Classics and New Trends in Robotics

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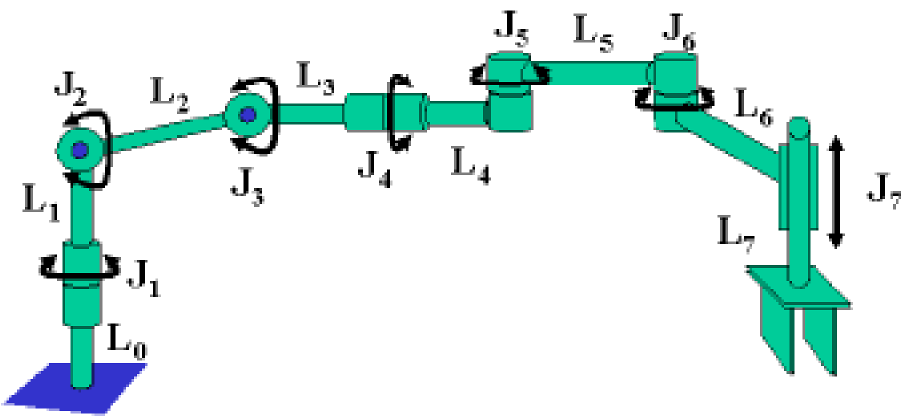
Classical Problems in Robotics



Joint Space→Workspace

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Piano-mover problem ←

Control Theory in Robotics

Dynamic of manipulator:

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\pi = \tau$$

II order ODE ->$x' = f(x,u)$, $x \in M$, $u \in U$.

- Controllability
- Motion Planning
- Stability
- Optimal Control

Controllability and Differential Geometry

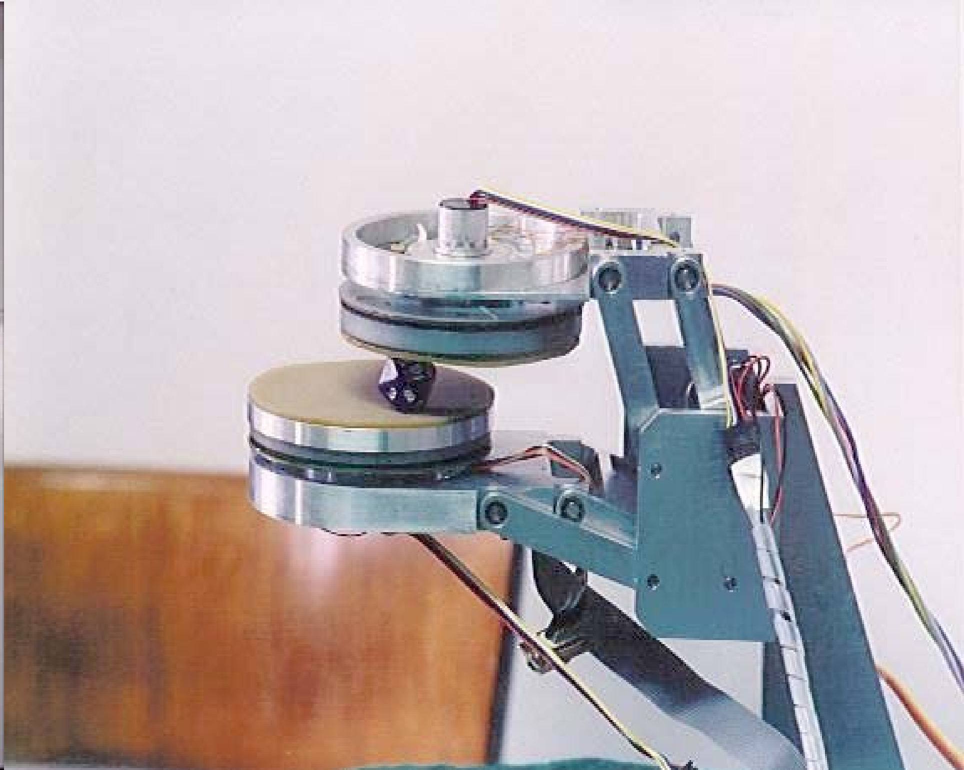
- Nonholonomic systems

Marigo Bicchi, TAC 2000

Example: manipulation by rolling

Bicchi Chitour Marigo, TAC 2004

- Frobenius Theorem characterizes the



Motion Planning and Normal Forms

Linear systems x’=Ax+Bu

Nilpotent systems

Triangular systems

Steering Algorithm;
(cf. [Marigo-Bicchi, CDC’98, IJ

$$\dot{x}_1 = f^1(x_2, ..., x \quad (x_2, \dots, x_n)\, u_i$$
$$\dot{x}_2 = f_0^2(x_3, ..., x \quad (x_3, \dots, x_n)\, u_i$$
$$\vdots$$
$$\dot{x}_{n-1} = f^{n-1}(x_n \quad {}^1(x_n)\, u_i$$
$$\dot{x}_n = f^n + \sum_{i=1}^{m}$$

