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A HP Fourier-Finite-Element Framework with Multiphysics Applications

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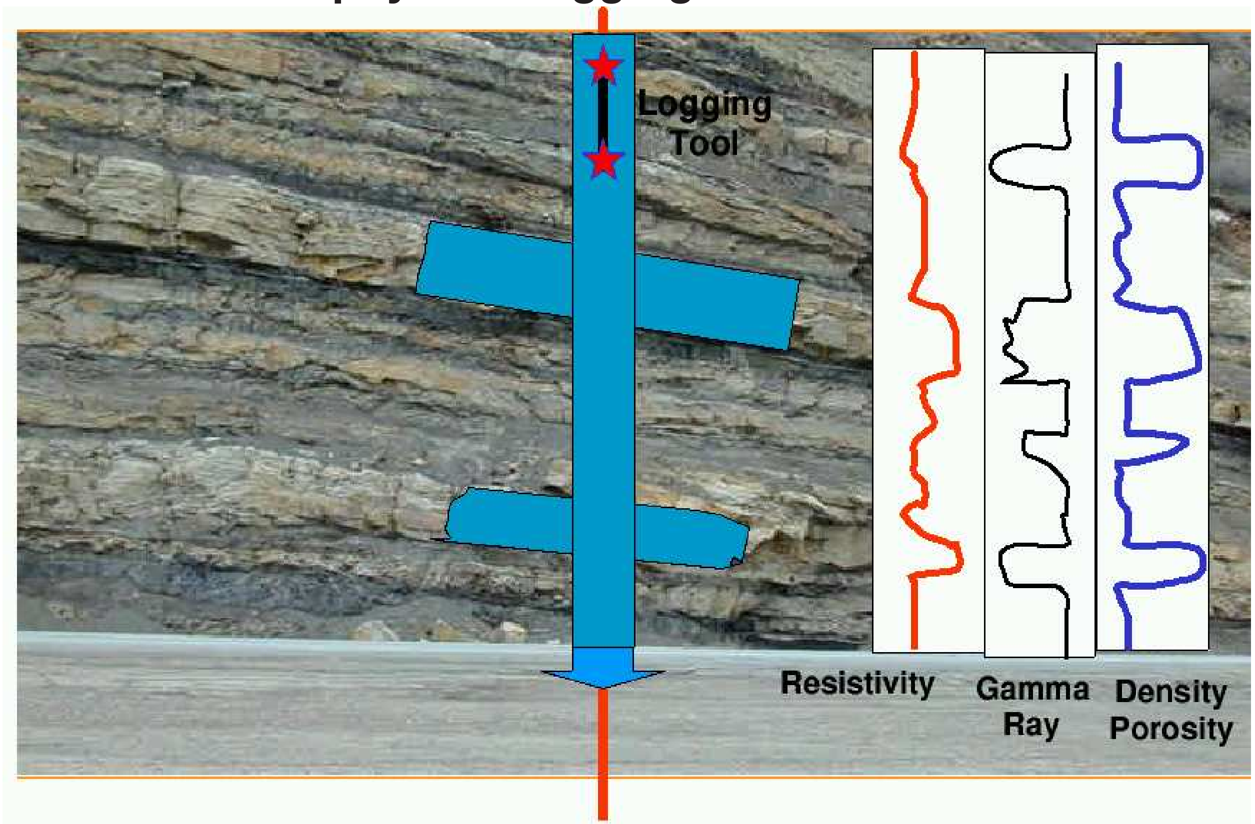
overview

1. **Motivation and Objectives: Joint Multiphysics Inversion.**
2. **Method:**
   - **Parallel Self-Adaptive Goal-Oriented hp Fourier Finite Element Method.**
   - **De Rham Diagram.**
   - **Electromagnetic and Sonic Applications.**
3. **Conclusions.**
4. **Future Work.**

motivation and objectives

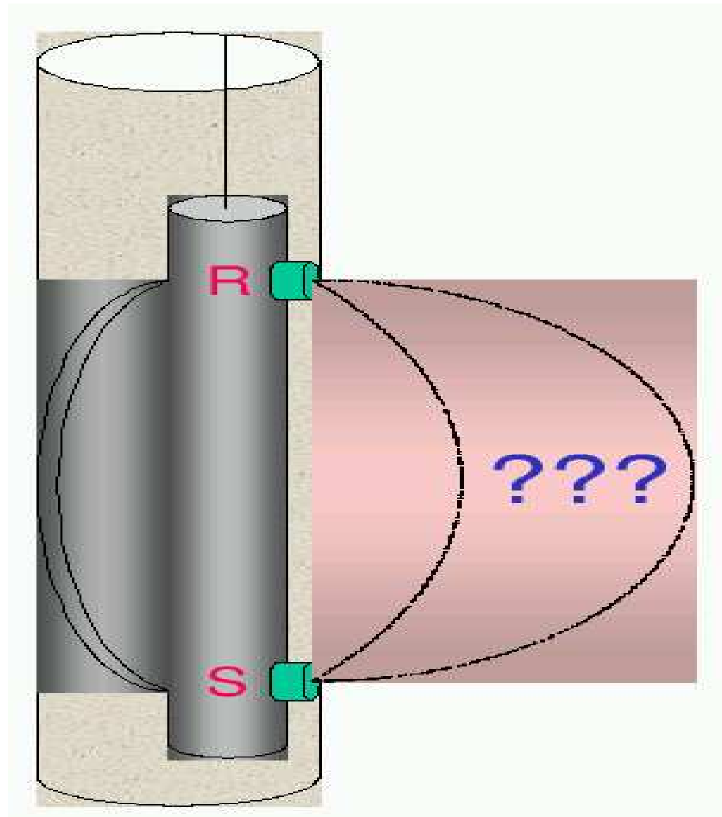
Multiphysics Logging Measurements



OBJECTIVES: To determine payzones (porosity), amount of oil/gas (saturation), and ability to extract oil/gas (permeability).

