Shell Presentation, 1:00 pm

A Self-Adaptive Goal-Oriented hp-Finite Element Simulation of Resistivity Cross-Well Measurements with One Steel Cased Well”

David Pardo (dzubiaur@yahoo.es),
Yang Wei, C. Torres-Verdin, L. Demkowicz

Collaborators: Science Department of Baker-Atlas,
L. Tabarovsky, J. Kurtz, M. Paszynski, D. Xue

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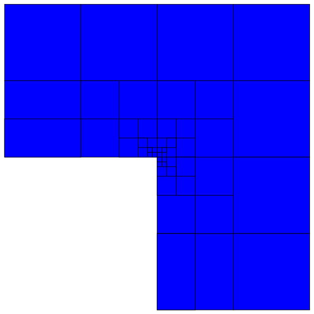
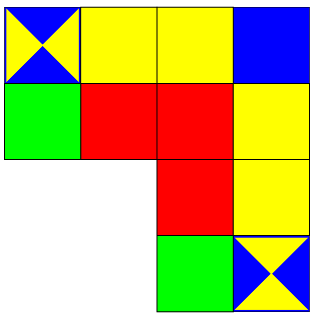
Department of Petroleum and Geosystems Engineering, and
Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

THE hp-FINITE ELEMENT METHOD



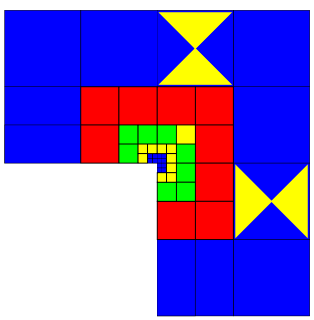
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal *h*-grids do NOT converge exponentially in real applications.
3. They may "lock" (100% error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.

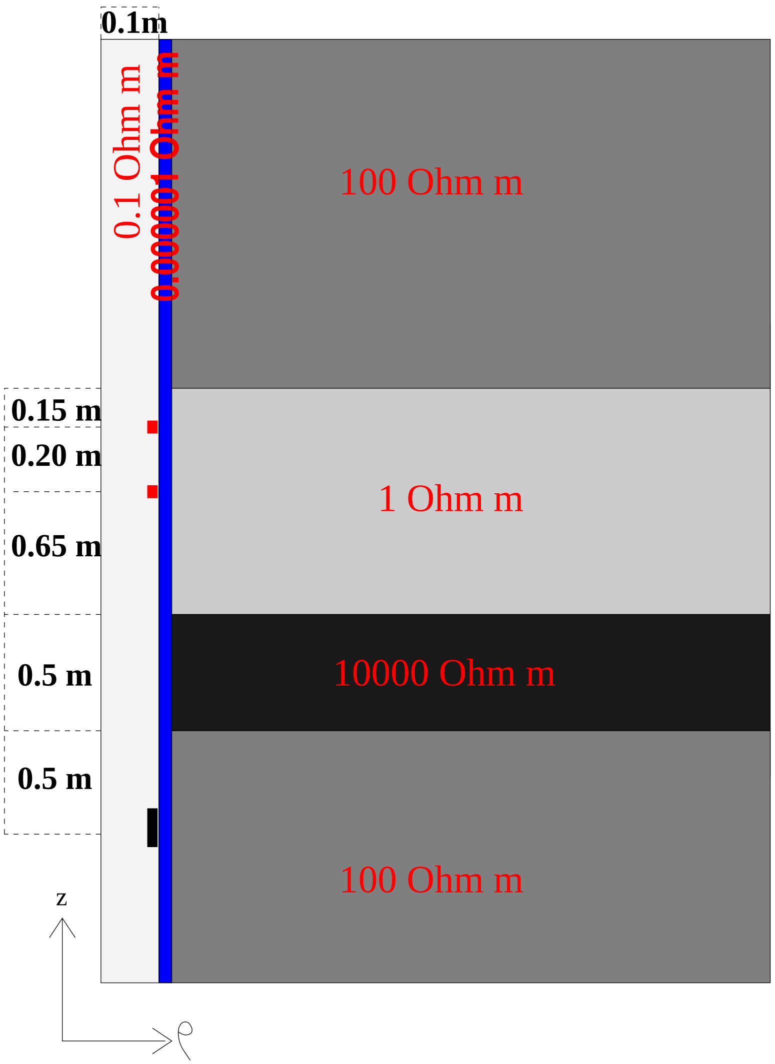


The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

THE hp-FINITE ELEMENT METHOD

Model Problem with Steel Casing



Frequency: 10 Hz.

Casing resistivity: 10^{-6} Ohm · m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 %

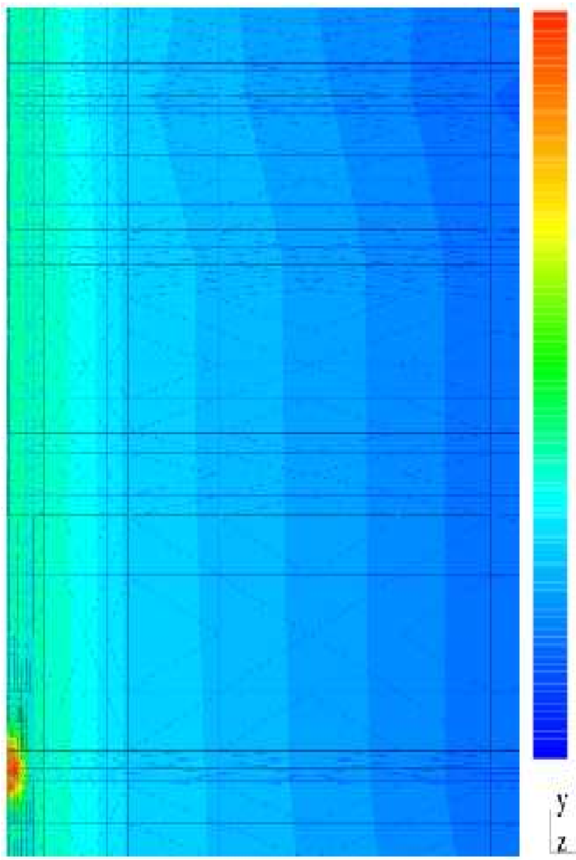
A variety of antennas

Size (domain): 500m x 4000m

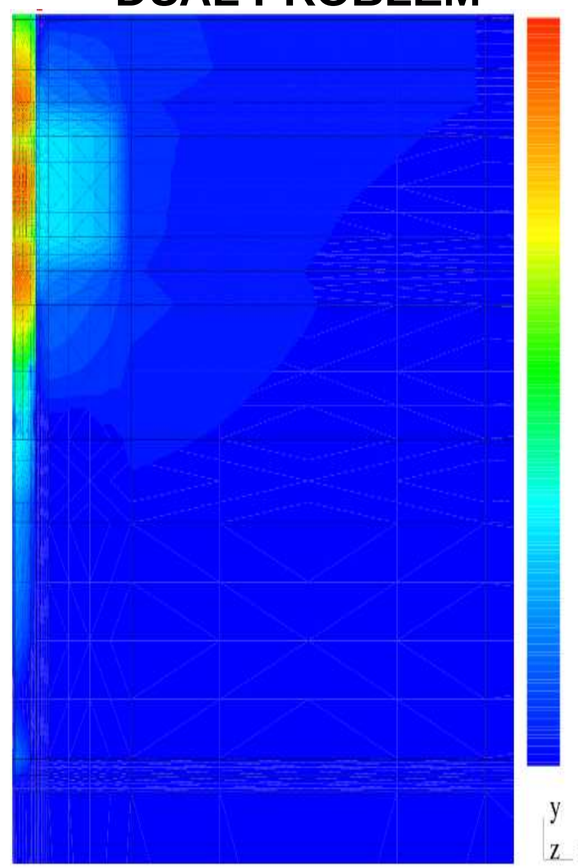
GOAL-ORIENTED ADAPTIVITY

Representation Formula

DIRECT PROBLEM



DUAL PROBLEM

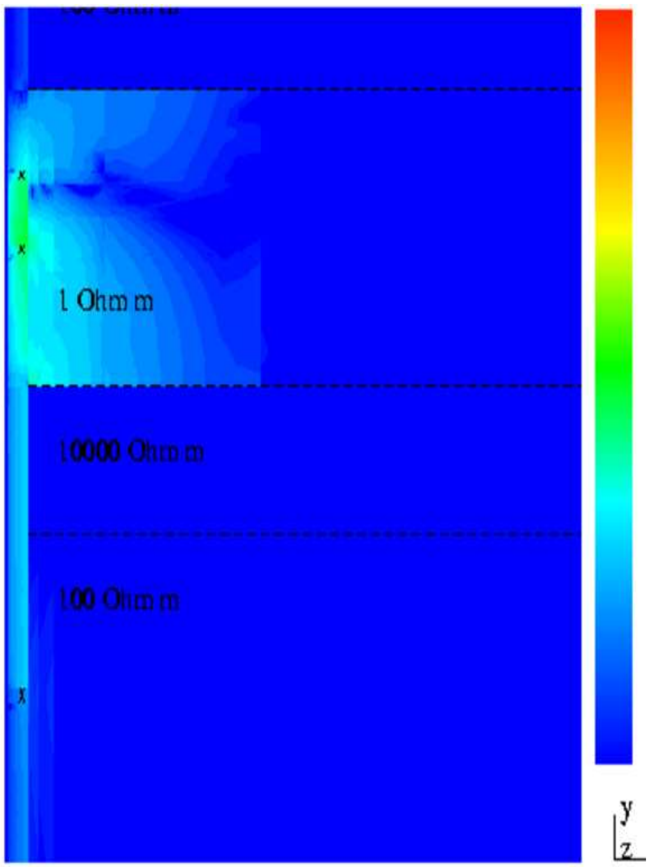


$$V = L(\Psi) = \int \bar{\bar{\sigma}} \nabla \Psi \cdot \nabla G$$

GOAL-ORIENTED ADAPTIVITY

Movie Presentation (Sensitivity Functions)

We want to study resolution and depth of investigation of a logging instrument.



We have: $|L(\Psi)| = |\int S\, dV| \leq \int |S|\, dV$.

In the next movies, we display: $\log_{10}|S|$.

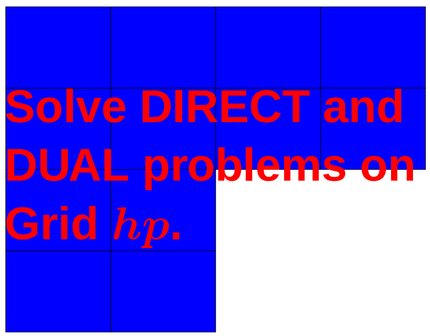
Scales:

- **Red** $\rightarrow |S| = |L(\Psi)| * 10^4$.
- **Blue** $\rightarrow |S| = |L(\Psi)| * 10^{-2}$.

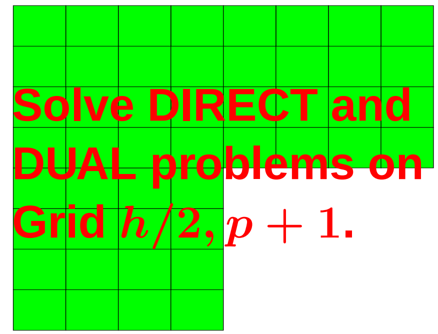
Direct Current

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Algorithm for Goal-Oriented Adaptivity



$\longrightarrow$

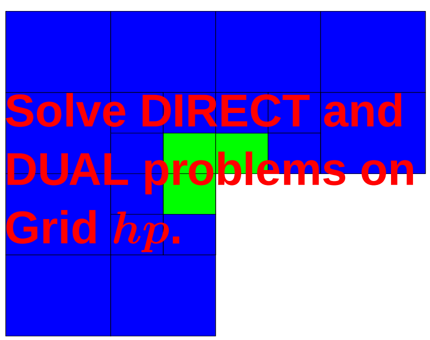


Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, **and** $\tilde{e} = \Psi_{h/2,p+1} - \Pi_{hp}\Psi_{h/2,p+1}$.

Compute $\epsilon = G_{h/2,p+1} - G_{hp}$, **and** $\tilde{\epsilon} = G_{h/2,p+1} - \Pi_{hp}G_{h/2,p+1}$.

$$|L(e)| = |b(e,\epsilon)| \sim |b(\tilde{e},\tilde{\epsilon})| \leq \sum_K |b_K(\tilde{e},\tilde{\epsilon})| \leq \sum_K \parallel \tilde{e} \parallel_{E,K} \parallel \tilde{\epsilon} \parallel_{E,K}.$$

Apply the fully automatic hp-adaptive algorithm.



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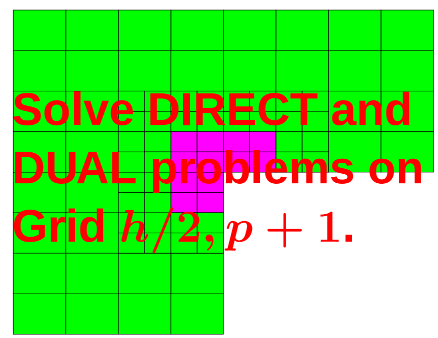
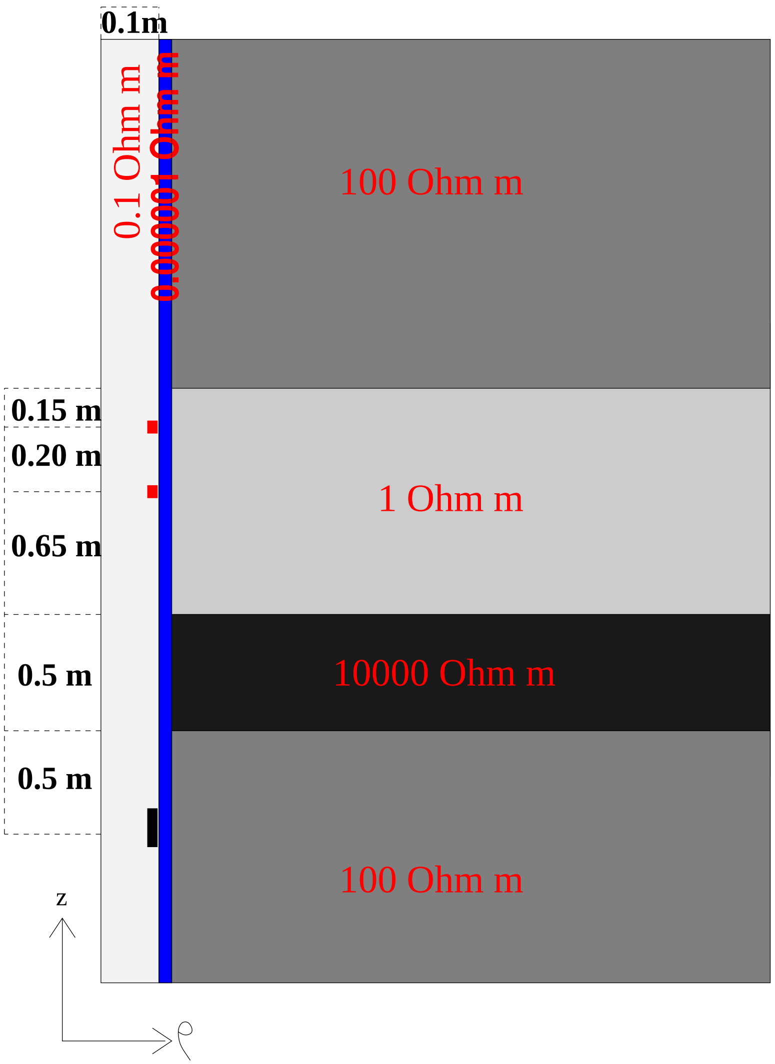


SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Model Problem with Steel Casing



Frequency: 10 Hz.

