A VOTING RULE FOR THE EU COUNCIL?
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- Naïve answer: give a weight proportional to the population.
  - Quota?
  - Definition of a voting rule

Let us consider a rule with \( n \) seats.

\[ N = \{1, 2, \ldots, n\}, \text{ set of labels.} \]

\( 2^n \) possible configurations of votes

\( S \subset N, \text{ vote configuration } S = \{i \mid i \text{ votes yes}\} \)

\( S \) is winning if it leads to the passage of the proposal.

\( W \) denotes the set of winning configurations
The adequate voting rule is the rule that guarantees that each citizen has the same “power index”

Power index = F (voting rule)

\[
Sh_i(W) = \sum_{S : i \in S \in W} \frac{(n - s)! (s - 1)!}{n!}
\]

\[
Bz_i(W) = \sum_{S : i \in S \in W} \frac{1}{2^{n-1}}
\]
LITERATURE: DEFINITION OF A POWER INDEX

- Axiomatic approach (Game theory)
- Ad hoc definition (Political Science)
- Probabilistic approach (Political Science)
- « Paradoxes » (Social Choice)

Too many indices to answer un unclear question?

- What is « Power »? What are we talking about?
VOTING SITUATION

- Situation where a group of people have to make a dichotomous decision with the help of a voting rule

- Examples: Parliament, Council, Jury, Referendum,…

- Relevant practical question:

  What is the most adequate voting rule for a committee in which each member acts on behalf of a group of individuals or a constituency of different sizes?
RELEVANT FEATURES

- Some important features of a voting situation:
  - The voting rule and the voters
  - Where does the proposal come from?
  - How does the vote take place?
  - Is there some room for modifying the proposal? For bargaining?
TWO EXTREME SITUATIONS

TAKE-IT-OR-LEAVE-IT COMMITTEE

- The proposal comes from outside
- Voters are asked to cast a Yes/No vote
- The final result is acceptance or rejection

BARGAINING COMMITTEE

- Some alternatives are available
- Voters have some preferences on these alternatives
- Voters bargain on the alternatives and pick one
TAKE-IT-OR-LEAVE-IT COMMITTEE

- Will the proposal be passed or not?
- Will the voters get their preferred outcome?
- How much utility will they get from the result?

DEPENDS ON THE VOTING RULE

DEPENDS ON THE VOTING BEHAVIOUR
VOTING BEHAVIOUR

Map $p : 2^N \rightarrow R$

$p(S) = \text{probability that } S \text{ emerges}$

$= \text{probability that voters in } S \text{ vote 'yes'}$

and voters in $N \setminus S$ vote 'no'.

$0 \leq p(S) \leq 1$ for any $S \subseteq N$ and $\sum_{S \subseteq N} p(S) = 1$
VOTER i’S UTILITY FOR A GIVEN ISSUE

VOTER i

i is in favour

i votes YES

S ∈ W

i votes YES

the outcome is YES

Y +

S ∉ W

i votes YES

the outcome is NO

N +

i is against

i votes NO

S ∉ W

i votes NO

the outcome is NO

N -

S ∈ W

i votes NO

the outcome is YES

Y -
MOST ADEQUATE VOTING RULE?

- Two principles
  - Egalitarianism
    Equal (a priori) expected utility for all voters
  - Utilitarianism
    The sum of (a priori) expected utility should be maximum

- Two cases
  - Direct committees
  - Committees of representatives
RESULTS: DIRECT COMMITTEES

- Egalitarianism: choose a q-majority rule

- Utilitarianism: choose a q-majority rule with

\[ q = \frac{\Delta^-}{\Delta^- + \Delta^+} \]

\[ \Delta^- = N^- - Y^- \]

\[ \Delta^+ = Y^+ - N^+ \]
COMMITTEES OF REPRESENTATIVES

- Model

COUNCIL OF REPRESENTATIVES

SIMPLE MAJORITY

SIMPLE MAJORITY

SIMPLE MAJORITY

SIMPLE MAJORITY
RESULTS: COMMITTEES OF REPRESENTATIVES

- **Egalitarianism:**
  - If the sizes of the represented groups are large, any rule

- **Utilitarianism:**
  - Weighted majority
    - Weight = Square root of the represented group’s size
    - Quota = depending on the ratio $\frac{\Delta^-}{\Delta^+}$
A BARGAINING COMMITTEE

BARGAINS IN ORDER TO FIND A CONSENSUS

WHAT GENERAL AGREEMENT IS LIKELY TO ARISE?

DEPENDS ON THE VOTING RULE

DEPENDS ON THE PREFERENCES
THE VOTERS’ PREFERENCES

BARGAINING PROBLEM

B = (D, d)

D = set of feasible payoffs

d = disagreement point
What is the **level of utility** a rational player can expect in a committee that looks for consensus **under a voting rule**?

**Game theoretic solution**

- Cooperative approach
  - Axiomatic approach

- Non cooperative approach
  - Definition of a game (protocol)
CRITERION TO CHOOSE A VOTING RULE

In committees of representatives

A voting rule such that any voter (citizen) would be indifferent between bargaining directly by himself and leaving the bargaining in the hands of his representative

FOR WHICH PREFERENCES?
Citizens with symmetric preferences

RECOMMENDATION
CONCLUSION – WORK IN PROGRESS

- Importance of the choice of the model
- Possibility to abstain or not to go
- Why do people vote? Network effect