$$\dot{x}_1 = u_1$$
$$\dot{x}_2 = u_2$$
$$\dot{x}_3 = x_2 u_1$$
$$\dot{x}_4 = x_3 u_1$$
$$\vdots$$
$$\dot{x}_n = x_{n-1} u_1$$

Normal Forms

- Feedback Linearization (if possible)
- Linear Approximation (non controllability preserving)
- Nilpotent approximation (controllability preserving)
- Reparametrization to Triangular form (if possible)

$\mathcal{G}$ solvable
(Marigo '99)

Triangular

Change of Variables

- Feedback Linearization
- Linear Approximation
- Nilpotent approximation

Nilpotent approximation and Rigid Carnot Algebras

1. M, n–dimensional smooth manifold;
   $\mathcal{F} = \{f_1, \ldots, f_d\} \subset VecM$;

Definition 1 $Lie_q\mathcal{F}$ *is rigid if there exists* $\{f_1, \ldots, f_d\}$, *generators of* $\bar{\mathcal{F}}$, *such that* $Lie_q\{f_1 \ldots f_d\}$ *is isomorphic to* $Lie_q\{f'_1 \ldots f'_d\}$ *for all* f'_i *sufficently close to* f_i *in the* C^∞ *topology.*

(d, n) *is rigid bi–dimension if there exists at least a set* $\{f_1, \ldots, f_d\} \subset VecM$ *such that* $Lie_q\{f_1, \ldots, f_d\}$ *is rigid.*

Classification results:
3 infinite series of rigid bi–dimensions

$+$

16 exceptional rigid bi–dimensions. The biggest being $(7, 26)$.

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Nilpotent approximation and Rigid Carnot Algebras

Remark 1 $Lie_{q_0}\mathcal{F}$ contains all the informations relative to the nilpotent approximation of $\mathcal{F}$.