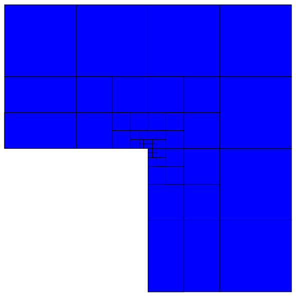
motivation and objectives

Main Objective: To Solve a Multiphysics Inverse Problem



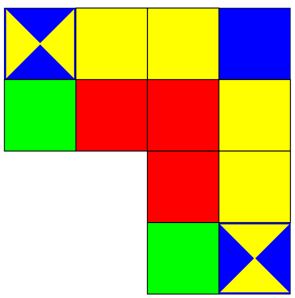
Software to solve the DIRECT problem is essential in order to solve the INVERSE problem.

hp finite element method



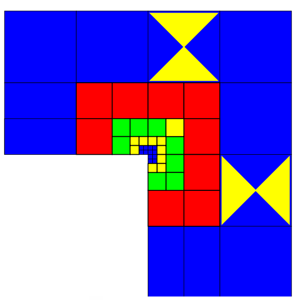
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal h-grids do NOT converge exponentially in real applications.
3. They may “lock” (100$\%$ error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical (“nice”) solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.

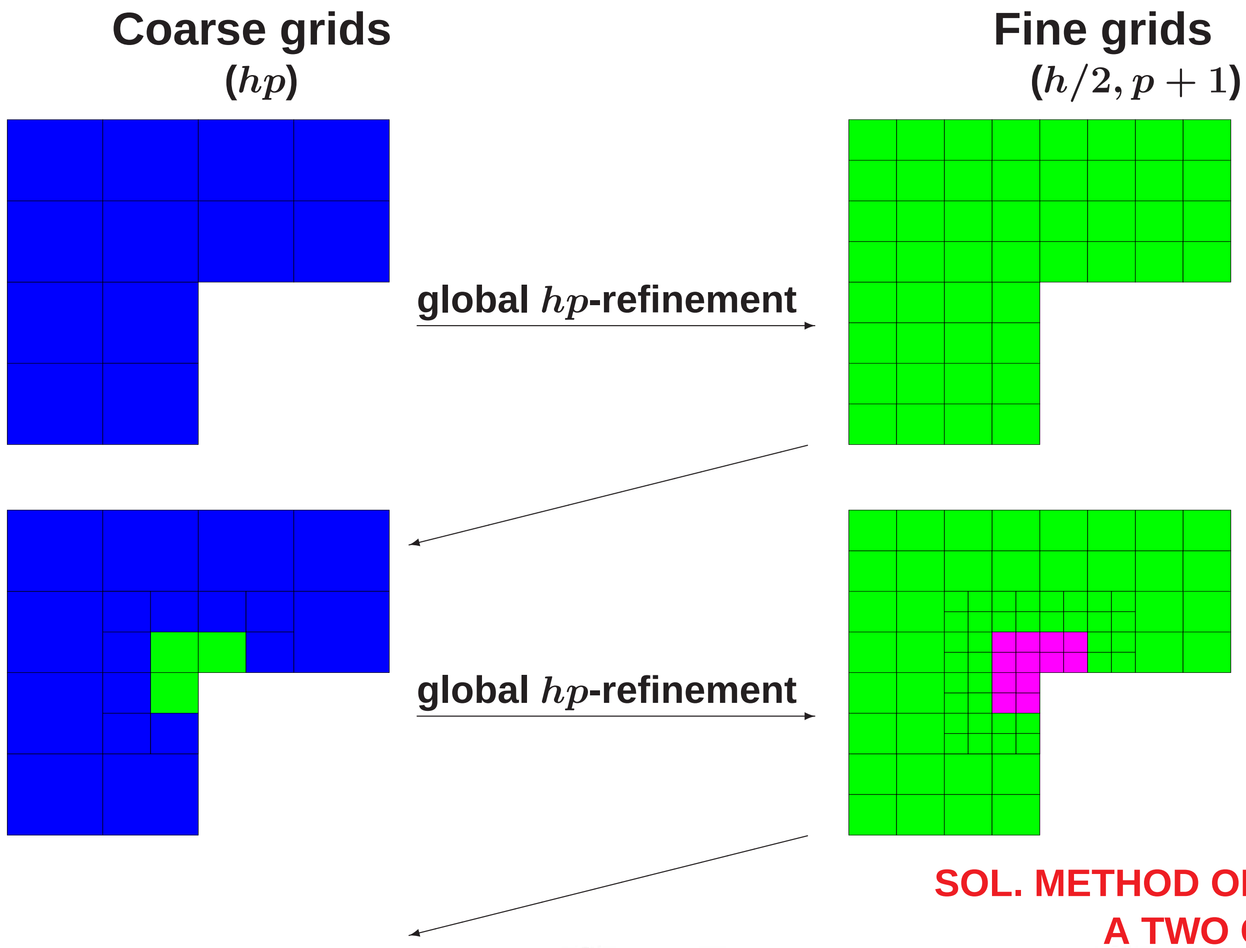


The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

hp adaptivity

Energy norm based fully automatic *hp*-adaptive strategy



SOL. METHOD ON FINE GRIDS:
A TWO GRID SOLVER

hp goal oriented adaptivity

Mathematical Formulation (Goal-Oriented Adaptivity)

We consider the following problem (in variational form):

$$\begin{cases} \text{Find } L(\Psi)\text{, where } \Psi \in V \text{ such that :} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \ . \end{cases}$$

We define residual $r_e(\xi) = b(e, \xi)$. We seek for solution G of:

$$\begin{cases} \text{Find } G \in V'' \sim V \text{ such that :} \\ G(r_e) = L(e) \ . \end{cases}$$

