frank Redig and Stefan Wagner.

Shahnaz Farhat, Constructor University

Title: *Quantum-classical motion of charged particles interacting with scalar fields*
Schedule: 10h20-10h50
Abstract: The goal of this talk is to investigate the dynamics of relativistic or non-relativistic charged particles in interaction with a scalar meson field. Our main contribution is the derivation of the classical dynamics of a particle-field system as an effective equation of the quantum microscopic Nelson model, in the classical limit where the value of the Planck constant approaches zero ($\hbar \to 0$). We use a Wigner measure approach to study such transition. Then, as a consequence of this interplay between classical and quantum dynamics, we establish the global well-posedness of the classical particle-field interacting system, despite the low regularity of the related vector field, which prevents the use of a fixed point argument.

Severin Schraven, Technical University of Munich

Title: *Counting eigenvalues of Schr¨odinger operators using the landscape function*
Schedule: 10h55-11h25
Abstract: We prove an upper and a lower bound on the rank of the spectral projections of the Schrödinger operator $-\Delta + V$ in terms of the volume of the sublevel sets of an effective potential u . Here, u is the 'landscape function' of David-Filoche-Mayboroda (2021), namely a solution of $(-\Delta + V)u = 1$ in $\mathbb{R}^d$. We prove the result for non-negative potentials satisfying a Kato-type and a doubling condition, in all spatial dimensions, in infinite volume, and show that no coarse graining is required. Our result yields in particular a necessary and sufficient condition for discreteness of the spectrum. This is joint work with S. Bachmann and R. Froese.

Claudio Landim, IMPA

Title: *Metastability from two different point of views*
Schedule: 11h30-12h25
Abstract: In this lecture I will discuss the relation between Gamma-convergence and the asymptotic behavior of the solutions of resolvent equations with metastability.