Casing resistivity: 10^{-6} Ohm · m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 %

A variety of antennas

Size (domain): 500m x 4000m

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

Time-Harmonic Maxwell's Equations

$\nabla \times \mathrm{H} = (\bar{\bar{\sigma}} + j\omega\bar{\bar{\epsilon}})\mathrm{E} + \mathrm{J}^{imp}$	**Ampere's law**
$\nabla \times \mathrm{E} = -j\omega\bar{\bar{\mu}}\mathrm{H} - \mathrm{M}^{imp}$	**Faraday's law**
$\nabla \cdot (\bar{\bar{\epsilon}}\mathrm{E}) = \rho$	**Gauss' law of Electricity**
$\nabla \cdot (\bar{\bar{\mu}}\mathrm{H}) = 0$	**Gauss' law of Magnetism**

E-VARIATIONAL FORMULATION:

$$\left\{ \begin{array}{l} \textbf{Find } \mathrm{E} \in \mathrm{E}_D + H_D(\mathrm{curl};\Omega) \textbf{ such that:} \\ \displaystyle\int_\Omega (\bar{\bar{\mu}}^{-1}\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\bar{\bar{k}}^2 \mathrm{E}) \cdot \bar{\mathrm{F}}\, dV = -j\omega \int_\Omega \mathrm{J}^{imp} \cdot \bar{\mathrm{F}}\, dV \\ \displaystyle\quad + j\omega \int_{\Gamma_N} \mathrm{J}^{imp}_{\Gamma_N} \cdot \bar{\mathrm{F}}_t\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}\mathrm{M}^{imp}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV \quad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega) \end{array} \right.$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

AXISYMMETRIC PROBLEMS

E_ϕ **-Variational Formulation (Azimuthal)**

$$\begin{cases} \text{Find } E_\phi \in E_{\phi,D} + \tilde{H}^1_D(\Omega) \text{ such that:} \\ \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \nabla \times \mathrm{E}_\phi) \cdot (\nabla \times \bar{\mathrm{F}}_\phi)\, dV - \int_\Omega (\bar{\bar{k}}^2_\phi \mathrm{E}_\phi) \cdot \bar{\mathrm{F}}_\phi\, dV = -j\omega \int_\Omega J^{imp}_\phi\, \bar{F}_\phi\, dV \\ \quad + j\omega \int_{\Gamma_N} J^{imp}_{\phi,\Gamma_N}\, \bar{F}_\phi\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \mathrm{M}^{imp}_{\rho,z}) \cdot \bar{\mathrm{F}}_\phi\, dV \qquad \forall\, F_\phi \in \tilde{H}^1_D(\Omega) \end{cases}$$

$\mathrm{E}_{\rho,z}$ **-Variational Formulation (Meridian)**

$$\begin{cases} \text{Find } (E_\rho, E_z) \in \mathrm{E}_D + \tilde{H}_D(\mathrm{curl}; \Omega) \text{ such that:} \\ \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \nabla \times \mathrm{E}_{\rho,z}) \cdot (\nabla \times \bar{\mathrm{F}}_{\rho,z})\, dV - \int_\Omega (\bar{\bar{k}}^2_{\rho,z} \mathrm{E}_{\rho,z}) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV = \\ \quad -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z\, dV + j\omega \int_{\Gamma_N} J^{imp}_{\rho,\Gamma_N} \bar{F}_\rho + J^{imp}_{z,\Gamma_N} \bar{F}_z\, dS \\ \quad - \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \mathrm{M}^{imp}_\phi) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV \qquad \forall\, (F_\rho, F_z) \in \tilde{H}_D(\mathrm{curl}; \Omega) \end{cases}$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

- **Physical Devices: Casing, Casing Imperfections, Materials with Different Magnetic Permeabilities, Insulators, Displacement Currents, Combination of All, etc.**
- **Materials: Isotropic, Anisotropic*.**
- **Sources: Toroidal Antennas, Solenoidal Antennas, Dipoles in Any Direction, Electrodes, Finite Size Antennas, Combination of All, etc.**
- **Logging Instruments: Logging While Drilling (LWD), Laterolog, Normal, Induction, Dielectric Instruments, Cross-well, etc.**
- **Any Frequency (0-10 Ghz).**

ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
  2. Exact solution in a homogeneous media with casing.
- **Verification of Physical Properties.**
  1. Reciprocity principle (Gregory Itskovich).
  2. Discrete divergence free approximation for edge elements.
- **Numerical Verifications.**
  1. Different size of domain and antennas.
  2. Comparison against other numerical software (Yang Wei).
  3. Error control provided by the fine grid solution.
  4. Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

Continuous Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516098429E-08	-0.1456374493E-08
FINE GRID	0.1516094029E-08	-0.1456390824E-08

Edge Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516060872E-08	-0.1456337248E-08
FINE GRID	0.1516093804E-08	-0.1456390864E-08

Error control provided by the fine grid solution.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

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Comparison between continuous elements vs. edge elements.

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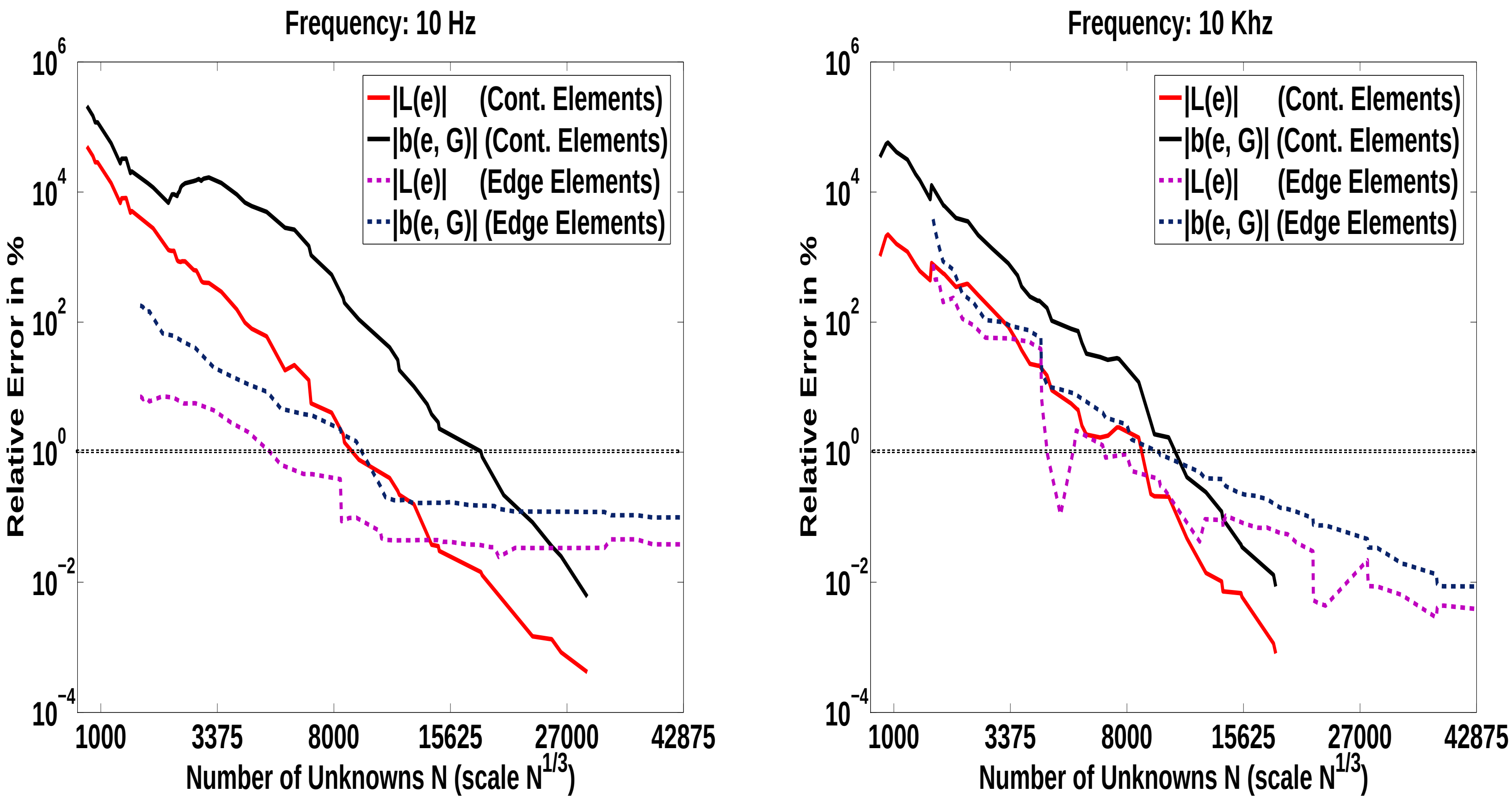
HIGHLY RELIABLE SOFTWARE

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Accuracy (Are the Solutions Accurate?)

Problem with Casing (Convergence Curve)



EXTREMELY ACCURATE SOFTWARE

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Performance (How Fast Can We Solve the Problems?)

80 Vert. Pos.	$10^{-6}\Omega \cdot m$	$10^{-5}\Omega \cdot m$
Toroid (10 Khz)	19’ 46”	16’ 28”
Ring of Vert. Dipoles (10 Khz)	22’ 47”	17’ 02”
Ring of Horiz. Dipoles (10 Khz)	19’ 25”	13’ 25”
Electrodes (0 Hz)	10’ 10”	8’ 35”

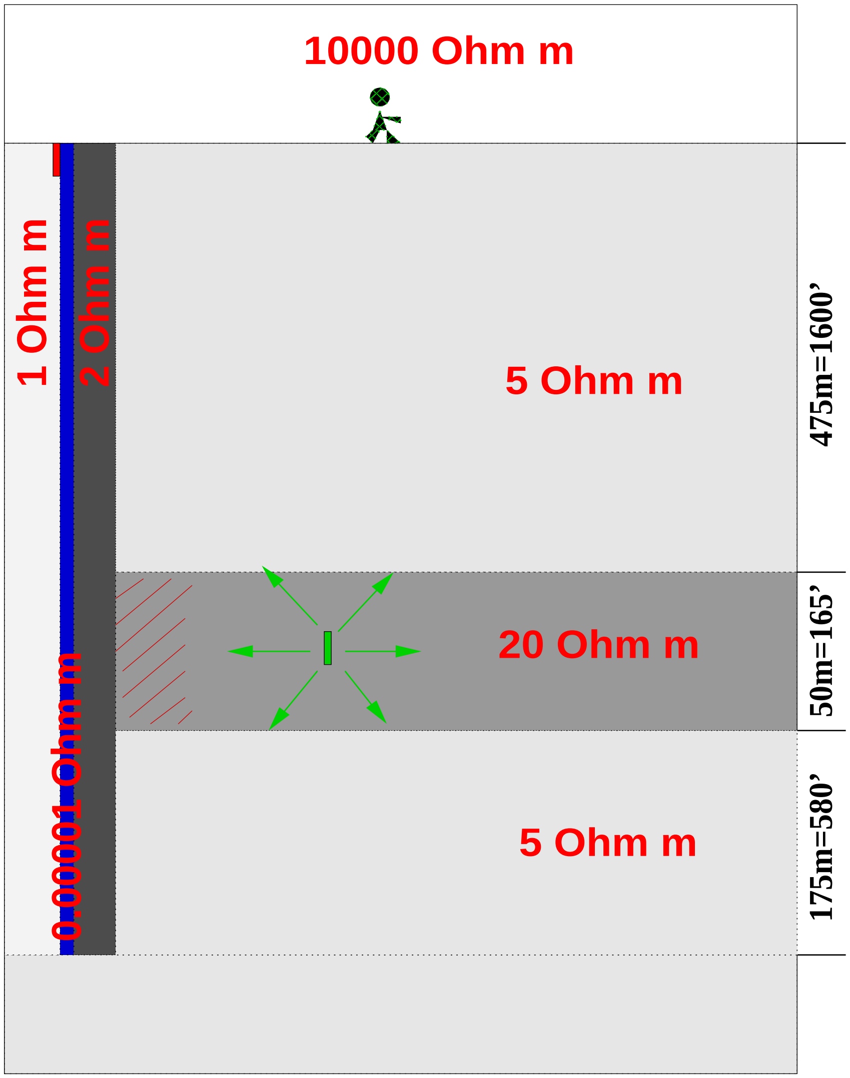
IBM Power 4 compiler 1.3 Ghz (4 years old)

Possible improvements in performance:

- To use a 3.4 Ghz processor.
- To execute the code in 8 processors (10 positions per processor).
- To improve implementation.

HIGH PERFORMANCE SOFTWARE

SIMULATION OF LOGGING INSTRUMENTS



5.5" Borehole radio ; 0.5" Casing ; 2" Cement

Axisymmetric 3D problem.

Five different materials.

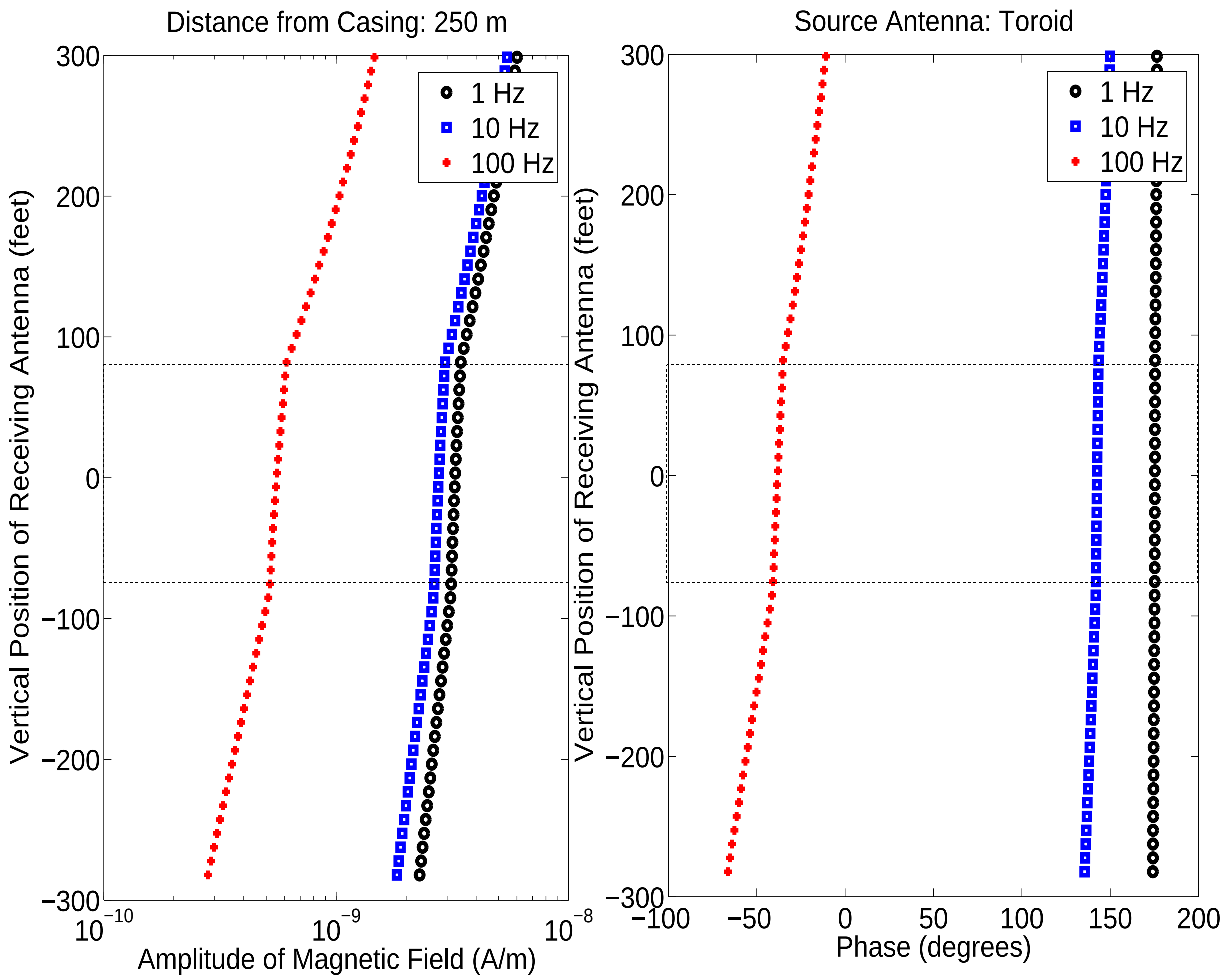
Different positions of receiving antenna.