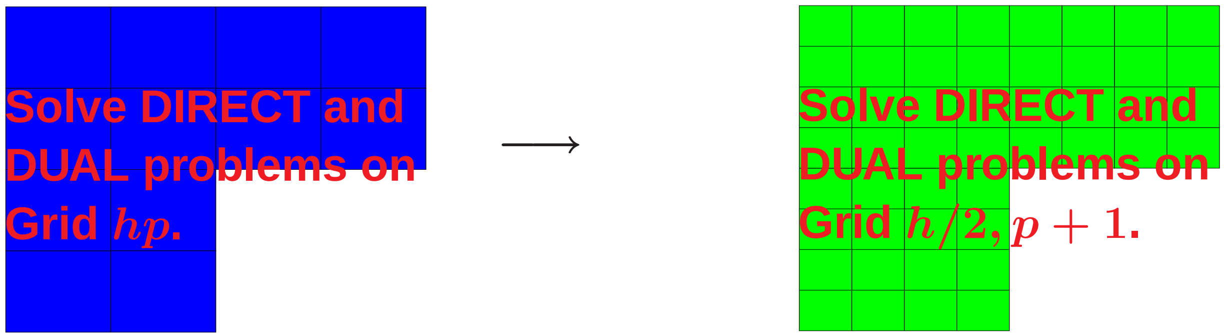
This is necessarily solved if we find the solution of the *dual* problem:

$$\begin{cases} \text{Find } G \in V \text{ such that :} \\ b(\Psi, G) = L(\Psi) \quad \forall \Psi \in V \ . \end{cases}$$

Notice that $L(e) = b(e, G)$.

hp goal oriented adaptivity

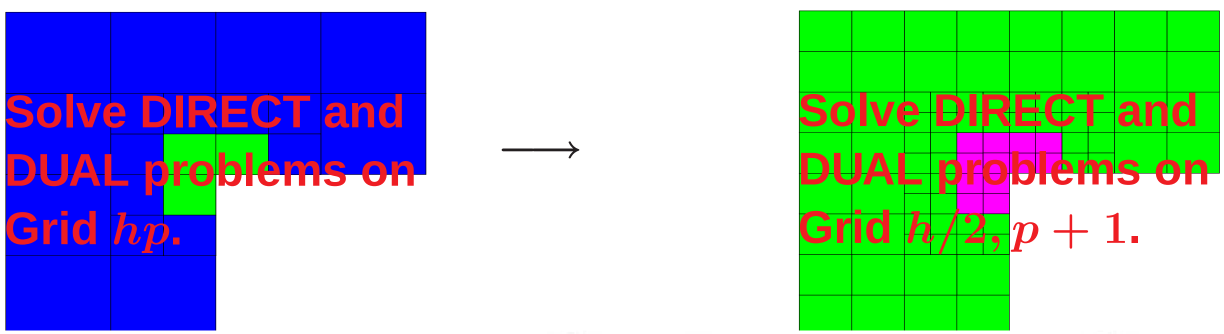
Algorithm for Goal-Oriented Adaptivity



Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, and $\epsilon = G_{h/2,p+1} - G_{hp}$.

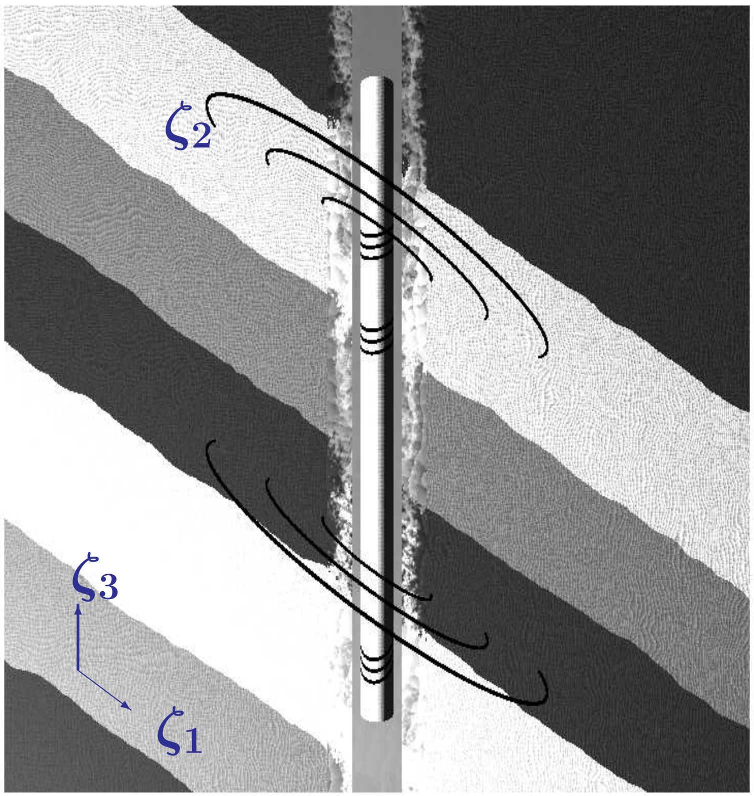
Represent the error as: $|L(e)| = |b(e,\epsilon)| \leq \sum_K |b_K(e,\epsilon)|$.

Apply the fully automatic hp-adaptive algorithm.



Fourier finite element method

Non-Orthogonal System of Coordinates



Fourier Series Expansion in ζ_2

DC Problems: $-\nabla \sigma \nabla u = f$

$$u(\zeta_1, \zeta_2, \zeta_3) = \sum_{l=-\infty}^{l=\infty} u_l(\zeta_1, \zeta_3) e^{jl\zeta_2}$$

$$\sigma(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-\infty}^{m=\infty} \sigma_m(\zeta_1, \zeta_3) e^{jm\zeta_2}$$

$$f(\zeta_1, \zeta_2, \zeta_3) = \sum_{n=-\infty}^{n=\infty} f_n(\zeta_1, \zeta_3) e^{jn\zeta_2}$$

Fourier modes $e^{jl\zeta_2}$ are orthogonal high-order basis functions that are (almost) invariant with respect to the gradient operator.

de Rham diagram

De Rham diagram

De Rham diagram is critical to the theory of FE discretizations of multi-physics problems.