A.Agrachev, A. Marigo: ERA AMS, 2003

A.Agrachev, A. Marigo: JDCS, 2005

$\Downarrow$

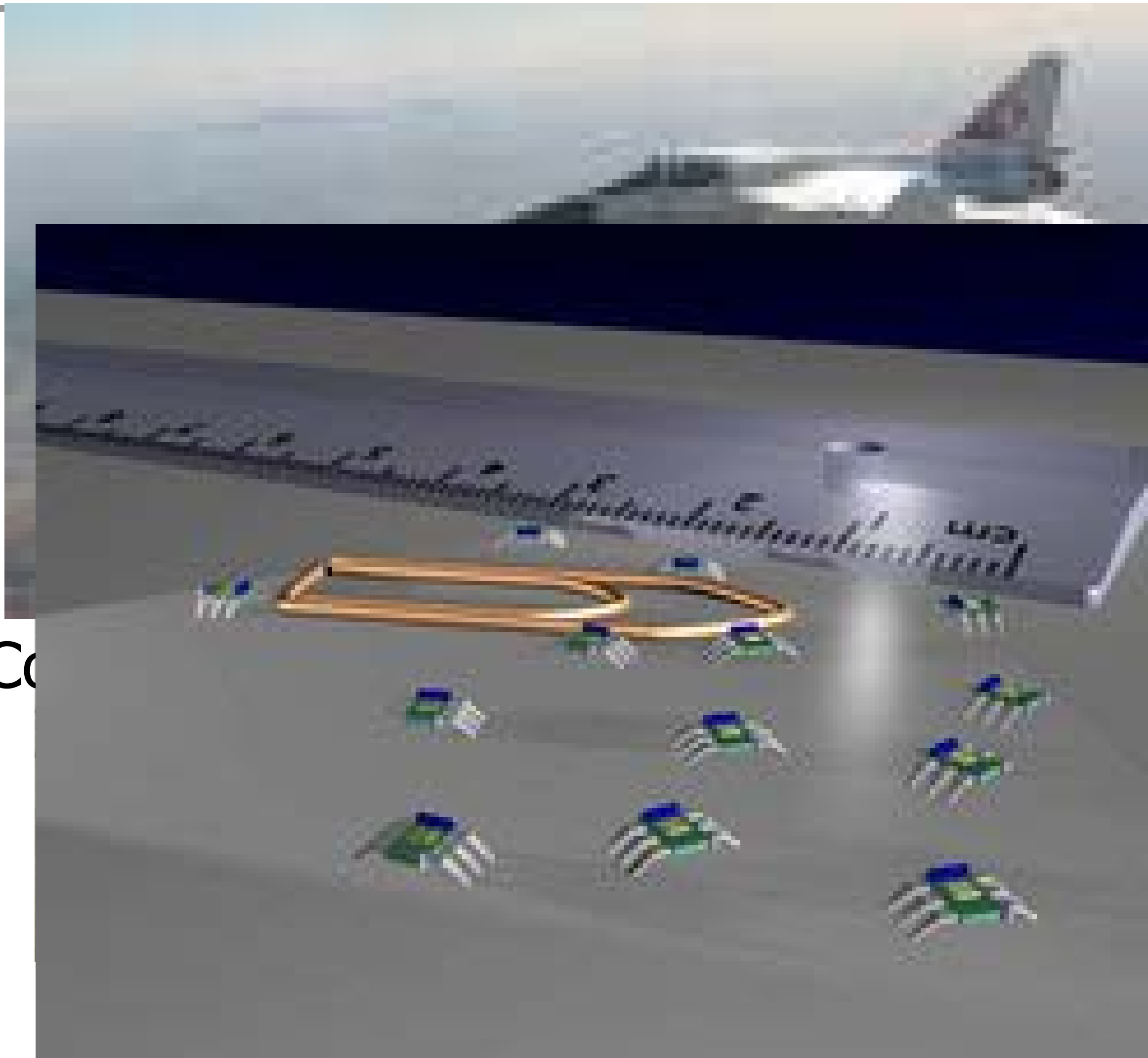
A. Marigo: JDCS, 2007

Giving normal forms for $Lie_{q_0}\mathcal{F}$ is equivalent to giving normal form of nilpotent approximations of $\mathcal{F}$.

Emerging Robotic Systems

Networked and Embedded Systems.

- Humanoid
- Airtraffic Decentralized Co
- Flocks

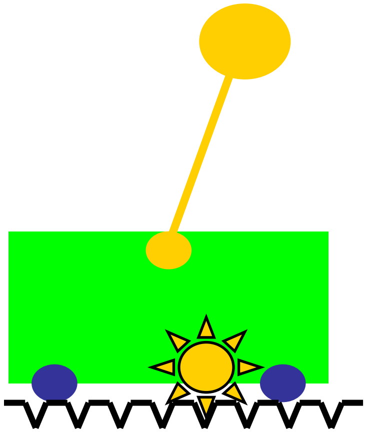


New Problems

Communication constraints

- Planning motion under limited information exchange among agents (avoid crashes)
- Planning motion under limited capacity of communication channels among sensors, controller and actuators.

Quantization



•An inverted pendulum on a cart – with a stepping motor

$$u \in U = \{0, \pm pulse\}$$

•A rolling object – with polyhedral shape

$$u_x \in U_x \subset \{1,2,3,4,5,6\}$$

Quantization and Motion Planning

The set of reachable points for a quantized system is countable.
Under certain assumption it is either a lattice (☺) or dense.

ε -approachability: exists $U(\varepsilon)$ such that reachable sets are lattices of mesh size ε

Motion planning solution:
- find the generators of the lattice,
- solve an Integer Programming problem

Bicchi Marigo Piccoli (TAC 2002): Chained form systems are ε–approachable.

Quantization: Optimal choice of controls

$$x^+ = x + u \quad x \in \mathbb{Z}^n$$
$$u \in \mathcal{U} = \{0, \pm u_1, \dots, \pm u_m\} \subset \mathbb{Z}^n$$

(***P***) Fix *m* and *K* integers. Choose the control set U with *m* elements in order to reach every integer point in a interval of maximal size N in at most *K* steps.

Postage stamp problem *(solution known only for m=2,3):*

Marigo: "Optimal Input Sets for Steering Quantized Systems" MCSS 2009

$u_1, \dots, u_m$ stamps;

Which is the minimum value N which cannot be obtained by using at most K stamps?

Future Plans

- Robots' swarms: what are the individual behaviors to obtain desired overall behavior.
- Study Insect swarm and find inspiration.
- Study Social Networks and find inspiration.
- Is a kind of "Quantized Feedback" present in nature?