Transmitter antenna located 3 m. below surface (inside and outside borehole).

Objective: Determine Electromagnetic fields at the receiver antennas.

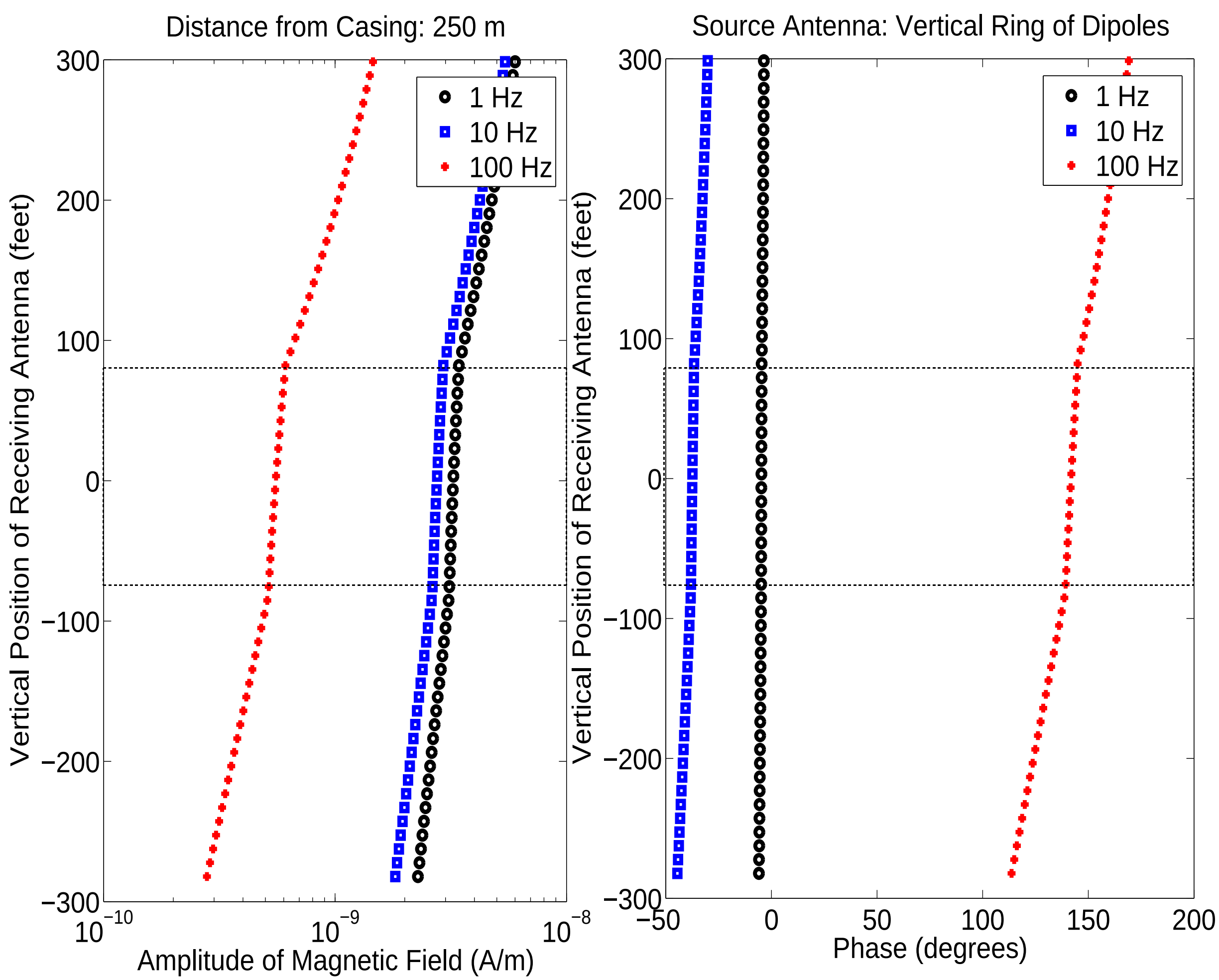
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study with One Cased Well: Toroid Antennas



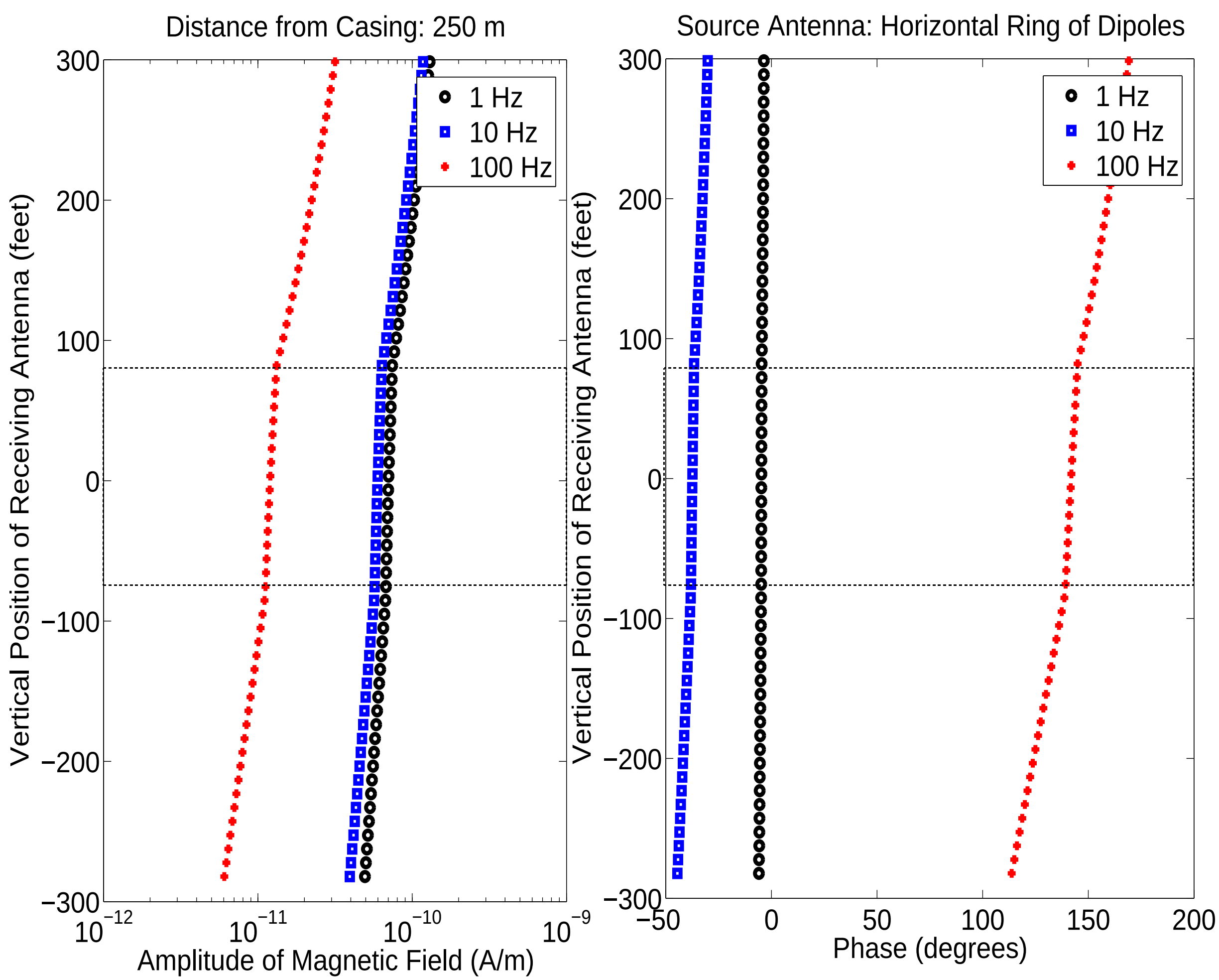
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Vertical Dipoles



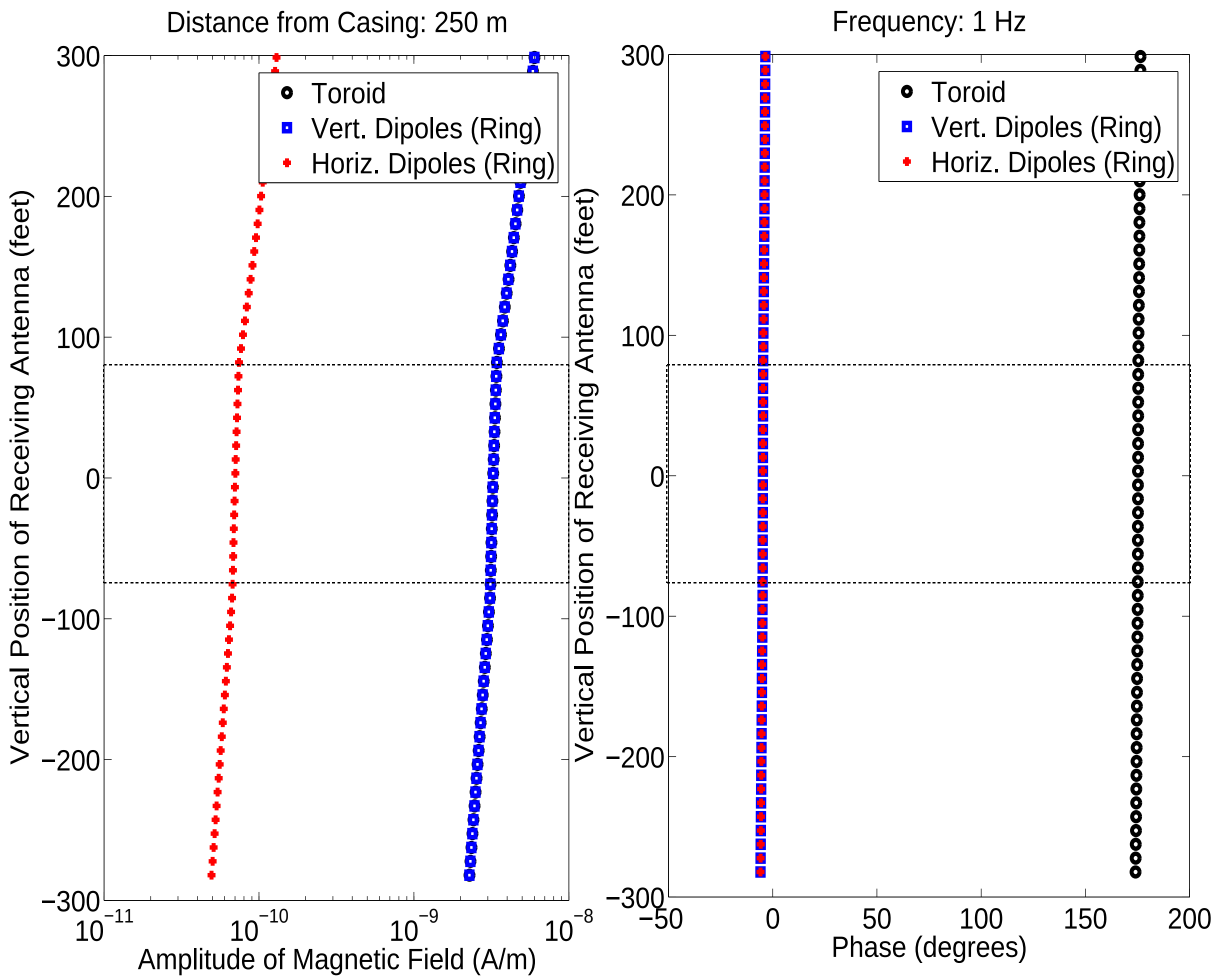
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Horizontal Dipoles



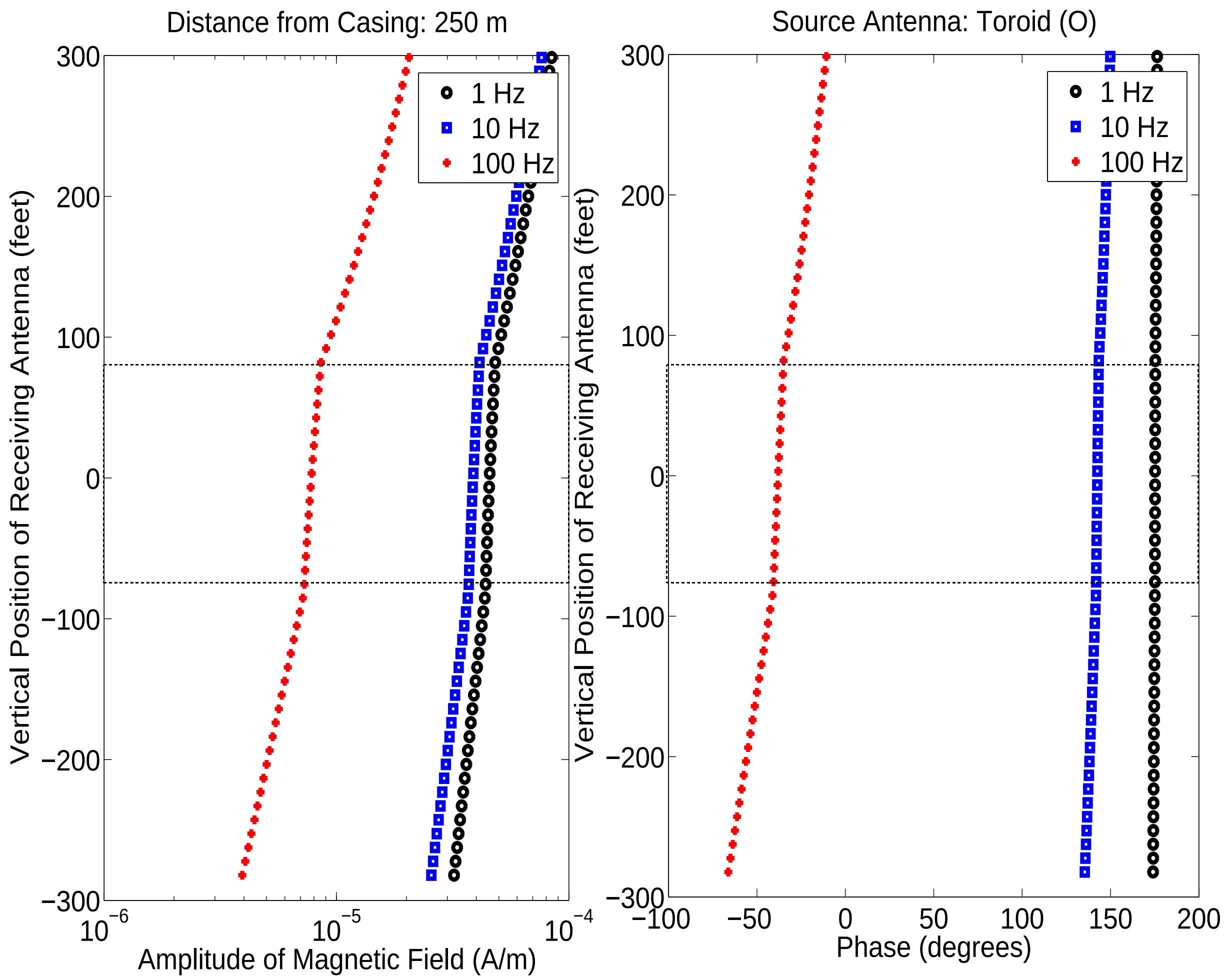
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Different Antennas