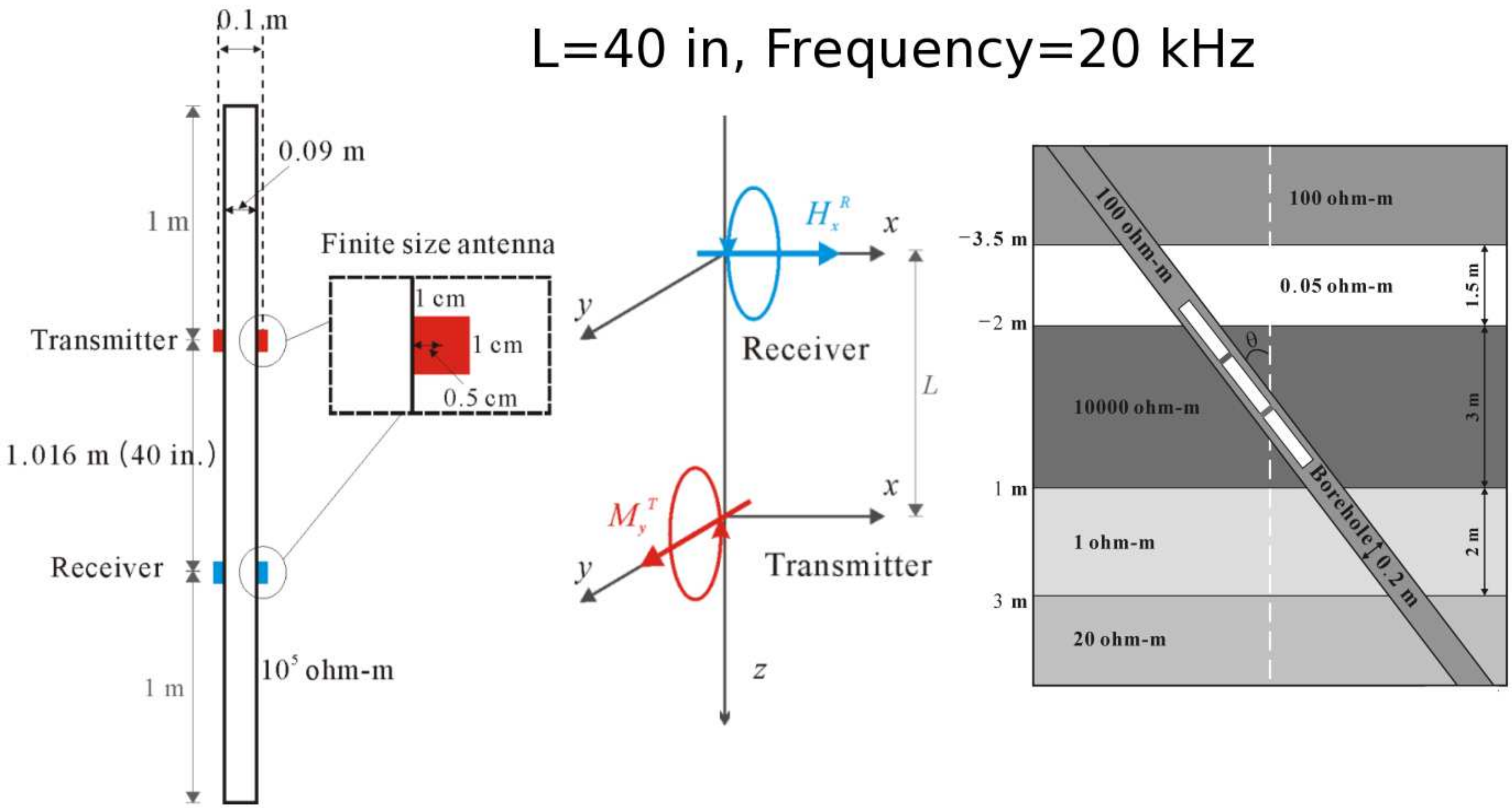
$$
\begin{array}{ccccccccccc}
\mathbb{R} & \longrightarrow & W & \xrightarrow{\nabla} & Q & \xrightarrow{\nabla\times} & V & \xrightarrow{\nabla\circ} & L^2 & \longrightarrow & 0 \\
\Big\downarrow id & & \Big\downarrow \Pi & & \Big\downarrow \Pi^{\mathrm{curl}} & & \Big\downarrow \Pi^{\mathrm{div}} & & \Big\downarrow P & & \\
\mathbb{R} & \longrightarrow & W^{\mathrm{p}} & \xrightarrow{\nabla} & \mathrm{Q}^{\mathrm{p}} & \xrightarrow{\nabla\times} & \mathrm{V}^{\mathrm{p}} & \xrightarrow{\nabla\circ} & W^{\mathrm{p}-1} & \longrightarrow & 0\,.
\end{array}
$$

This diagram relates two exact sequences of spaces, on both continuous and discrete levels, and corresponding interpolation operators.

electromagnetic applications

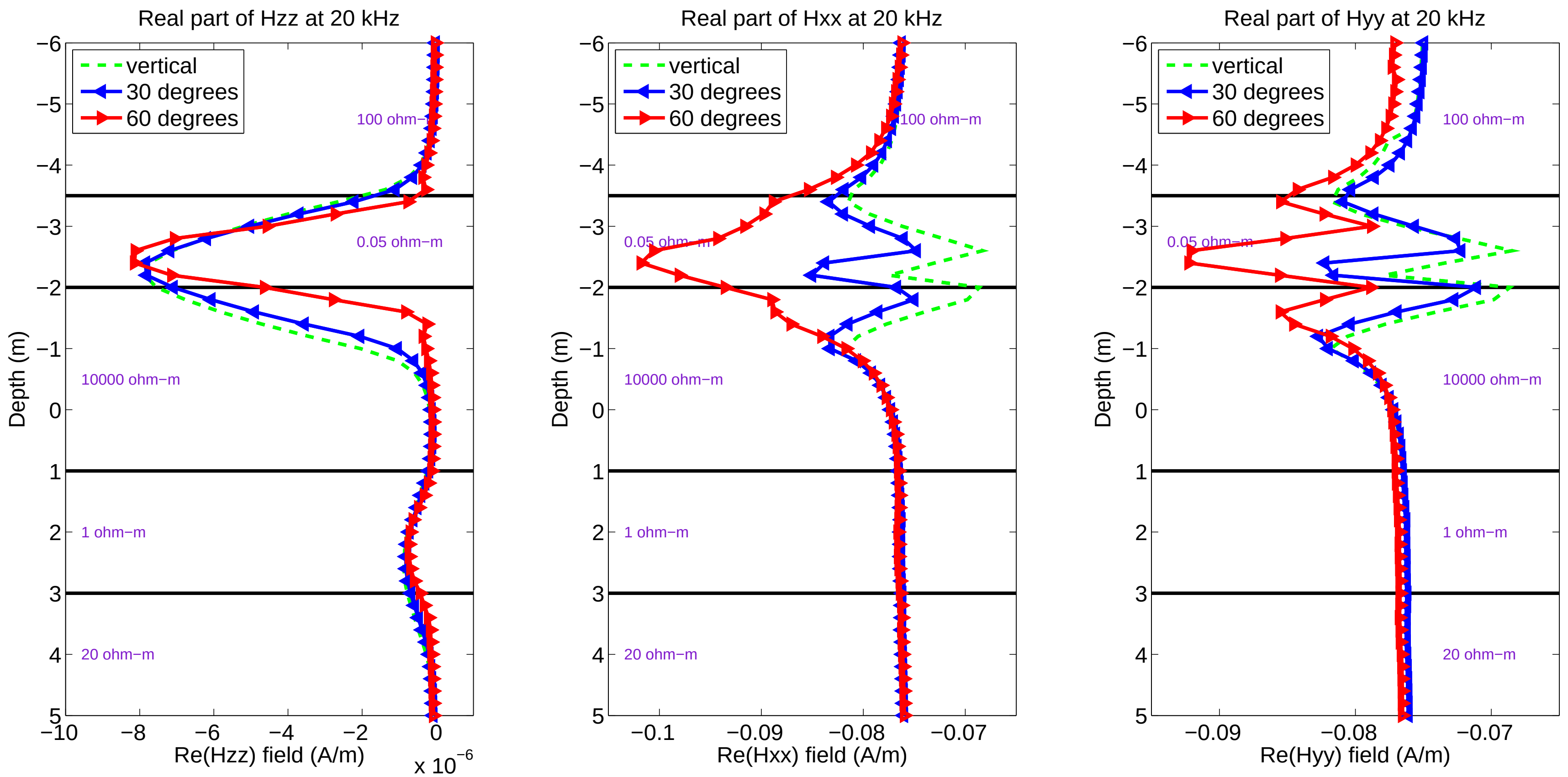
Tri-Axial Induction Tool

L=40 in, Frequency=20 kHz



electromagnetic applications

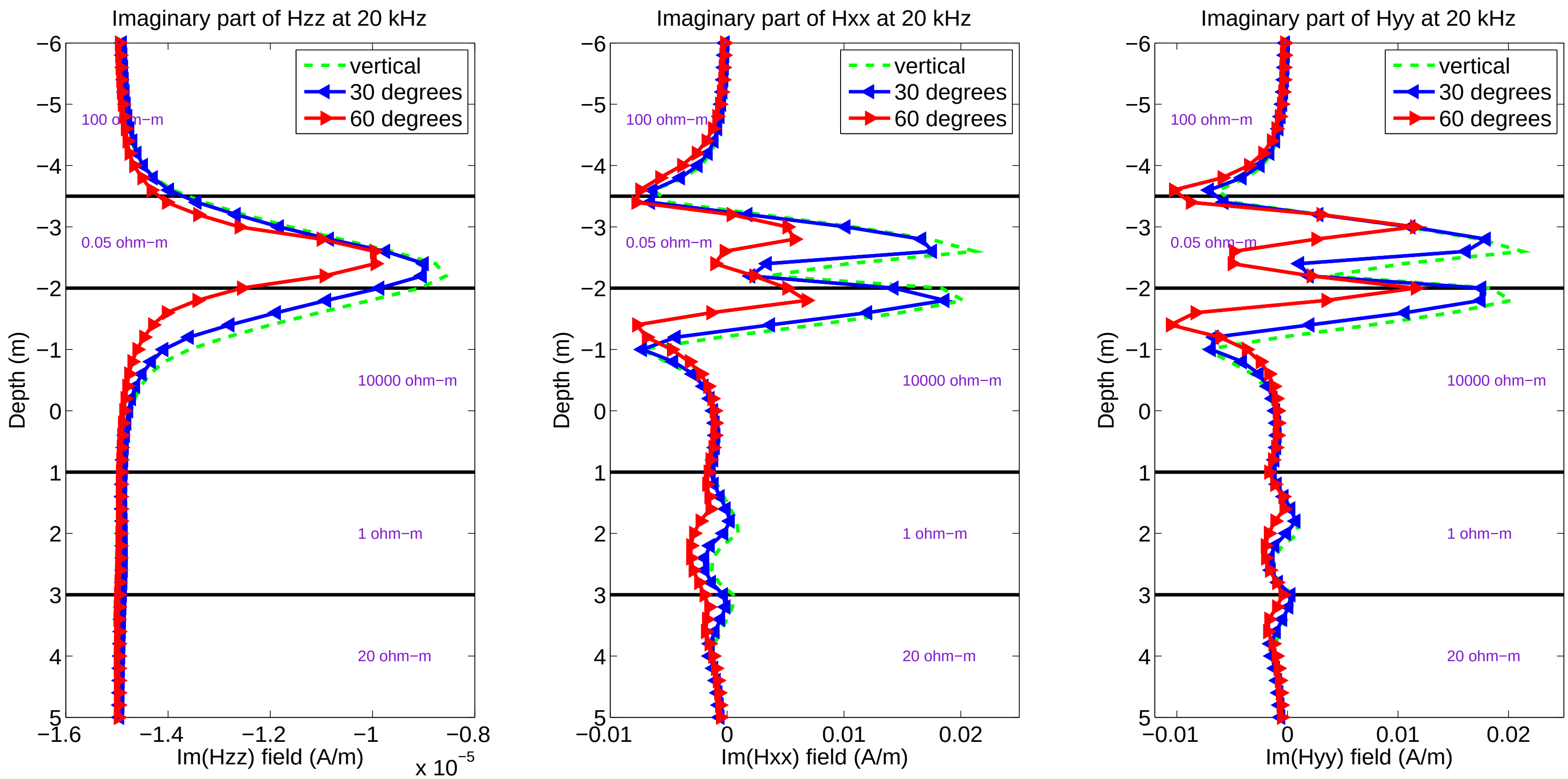
Tri-Axial Induction Tools in Deviated Wells (0, 30, and 60 degrees)