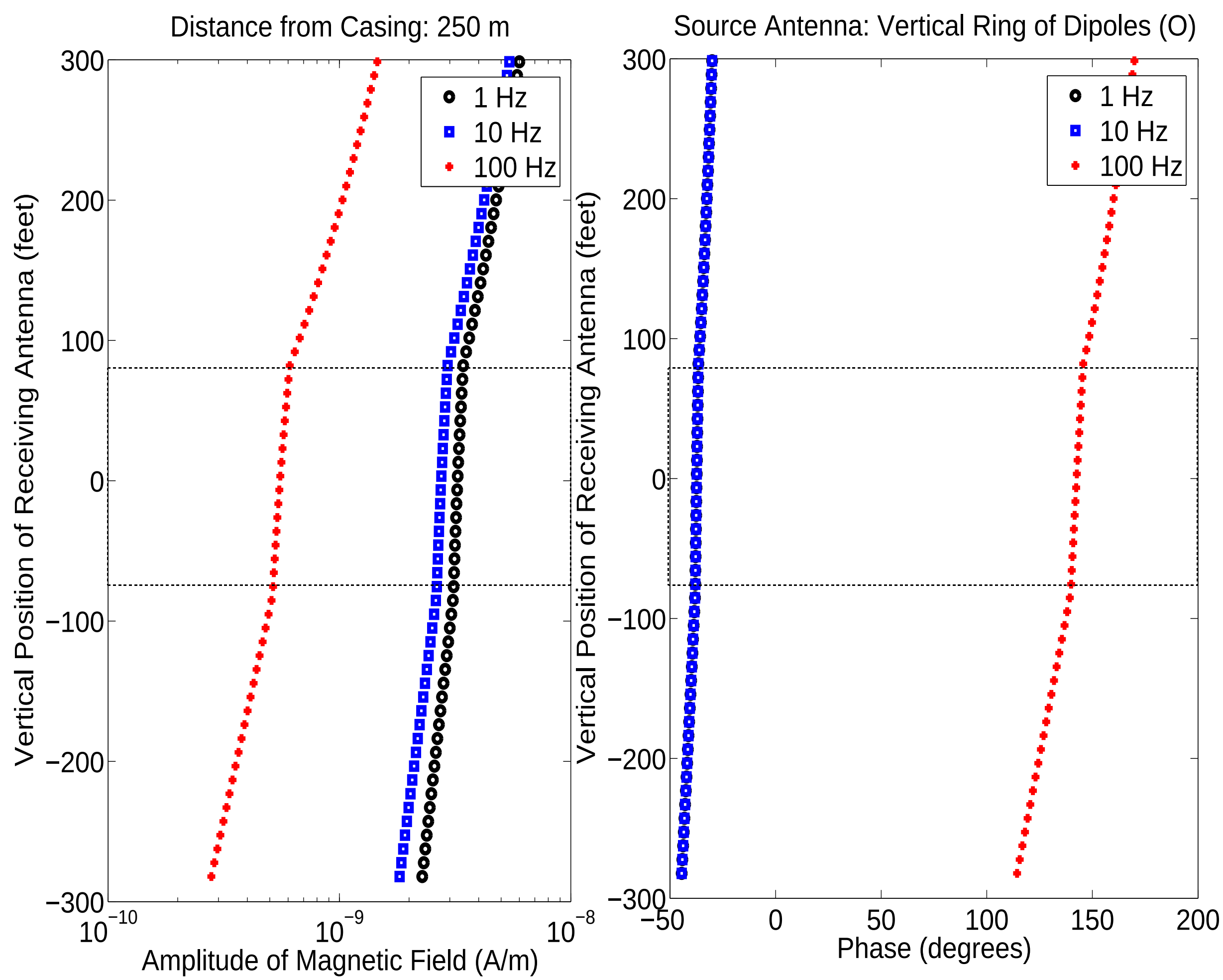
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Toroid Antennas (Outside Borehole)



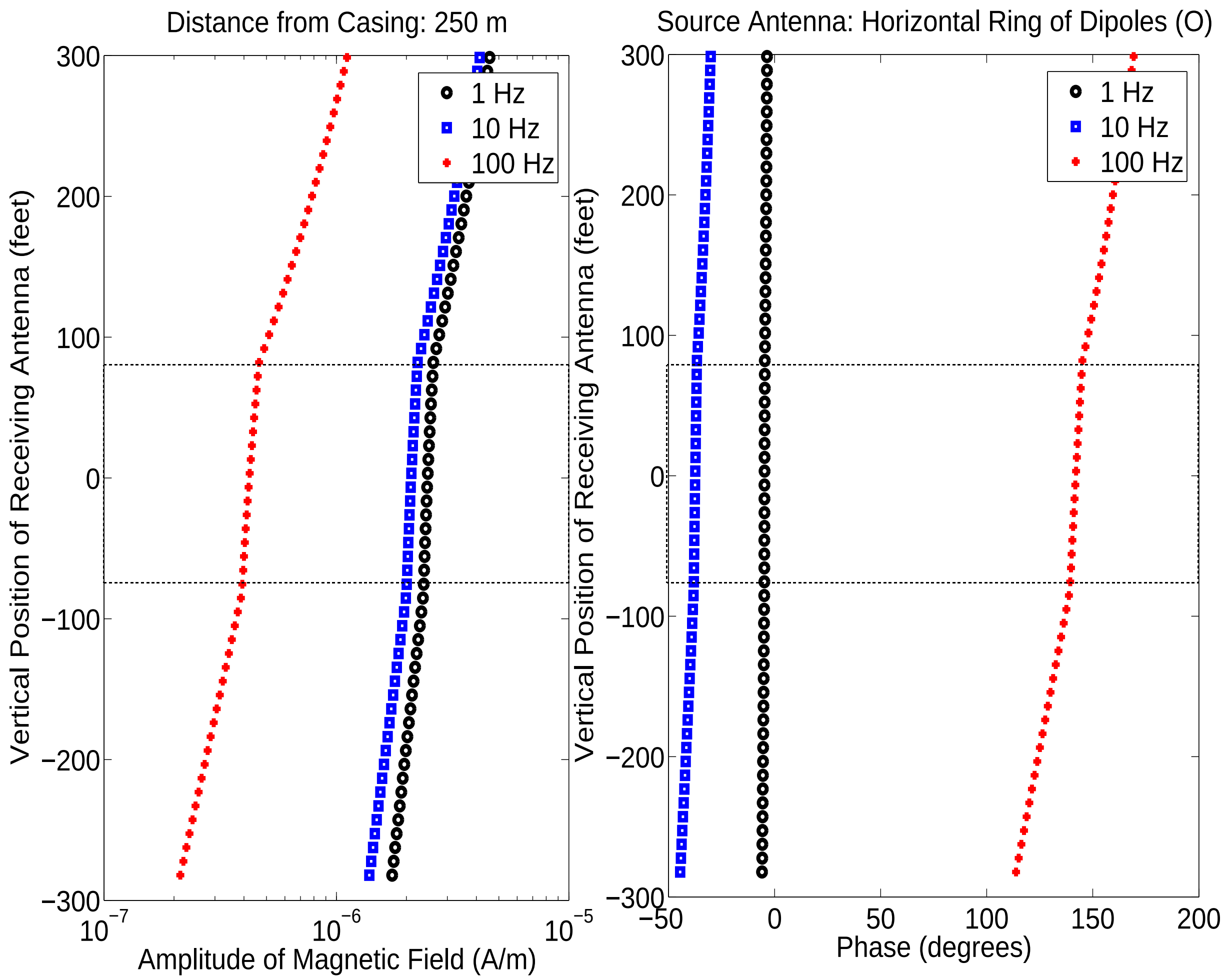
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Vertical Dipoles (Outside Borehole)



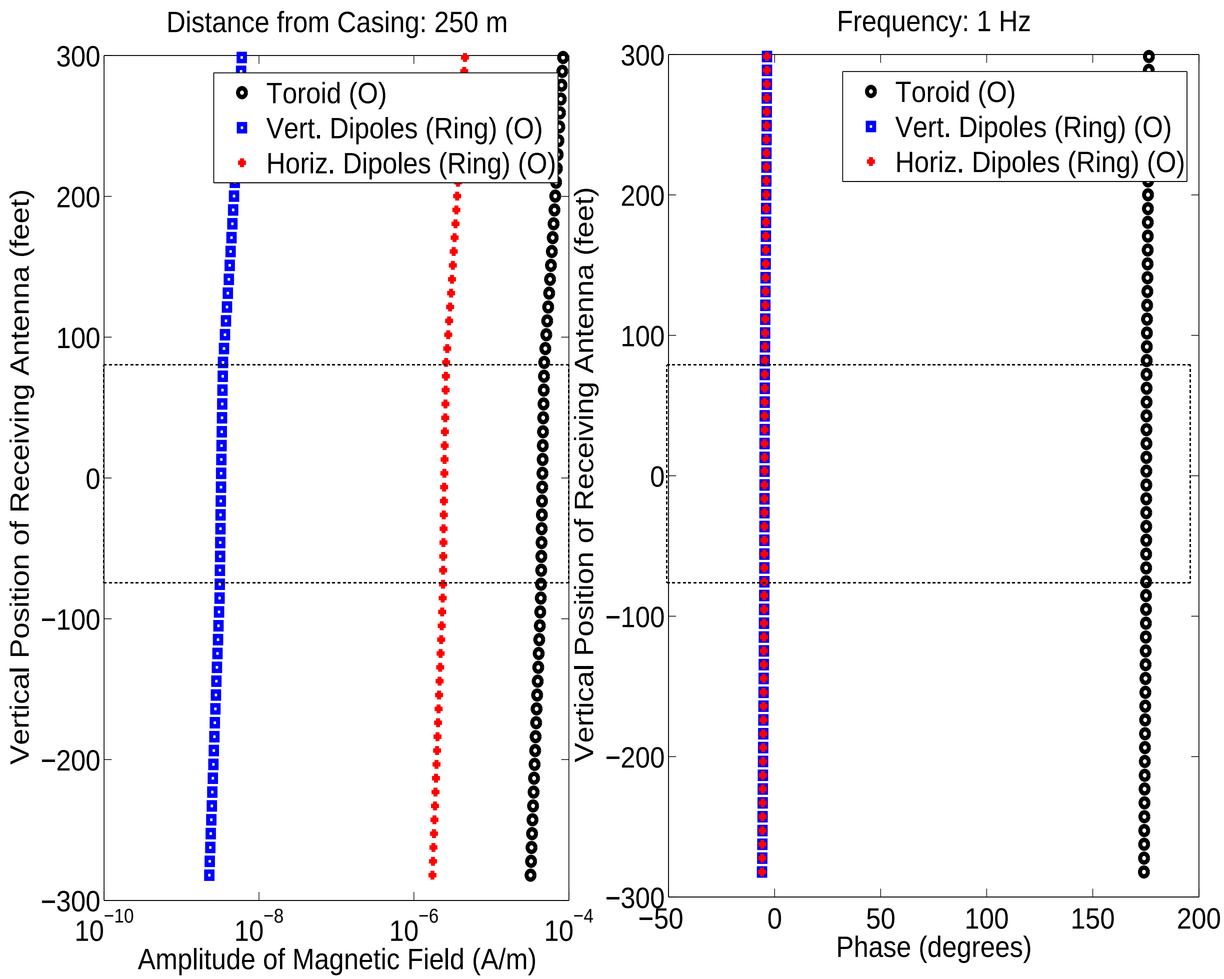
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Horizontal Dipoles (Outside Borehole)



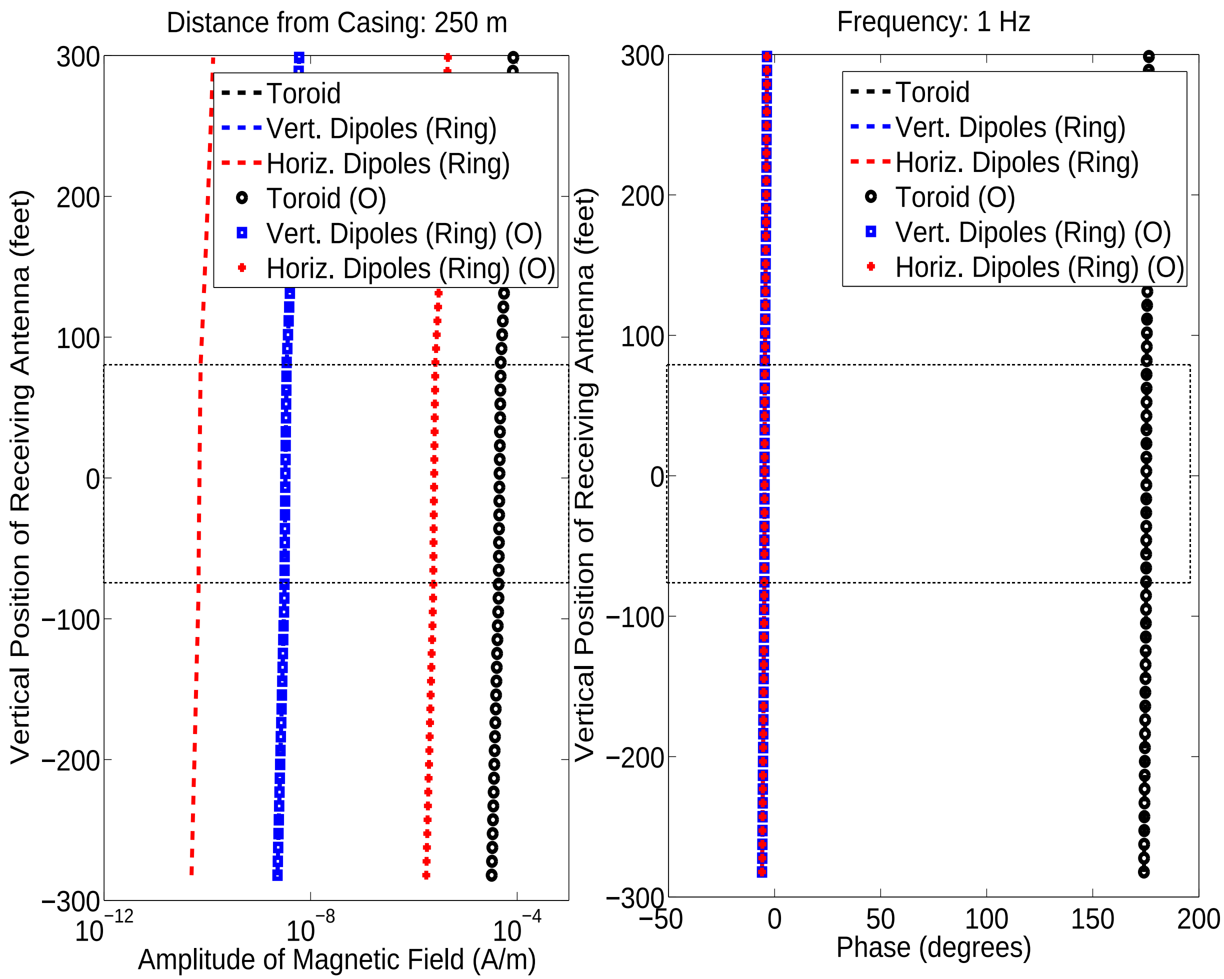
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Different Antennas (Outside Borehole)