Triaxial tools are more sensitive to dip angle effects

electromagnetic applications

Tri-Axial Induction Tools in Deviated Wells (0, 30, and 60 degrees)



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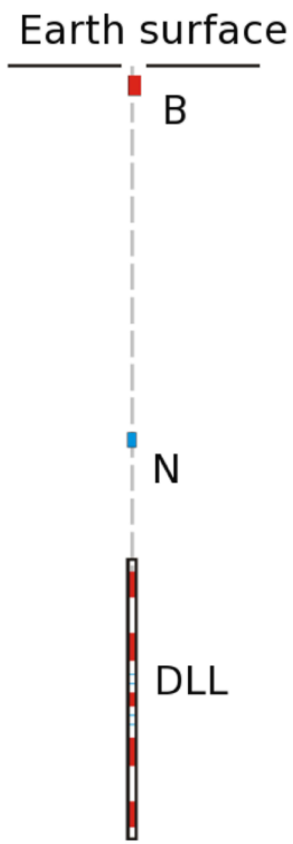
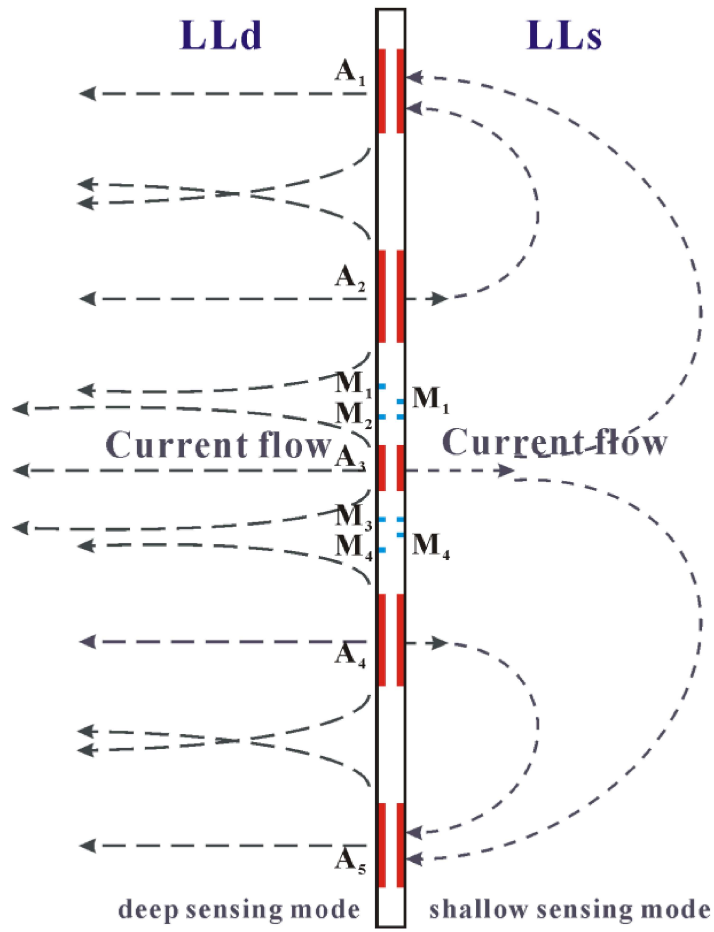
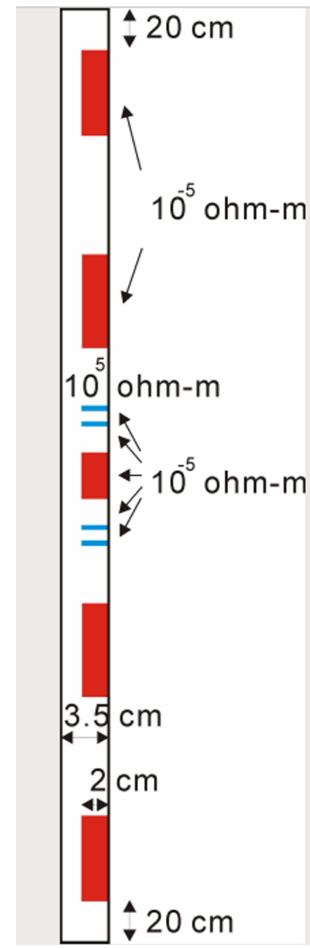
electromagnetic applications