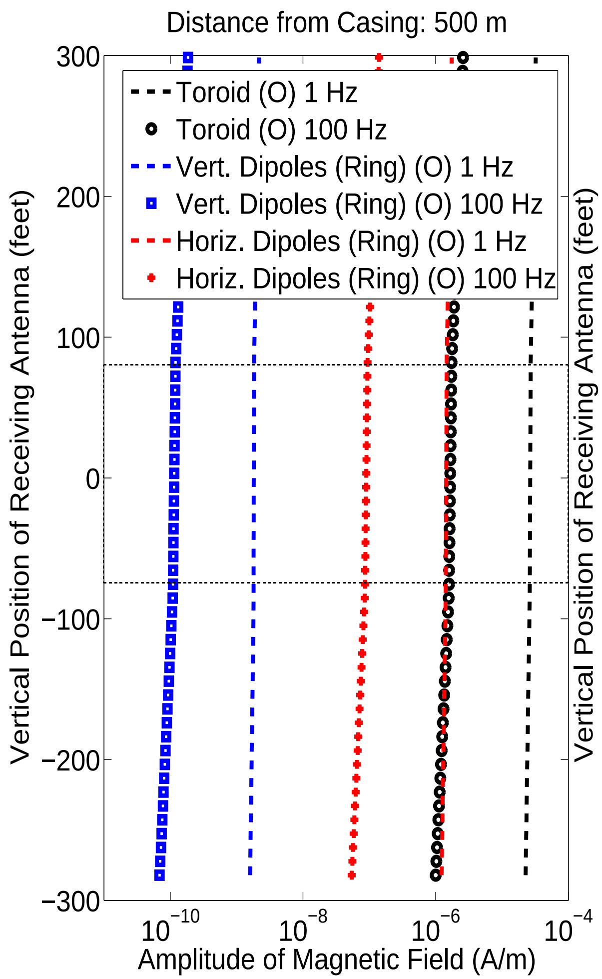
SIMULATION OF LOGGING INSTRUMENTS

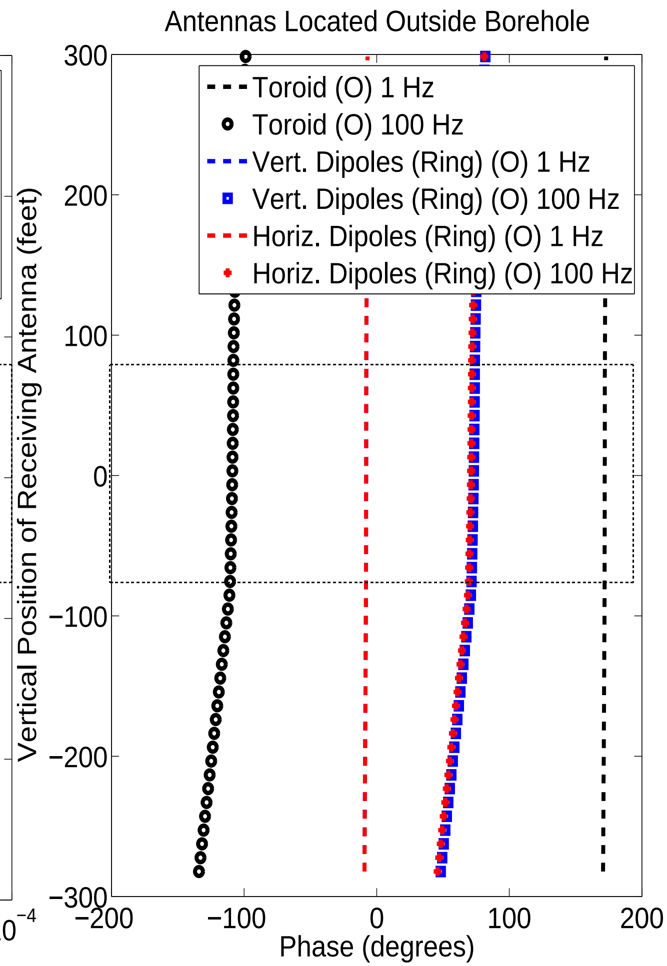
A Cross-Well Study: Antennas Inside and Outside Borehole



SIMULATION OF LOGGING INSTRUMENTS

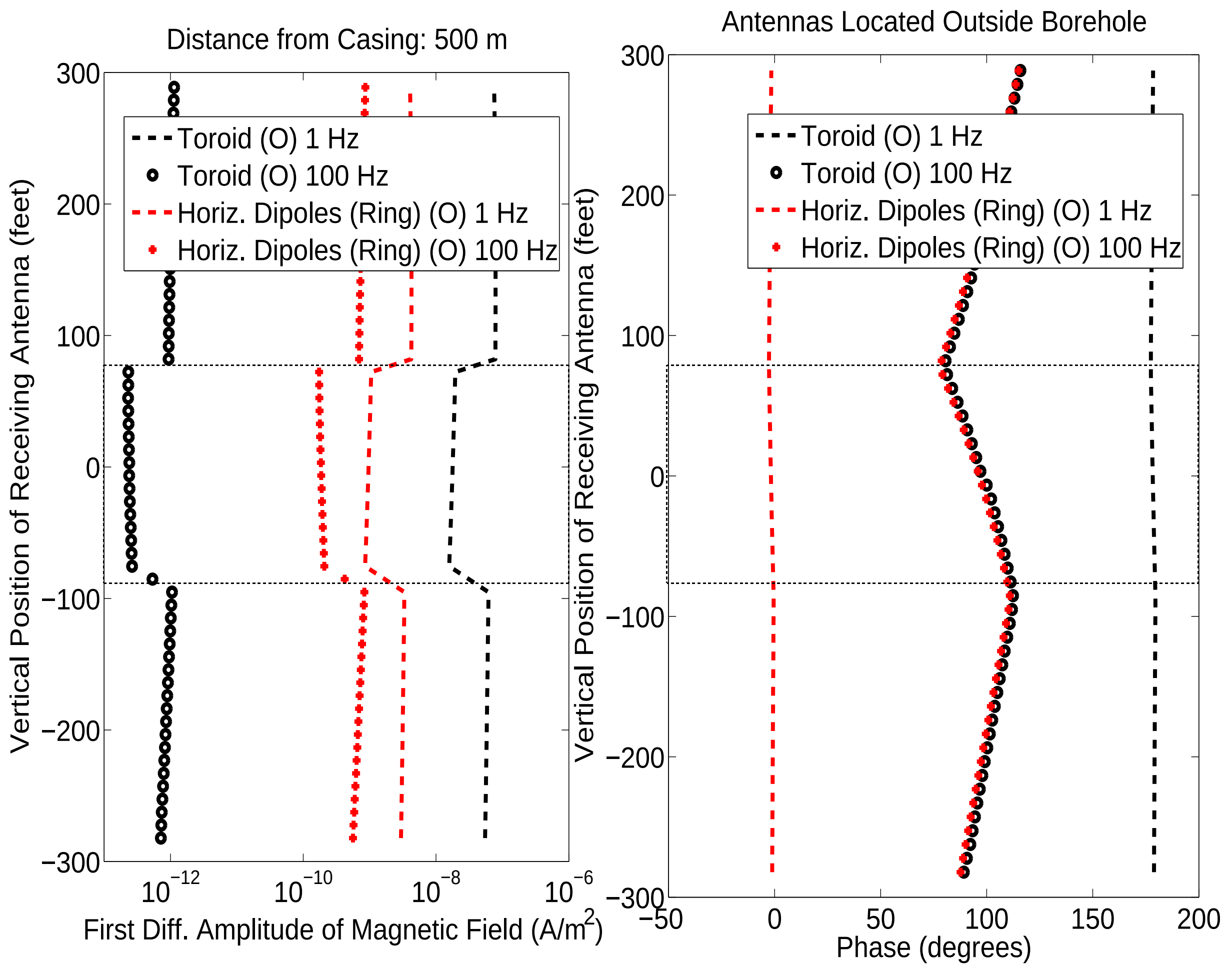
A Cross-Well Study: Receivers at 500 m (Horizontal Distance)





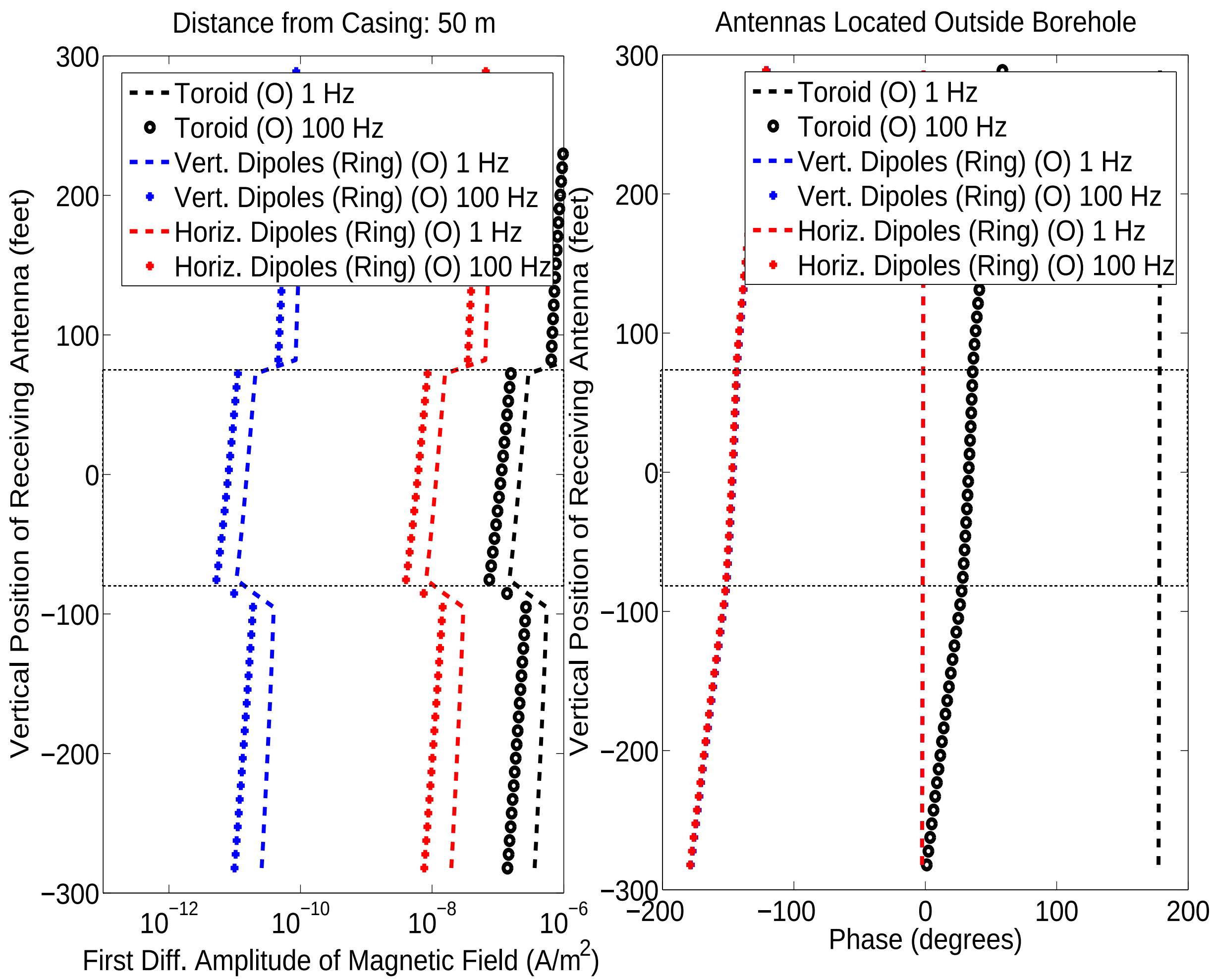
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: First Vertical Diff. of Magnetic Field



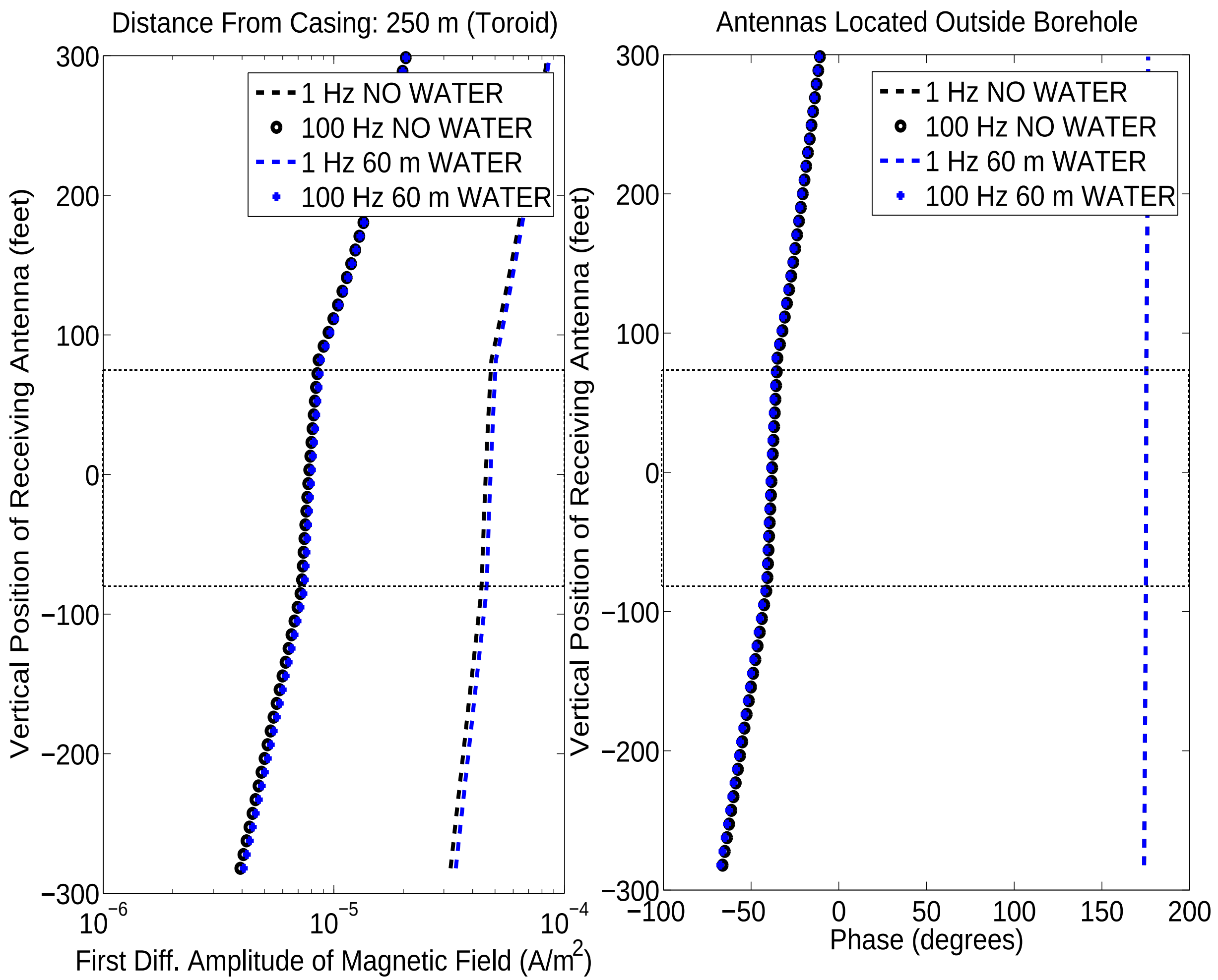
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: First Vert. Diff. Magnetic Field (50 m)



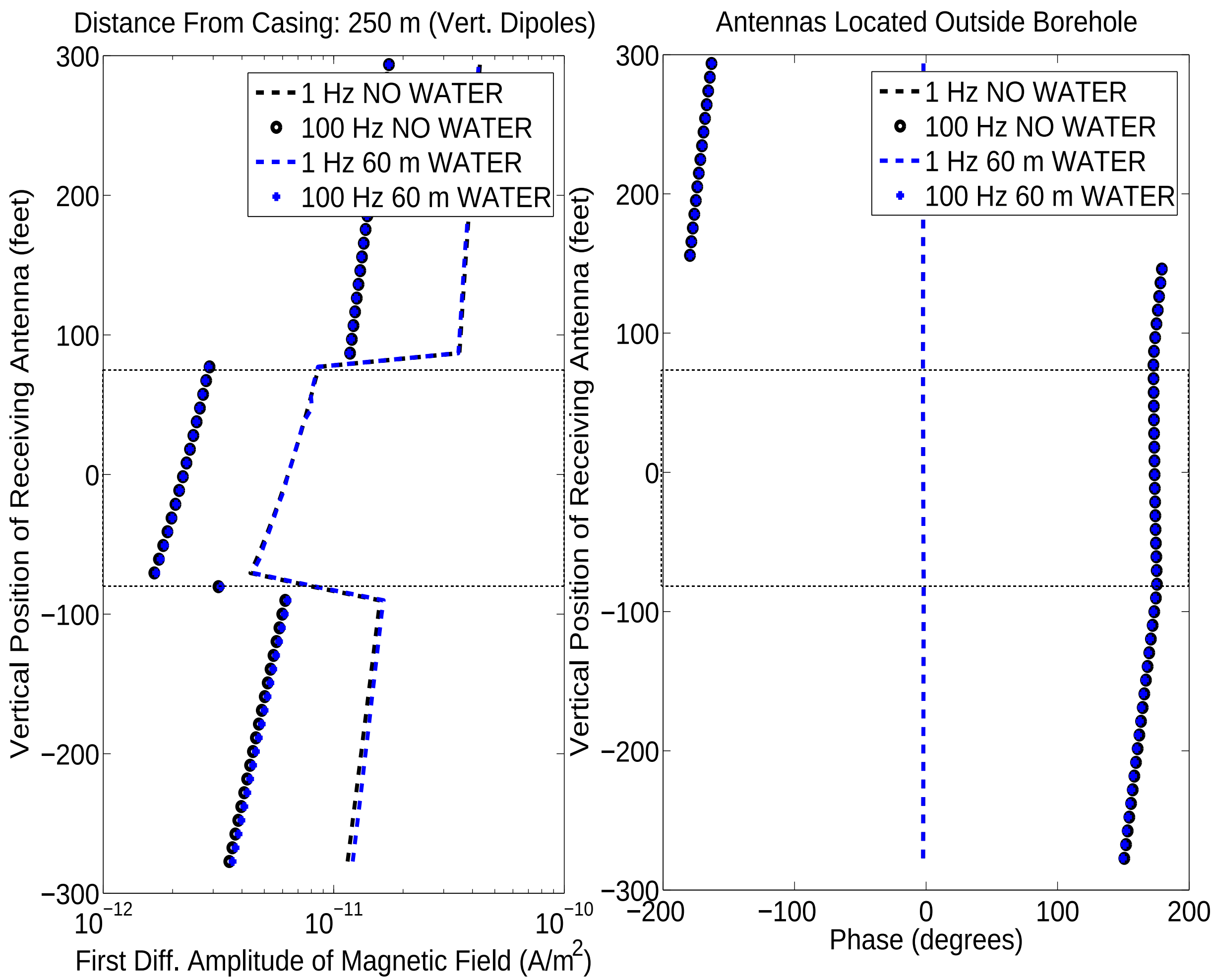
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Water Invasion with Toroids (250 m)



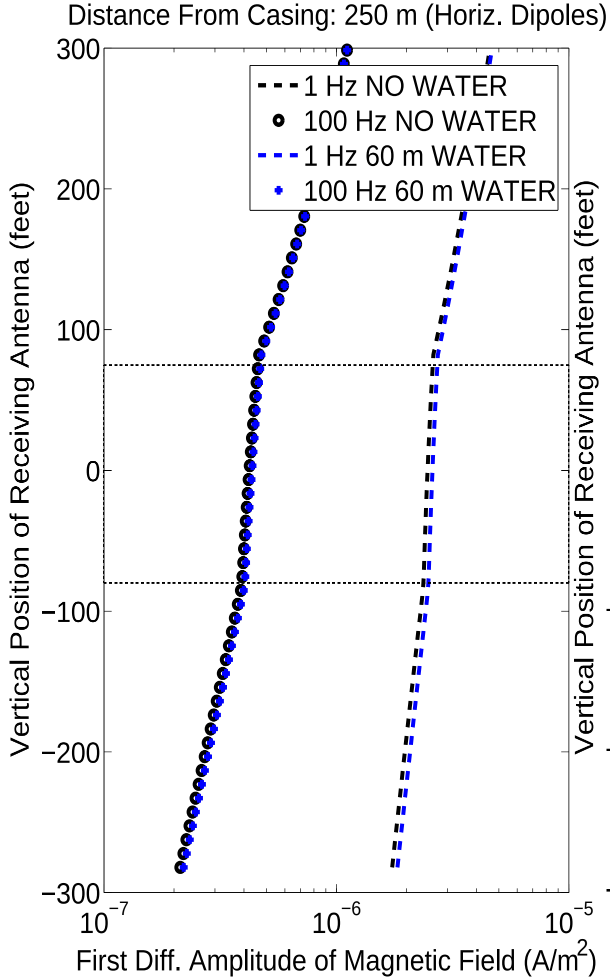
SIMULATION OF LOGGING INSTRUMENTS

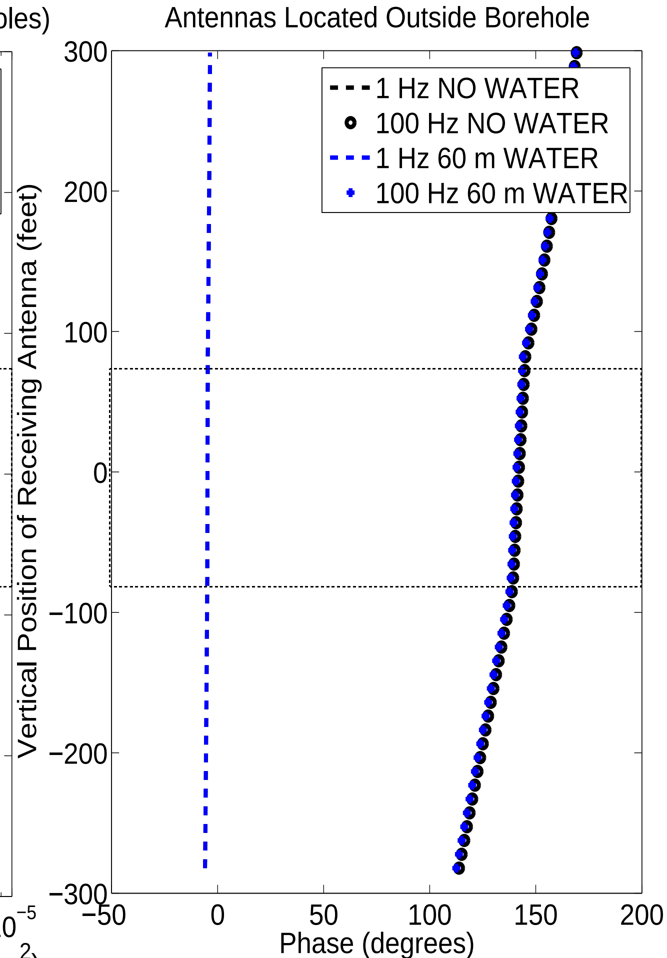
A Cross-Well Study: Water Invasion, Vert. Dipoles (250 m)



SIMULATION OF LOGGING INSTRUMENTS

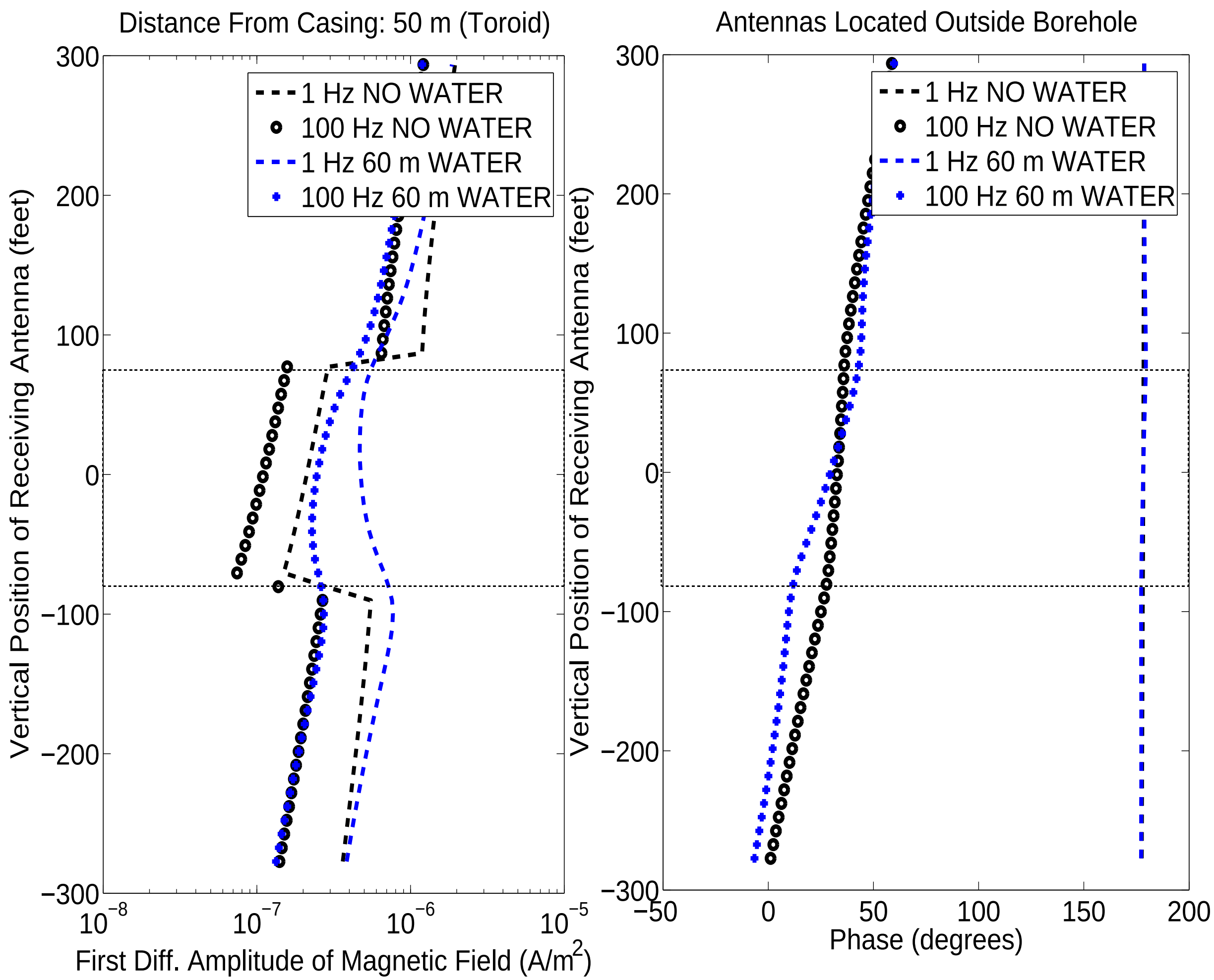
A Cross-Well Study: Water Invasion, Horiz. Dipoles (250 m)





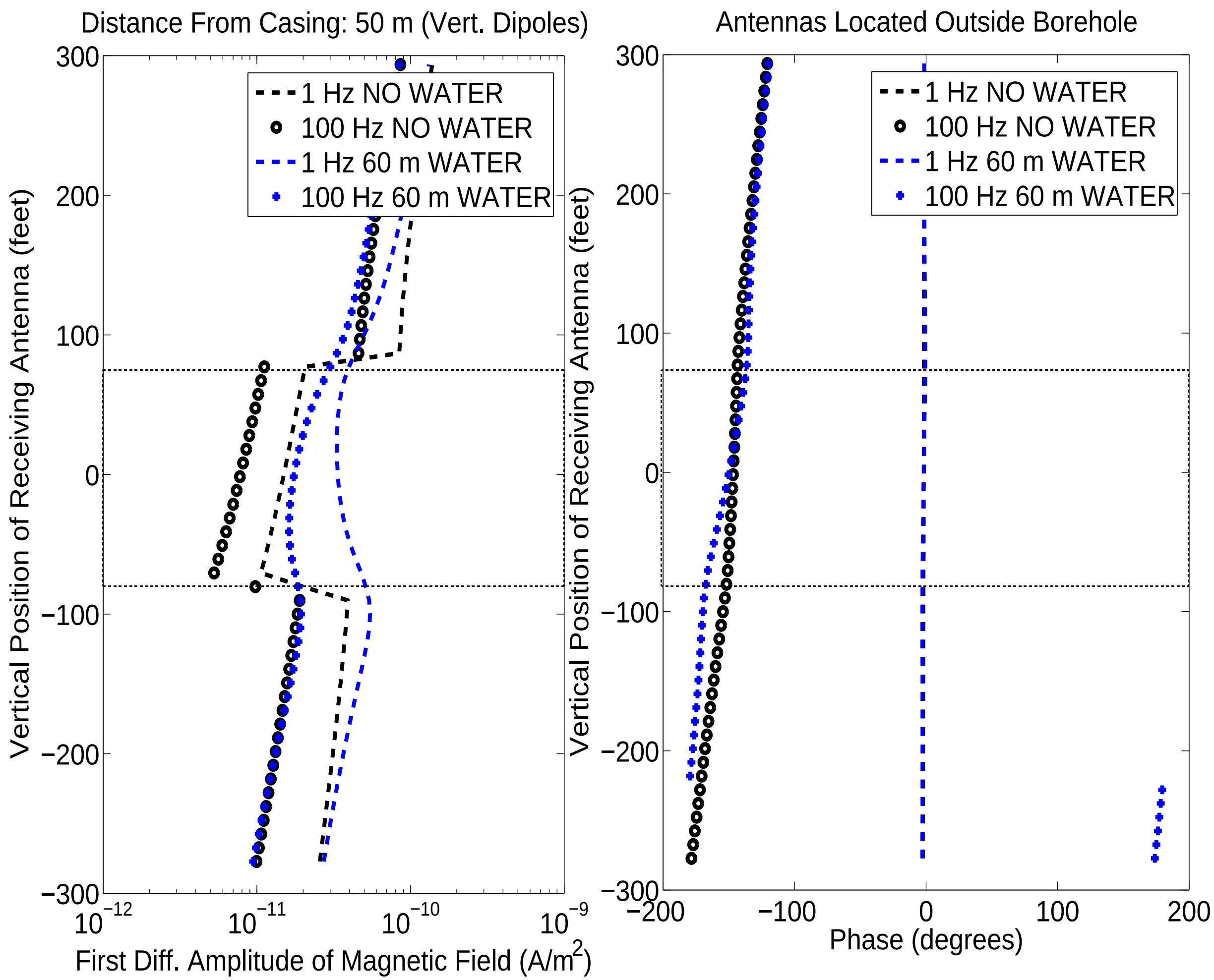
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Water Invasion, Toroid (50 m)