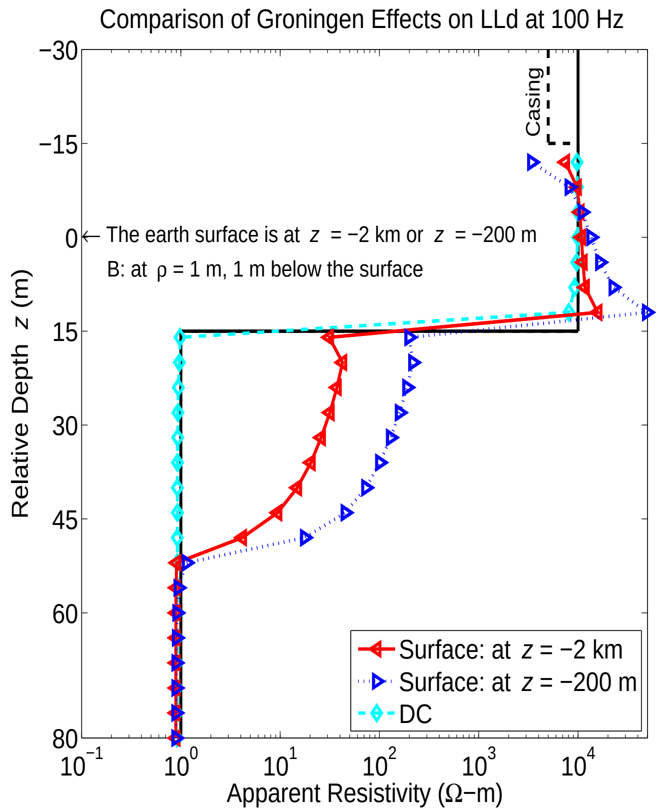
DLL Tool

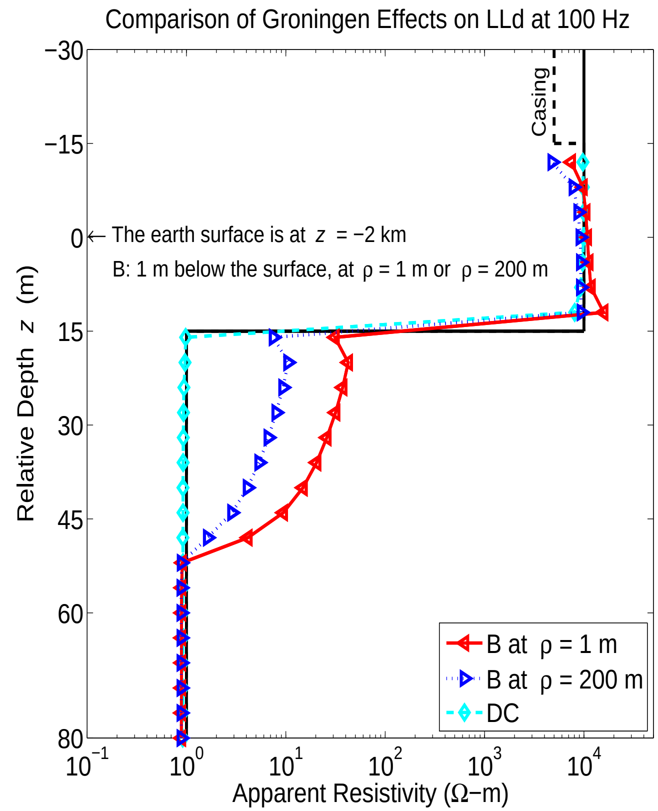




electromagnetic applications

Groningen Effect

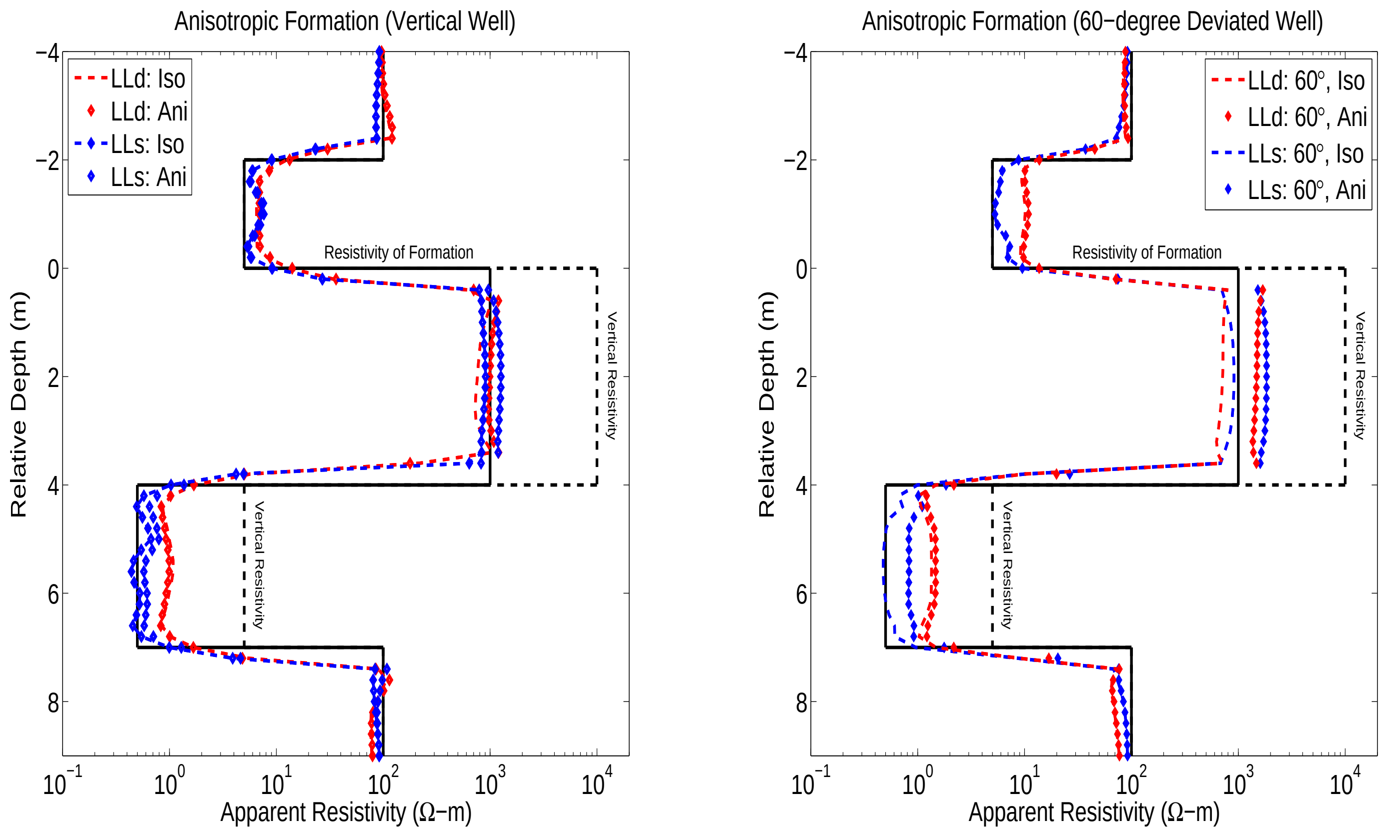




As we place the current return electrode B farther from the logging instrument, the Groningen effect diminishes

electromagnetic applications

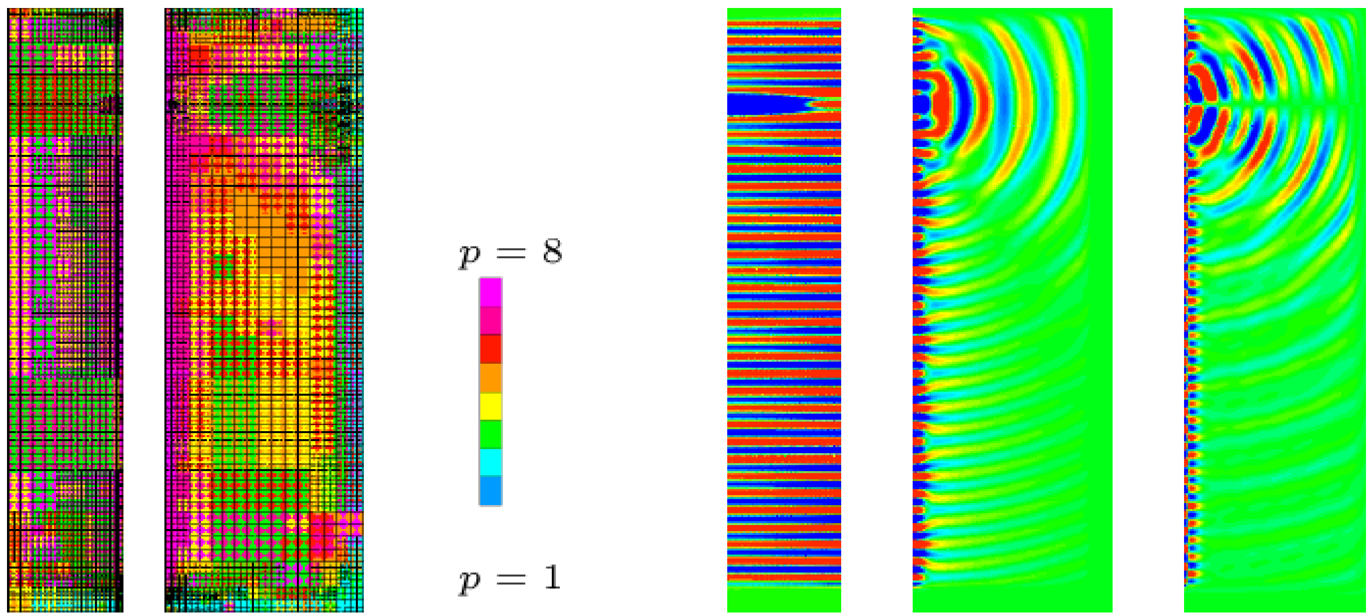
DC DLL in Deviated Wells



Anisotropy is better identified when using deviated wells

acoustic applications

Final hp-grid and solution



acoustic hp-mesh | elastic hp-mesh | acoustic p_{acoust} | elastic u_r | elastic u_z

8 KHz, acoustics, open borehole setting (no logging instrument).

conclusions

- We have described an efficient numerical method for solving PDE's based on a self-adaptive goal-oriented hp refinement strategy.
- We are developing a multiphysics version of the code using the *de Rham* diagram.
- Our main objective is to create a software infrastructure enabling solution of challenging multiphysics inverse problems with applications to geophysics (hydrocarbon detection and monitoring, etc.), aeronautics and medicine.
- To achieve this objective, we need Ph.D. students, post-doctoral fellows, experienced researchers, and collaborators in different areas (inversion, solvers, etc).

future work

I. Garay

Development of algorithms for solving multiphysics inverse problems.

F. de la Hoz

Development of fast iterative solvers.

M. Paszynski

Parallel computations.

M.J. Nam

Simulations of resistivity logging instruments.

future work

L.E. García-Castillo

Electromagnetic computations.

E. Pérez

Visualization.

I. Gómez

Three-dimensional computations.

C. Torres-Verdín

Contacts with the oil industry.