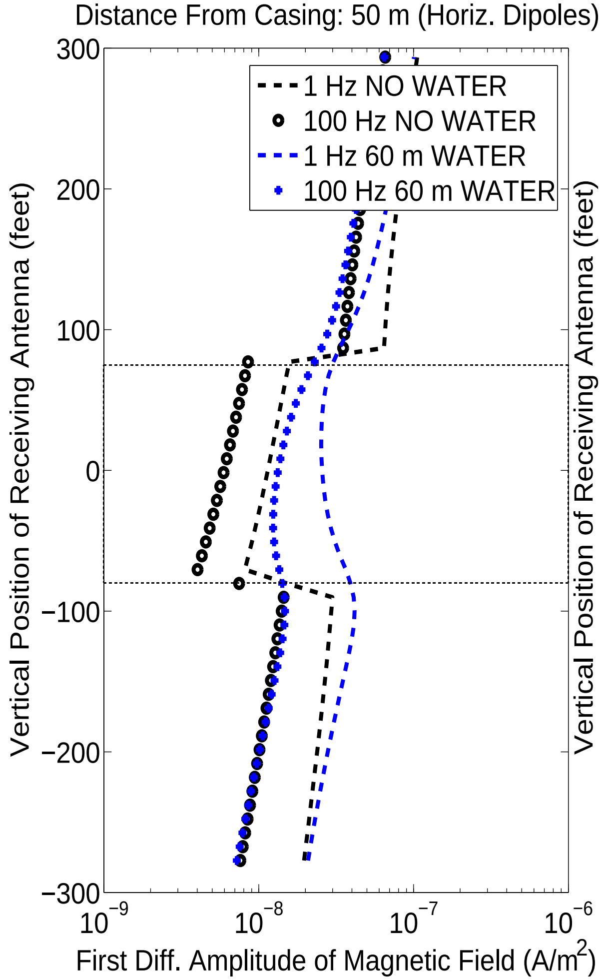
SIMULATION OF LOGGING INSTRUMENTS

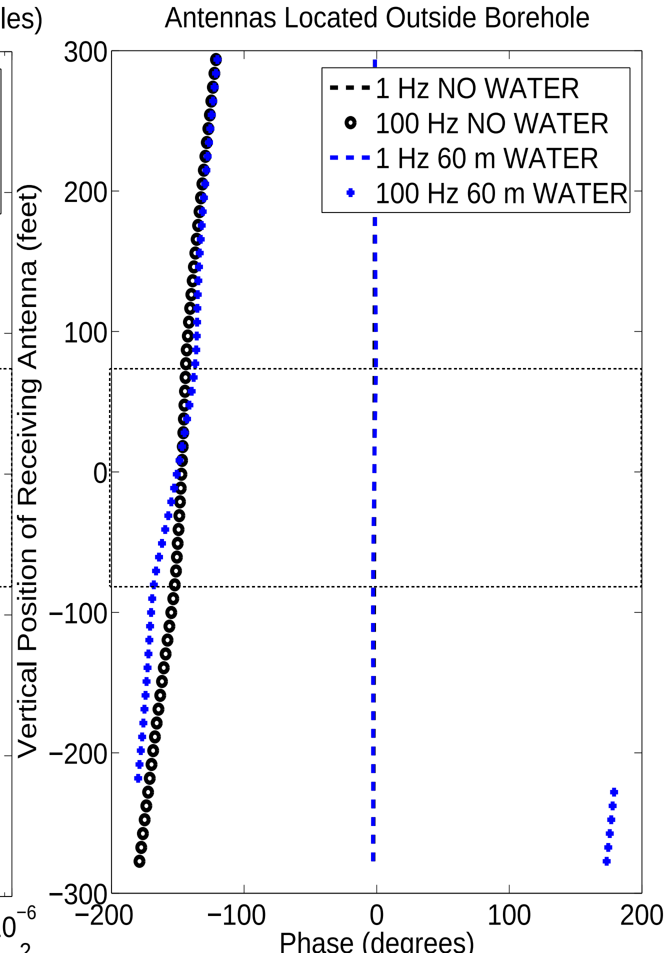
A Cross-Well Study: Water Invasion, Vert. Dipoles (50 m)



SIMULATION OF LOGGING INSTRUMENTS

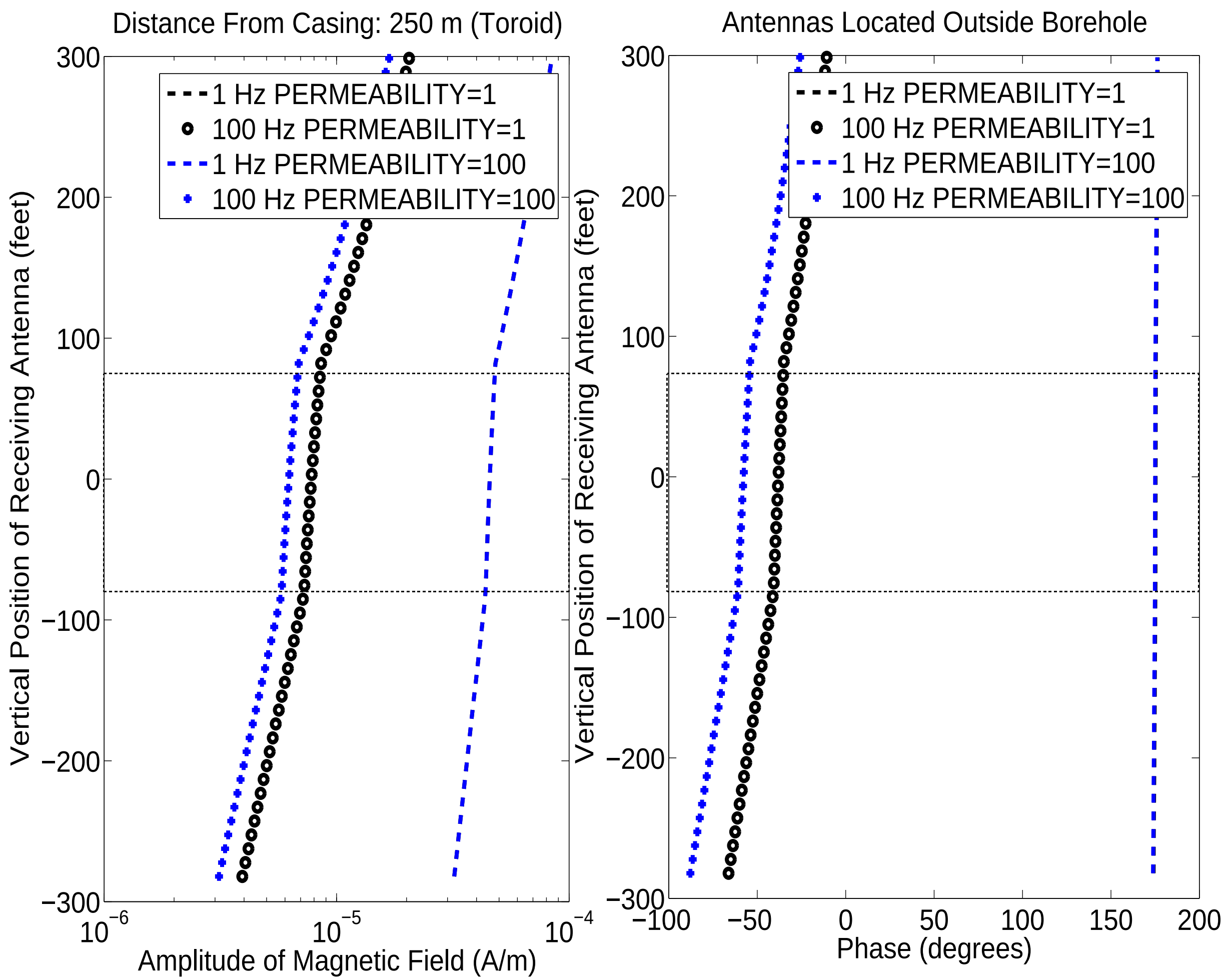
A Cross-Well Study: Water Invasion, Horiz. Dipoles (50 m)





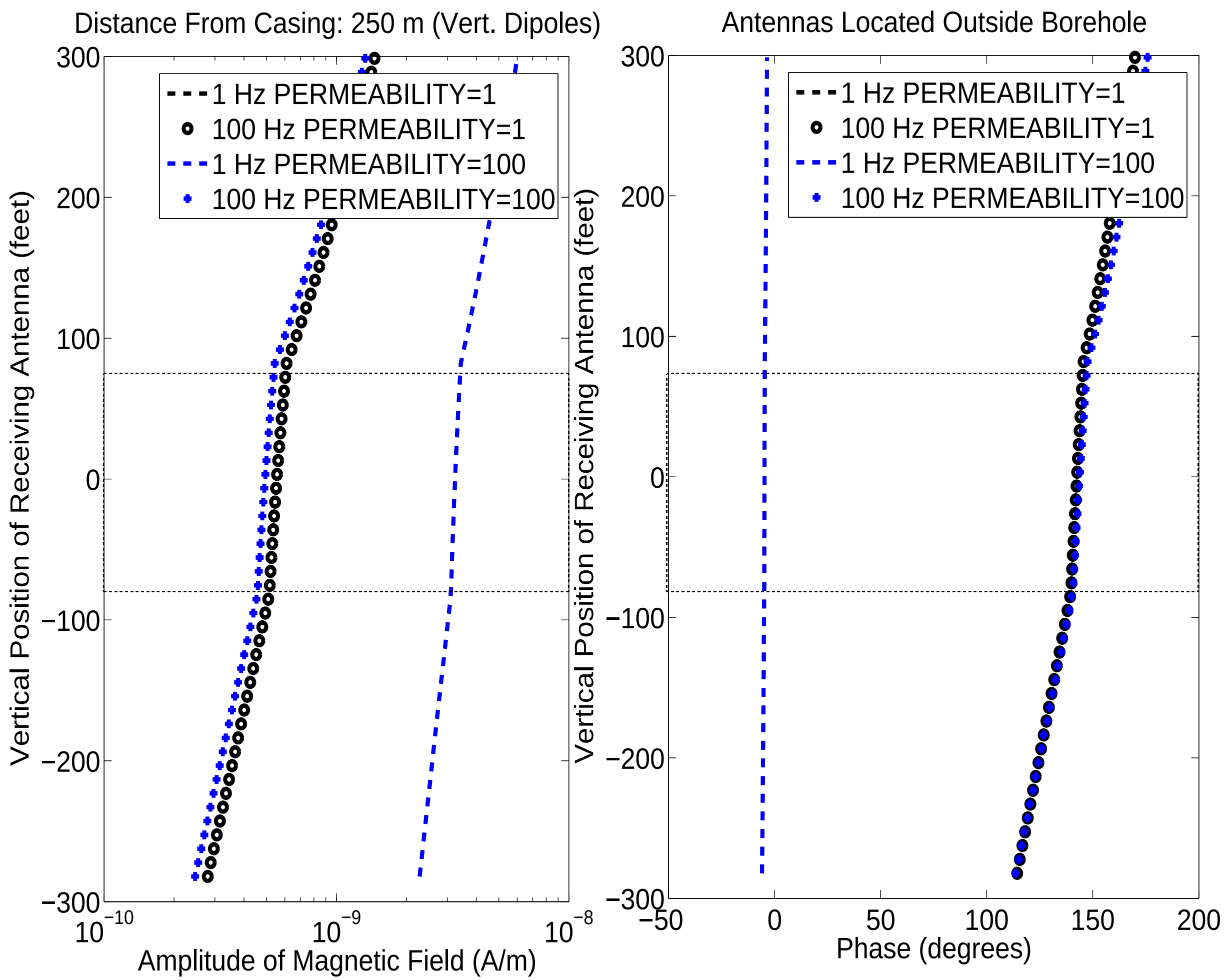
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Magnetic Perm., Toroid (250 m)



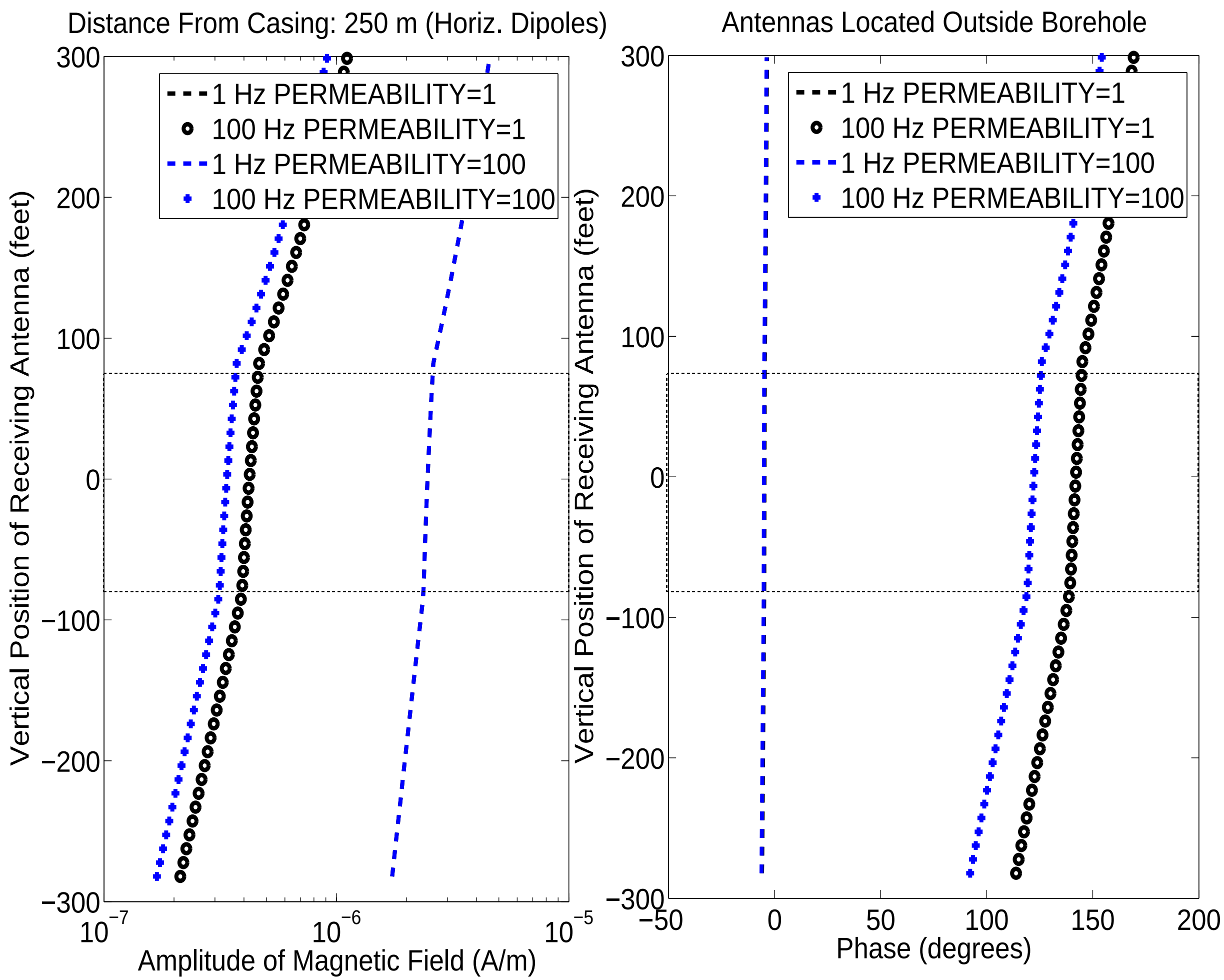
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Magnetic Perm., Vert. Dipoles (250 m)



SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Magnetic Perm., Horiz. Dipoles (250 m)



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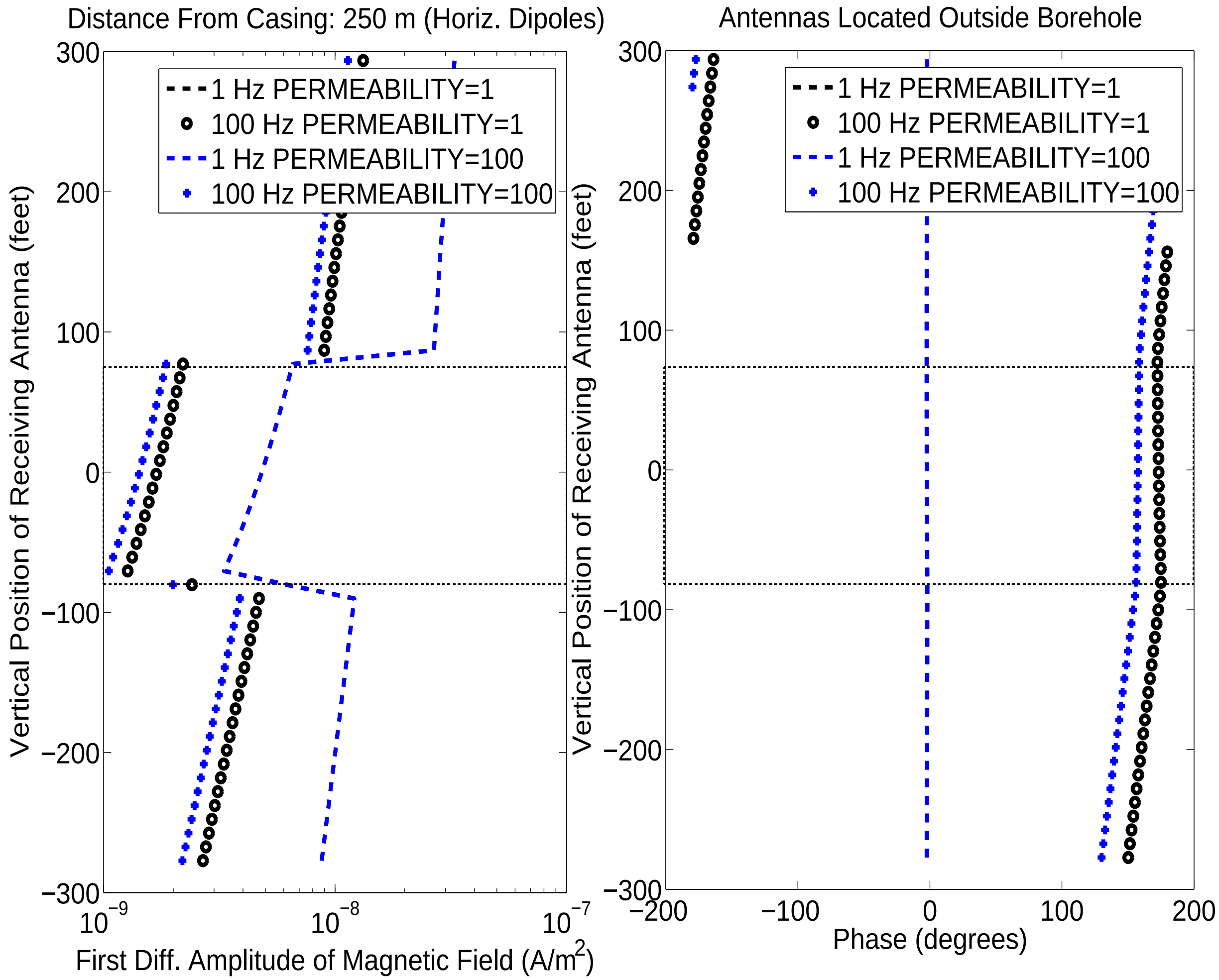
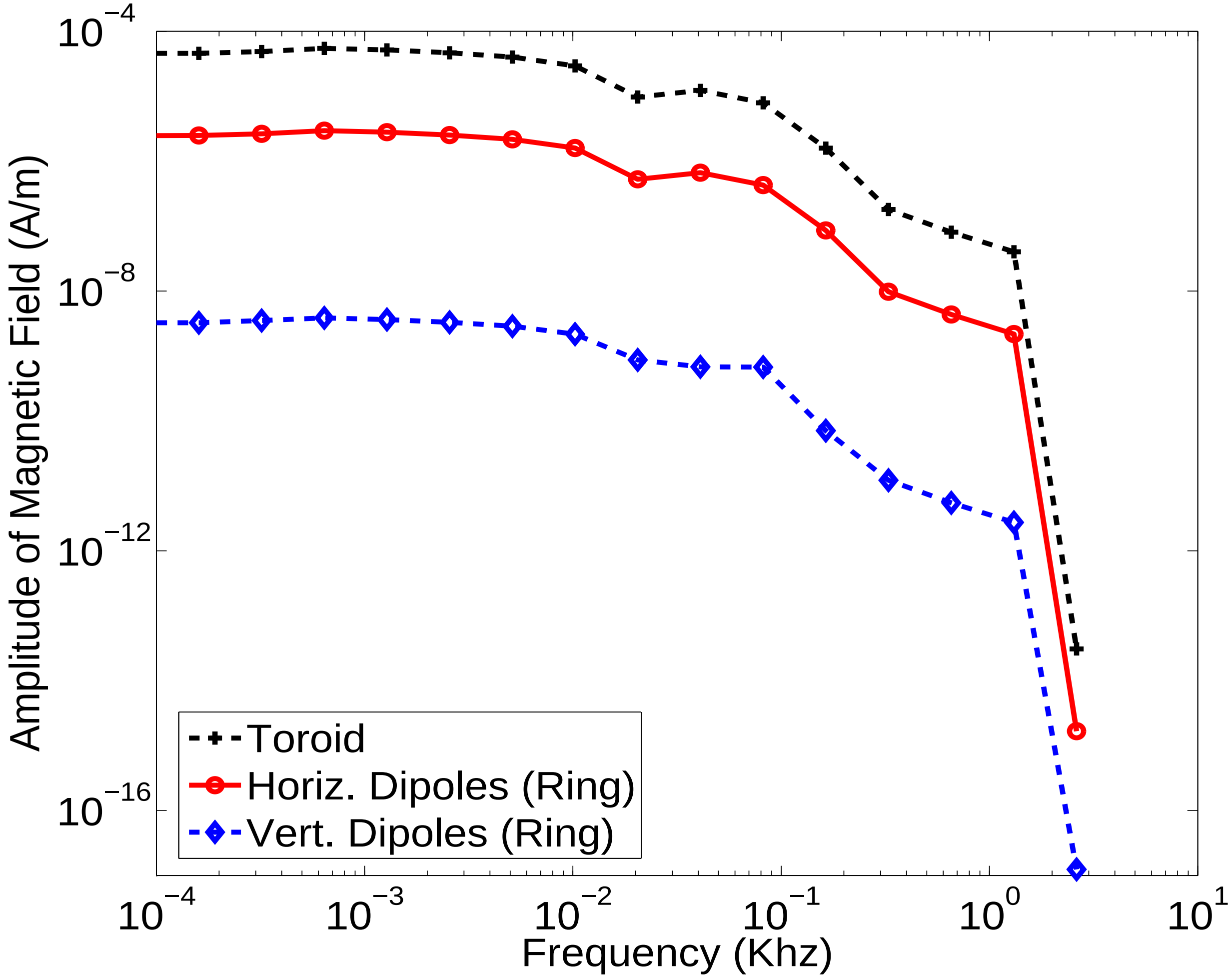


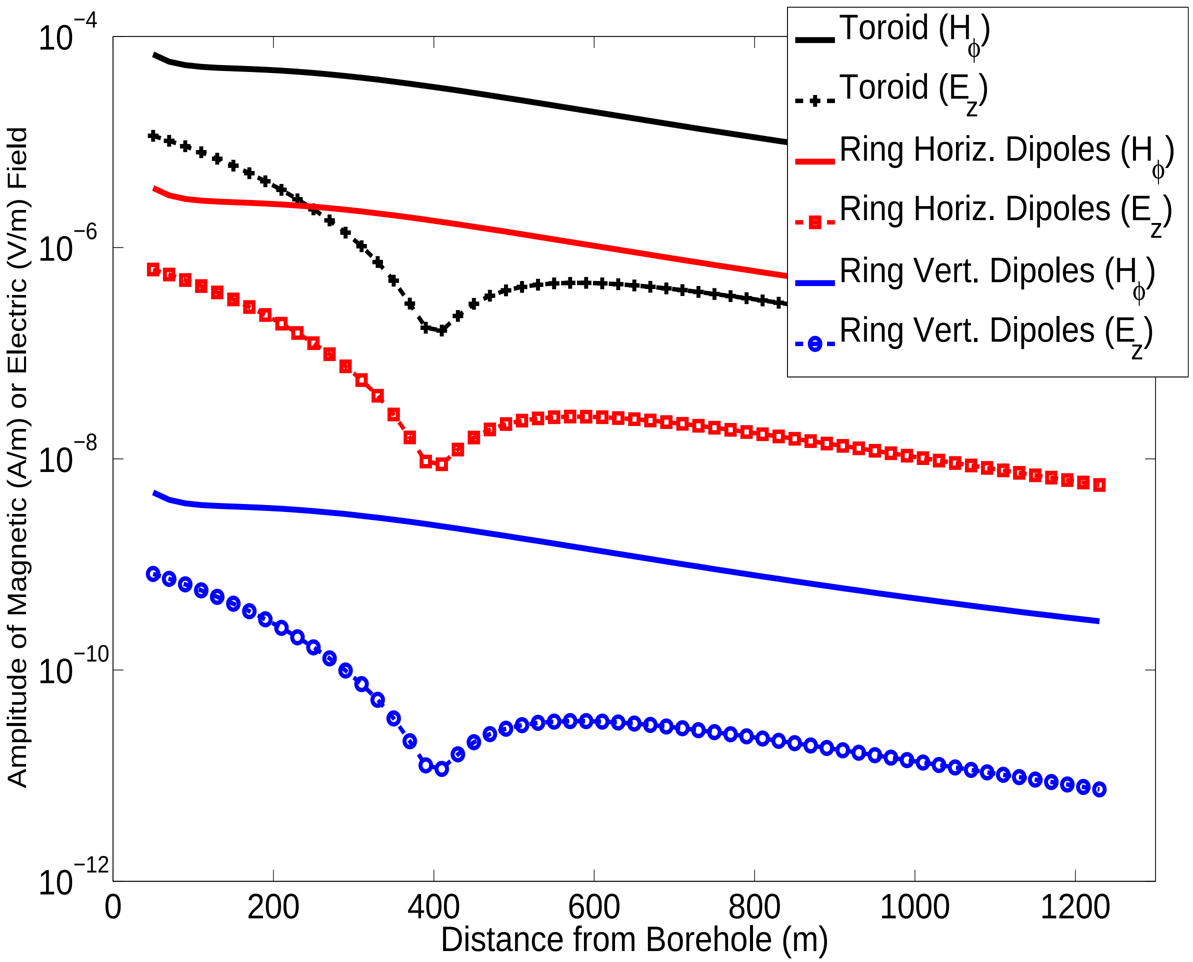
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Frequency Dependance at 250 m



SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Distance Dependance at 1 Hz



CONCLUSIONS AND FUTURE WORK

- **It is possible to accurately simulate cross-well configurations with one steel cased well by using the self-adaptive goal-oriented hp-FEM.**

- **For the discussed cross-well problem, numerical results suggest to:**

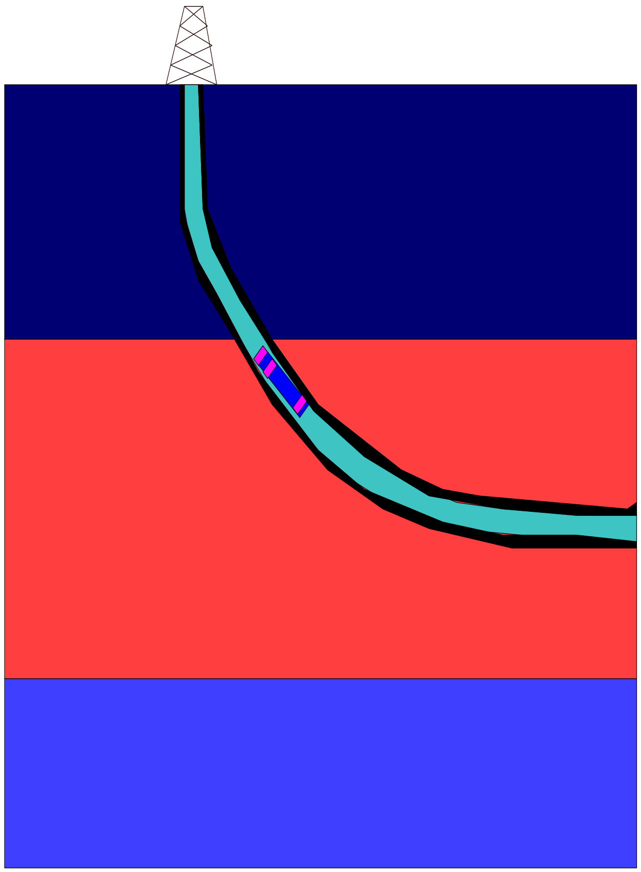
  1. **Use a ring of horizontal dipoles surrounding the casing. If this is not possible, place a ring of vertical dipoles inside the borehole.**

  2. **Use low frequencies (below 200 Hz).**

  3. **Use downhole antennas (if possible) for cross-well EM measurements.**

FUTURE WORK

Simulation of 3D Resistivity Logging Problems



- **PART I: Simulate 3D DC Resistivity Logging Problems.**
  - **Estimated completion time: 8-10 months (40 hours/week).**
  - **Main challenge: Speed.**
  - **Expected results: Similar results as in 2D.**
- **PART II: Simulate 3D AC Resistivity Logging Problems.**
  - **Estimated completion time: 8-10 months (40 hours/week).**
  - **Main challenge: Speed and Implementation.**
  - **Expected results: Similar results as in 2D.**